Obtaining Ground States of Ising Spin Glasses via Optimizing Bonds Instead of Spins

Martin Pelikan
Missouri Estimation of Distribution Algorithms Lab (MEDAL)
University of Missouri-St. Louis
E-mail: pelikan@cs.umsl.edu
WWW: http://medal.cs.umsl.edu/

Alexander K. Hartmann
Computational Theoretical Physics
Universität Oldenburg, Germany
E-mail: a.hartmann@uni-oldenburg.de
WWW: http://www.theorie.physik.uni-goettingen.de/~hartmann/

Motivation

- Ising spin glasses are prototypical models for disordered systems.
- Ising spin glasses are also challenging class of optimization problems
  - Fast growth of the number of local optima.
  - High-quality solutions divided by barriers of low-quality ones.
  - High-order interactions.
  - Cannot be factored into subproblems of bounded order.
- Hierarchical BOA (hBOA) performs very well on Ising spin glasses and other difficult combinatorial and constraint satisfaction problems.
- However, standard GAs and local search perform poorly.
- Questions
  - Can we improve performance of standard evolutionary algorithms on spin glasses by transforming the problem in some way?
  - Can we generalize this problem transformation to apply to other important classes of constraint satisfaction problems (CSPs), such as MAXSAT?

Goals

- Transform the Ising spin glass problem to optimize bonds instead of spins in order to simplify the problem.
- Test various evolutionary algorithms on the transformed problem and analyze the effects of the transformation.

Ising spin glass

- Ising spin glass
  - Spins arranged on a 2D or 3D grid.
  - Spins $\{s_i\}$ can obtain values 2 values: +1 or -1.
  - Neighbors connected (+ periodic boundary conditions)
  - Each connection (i,j) has a weight $J_{i,j}$
  - Spin glass instance specified by all $J_{i,j}$
- Energy of a spin glass

$$E(C) = \sum_{\langle i,j \rangle} s_i J_{i,j} s_j$$

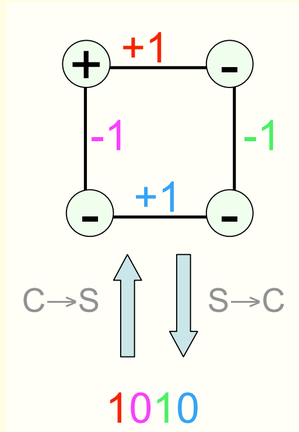
- Optimization problem: Find ground state
  - Given all coupling constants $J_{i,j}$.
  - Find values of spins so that energy is minimized.

Optimizing bonds instead of spins

- Candidate solutions represented by binary vectors (1 bit = 1 spin).
- After selection, spin vectors are transformed into bond vectors (another representation).
- Variation is applied to the transformed solutions.
- The new solutions are transformed back into spin vectors.

Transforming spins to bonds and vice versa

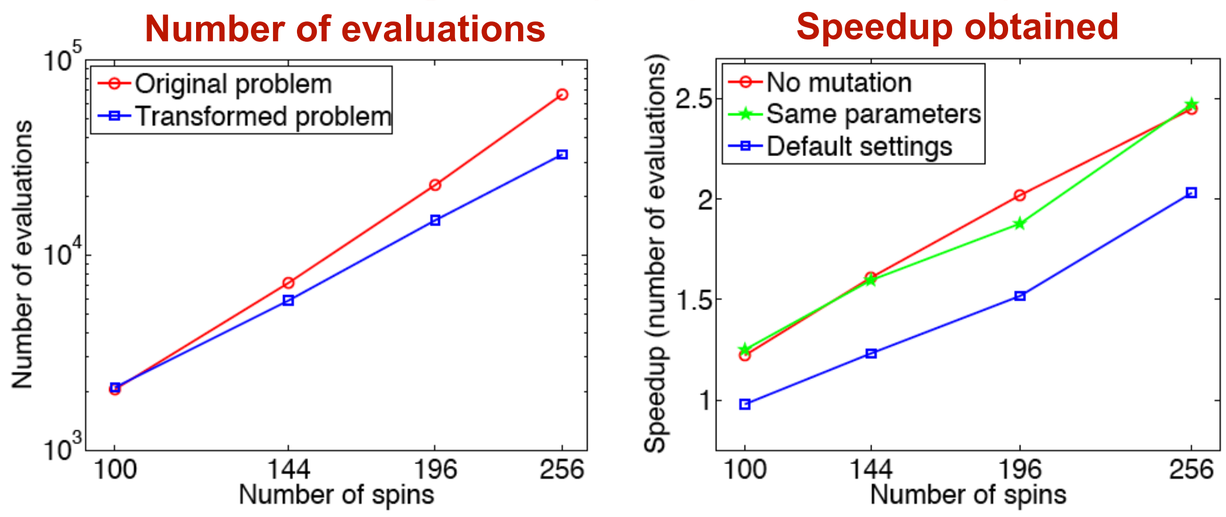
- Spins to couplings (S → C)
  - Each coupling maps to a bit.
  - 1: Satisfied constraint (negative energy)
  - 0: Unsatisfied constraint (positive energy)
- Couplings to spins (C → S)
  - Start in a random spin, set it to random value.
  - In each step choose a spin to set to maximize the number of consistent couplings (ties are resolved randomly).



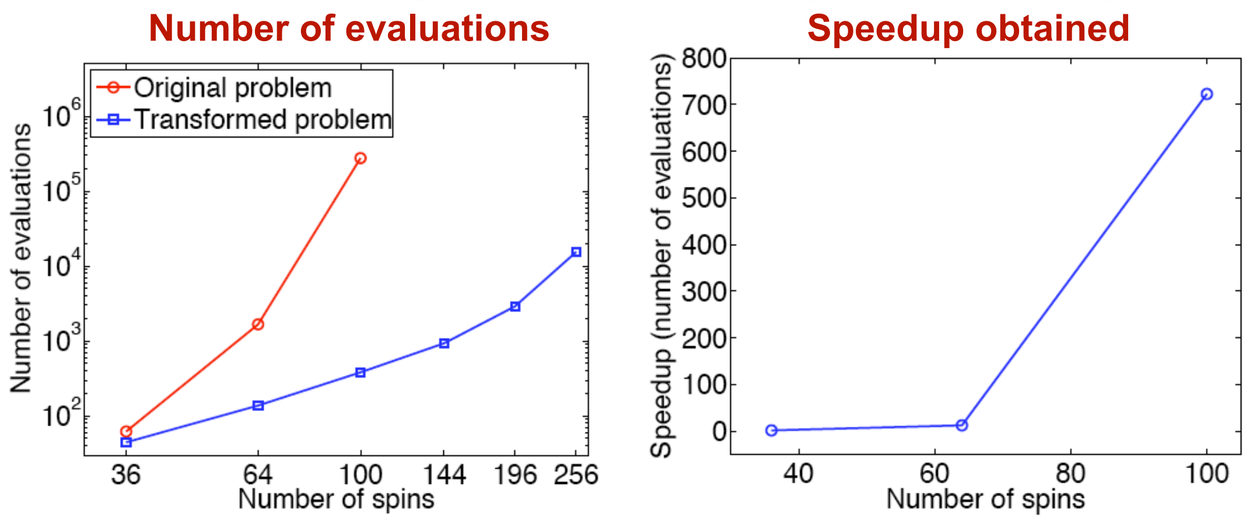
Experiments

- Test problems
  - 2D spin glasses of size 6x6 to 16x16.
  - 1000 random instances for each size (couplings are +1 or -1)

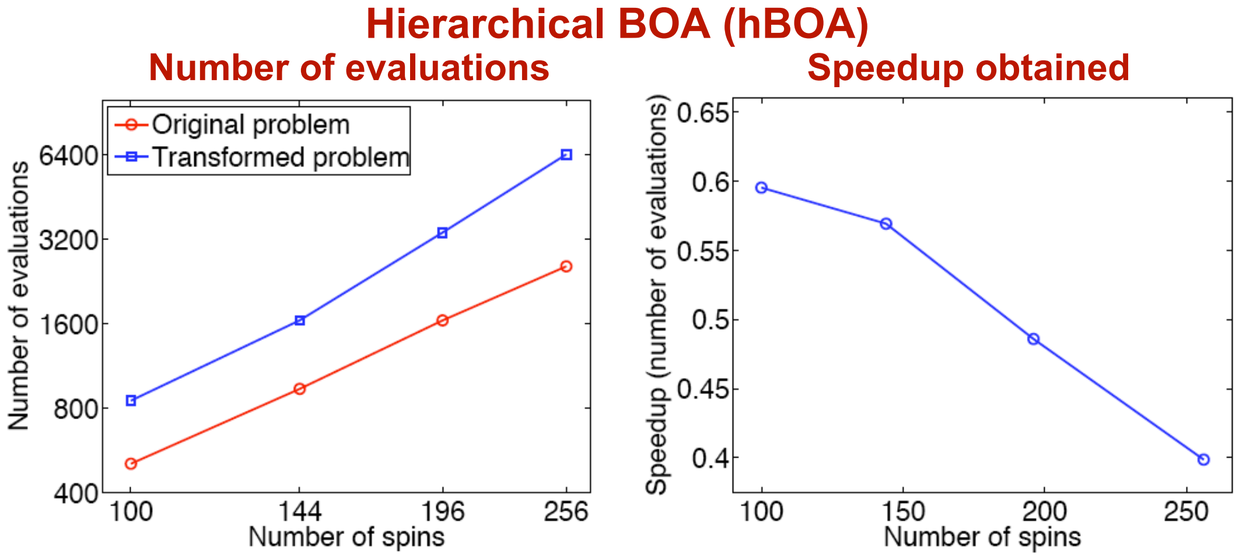
Genetic algorithm (two-point crossover)



Univariate marginal distribution algorithm (UMDA)



Hierarchical BOA (hBOA)



Conclusions

- Transformation significantly helps simple evolutionary algorithms like GA and UMDA.
- However, hBOA works better without transformation.
- Can we generalize these results?

Acknowledgments

- NSF; NSF CAREER grant ECS-0547013
- VolkswagenStiftung (Germany) within the program Nachwuchsgruppen an Universitäten
- University of Missouri; High Performance Computing Collaboratory sponsored by Information Technology Services; Research Award; Research Board