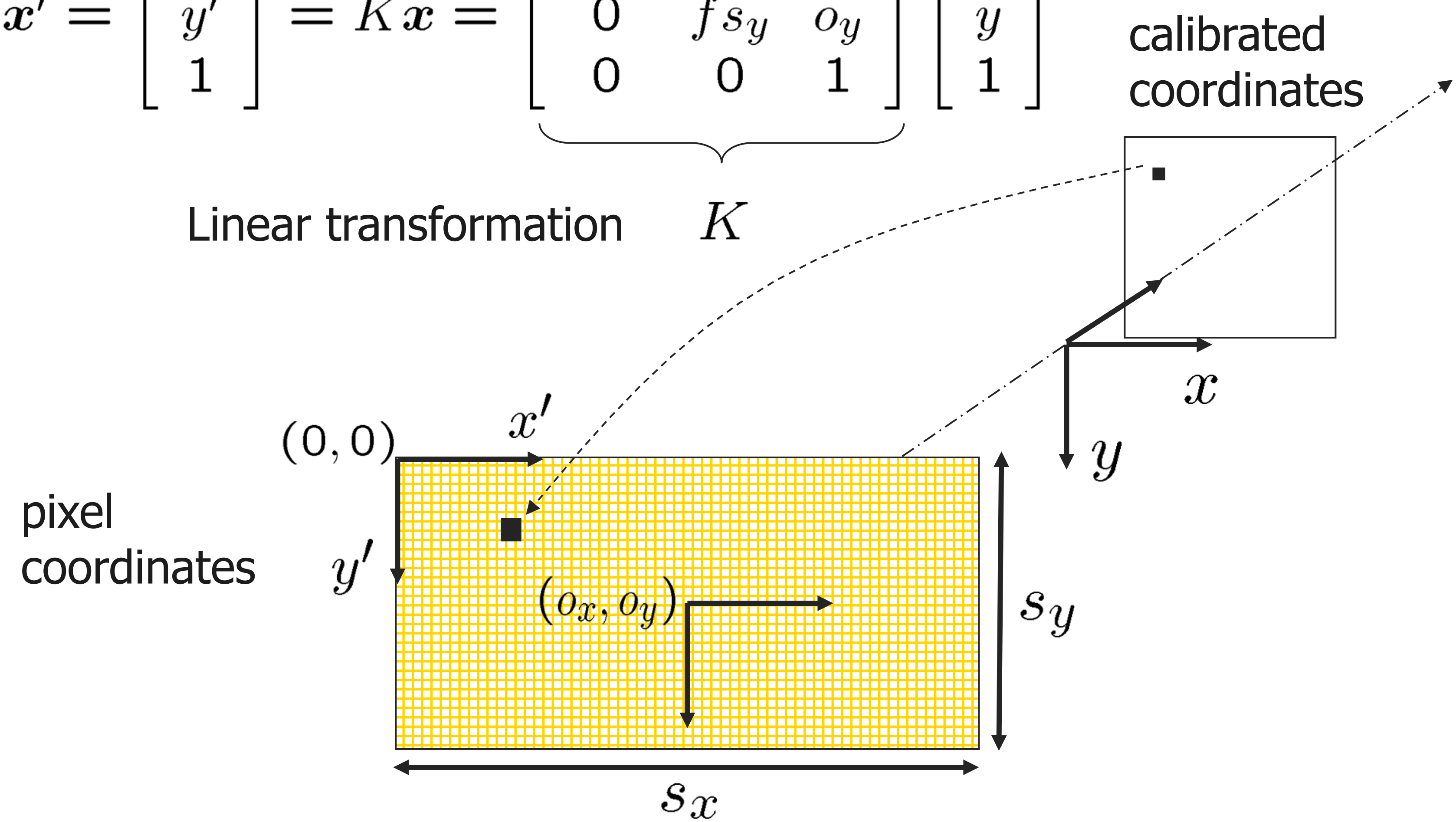
Camera Calibration

Uncalibrated Camera

$$\boldsymbol{x}' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K\boldsymbol{x} = \underbrace{\begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Linear transformation K



Overview

- Calibration with a rig
- Uncalibrated epipolar geometry
- Ambiguities in image formation
- Stratified reconstruction
- Autocalibration with partial scene knowledge

Calibration with a Rig

Use the fact that both 3-D and 2-D coordinates of feature points on a pre-fabricated object (e.g., a cube) are known.

Calibration with a Rig

- Given 3-D coordinates on known object

$$\lambda \mathbf{x}' = [KR, KT]\mathbf{X} \Rightarrow \lambda \mathbf{x}' = \Pi \mathbf{X}$$

$$\lambda \begin{bmatrix} x^i \\ y^i \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} \begin{bmatrix} X^i \\ Y^i \\ Z^i \\ 1 \end{bmatrix}$$

- Eliminate unknown scales

$$\begin{aligned} x^i(\pi_3^T \mathbf{X}) &= \pi_1^T \mathbf{X}, \\ y^i(\pi_3^T \mathbf{X}) &= \pi_2^T \mathbf{X} \end{aligned}$$

- Recover projection matrix $\Pi = [KR, KT] = [R', T']$

$$\min \|M\Pi^s\|^2 \quad \text{subject to} \quad \|\Pi^s\|^2 = 1$$

$$\Pi_s = [\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}, \pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}]^T$$

- Factor the *KR* into $R \in SO(3)$ and *K* using QR decomposition
- Solve for translation $T = K^{-1}T'$

More details

- Direct calibration by recovering and decomposing the projection matrix

$$\lambda \begin{bmatrix} x^i \\ y^i \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} \begin{bmatrix} X^i \\ Y^i \\ Z^i \\ 1 \end{bmatrix} \rightarrow Z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x_i = \frac{\pi_{11} X_i + \pi_{12} Y_i + \pi_{13} Z_i + \pi_{14}}{\pi_{31} X_i + \pi_{32} Y_i + \pi_{33} Z_i + \pi_{34}} \qquad y_i = \frac{\pi_{21} X_i + \pi_{22} Y_i + \pi_{23} Z_i + \pi_{24}}{\pi_{31} X_i + \pi_{32} Y_i + \pi_{33} Z_i + \pi_{34}}$$

$$x_i(\pi_{31} X_i + \pi_{32} Y_i + \pi_{33} Z_i + \pi_{34}) = \pi_{11} X_i + \pi_{12} Y_i + \pi_{13} Z_i + \pi_{14}$$
$$y_i(\pi_{31} X_i + \pi_{32} Y_i + \pi_{33} Z_i + \pi_{34}) = \pi_{21} X_i + \pi_{22} Y_i + \pi_{23} Z_i + \pi_{24}$$

$$\begin{aligned} x^i(\pi_3^T \mathbf{X}) &= \pi_1^T \mathbf{X}, \\ y^i(\pi_3^T \mathbf{X}) &= \pi_2^T \mathbf{X} \end{aligned}$$

2 constraints per point

$$[X_i, Y_i, Z_i, 1, 0, 0, 0, 0, -x_i X_i, -x_i Y_i, -x_i Z_i, -x_i]\Pi_s = 0$$
$$[0, 0, 0, 0, X_i, Y_i, Z_i, 1, -y_i X_i, -y_i Y_i, -y_i Z_i, -y_i]\Pi_s = 0$$

$$\Pi_s = [\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}, \pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}]^T$$

More details

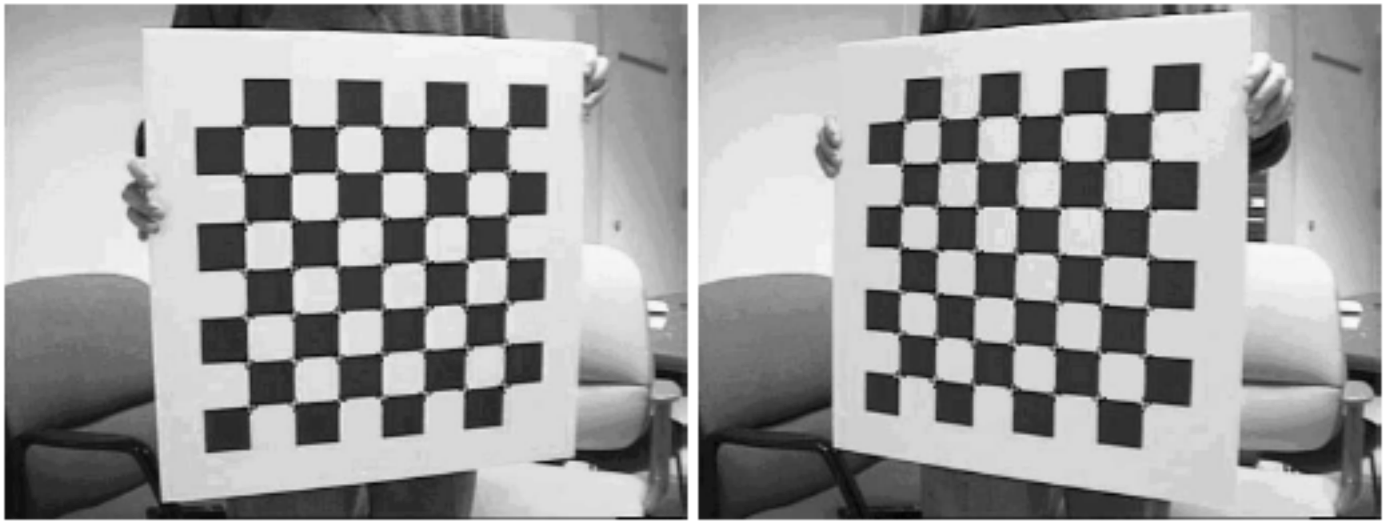
- Recover projection matrix $\Pi = [KR, KT] = [R', T']$

$$\min \|M\Pi^s\|^2 \quad \text{subject to} \quad \|\Pi^s\|^2 = 1$$

$$\Pi^s = [\pi_{11}, \pi_{21}, \pi_{31}, \pi_{12}, \pi_{22}, \pi_{32}, \pi_{13}, \pi_{23}, \pi_{33}, \pi_{14}, \pi_{24}, \pi_{34}]^T$$

- Collect the constraints from all N points into matrix M (2N x 12)
- Solution eigenvector associated with the smallest eigenvalue $M^T M$
  [u,s,v] = svd(M) take v(:,12)
- Unstack the solution and decompose into rotation and translation
- Factor the *R'* into $R \in SO(3)$ and *K* using QR decomposition (qr matlab function)
- Solve for translation $T = K^{-1}T'$

Calibration with a planar pattern



$$H \doteq K[r_1, r_2, T] \quad \in \mathbb{R}^{3\times 3} \qquad \lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K[r_1, r_2, T] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix},$$

To eliminate unknown depth, multiply both sides by $\widehat{x'}$

$$\widehat{x'} H [X, Y, 1]^T = 0.$$

Two constraints per point at least 8 points, solve system of homogeneous equations

Review: Perspective Projection from 3D to 2D

- Relationship between coordinates in the world frame and image

$$\lambda \boldsymbol{x}' = \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

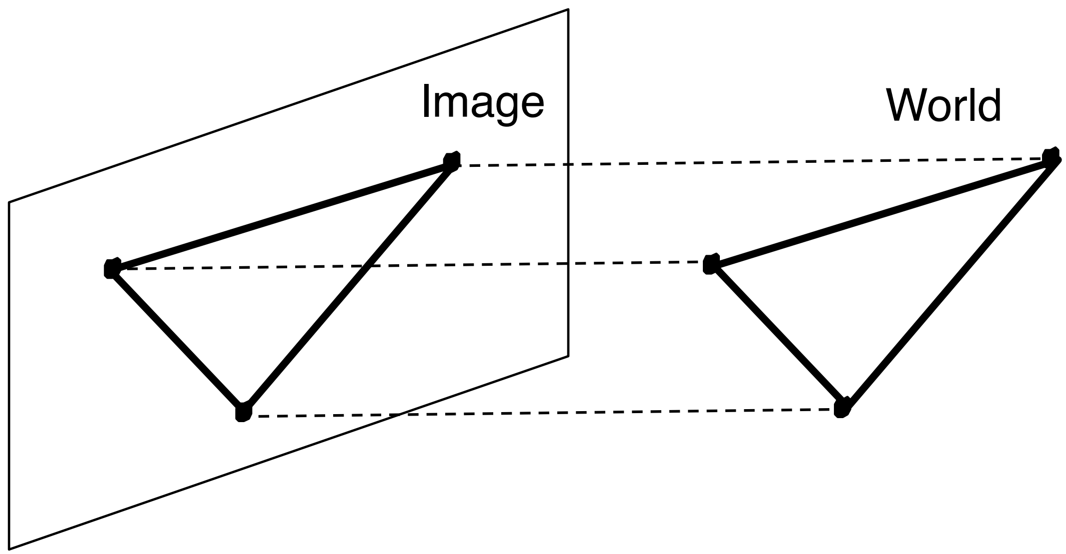
More compactly

$$\lambda \boldsymbol{x} = K_f \Pi_0 g \boldsymbol{X} = \Pi \boldsymbol{X}$$

Transformation between camera coordinate
Systems and world coordinate system

Review: Orthographic Projection from 3D to 2D

- Special case of perspective projection
  - Distance from center of projection to image plane is infinite



  - Also called “parallel projection”
  - What’s the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

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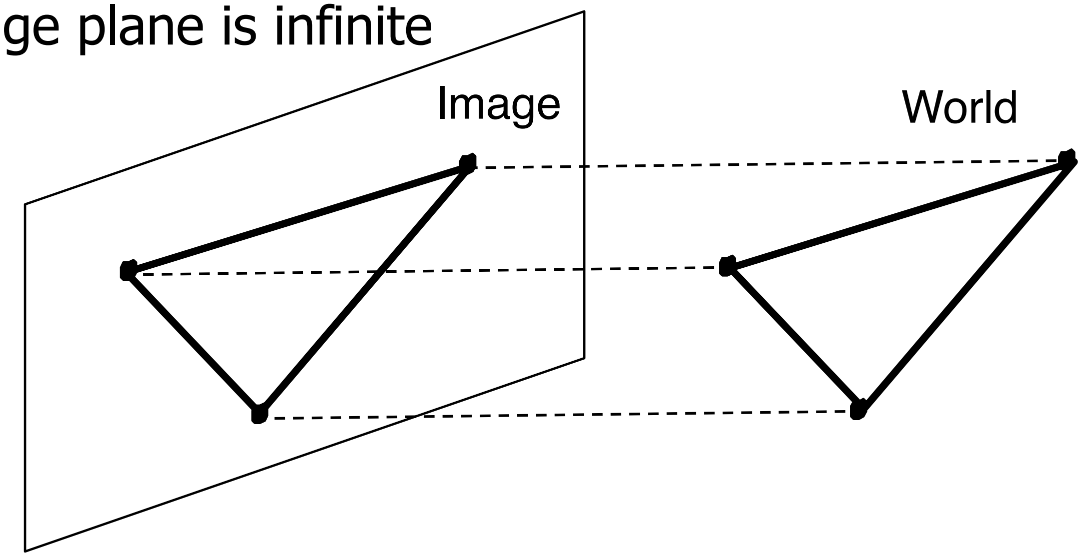
Scaled Orthographic Projection from 3D to 2D

- Special case of perspective projection
  - Distance from center of projection to image plane is infinite



  - Scaled orthographic

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = s \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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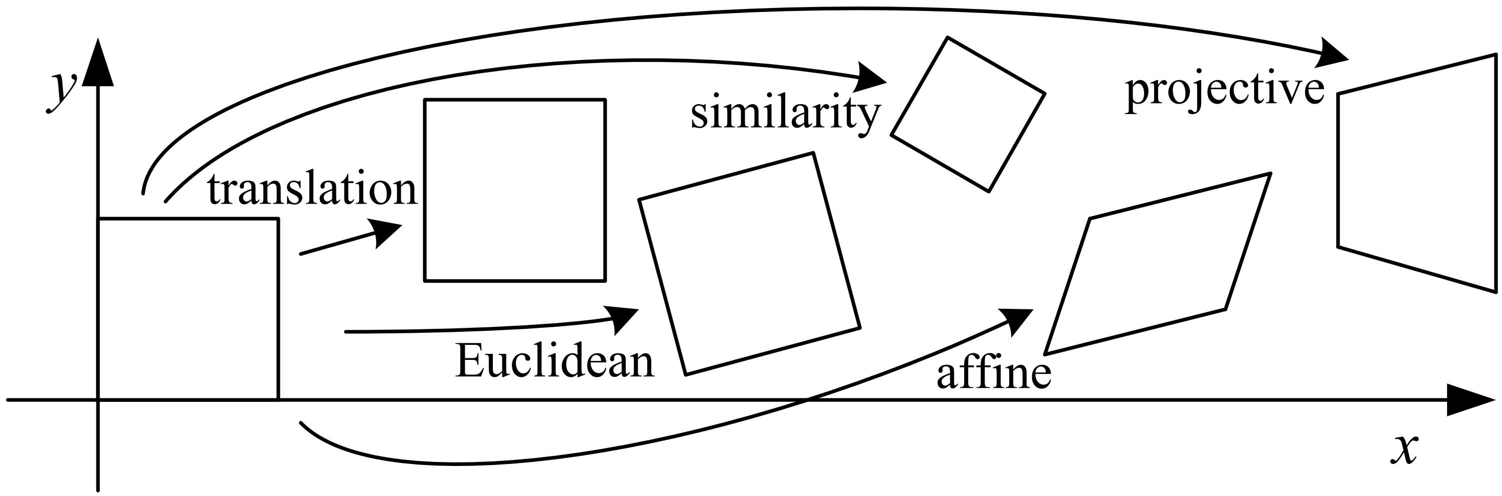
Different types of transformations 2D -> 2D



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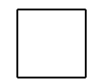
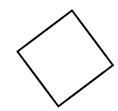
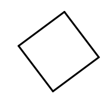
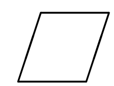
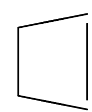
Different types of transformations 2D -> 2D

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

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