ELEC546 – Wireless Fading Channels

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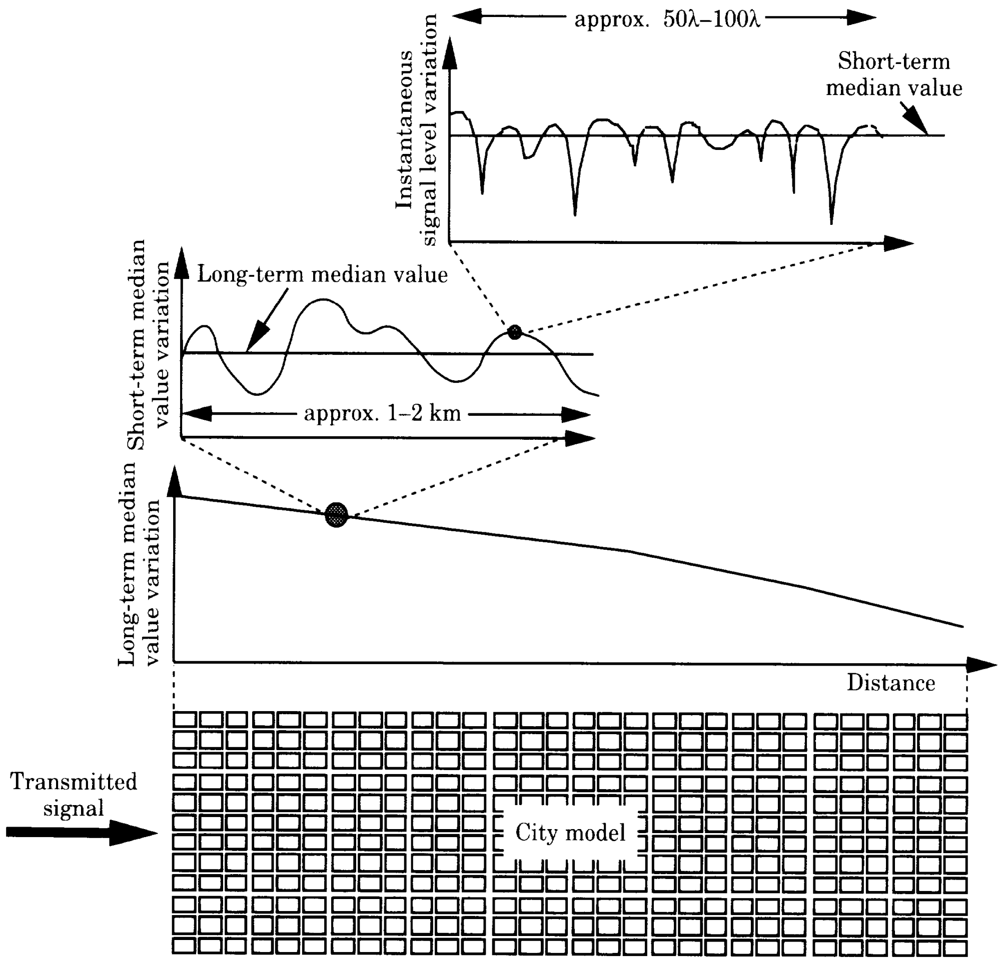
General Model for Wireless Channels

- Multipath Fading
  - constructive and destructive interference caused by multiple TX-RX paths with diff lengths arriving from diff directions
  - Signal envelope varies widely over 30 dB in the span of a few wavelengths in distance (e.g. λ = 1 ft when f_c=1GHz)
- Shadowing
  - Short-term average variation or large-scale signal variation
  - obtain by averaging over 50-100 wavelengths in distance
  - caused by local changes in terrain features or man-made obstacles (e.g. blockage)
- Path Loss Model
  - Long-term or large-scale average signal level
  - depends on the distance between TX and RX

General 3-level Model



Sampei, p. 16, Fig 2.1

General 3-level Model

- Path loss model is used for
  - system planning, cell coverage
  - link budget (what is the frequency reuse factor?)
- Shadowing is used for
  - power control design
  - 2nd order interference and TX power analysis
  - more detailed link budget and cell coverage analysis
- Multipath fading is used for
  - physical layer modem design --- coder, modulator, interleaver, etc

Ideal Path Loss Model

$$P_r = \frac{c^2 P_t G_t G_r}{16\pi^2}\frac{1}{d^2}\frac{1}{f^2}$$

$$P_r(\text{in } dBm) = 10\log_{10} P_r$$

$$= 10\log_{10} P_t + C - 20\log_{10} f_c - 20\log_{10} d$$

dBm if P_r is in mWatt & dBW if P_r is in Watt

$$PL_{\text{freespace}}(\text{in } dB) = P_t(\text{in } dBm) - P_r(\text{in } dBm)$$

$$= -C + 20\log_{10} f + 20\log_{10} d$$

Path Loss Exponent = 2

$$PL_{\text{freespace}}(\text{in } dB) = PL(d_0) + 20\log\frac{d}{d_0}$$

– Path Loss Exponent indicates how fast signal power drops with Tx-Rx separation
  - 2 means 6dB drop per doubling of the distance

Path Loss Exponent

- Path loss in dB depends on TX-RX distance via PL exponent, n.

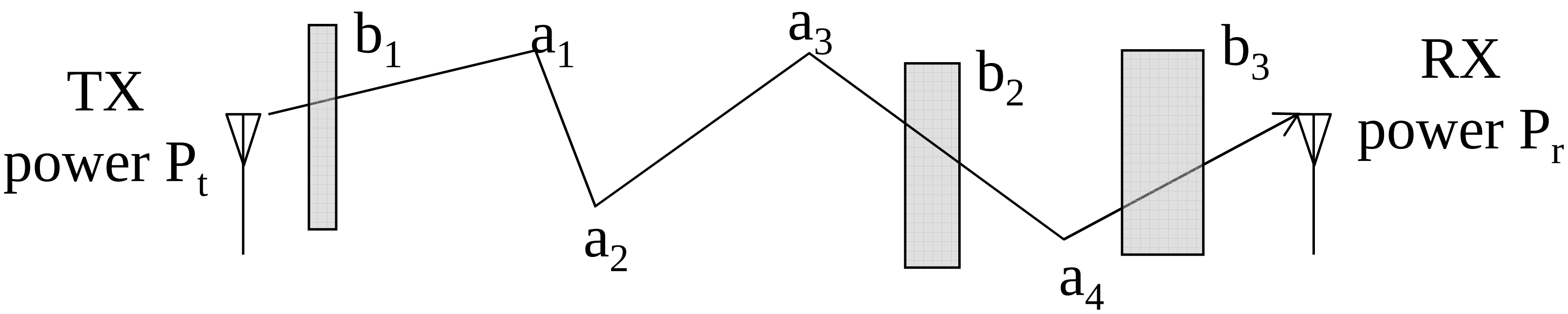
$$PL(d) \propto \left(\frac{d}{d_0}\right)^n$$

$$PL(d) = PL(d_0) + 10n\log\left[d / d_0\right]$$

Environment	Path Loss Exponent
Free Space	2
Urban area cellular	2.7 to 3.5
Shadowed urban cellular	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Shadowing Effect

- Variations around the path loss predication due to buildings, hills, trees, etc.



  - Consider a signal undergoes multiple reflections (each with a power attenuation factor a_i) and passes through multiple obstacles (with factors b_i).

$$P_r = \prod_{i=1}^{4} a_i \prod_{i=1}^{3} b_i P_t$$

$$P_r(\text{in } dBm) = \sum_{i=1}^{4} 10\log(a_i) + \sum_{i=1}^{3} 10\log(b_i) + P_t(\text{in } dBm)$$

$$= \sum_i \alpha_i(\text{in } dB) + P_t(\text{in } dBm)$$

Shadowing Effect

- Each term introduces a random attenuation of α_i dB and they are assumed to be *statistically independent*
- As the number of these factors increases, by the central limit theorem, the sum, S, approaches a Gaussian (normal) random variable

$$P_r(\text{in } dBm) = S(\text{in } dB) + P_t(\text{in } dBm)$$

$$= m(\text{in } dB) + X(\text{in } dB) + P_t(\text{in } dBm)$$

where $S \sim N(m, \sigma^2)$ and $X \sim N(0, \sigma^2)$

- the mean m is generally included in the Path loss model (that's why the path loss exponent can be larger than 2 as the number of terms generally increases with the TX-RX separation)

Shadowing Effect

– When we study only the Shadowing effect, we have

$$P_r(\text{in } dBm) = X(\text{in } dB) + P_t(\text{in } dBm)$$

where X is a zero mean Gaussian random variable with variance σ^2

– Expressing in linear scale, we have

$$P_r = 10^{(X/10)} P_t = A_s P_t$$

where A_s is the attenuation factor due to shadowing effect

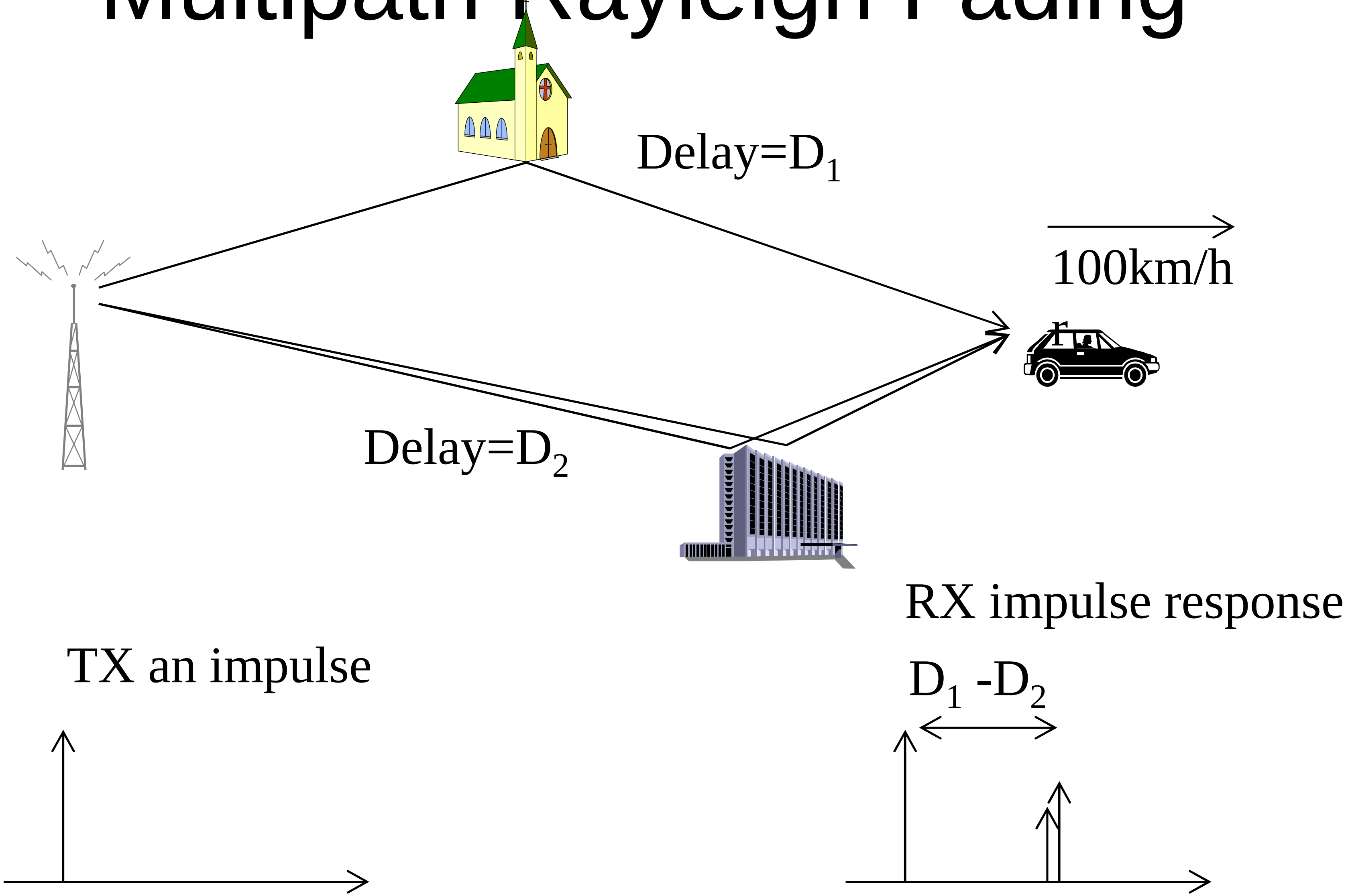
– Note that $\log(A_s) = X/10$ is normally distributed; hence, the distribution of A_s is known as the “Lognormal” distribution

– σ is called the standard deviation and has a unit of dB

Shadowing Effect

- Variations around the median path loss line due to buildings, hills, trees, etc.
  - Individual objects introduces random attenuation of x dB, after pass through so many objects the attenuation factors multiply (or add in dB scale)
  - As the number of these x dB factors increases, the combined effects becomes Gaussian (normal) distribution (by central limit theorem) in dB scale: “Lognormal”
- $PL(dB) = PL_{avg}(dB) + X$ where X is $N(0,\sigma^2)$ where
  - PL_{avg} (dB) is obtained from the path loss model
  - σ is the standard deviation of X in dB

Multipath Rayleigh Fading



Multipath Rayleigh Fading

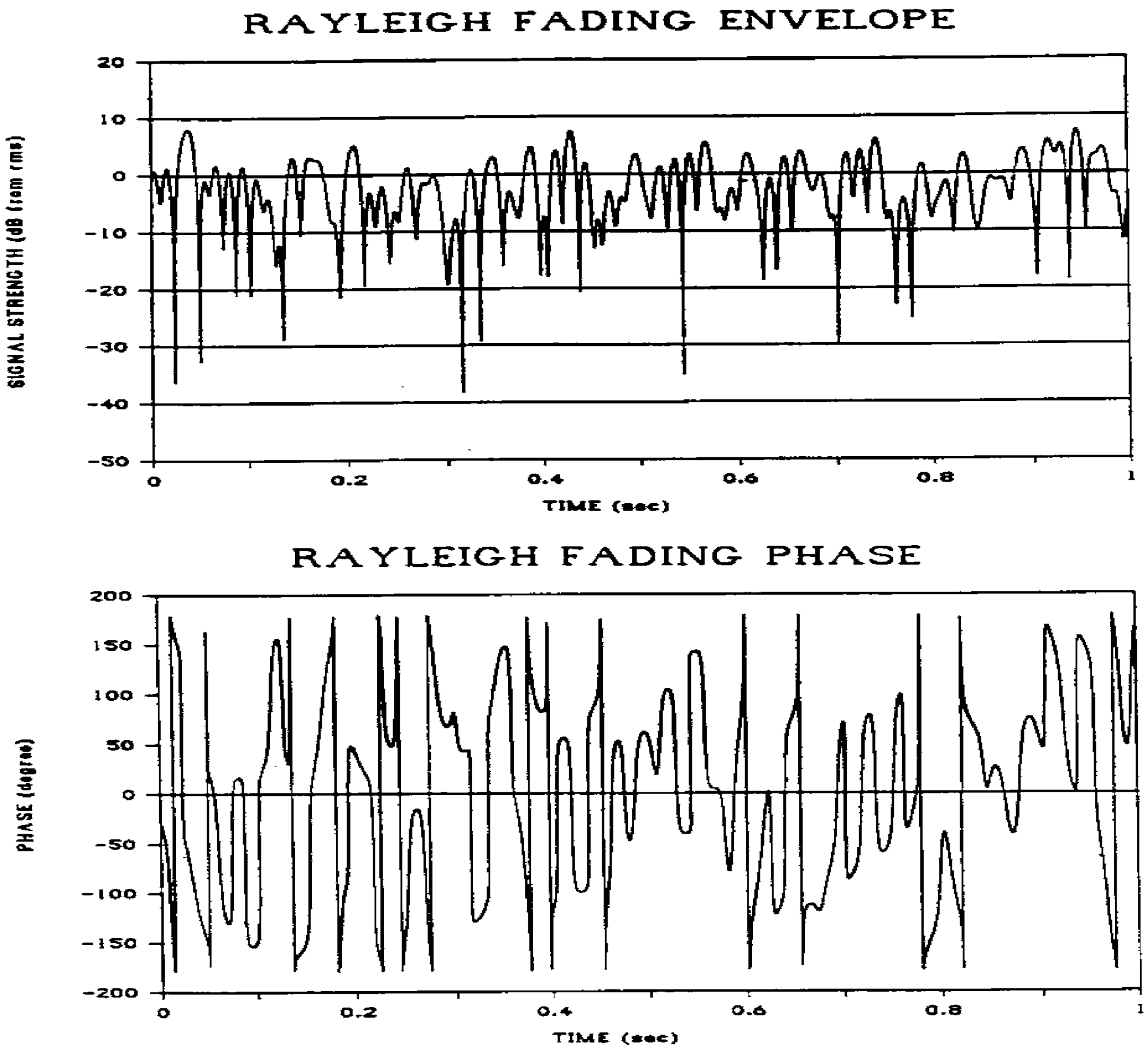
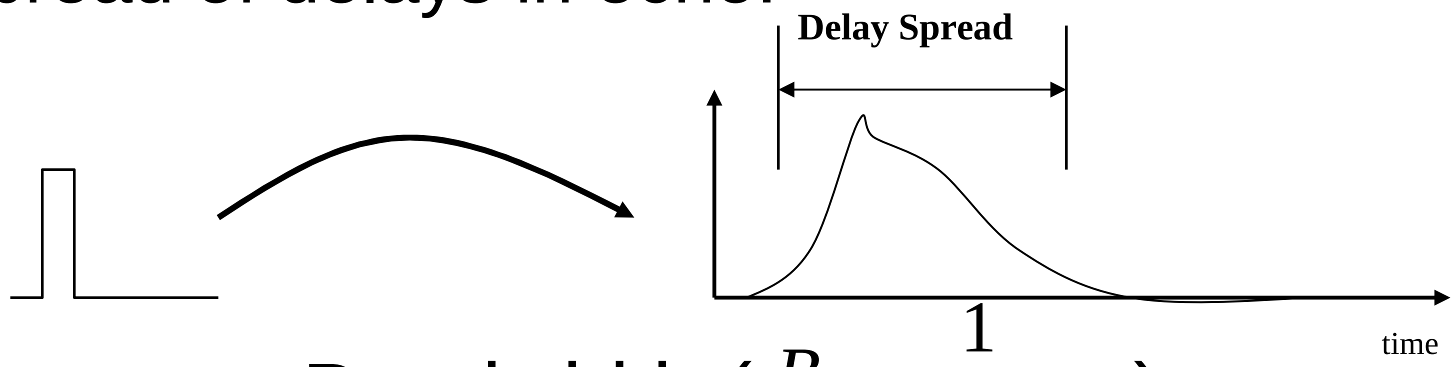


Figure 1.1: Typical Profile of the received signal's Rayleigh fading envelope and phase. Vehicular MS speed of 30 mph, carrier frequency of 900 MHz.

Microscopic Fading – Multipath Dimension

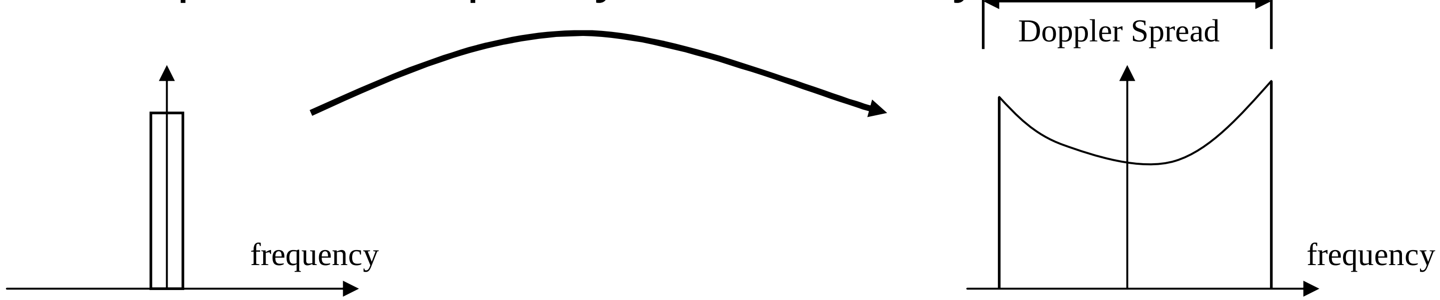
- Delay Spread (σ_τ):
  - spread of delays in echo.



- Coherence Bandwidth ($B_c \approx \frac{1}{5\sigma_\tau}$):
  - min separation of frequency for uncorrelated fading.
- Typical values
  - Indoor: Bc ~ 1MHz
  - Outdoor: Bc ~ 100 kHz.

Microscopic Fading – Time Dimension

- Doppler Spread ($f_d = \frac{v}{\lambda}$):
  - spread of frequency due to mobility

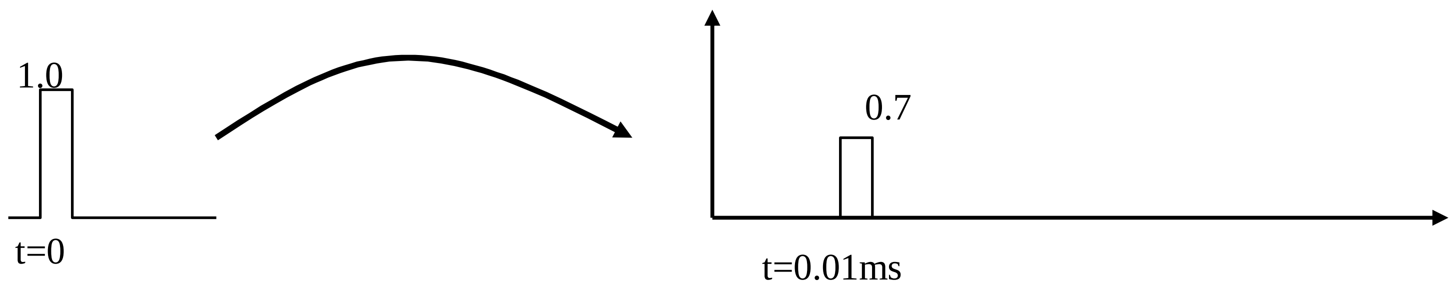


- Coherence Time ($T_C \approx \frac{9}{16\pi f_d}$):
  - min separation of time for uncorrelated fading.
- Typical Values
  - Pedestrian (~ 5 km / hr) → fd ~ 14 Hz (at 2.4 GHz)
  - Vehicular (~ 100 km/hr) → fd ~ 300 Hz (at 2.4 GHz)

Flat Fading Channels

- "Narrowband Transmission"
  - coherence bandwidth of channel > signal BW.
- "Single path" channel model:

$$\underbrace{y(t)}_{received\_signal} = \underbrace{\alpha_1 x(t-\tau_1)}_{\text{info\_signal}} + \underbrace{\eta(t)}_{\substack{channel\\noise}}$$



Baseband Representation of Digitally Modulated Passband Signals

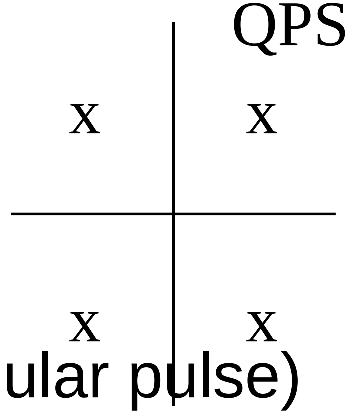
- TX passband signals

$$s(t) = s_I(t)\cos(\omega t) - s_Q(t)\sin(\omega t)$$
$$= \mathrm{Re}\left\{\left[s_I(t) + js_Q(t)\right]e^{j\omega t}\right\}$$

- Complex Representation of passband signals

$$\tilde{s}(t) = s_I(t) + js_Q(t)$$

QPSK

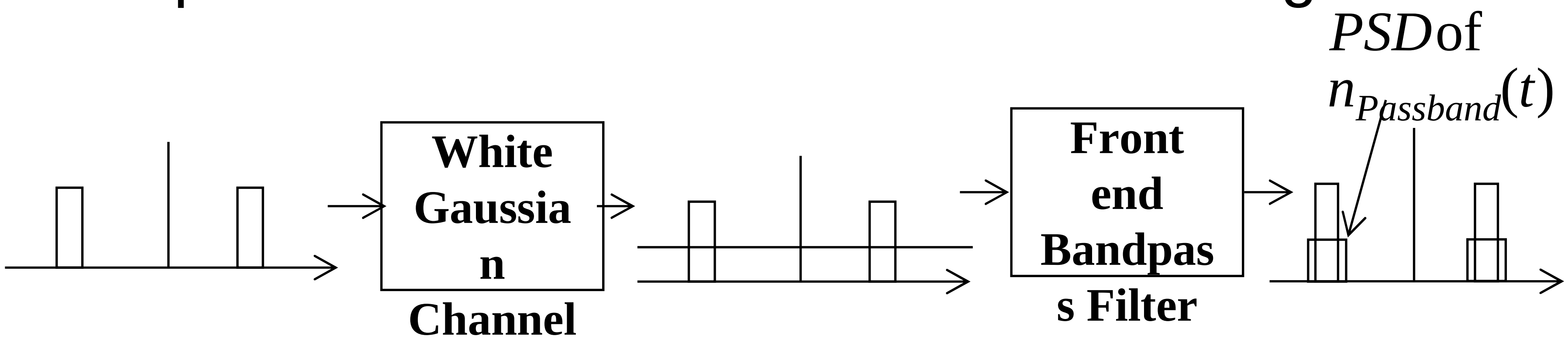


- For Digital Communications:

$$s_I(t) = \sum_n s_{n,I}\, p(t - nT)$$
$$s_Q(t) = \sum_n s_{n,Q}\, p(t - nT)$$

  – where p(t) is the pulse shape (e.g. rectangular pulse)
  – $s_n = s_{n,I} + js_{n,Q}$ is one of the points on the signal constellation

Representation of Received Passband Signals



$$y(t) = x(t) * \alpha\delta(t-\tau) + n_{Passband}(t)$$

$$= \mathrm{Re}\left\{\alpha\left[s_I(t-\tau) + js_Q(t-\tau)\right]e^{j\omega(t-\tau)} + \tilde{n}_{baseband}(t)e^{j\omega t}\right\}$$

$$= \mathrm{Re}\left\{\alpha e^{-j\omega\tau}\left[s_I(t-\tau) + js_Q(t-\tau)\right]e^{j\omega t} + \tilde{n}_{baseband}(t)e^{j\omega t}\right\}$$

Since the phase of α is uniformly distributed and if we assume that the pdf of τ is fairly constant over any interval with width $1/\omega$, we find that α and $\alpha e^{j\omega\tau}$ has the same statistics. Hence, renaming $\alpha e^{j\omega\tau}$ by α, we have

$$y(t) = \mathrm{Re}\left\{\left[\alpha\left[s_I(t-\tau) + js_Q(t-\tau)\right] + \tilde{n}_{baseband}(t)\right]e^{j\omega t}\right\}$$

Baseband Representation of Received Passband Signals

- Using the complex representation, we have

$$\tilde{y}(t) = \alpha\tilde{s}(t) + \tilde{n}_{baseband}(t)$$
$$= \alpha_I s_I(t) - \alpha_Q s_Q(t) + n_I(t) + j\left[\alpha_Q s_I(t) + \alpha_I s_Q(t) + n_Q(t)\right]$$
$$\text{where} E n_I(t) n_I(\tau) = E n_Q(t) n_Q(\tau) = N_0 \delta(t-\tau)$$

- Using the matched filter, the sampled output becomes, assuming p(t) has unit energy,

$$\tilde{y}_n = \int_{nT}^{(n+1)T} \tilde{y}(t)\, p(t-nT)\, dt$$
$$= \alpha_I s_{n,I} - \alpha_Q s_{n,Q} + n_{n,I} + j\left[\alpha_Q s_{n,I} + \alpha_I s_{n,Q} + n_{n,Q}\right]$$
$$= \alpha\tilde{s}_n + \tilde{n}_n$$
$$\text{where } E n_{n,I}^2 = E n_{n,Q}^2 = N_0$$

BER Analysis on Flat Fading

- Consider the uncoded performance:
- Assuming BPSK and coherent demodulation, the conditional error probability (conditioned on fading α) is given by:

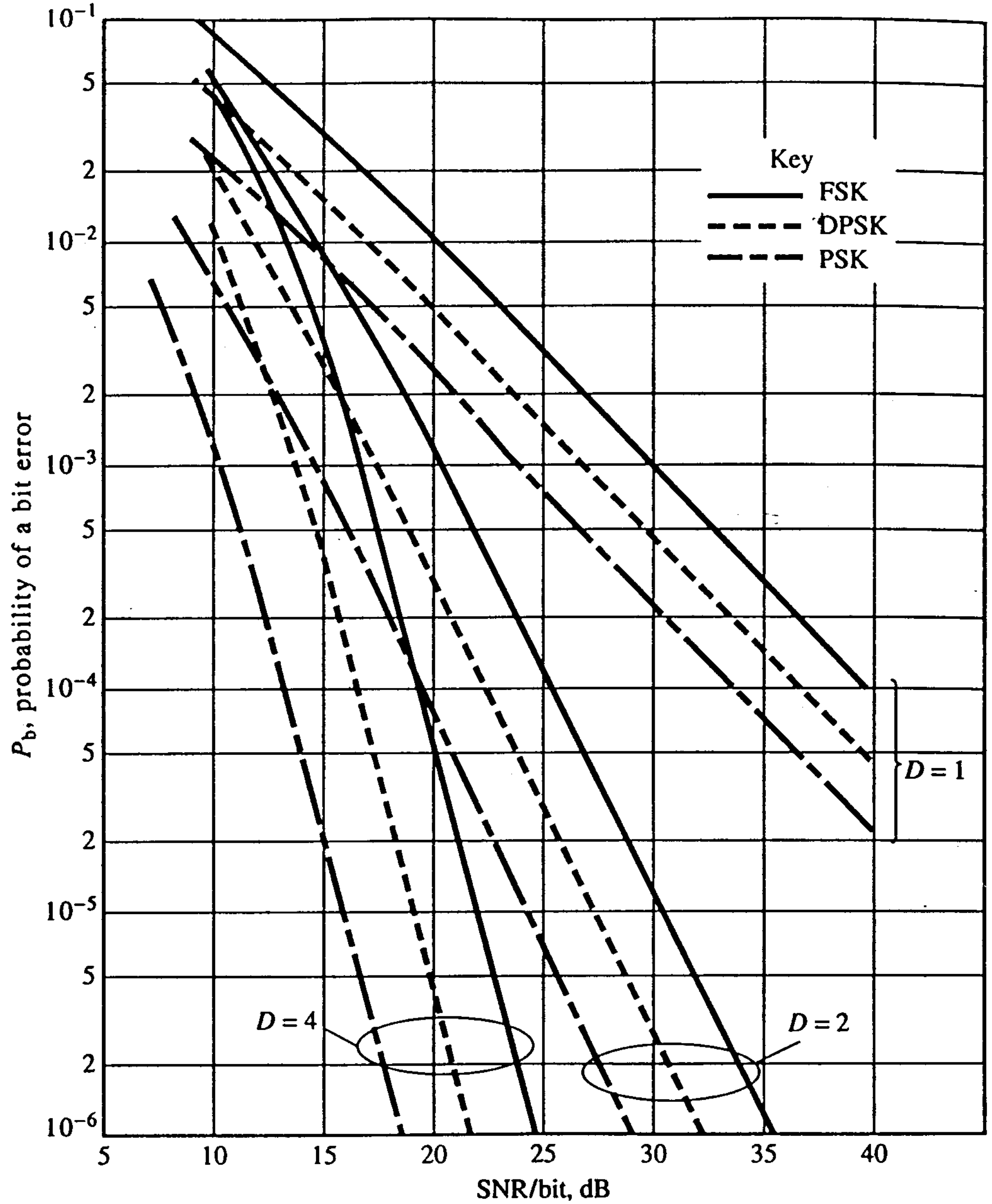
$$P_e(\alpha) = Q\left(\sqrt{\frac{2|\alpha|^2 E_s}{N_0}}\right) \quad \text{where } E_s = \int_0^{T_s} s^2(t)dt$$

- The average error probability is given by

$$\overline{P}_e = E\left[P_e(\alpha)\right] \approx \left(\frac{1}{E_s / N_0}\right) \quad \text{for large SNR}$$

Effect of Flat Fading Channels

- Flattening of BER Curves
- At BER = 10^(-3), the SNR penalty = 15dB!!
- Solution → Diversity



Wideband TX: Frequency Selective Channel

- Signal Bandwidth is too large to see a flat fading channel
  - Different frequency components in the wideband signal undergo different fades

- Equivalently, the symbol period is short when compared to time delay spread
  - suffer from inter-symbol interference (ISI)
  - the ISI characteristic is time-varying as well

- Channel is modeled by a multi-path model

$$h(t,\tau) = \alpha_1(t)\delta(\tau-\tau_1) + \alpha_2(t)\delta(\tau-\tau_2) + \cdots + \alpha_N(t)\delta(\tau-\tau_N)$$

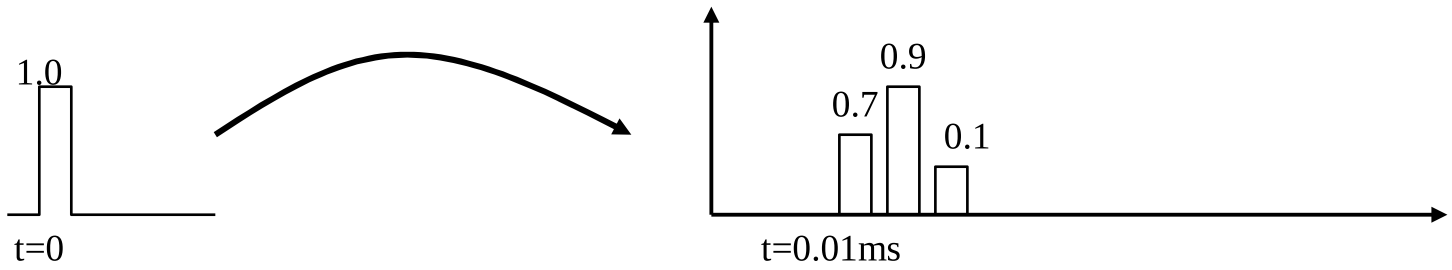
$$y(t) = \alpha_1(t)s(t-\tau_1) + \alpha_2(t)s(t-\tau_2) + \cdots + \alpha_N(t)s(t-\tau_N)$$

  - where N is the number of paths

Frequency Selective Fading Channels

- Wideband Transmission:
  - Coherent BW < Signal BW.
- "multipath" channel model.

$$\underbrace{y(t)}_{received\_signal} = \underbrace{\alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2) + \ldots + \alpha_L x(t-\tau_L)}_{\text{info\_signal}} + \underbrace{z(t)}_{\substack{channel \\ noise}}$$



- "equivalently"
  - uncorrelated fading across the signal bandwidth
- Number of possible paths is approximately BW/B_c
- Paths are often assumed to be independently faded.

Frequency Selectivity

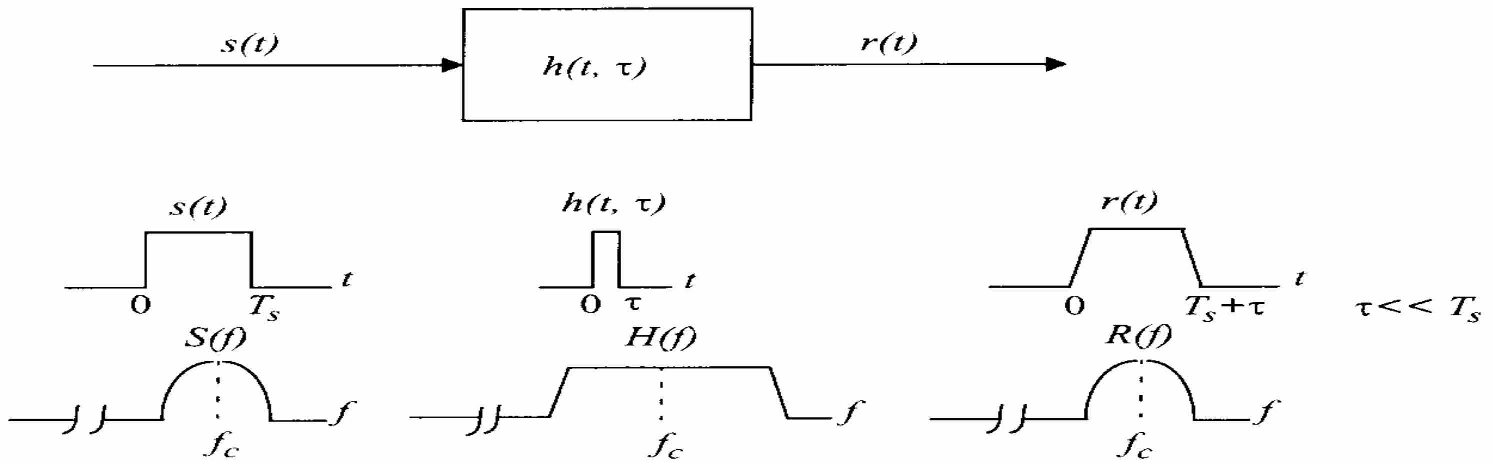


Figure 4.12
Flat fading channel characteristics.

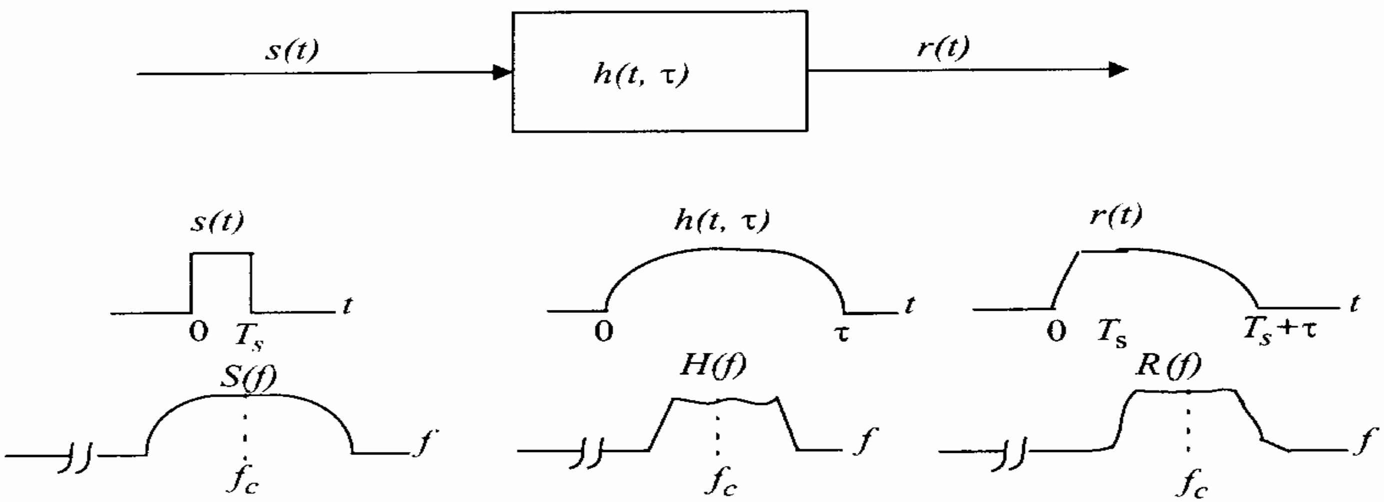


Figure 4.13
Frequency selective fading channel characteristics.

Effect of Frequency Selective Fading

- Multipath → Inter-symbol interference (ISI)
- In addition to flattening of BER curves, we have ***irreducible error floor.***
- Solution
  - Diversity → take care of the flattening
  - Equalization → take care of error floor.

Fast and Slow Fading

Very Fast Fading (Very rare in practical systems)

- Coherence time < Symbol period
- Channel variations faster than baseband signal variations

Fast Fading

- Coherence time ~ 10 to a few hundred symbol periods

Slow Fading

- Coherence time ~ a thousand or more symbol periods

Summary of Main Points

- Wireless Channels
  - Path loss
    - Variation of signal strength due to distance variation.
    - Long term variation ~ secs or minutes
  - Shadowing
    - Variation of signal strength due to terrain change
    - Medium term variation ~ secs
  - Microscopic Fading
    - Variation of signal strength due to multipath
    - Short term variation ~ ms
    - Multipath dimension
      - Delay spread / Coherence BW
      - # of resolvable multipaths = Tx BW/Bc
      - Flat fading (TxBW < Bc) or Frequency selective fading (TxBW > Bc)
    - Time Variation Dimension
      - Doppler spread / Coherence Time
      - Fast fading (Tc < Tframe), slow fading (Tc > Tframe)
- Flat Fading Effect on BER
  - Flattening on the BER curve
  - Solution
    - Diversity (Time, frequency, spatial)
      - Male multiple independent observations before making the hard decision.
- Frequency Selective Fading on BER
  - Flattening of BER curve + ISI (Inter-symbol interference) due to echos / multipaths
  - Solution
    - Diversity + Equalization.