Physics 1501
Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 19: System of Particles III

Recap: motion of the center of mass

- The center of mass obeys Newton's second law:

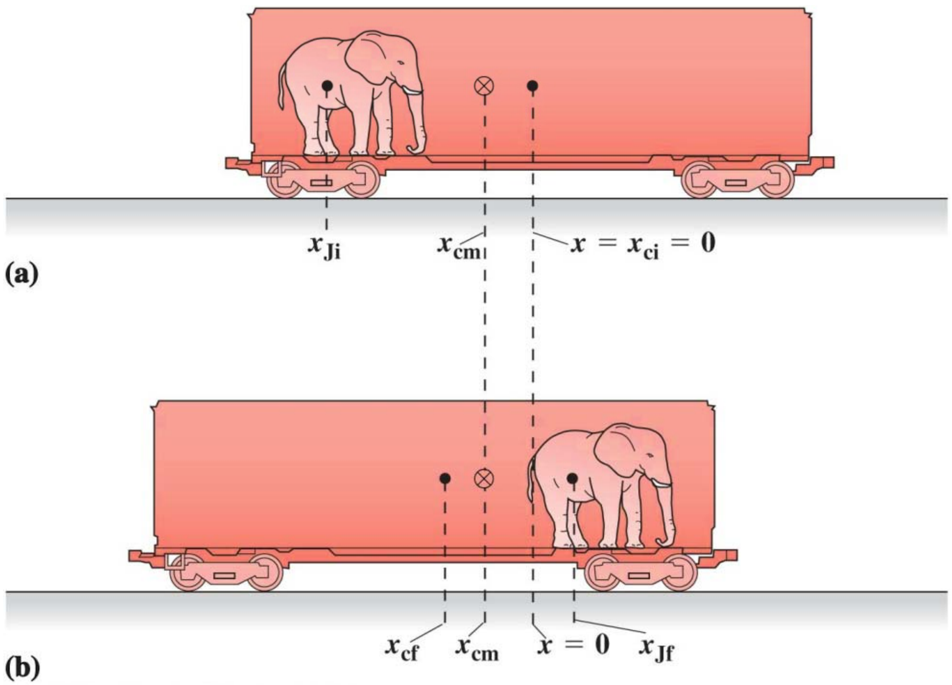
$$\vec{F}_{\text{net external}} = M\vec{a}_{\text{cm}}$$

- Here most parts of the skier's body undergo complex motions, but his center of mass describes the parabolic trajectory of a projectile:

Motion of the center of mass

- Absent any *external* forces on a system, the center of mass motion remains unchanged; if it's at rest, it remains in the same place—no matter what *internal* forces may act.

Here Jumbo walks, but the CM of the rail car plus elephant doesn't move. This allows us to find the car's final position:



$$x_{cm} = \frac{m_J x_{Jf} + m_c x_{cf}}{M} = \frac{m_J(x_{Ji} + 19\text{ m} + x_{cf}) + m_c x_{cf}}{M}$$

$$x_{cm} = -\frac{(19\text{ m})m_J}{(m_J + m_c)} = -\frac{(19\text{ m})(4.8\text{ t})}{(15\text{ t} + 4.8\text{ t})} = -4.6\text{ m}$$

Momentum and the center of mass

- The center of mass obeys Newton's law, which can be written $\vec{F}_{\text{net external}} = M\vec{a}_{\text{cm}}$ or, equivalently,

$$\vec{F}_{\text{net external}} = \frac{d\vec{P}}{dt}$$

where $\vec{P}$ is the total momentum of the system:

$$\vec{P} = \sum m_i \vec{v}_i = M\vec{v}_{\text{cm}}$$

with $\vec{v}_{\text{cm}}$ the velocity of the center of mass.

question

A 500-g fireworks rocket is moving with velocity $\vec{v} = 60\hat{\boldsymbol{j}}$ m/s at the instant it explodes. If you were to add the momentum vectors of all its fragments just after the explosion, what would be the result?

A. $\vec{v} = 60\hat{\boldsymbol{j}}$ kg·m/s

B. $\vec{v} = 30\hat{\boldsymbol{j}}$ kg·m/s

C. $\vec{v} = 60000\hat{\boldsymbol{j}}$ kg·m/s

D. $\vec{v} = 30000\hat{\boldsymbol{j}}$ kg·m/s

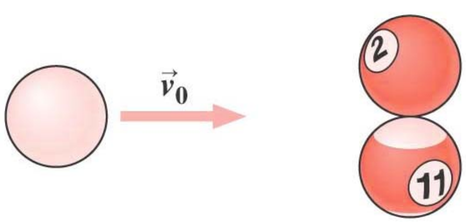
Conservation of momentum

- When the net external force is zero, $d\vec{P}/dt = 0$.
- Therefore the total momentum of the system is unchanged:

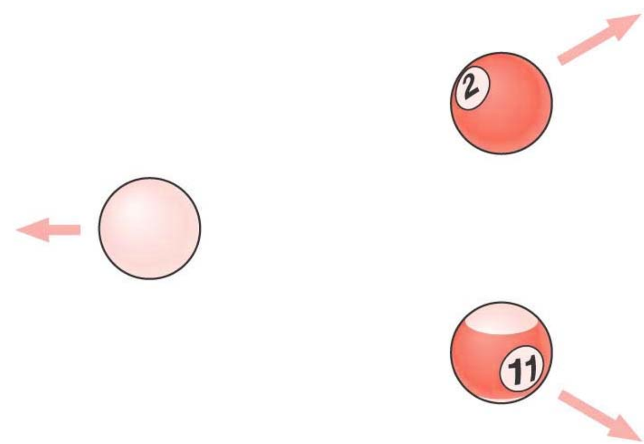
$$\vec{P} = \text{constant}$$

This is the conservation of linear momentum.

- A system of three billiard balls:
  - Initially two are at rest; all the momentum is in the left-hand ball:



- Now they're all moving, but the total momentum remains the same:



Collisions

- A collision is a brief, intense interaction between objects.
  - The collision time is short compared with the timescale of the objects' overall motion.
  - Internal forces of the collision are so large that we can neglect any external forces acting on the system during the brief collision time.
  - Therefore linear momentum is essentially conserved during collisions.

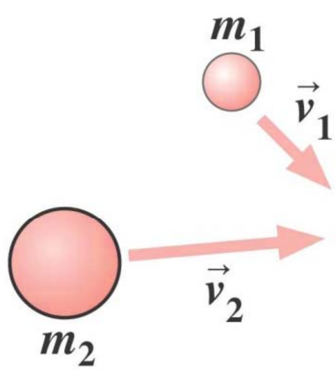
Elastic and inelastic collisions

- In an elastic collision, the internal forces of the collision are conservative.
  - Therefore an elastic collision conserves kinetic energy as well as linear momentum.
- In an inelastic collision, the forces are not conservative and mechanical energy is lost.
  - In a totally inelastic collision, the colliding objects stick together to form a single composite object.
  - But if a collision is totally inelastic, that doesn’t necessarily mean that all kinetic energy is lost.

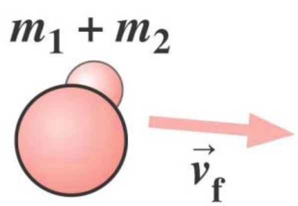
Totally inelastic collisions

- Totally inelastic collisions are governed entirely by conservation of momentum.
  - Since the colliding objects join to form a single composite object, there's only one final velocity:

Before collision



After collision



- Therefore conservation of momentum reads

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

Elastic collisions

- Elastic collisions conserve both momentum and kinetic energy:

- Therefore the conservation laws read

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2$$

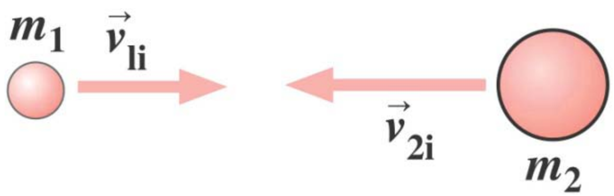
question

Two skaters toss a basketball back and forth on frictionless ice. Which one of the following does not change?

A. The momentum of an individual skater
B. The momentum of the system consisting of one skater and the basketball
C. The momentum of the basketball
D. The momentum of the system consisting of both skaters and the basketball

Elastic collisions in one dimension

- In general, the conservation laws don't determine the outcome of an elastic collision.
  - Other information is needed, such as the direction of one of the outgoing particles.
- But for one-dimensional collisions, when particles collide head-on, then the initial velocities determine the outcome:



- Solving both conservation laws in this case gives

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

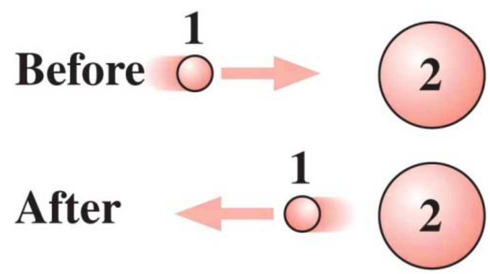
question

Which one of the following qualifies as an inelastic collision?

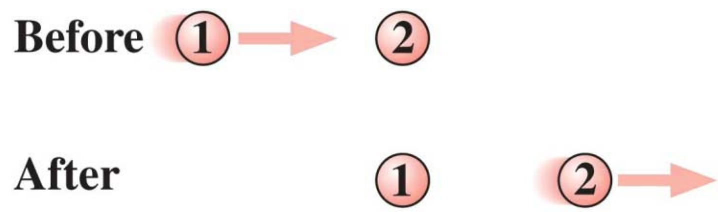
A. Two magnets approach, their north poles facing; they repel and reverse direction without touching.
B. A truck strikes a parked car and the two slide off together, crumpled metal hopelessly entwined.
C. A basketball flies through the air on a parabolic trajectory.
D. A basketball rebounds off the backboard.

Special cases: 1-D elastic collisions; m_2 initially at rest

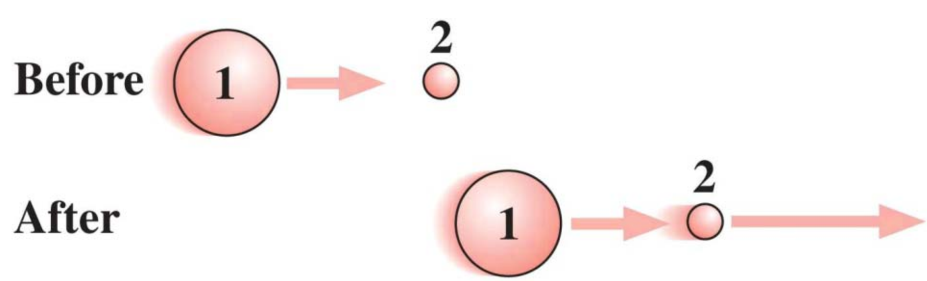
1) $m_1 << m_2$
Incident object rebounds with essentially its incident velocity



2) $m_1 = m_2$
Incident object stops; struck object moves away with initial speed of incident object



3) $m_1 >> m_2$
Incident object continues with essentially its initial velocity; struck object moves away with twice that velocity



question

Ball A is at rest on a level floor. Ball B collides elastically with Ball A, and the two move off separately, but in the same direction. What can you conclude about the masses of the two balls?

A. Ball A and Ball B have the same mass.

B. Ball B has a greater mass than Ball A.

C. Ball A has a greater mass than Ball B.

D. You cannot conclude anything without more information.

Summary

- A composite system behaves as though its mass is concentrated at the **center of mass:**

$$\vec{r}_{\text{cm}} = \frac{\sum m_i \vec{r}_i}{M} \quad \text{(discrete particles)} \qquad \vec{r}_{\text{cm}} = \frac{\int \vec{r}\, dm}{M} \quad \text{(continuous matter)}$$

- The center of mass obeys Newton's laws, so

$$\vec{F}_{\text{net external}} = M\vec{a}_{\text{cm}} \text{ or, equivalently, } \vec{F}_{\text{net external}} = \frac{d\vec{P}}{dt}$$

- In the absence of a net external force, a system's linear momentum is conserved, regardless of what happens internally to the system.
- Collisions are brief, intense interactions that conserve momentum.
  - Elastic collisions also conserve kinetic energy.
  - Totally inelastic collisions occur when colliding objects join to make a single composite object.