Performance and Turnover in a Stochastic Partnership

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Washington University in St. Louis, May 2009

Motivation: endogenous stability

A long-lasting ("stable") relationship is essential for effective informal incentives.

Stability is often treated as exogenous, but empirical research has found that real-world partnerships display endogenous stability.

- **Stylized fact**: older partnerships tend to be more stable and more productive.

I shall develop a theory of **endogenous stability** and **dynamic performance** in partnerships, when payoffs evolve over time according to a controlled stochastic process.

As an application, I shall derive the value and steady-state distribution of a **partnership economy** with anonymous re-matching.

Motivating example: Shanghai General Motors

- General Motors Shanghai is a 50-50 owned joint-venture of GM and Shanghai Automotive Industry Company (SAIC).
- GM sales in China have grown from 20K in 1999 to over one million cars sold in 2007

In this venture, GM decides how much to invest and share while SAIC decides how much to invest and whether/when to develop copy-cat products.

Shanghai GM: future exit?

One key consideration in this game is uncertainty about the future of the partnership, e.g.

- Will SAIC have reason to end the partnership?
  - competition with own line
  - political pressure to expand own capacity / capability
  - another partner might become better match [at most two]
- Will GM have reason to end the partnership?
  - intellectual property fears
  - constraints on exporting earnings
  - waning Shanghai political power
  - prospect of wholly-owned foreign entities

How will anticipation of such uncertainty affect GM and SAIC's willingness to cooperate today?

Shanghai GM: cooperation breakdown?

> *"General Motors announced ... that it would build a [solely-operated] advanced research center in Shanghai to develop hybrid technology and other designs, in the latest research investment in China by a foreign automaker despite chronic problems with purloined car designs."*
>
> *"Kevin Wale, the president of GM's China operations, said that the company remained 'very comfortable' with the partnership, and that the Shanghai company's recent introduction of its own sedans in competition had shown 'no significant impact' on GM's own sales."*
> *– Int Herald Tribune, October 29, 2007.*

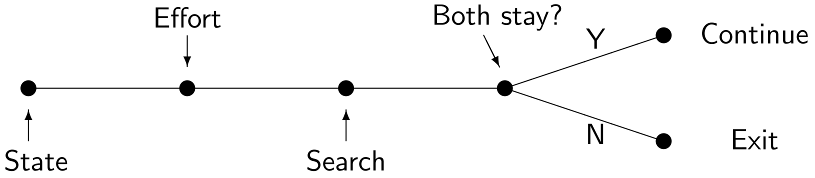
Should we expect the partnership to dissolve soon, given GM and SAIC's apparent recent failure to cooperate fully?

Outline of talk

- **Model & preview of results**
- Welfare-maximizing symmetric equilibrium
- Comparative statics
- Simple example: dynamic Prisoners' Dilemma
- Partnership economy with anonymous rematching
- Related literature
- Directions for future work

Model

Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



Model

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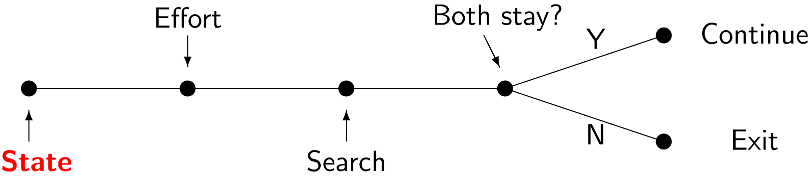
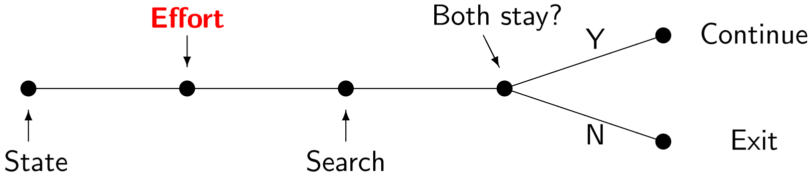


- State $x_t \in \mathcal{X}_t$ commonly observed; $(\mathcal{X}_t, \succeq)$ partially ordered.
- Distribution of X_t depends on history of past states x_{t-1} and past efforts $\mathbf{e}_{t-1}$, but *not* on past search.
- $x'_{t-1} \succeq x_{t-1} \Rightarrow \Pr(X_t \in A | x'_{t-1}, \mathbf{e}_{t-1}) \geq \Pr(X_t \in A | x_{t-1}, \mathbf{e}_{t-1})$ for all increasing subsets $A \in \mathcal{X}_t$ and all $\mathbf{e}_{t-1}$.

Model

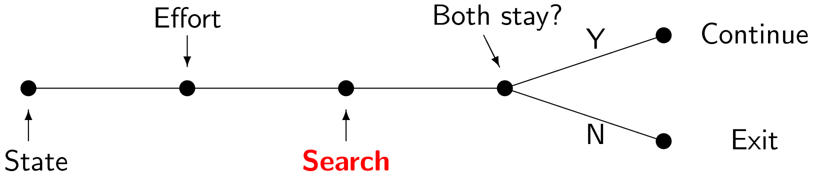
Each period $t = 0, 1, 2, \dots$ in an active partnership proceeds as:



- Effort $e_{it} \in \mathcal{E}_t$ simultaneous then observed; $(\mathcal{E}_t, \succeq)$ partially ordered with minimal element "0".
- Stage-game payoff $\pi_{it}(e_t; x_t)$ from efforts $e_t = (e_{it}, e_{jt})$.
- π_{it} symmetric; $e_{it} = 0$ weakly dominant in effort stage-game.

Model

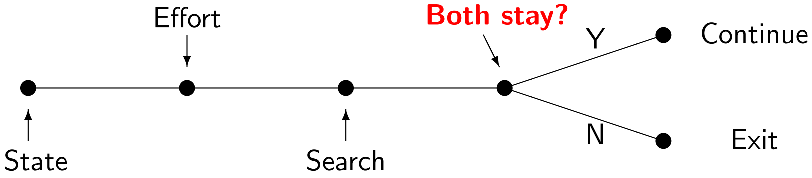
Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



- Search $s_{it} \in \mathcal{S}_t$ unobserved; costs $c_t(s_{it})$; $(\mathcal{S}_t, \geq)$ lattice.
- Player i then observes outside option $v_{it} \in \mathbf{R}_+$ should partnership end this period.
- Distribution of V_{it} depends only on i's current search intensity.
- $s'_{it} \geq s_{it} \Rightarrow V_{it}|s'_{it}$ FOSD $V_{it}|s_{it}$.

Model

Each period $t = 0, 1, 2, \ldots$ in an active partnership proceeds as:



- The partnership ends with exogenous probability $\lambda \in [0, 1]$ or endogenously if *either* player chooses to quit.
- In that case, each player enjoys outside option v_{it} and zero payoffs thereafter.
- Players share common discount factor $\delta \in (0, 1)$.

Preview of Results

Welfare-maximizing equilibrium construction. Focus on symmetric pure-strategy subgame-perfect equilibrium (SPSPE).

Comparative statics in this eqm. Welfare is increasing in the state. Under additional assumptions, the following hold true in higher states after higher effort histories:

- welfare is higher
- effort is higher
- search intensity is lower
- stopping time of the partnership is FOSD-higher

Partnership economy. Steady state of a partnership economy is derived, in a special case corresponding to "anonymous re-matching".

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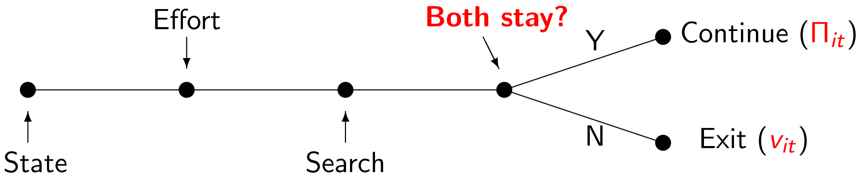
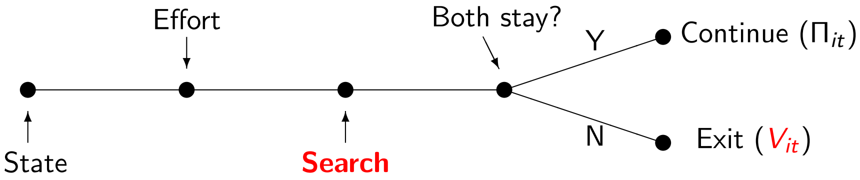
Welfare-maximizing symmetric equilibrium



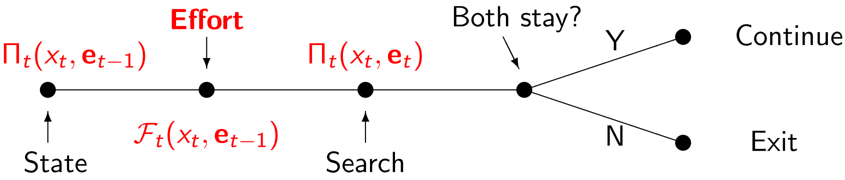
- Exit ends the game $\Rightarrow$ We may view exit decisions as an isolated one-shot game.
- In any equilibrium, the partnership must end when $v_{it} > \Pi_{it}$.
- Exit imposes a negative externality on the other player.
- $\Rightarrow$ Welfare-maximizing equilibrium has each player i exit iff $v_{it} > \Pi_{it}$.
- Welfare-minimizing equilibrium has each player i always exit.

Welfare-maximizing symmetric equilibrium



- Search is unobserved and does not affect future payoffs $\Rightarrow$ We may view search decisions as an isolated one-shot game.
- Search imposes a negative externality on the other player.
- Search intensities are strategic complements, with decreasing differences in $(\Pi_{it}, \Pi_{jt}) \Rightarrow$ by Milgrom Roberts (1990):
  - $\exists$ minimal & maximal equilibria that are welfare-maximizing & welfare-minimizing, respectively.
  - An increase in (Π_{it}, Π_{jt}) decreases search intensity and increases welfare in these equilibria.
  - If $\Pi_{it} = \Pi_{jt}$, these equilibria are in symmetric pure strategies.

Welfare-maximizing symmetric equilibrium



- Efforts are observable and control future payoffs directly.
- $\Rightarrow$ Symmetric continuation payoffs after time-t efforts, $\Pi_t(x_t, \mathbf{e}_{t-1}, e_t)$, depend on effort e_t.
- Feasible effort $e_t \in \mathcal{F}_t(x_t, \mathbf{e}_{t-1})$ must satisfy IC constraint:

$$\Pi_t(x_t, \mathbf{e}_{t-1}, e_t) - \Pi_t^{exit} \geq \pi_{it}(0, e_t; x_t) - \pi_{it}(e_t, e_t; x_t)$$

where Π_t^{exit} is continuation payoff in welfare-minimizing equilibrium of search & exit game to follow.

Construction of welfare-maximizing SPSPE

$\overline{\boldsymbol{\Pi}}^0_t(x_t, \mathbf{e}_{t-1})$ $\overline{\boldsymbol{\Pi}}^0_{t+1}(x_{t+1}, \mathbf{e}_t)$

- Suppose that there exist upper bounds $\overline{\boldsymbol{\Pi}}^0_t(x_t, \mathbf{e}_{t-1})$ on SPSPE payoffs that are (weakly) increasing in x_t.
- Exist trivially as long as stage-game payoffs from effort and outside options are uniformly bounded.

Construction of welfare-maximizing SPSPE

$\longleftarrow \quad \overline{\boldsymbol{\Pi}}^0_t(x_t, \mathbf{e}_{t-1}) \qquad \overline{\boldsymbol{\Pi}}^0_t(x_t, \mathbf{e}_t) \longleftarrow \overline{\boldsymbol{\Pi}}^0_{t+1}(x_{t+1}, \mathbf{e}_t)$

- Previously shown: given welfare-maximizing search & exit, payoffs after time-t effort $\overline{\boldsymbol{\Pi}}^0_t(x_t, \mathbf{e}_t)$ are increasing in expected time-$(t+1)$ inside continuation payoff.
- $E[\overline{\boldsymbol{\Pi}}^0_{t+1}(X_{t+1}, \mathbf{e}_t)|x_t, \mathbf{e}_t] = \int_0^\infty \Pr(\overline{\boldsymbol{\Pi}}^0_{t+1}(X_{t+1}, \mathbf{e}_t) > \Pi | x_t, \mathbf{e}_t) \mathrm{d}\Pi$ increasing in x_t:
  1. Previously shown: $\overline{\boldsymbol{\Pi}}^0_{t+1}(x_{t+1}, \mathbf{e}_t)$ increasing in x_{t+1}.
  2. Thus, $\{x_{t+1} : \overline{\boldsymbol{\Pi}}^0_{t+1}(x_{t+1}, \mathbf{e}_t) \geq \Pi\}$ is an increasing subset of $\mathcal{X}_{t+1}$, for all $\Pi \geq 0$.
  3. $X_{t+1}|(x_t, e_t)$ is increasing in x_t, relative to our generalized notion of FOSD.

Construction of welfare-maximizing SPSPE

$$\begin{tikzcd} & \mathcal{F}^1_t(x_t, \mathbf{e}_{t-1}) & \\ \longleftarrow \quad \overline{\mathbf{\Pi}}^0_t(x_t, \mathbf{e}_{t-1}) & \overline{\mathbf{\Pi}}^0_t(x_t, \mathbf{e}_t) \arrow[u] & \overline{\mathbf{\Pi}}^0_{t+1}(x_{t+1}, \mathbf{e}_t) \arrow[l] \end{tikzcd}$$

- Set of (potentially) IC efforts is increasing in x_t:
  1. By assumption: short-term incentive to deviate $\pi_{it}(0, e_t; x_t) - \pi_{it}(e_t, e_t; x_t)$ decreasing in x_t.
  2. Previously shown: upper bound on continuation payoffs after any effort-level are increasing in x_t
  3. $\Rightarrow$ IC constraint slackens as x_t increases.

Construction of welfare-maximizing SPSPE

$$
\begin{tikzcd}
{} & & \mathcal{F}_t^1(x_t, \mathbf{e}_{t-1}) \arrow[dl] & & & & {} \arrow[dl] \\
{} & \overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_{t-1}) \arrow[l] & & \overline{\mathbf{\Pi}}_t^0(x_t, \mathbf{e}_t) \arrow[ll] \arrow[ul] & & \overline{\mathbf{\Pi}}_{t+1}^1(x_{t+1}, \mathbf{e}_t) \arrow[ll] & {} \arrow[l]
\end{tikzcd}
$$

$$\overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_{t-1}) = \max_{e_t \in \mathcal{F}_t^1(x_t, \mathbf{e}_{t-1})} \left(\pi_{it}(e_t, e_t; x_t) + \overline{\mathbf{\Pi}}_t^0(x_t, \mathbf{e}_{t-1}, e_t) \right)$$

- By assumption: stage-game payoffs π_{it} increasing in x_t.
- Previously shown: $\overline{\mathbf{\Pi}}_t^0(x_t, \mathbf{e}_{t-1}, e_t)$ increasing in x_t for all e_t.
- Previously shown: IC efforts in low state are IC in high state.
- $\Rightarrow \overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_{t-1})$ is increasing in x_t: players in high state get higher payoff when "mimicking" optimal effort in low state.

Construction of welfare-maximizing SPSPE

$$
\begin{tikzcd}
 & & \mathcal{F}_t^2(x_t, \mathbf{e}_{t-1}) \arrow[dl] & & & \\
 & \overline{\mathbf{\Pi}}_t^2(x_t, \mathbf{e}_{t-1}) \arrow[l] & & \overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_t) \arrow[ll] \arrow[ul] & \overline{\mathbf{\Pi}}_{t+1}^1(x_{t+1}, \mathbf{e}_t) \arrow[l] & \arrow[l] \arrow[ul, phantom] \\
\end{tikzcd}
$$

By induction, these sequences of payoff-bounds (and IC effort-set bounds) are monotone decreasing.

- Obviously: $\overline{\mathbf{\Pi}}_{t+1}^1(x_{t+1}, \mathbf{e}_t) \leq \overline{\mathbf{\Pi}}_{t+1}^0(x_{t+1}, \mathbf{e}_t)$.
- $\Rightarrow \overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_t) \leq \overline{\mathbf{\Pi}}_t^0(x_t, \mathbf{e}_t)$ and $\mathcal{F}_t^2(x_t, \mathbf{e}_{t-1}) \subset \mathcal{F}_t^1(x_t, \mathbf{e}_{t-1})$.
- $\Rightarrow \overline{\mathbf{\Pi}}_t^2(x_t, \mathbf{e}_{t-1}) \leq \overline{\mathbf{\Pi}}_t^1(x_t, \mathbf{e}_{t-1})$

Last step of proof: Limiting payoffs can be achieved in a SPSPE in which effort at each history is the best possible from the limiting set of IC efforts. This SPSPE is welfare-maximizing among all SPSPE.

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Comparative statics in the optimal SPSPE

Payoff in state: Players' equilibrium payoff is (weakly) increasing in x_t, for any fixed $\mathbf{e}_{t-1}$.

Search in state: Players' search intensity is decreasing in x_t, for any fixed $\mathbf{e}_{t-1}$, *if* we also hold fixed time-t effort e_t.

Immediate exit in state: Players' likelihood of exiting is decreasing in x_t, for any fixed $\mathbf{e}_{t-1}$, *if* we also hold fixed time-t effort e_t.

Exit % need not fall with state

Higher state need not translate to lower search intensity / lower likelihood of exit this period.

Example: Co-authoring a paper. After several breakthroughs, co-authors who complete a research paper may then sever their relationship as they have nothing left to do.

What's going on here? In the highest state when the co-authors are ready to complete the paper, exerting the effort needed to optimally finish the paper causes next period's state to be very low.

One way to rule out this sort of example is by assuming an *exogenous* stochastic process.

Comparative statics given exogenous process

$\{X_t : t \geq 0\}$ is an exogenous process if X_t depends only on x_{t-1} for all t.

Under this extra assumption, I derive the following comparative statics in the optimal SPSPE:

Payoff in state: Players' payoff is (weakly) increasing in x_t.

Search in state: Players' search intensity is decreasing in x_t.

Stopping time in state: The distribution of the partnership's stopping time is increasing in x_t, in the sense of FOSD.

Effort need not rise with state

In the general model, higher states need not induce higher effort.

Example: Marriage. Some couples may only need to "work on their marriage" when the state is low and near-future exit would otherwise be likely.

What's going on here? Although it was feasible to work during happy times, the couple prefers not to because doing so is costly.

Another way to rule out this sort of example (apart from assuming exogeniety) is to impose the following set of extra assumptions:

- **Positive feedback from effort:** $X_t | (x_{t-1}, \mathbf{e}_{t-1})$ is increasing in $\mathbf{e}_{t-1}$, relative to our generalized notion of FOSD.
- **Immediate reward from effort:** $\pi_{it}(e_t, e_t; x_t)$ is increasing in e_t (as in Prisoners' Dilemma).
- **Effort one-dimensional.**

I will maintain these assumptions for the rest of the presentation.

Comparative statics given positive feedback & immediate rewards

Under these extra assumptions, I derive the following comparative statics in the optimal SPSPE:

Payoff in state and effort: Players' payoff is (weakly) increasing in $(x_t, \mathbf{e}_{t-1})$.

Effort in state and effort: Players' effort is increasing in $(x_t, \mathbf{e}_{t-1})$.

Search in state: Players' search intensity is decreasing in $(x_t, \mathbf{e}_{t-1})$.

Stopping time in state: The distribution of the partnership's stopping time is increasing in $(x_t, \mathbf{e}_{t-1})$, in the sense of FOSD.

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Example: dynamic Prisoners' Dilemma

Non-random outside option $v \geq 0$ [search is trivial].

Stage-game payoffs from effort take the form:

	Work	Shirk
Work	w_t, w_t	$-w_t - d_t, w_t + d_t$
Shirk	$w_t + d_t, -w_t - d_t$	0,0

$(w_t, d_t) \in \mathbf{R}^2_+$ follows an exogenous stochastic process.

Example 1: w_t iid, $d_t = d > 0$ [Ramey Watson 97]

Example 2: $\log(w_t)$ random walk, $d = 0$ [Levinthal 91]

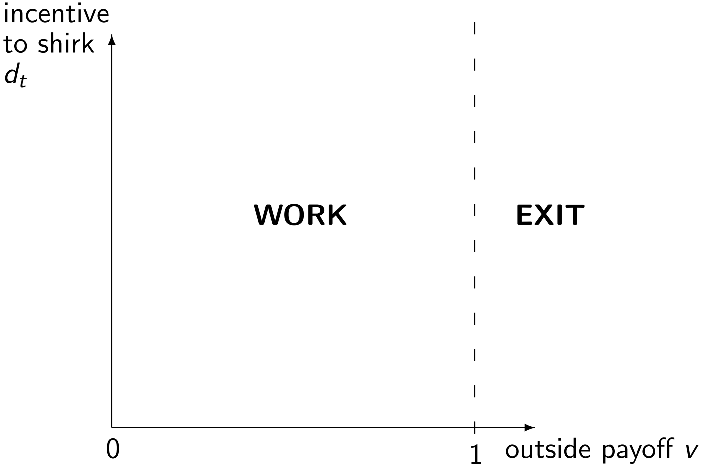
Example 3: $w_t = \frac{\sum_{s \leq t} x_s}{t}$, $x_s \sim N(w, \epsilon)$ iid, $d = 0$ [Jovanovic 79a]

For simplicity, I will focus here on special case in which $\delta = \frac{1}{2}$ and

- $w_t = 1$ for all t
- $\log(d_t)$ symmetric random walk

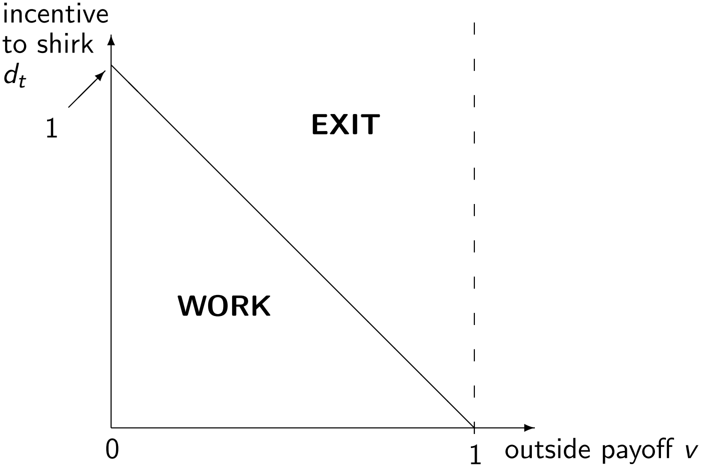
Summary of findings

Efficient benchmark: work forever regardless of the state.



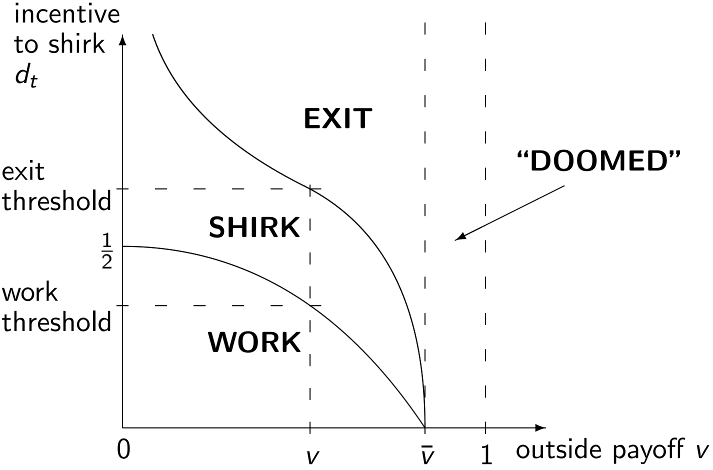
Summary of findings

Unchanging benchmark: either work forever or quit immediately.



Summary of findings

Dynamic game: $\log(d_t)$ evolves according to a random walk.

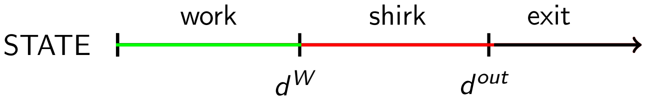


Structure of the “optimal” SPE.

Corollary. Suppose that $v \in [0,1)$. There exist $d^W \le d^{out}$ such that, in a SPE that maximizes joint (discounted) payoffs:

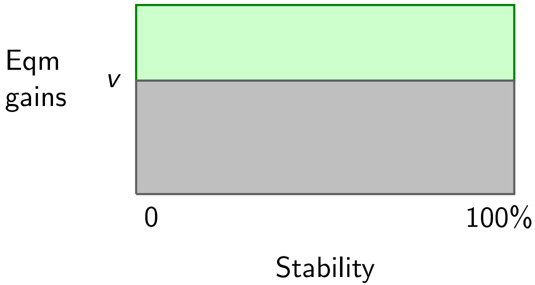
1. [**good times**] both work and stay when $d_t \le d^W$
2. [**hard times**] both shirk and stay when $d^W < d_t \le d^{out}$
3. [**exit**] both shirk and quit when $d_t > d^{out}$
4. [**after any deviation**] both shirk and quit

[In companion paper, d^W, d^{out} are characterized for any Markov process.]



Cooperation is harder in the changing game

Players cooperate in equilibrium in an even smaller set of states, compared to the unchanging case, for two main reasons:



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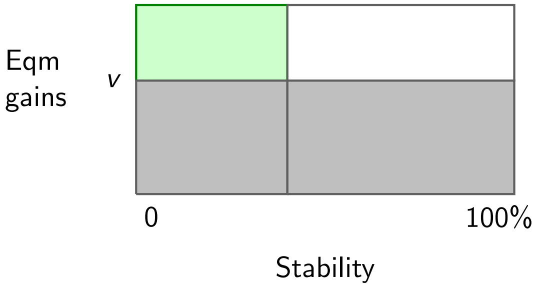
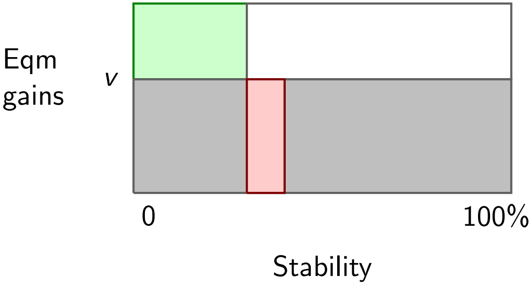
1. **Exit:** fewer future periods in which to cooperate.



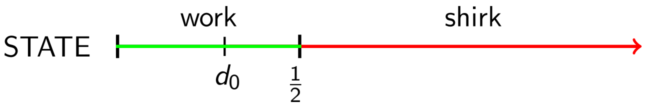
Cooperation is harder in the changing game

Players cooperate in equilibrium in an even smaller set of states, compared to the unchanging case, for two main reasons:

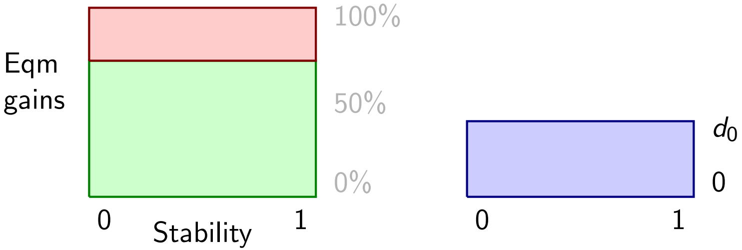
1. **Exit:** fewer future periods in which to cooperate.
2. **Hard times:** losses in periods without cooperation.



Graphical intuition why $d^{W*} = \frac{1}{2}$ when $v = 0$



If $d_0 \leq \frac{1}{2}$, cooperation can be supported **both now and frequently in the future**.



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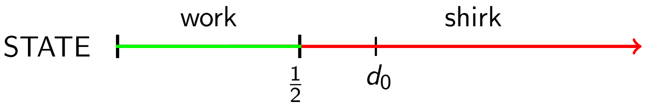
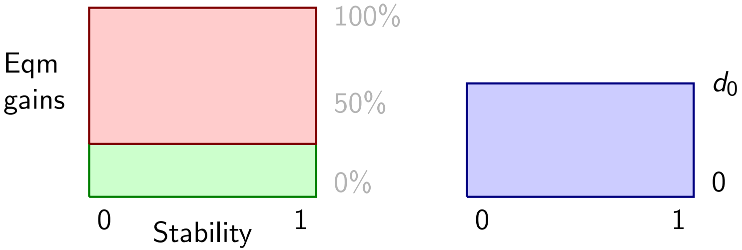


If $d_0 > \frac{1}{2}$, cooperation can be supported **neither now nor frequently in the future**.



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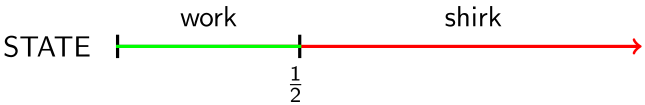
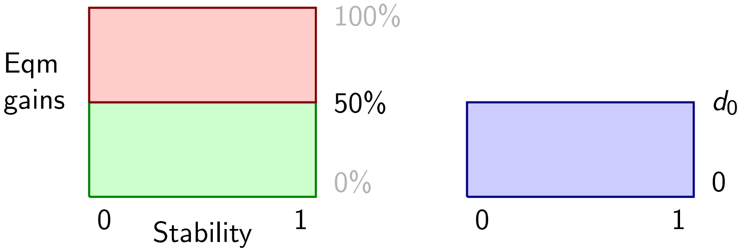
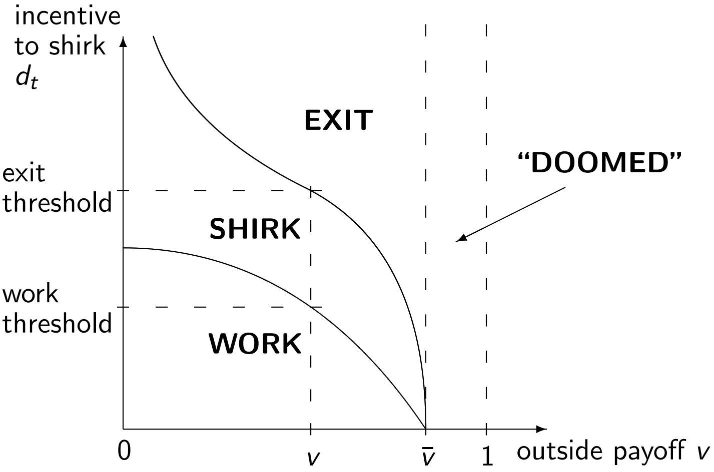


If $d_0 = \frac{1}{2}$, future gains equal current incentive to shirk.

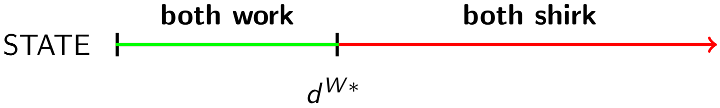


Summary of behavior in the optimal SPE

Example: $\log(d_t)$ follows random walk



Dynamics of partnership performance



Regime persistence: *partners who have enjoyed several periods of cooperation are more likely to remain cooperative.*

Period	2	3	4	5	10	25
% work resumes	25%	16.7%	12.8%	9.8%	5.0%	2.0%

Table: Probability that cooperation *first* breaks down in period t, conditional on $d_0 = d^{W*}$ and cooperation in periods $1, ..., t-1$. [Assuming $v = 0$ and $\log(d_{t+1}) = \log(d_t) + \epsilon_t$ where $\epsilon_t \sim U[-1, 1]$ iid.]

Partnership persistence: *when $v > 0$, similarly, hazard of exit monotonically decreasing in age once partnership is old enough.*

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Model: partnership economy

Unit mass of players, with flow $(1-\delta)$ of births and deaths. Each player dies with iid probability $(1-\delta)$ each period, seeks to maximize expected lifetime earnings.

Each period that a player is not already matched, he is automatically and costlessly matched with a new partner.

Each new match is a "fresh start" in two senses:

1. The stochastic state process is iid across partnerships (and controlled only by efforts in the current partnership).
2. Neither player knows anything about his partner's prior history, including age, number of past partnerships, etc.

Thus, any welfare-maximizing equilibrium of the overall economy specifies welfare-maximizing play in each partnership, taking into account the endogenous outside option $v^* \geq 0$ from re-matching.

Fit within stochastic partnership model

Each individual partnership can be interpreted as an instance of our stochastic partnership game, under the following parameter restrictions:

- *Discount factor δ:* Given death-rate $(1-\delta)$, each player acts as if maximizing discounted earnings.
- *Exogenous separation probability $\lambda = (1-\delta)$:* Conditional on own survival, each player's partner dies with prob. $(1-\delta)$.
- *Trivial search and non-random outside option $v^* \geq 0$:* v^* will be determined endogenously as the expected continuation payoff from being re-matched next period.

Extra assumptions for upcoming analysis

The results to be presented in the remainder of the talk have been proven under the following additional assumptions:

- State x_t is one-dimensional.
- Next-period state $X_t|(x_{t-1}, \mathbf{e}_{t-1})$ is atomless.
- The set of efforts is finite and totally ordered.

Maximal economy-wide equilibrium welfare

Theorem 8 (Welfare-maximizing payoffs in partnership economy).
There is a unique equilibrium of the overall partnership economy with welfare-maximizing SPSPE play within each partnership. Each player's ex ante expected payoff at birth in this equilibrium is v^*/δ*, where* $v^* \geq 0$ *is the* unique *solution to*

$$v = E[\delta \overline{\Pi}_0^{eqm}(X_0; v)] \geq 0 \tag{1}$$

Uniqueness arises as LHS of (1) rises more quickly than RHS.
Intuition:

- Holding equilibrium efforts fixed, marginal benefit from higher outside option is at most one-for-one.
- Set of IC efforts shrinks as v increases $\Rightarrow$ performance inside the partnership deteriorates with the outside option.

Steady-state distribution of partnership histories

Some notation:

- $h_t = (x_t, \mathbf{e}_{t-1})$: payoff-relevant history at start of period t.
- $e_t(h_t)$: symmetric effort chosen at history h_t.
- $p_t^{exit}(h_t)$: probability at least one player exits.
- $X_{t+1}(h_t) \sim X_{t+1}|(h_t, e_t(h_t))$: random next-period state.

Transition probabilities among histories are fully described by:

- With probability $1 - \delta^2$, each partnership will end by death, replaced by a new one having random initial history $H_0 = X_0$.
- With probability $\delta^2 p_t^{exit}(h_t)$, it will end due to some partner's endogenous departure, and also be replaced by a new one.
- Otherwise, the partnership will continue to time $t+1$, with augmented random history $H_{t+1} = (h_t; e_t(h_t); X_{t+1}(h_t))$.

Steady-state distribution of partnership histories

Through the process of death and re-birth, all histories reached on the equilibrium path communicate and are positively recurrent.

$\Rightarrow$ the Markov chain of partnership histories is ergodic.

Corollary to Theorem 8 (Steady-state distribution). *In a partnership economy with welfare-maximizing SPSPE play within each partnership, there exists a unique steady-state distribution over partnership histories.*

A "typical" life in the partnership economy

Play tends to pass through a few (mostly) distinct phases:

- Dating.
- Honeymoon.
- Good times.
- Hard times.
- Golden years.

with typical transitions

- Dating $\rightarrow$ Honeymoon
- Honeymoon $\rightarrow$ Good times or Hard times
- Hard times $\rightarrow$ Dating or Good times
- Good times $\rightarrow$ Golden Years or Hard times
- Golden years absorbing (until death do they part)

Dating

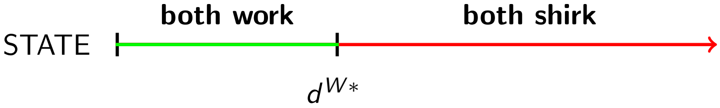
When searching, players will churn through a sequence of one-period, zero-effort partnerships.

Discussion: The assumption that X_0 is atomless is crucial for this result. Given a non-stochastic repeated partnership, welfare-maximizing equilibria exhibit an "incubation period" with the first person that you meet (Kranton (1996), Mailath Samuelson (2006)).

"Dating equilibria" also arise as welfare-maximizing if such players have access to a public randomization device.

Once there is non-trivial payoff-relevant variation in "initial fit", dating equilibria strictly outperform incubation equilibria.

Good times / Hard times

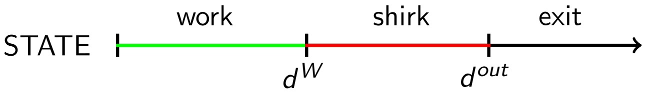


The partnership transitions, in equilibrium, between **"good times"** when both players work ($d_t \leq d^{W*}$) and **"hard times"** when both shirk ($d_t > d^{W*}$).

Given $v > 0$, **exit is typically preceded by hard times**.

Players remain in a rocky relationship because of the (endogenous) option value created by the possibility of future cooperation.

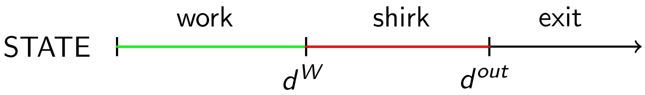
Honeymoon



Players only form an enduring partnership in good times, if being unmatched yields a positive payoff flow. (Otherwise, they are still "likely" to form a partnership in good times.)

Consistent with findings on the survival of organizations but *not* on the survival of marriages. ▸ How to reconcile this?

Survivorship bias



Partnerships that have survived a long time are likely to last even longer.

Consistent with findings on the survival of organizations (Levinthal 1991), employment relationships (Topel Ward 1992), and marriages (Stevenson Wolfers 2007).

Golden years

After high enough histories, the partnership becomes permanent (until death).

Golden years can arise for two distinct reasons:

1. Absorbing increasing subset of the state-space.
2. High efforts in high states, combined with overwhelming positive feedback from effort.

Outline of talk

- Model & preview of results
- Welfare-maximizing symmetric equilibrium
- Comparative statics
- Simple example: dynamic Prisoners' Dilemma
- Partnership economy with anonymous rematching
- **Related literature**
- Directions for future work

Dynamics in relationships: some classic models

Green Porter (1984) — imperfect monitoring

- *More precise prediction*: during non-cooperative regime, the rate at which cooperation resumes falls over time.

Ghosh Ray (1996), Watson (2002) — building trust (signaling)

- *Different implication*: trust not monotonically increasing in duration of partnership, though survivors tend to be those who have built up more trust.

Jovanovic (1979a,b), Pissarides (1994) — learning about match quality, match-specific investment, and on-the-job search

- *Key difference*: this paper adds two-sided moral hazard, as well as richer investment & search possibilities.

Some related (and unrelated) literature

Stochastic exit (and entry): e.g. **Jovanovic** (1979a,b), Dixit (1989), Levinthal (1991)

Stochastic games: e.g. **Amir** (1996), Curtat (1996)

Dynamic games: e.g. Rotemberg Saloner (1986), Haltiwanger Harrington (1991), **Bagwell Staiger** (1997)

... and more dynamic games: Abreu Pearce Stacchetti (1990), Roth (1996)

Self-enforcing contracts: e.g. **Ramey Watson** (1997), Levin (2003)

Doomed relationships: e.g. Baker Gibbons Murphy (1994), Kranton (1996), Di Tella McCullough (2002)

Exit (option) games: e.g. Grenadier (2002), Chassang (2007)

Dynamic IO: e.g. Hopenhayn (1992), Ericson Pakes (1995)

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Promising future directions

The tractability and flexibility of the model invite various extensions and suggest various explorations as future work. A few such possibilities are discussed in the paper:

- Macroeconomic volatility.
- Changing individuals.
- Poaching.
- Networking.
- Endogenous learning.

Macroeconomic volatility

Consider a partnership economy with costly re-matching and common productivity shocks, e.g. with stage-game payoffs

	Work	Shirk
Work	γ_t, γ_t	$-\gamma_t(1+d_t), \gamma_t(1+d_t)$
Shirk	$\gamma_t(1+d_t), -\gamma_t(1+d_t)$	0,0

where γ_t is a common to all partnerships and d_t is idiosyncratic.

Conjecture: In the welfare-maximizing equilibrium, the players' d_t-thresholds to exit and to work are decreasing in γ_t.

Implication #1: In a deep recession, players' outside option is unemployment. This makes them work harder in an existing partnership, dampening the pain of a deep recession.

Implication #2: In boom times, players will dissolve relationships more freely, and realize lower effort for any given d_t.

Poaching

The current partnership economy analysis does not exploit the possibility of on-the-job search, which is much richer than the zero-one search common in the job-matching literature, e.g.

- "search intensity" = the number of potential mates that you meet, in either a directed context ("bar") or undirected context ("street")
- "outside option" = payoff from leaving your current partner and matching with the most desirable of these potential mates, among those who are willing to match with you**

Conjecture: Welfare-maximizing search cost is non-zero as long as "initial fit" is sufficiently unimportant.

** *Aside:* Determine re-matchings via deferred acceptance.

Changing individuals / Networking

Assumptions on the stochastic process rule out interesting applications. Relaxing these assumptions will be an important direction for future work, e.g.

- *Stochastic partnerships are iid across states.* This rules out the possibility that individuals have stochastic characteristics (such as age, beauty, skills) that they carry with them.
- *Search today has no impact on future outside options.* This rules out the possibility that players may develop a network of contacts.

Endogenous learning

This paper's model can capture various sorts of exogenous public learning (à la Jovanovic (1979a)): let x_t = summary statistic of shared belief.

Yet "effort" does not encompass activities that generate public signals (or private signals), since these activities do not *increase* beliefs.

Nonetheless, the analysis suggests the following **speculation**, should players be able to influence the precision of their signal each period about the underlying (changing) state,

- Players will pay to observe more precise signals at first
- ... but seek to learn *less* about the match once in good times
- ... until their relationship is "on the rocks", when they will once again seek to learn more.

THANK YOU

More on the "honeymoon effect"

Stinchcombe (1965) argued that partnerships can be especially unstable when they are young if, among other reasons, players are uncertain about each other's type and *quickly learn* whether they are a good match.

This is consistent with the dynamics of my model, when one counts "dates" as relationships. Indeed, the model generates a large mass of quickly-dissolving partnerships, followed by a lull in break-ups. This pattern is found in data on American marriages.

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