Particle Swarm Optimization

(Kennedy & Eberhart, 1995)

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Motivation

- Originally a model of social information sharing
- Abstract vs. concrete spaces
  - cannot occupy same locations in concrete space
  - can in abstract space (two individuals can have same idea)
- Global optimum (& perhaps many suboptima)
- Combines:
  - private knowledge (best solution each has found)
  - public knowledge (best solution entire group has found)

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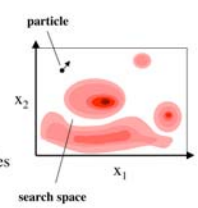
Particle Swarms

Idea

- moving points in the search space, which refine their knowledge by interaction

What is a particle?

- a particle consists of:
  - $\vec{x}_i$ position
  - $\vec{v}_i$ velocity
  - $\vec{p}_i$ best position found so far
  - velocity and position update rules

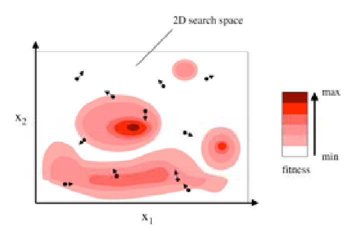


(Kennedy and Eberhart, 1995)

10/13/04 Fig. from EVALife site 3

Example

Particle Swarms



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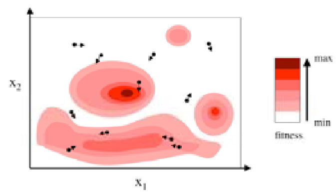
Example

Particle Swarms



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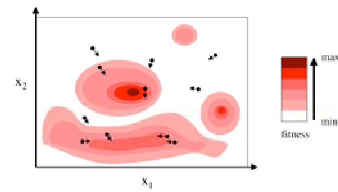
Example

Particle Swarms



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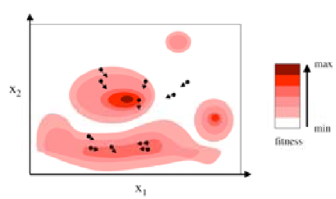
Example

Particle Swarms



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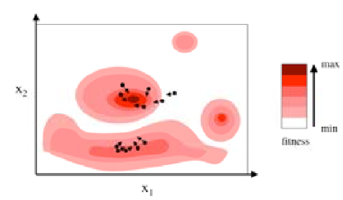
Example

Particle Swarms



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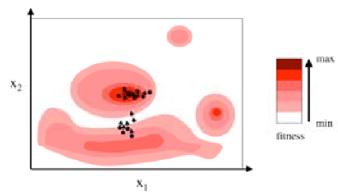
Example

Particle Swarms



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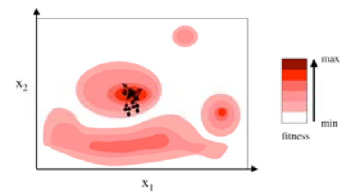
Example

Particle Swarms



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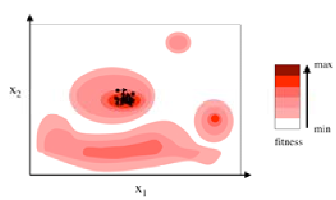
Example

Particle Swarms



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Variables

$\mathbf{x}_k$ = current position of particle k

$\mathbf{v}_k$ = current velocity of particle k

$\mathbf{p}_k$ = best position found by particle k

$Q(\mathbf{x})$ = quality of position $\mathbf{x}$

g = index of best position found so far

i.e., $g = \text{argmax}_k\, Q(\mathbf{p}_k)$

ϕ_1, ϕ_2 = random variables uniformly distributed over [0, 2]

w = inertia < 1

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Velocity & Position Updating

$\mathbf{v}_k' = w\,\mathbf{v}_k + \phi_1 (\mathbf{p}_k - \mathbf{x}_k) + \phi_2 (\mathbf{p}_g - \mathbf{x}_k)$

- $w\,\mathbf{v}_k$ maintains direction (*inertial* part)
- $\phi_1 (\mathbf{p}_k - \mathbf{x}_k)$ turns toward private best (*cognition* part)
- $\phi_2 (\mathbf{p}_g - \mathbf{x}_k)$ turns towards public best (*social* part)

$\mathbf{x}_k' = \mathbf{x}_k + \mathbf{v}_k$

- Allowing $\phi_1, \phi_2 > 1$ permits overshooting and better exploration (*important!*)
- Good balance of *exploration* & *exploitation*
- Limiting $\|\mathbf{v}_k\| < \|\mathbf{v}_{\max}\|$ controls resolution of search

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Improvements

- Alternative velocity update equation:
  $\mathbf{v}_k' = \chi\,[w\,\mathbf{v}_k + \phi_1 (\mathbf{p}_k - \mathbf{x}_k) + \phi_2 (\mathbf{p}_g - \mathbf{x}_k)]$
  χ = constriction coefficient (controls magnitude of $\mathbf{v}_k$)
- Alternative neighbor relations:
  - **star**: fully connected (each responds to best of all others; fast information flow)
  - **circle**: connected to K immediate neighbors (slows information flow)
  - **wheel**: connected to one axis particle (moderate information flow)

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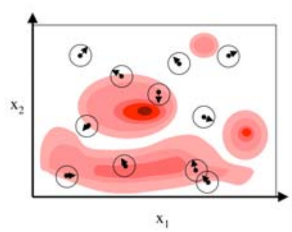
Spatial Extension



- Spatial extension avoids premature convergence
- Preserves diversity in population
- More like flocking/schooling models

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Some Applications of PSO

- integer programming
- minimax problems
  - in optimal control
  - engineering design
  - discrete optimization
  - Chebyshev approximation
  - game theory
- multiobjective optimization
- hydrologic problems
- musical improvisation!

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Millonas' Five Basic Principles of Swarm Intelligence

1. *Proximity principle:*
   pop. should perform simple space & time computations
2. *Quality principle:*
   pop. should respond to quality factors in environment
3. *Principle of diverse response:*
   pop. should not commit to overly narrow channels
4. *Principle of stability:*
   pop. should not change behavior every time env. changes
5. *Principle of adaptability:*
   pop. should change behavior when it's worth comp. price

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Kennedy & Eberhart on PSO

"This algorithm belongs ideologically to that philosophical school

that allows wisdom to emerge rather than trying to impose it,

that emulates nature rather than trying to control it,

and that seeks to make things simpler rather than more complex.

Once again nature has provided us with a technique for processing information that is at once elegant and versatile."

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Additional Bibliography

1. Camazine, S., Deneubourg, J.-L., Franks, N. R., Sneyd, J., Theraulaz, G.,& Bonabeau, E. *Self-Organization in Biological Systems*. Princeton, 2001, chs. 11, 13, 18, 19.
2. Bonabeau, E., Dorigo, M., & Theraulaz, G. *Swarm Intelligence: From Natural to Artificial Systems*. Oxford, 1999, chs. 2, 6.
3. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000, ch. 6.
4. Resnick, M. *Turtles, Termites, and Traffic Jams: Explorations in Massively Parallel Microworlds*. MIT Press, 1994, pp. 59-68, 75-81.
5. Kennedy, J., & Eberhart, R. "Particle Swarm Optimization," *Proc. IEEE Int'l. Conf. Neural Networks* (Perth, Australia), 1995. http://www.engr.iupui.edu/~shi/pso.html.

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