Analyzing fuzzy and contextual approaches to vagueness by semantic games

PhD Thesis

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Motivation

Vagueness

- ubiquitous in all natural languages
- still neglected by most natural language processing systems

Vagueness is modeled by logicians, philosophers, linguists, etc., but

- ... different aims and methodologies
- ... no systematic comparison of different approaches
- ... lack of interpretations

Outline

What is Vagueness?

Related phenomena

'Vagueness' in colloquial language may be used for:

Underspecificity / Unwanted generality

Someone said something.

Ambiguity

I'm at the bank.

'Genuine' vagueness

He is rather tall.

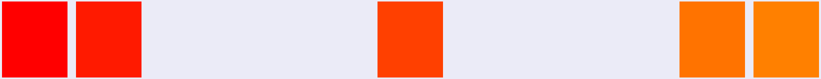
Vagueness due to imprecision

We'll meet at 12 o'clock.

What is Vagueness?

Characteristics of Vagueness

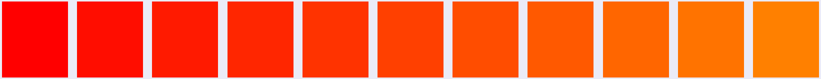
Borderline cases



red ??? *not red*

Is the patch in the middle considered *red* or *not red*?

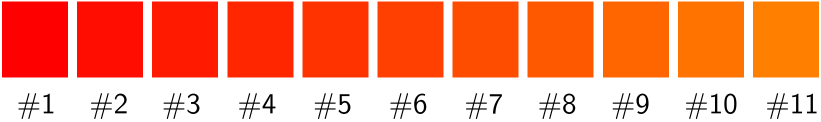
Blurry boundaries



Where do the red patches and the not-red patches start?

What is Vagueness?

Characteristics of Vagueness (contd.)



Sorites paradox

(Premise 1) Patch #1 is red.

(Premise 2) If a patch is red and you change it just a bit, it remains red.

(Conclusion) Patch #11 is red.

- Sorites series can be formed for all vague predicates

Fuzzy approaches to vagueness

Fuzzy approaches

- "*Truth comes in degrees.*": unit interval $[0,1]$
- truth-functional
- based on continuous t-norms

Interpretation of fuzzy models

- dialogue semantics (Giles 1970)
- characterize truth by evaluation games

Giles's game

Overview

Giles's game for Łukasiewicz logic $Ł_\infty$

- evaluation game: determine truth in a model
- each player asserts a multiset of formulas
- initially, I assert the formula in question
- dialogue part and betting part

- can be motivated as a generalization of Hintikka's evaluation game for classical logic

Giles's game

Dialogue part

- Decompose formulas according to dialogue rules

($R_{\to}$): If I assert $F \to G$ then you may attack by asserting F, which obliges me to defend by asserting G.

($R_{\forall}$): If I assert $\forall x.F(x)$, then you may attack by choosing c, which obliges me to assert $F(c)$.

(LLA): I can declare not to attack an assertion.

(LLD): I can just assert $\bot$ in reply to your attack.

- rules for your assertions are dual
- no further regulations

Giles's game

Betting part and correspondence to $Ł_\infty$

- dispersive binary experiments
- success probabilities correspond to v_M
- experiment fails: the asserting player pays 1€
- Risk: the expected amount of money I have to pay

Theorem (Giles)

The least final risk r I can enforce directly corresponds to $v_M(F)$:

- I have a strategy, that my risk is not greater than r
- you have a strategy, that my risk is not smaller than r

Generalizing Giles's game

Motivation

Generalize Giles's game

Payoff function: take some payoff function

Truth values: transformations between payoff and truth values

Dialogue rules: define a general format of dialogue rules

- conditions on payoff functions and dialogue rules
- derive truth functions from dialogue rules

Generalizing Giles's game
Results

Theorem

Let $\text{\textbackslash}\!\circlearrowleft$ be a game with discriminating payoff function and decomposing dual rules. Then for each connective $\diamond$ there is a function $f_\diamond$ such that $\langle \mid \diamond(F_1, \ldots, F_n)\rangle = f_\diamond(\langle\mid F_1\rangle, \ldots, \langle\mid F_n\rangle)$ for all formulas $F_1, \ldots, F_n$.

Logics which can be characterized in this framework:

- Łukasiewicz logic $\mathbf{Ł}_\infty$, $\mathbf{Ł}_n$, Abelian logic **A**, **CHL**
- $f_\diamond$ composed by min, max, and $\oplus$

Fuzzy Quantification

Motivation

- model vague natural language quantifiers like *about half*, *nearly a third*
- focus on semi-fuzzy quantifiers

Random witness selection

If I assert $\Pi x.\hat{F}(x)$ then I have to assert $\hat{F}(c)$ for a randomly picked c.

- uniform distribution over a finite domain $\mathcal{D}$

Modeling fuzzy quantification

Giles's game with random witness selection

- random witness selection
- blind choice and deliberate choice

Blind choice quantifiers

- Place bets for and against a fixed number of instances.
- Identities of random instances are revealed only afterwards.

Modeling fuzzy quantification

Blind choice quantifiers - Results

Theorem

All blind choice quantifiers can be expressed using quantifiers of the form L_m^k, G_m^k, disjunction $\vee$, conjunction $\wedge$, and $\perp$.

Theorem

The blind choice quantifiers G_m^k and L_m^k can be expressed in $\text{Ł}_\infty(\Pi)$ via the following reductions:

$$G_m^k x.\hat{F}(x) \equiv \neg((\neg\Pi x.\hat{F}(x))^{m+1}) \mathbin{\&} (\Pi x.\hat{F}(x))^{k-1}$$
$$L_m^k x.\hat{F}(x) \equiv \neg((\Pi x.\hat{F}(x))^{k+1}) \mathbin{\&} (\Pi x.\neg\hat{F}(x))^{m-1}$$

Corollary

All blind choice quantifiers can be expressed in $\text{Ł}_\infty(\Pi)$.

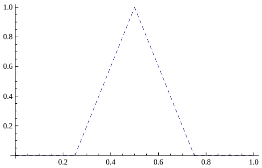
Modeling fuzzy quantification

Blind choice quantifiers - Example

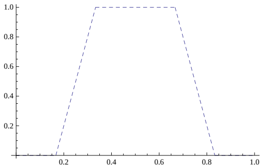
Example - Around half

Consider the quantifier $H^s_t x.\hat{F}(x) =_{df} G^{s+t}_{s-t} x.\hat{F}(x) \wedge L^{s+t}_{s-t} x.\hat{F}(x)$

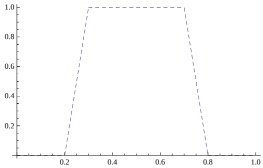
- s: determines sample size
- t: determines tolerance



$H^2_0 x.\hat{F}(x)$



$H^3_1 x.\hat{F}(x)$



$H^5_2 x.\hat{F}(x)$

Modeling fuzzy quantification

Deliberate choice quantifiers

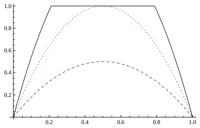
$(R_{\Pi^k_m})$:

- choose $k + m$ constants randomly
- defender bets for k and against m constants
- may also invoke LLD instead

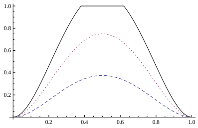
Proposition

The dialogue rule $R_{\Pi^k_m}$ matches the specification

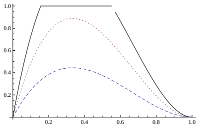
$$v_M(\Pi^k_m \hat{F}(x)) = \binom{k+m}{k} (Prop_x \hat{F}(x))^k (1 - Prop_x \hat{F}(x))^m$$



$W_i(\Pi^1_1)x.\hat{F}(x)$



$W_i(\Pi^2_2)x.\hat{F}(x)$



$W_i(\Pi^1_2)x.\hat{F}(x)$

- operator W_i to project towards 1

- linguistic dynamic, scale-based approach to vagueness

The 'Dynamics of vagueness'

- objects possess vague / gradable properties to some degree
- context is defined as a set of classical worlds
- meaning defined as context change potential

Barker's 'Dynamics of Vagueness'

Example Context

Example context C

w	$\delta(w)(^{\uparrow}\mathit{tall})$	$\mathrm{tall}(w)(\mathrm{b})$	$\delta(w)(^{\uparrow}\mathit{clever})$	$\mathrm{clever}(w)(\mathrm{b})$
w_1	170	175	100	105
w_2	160	170	120	125
w_3	170	180	100	95
w_4	180	175	105	100
w_5	170	165	110	115

- values for *tallness*, *cleverness*, Bill's height and cleverness

Barker's 'Dynamics of Vagueness'

Example Context

Example context C

w	$\delta(w)(^{\uparrow}tall)$	$\text{tall}(w)(\text{b})$	$\delta(w)(^{\uparrow}clever)$	$\text{clever}(w)(\text{b})$
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context update: filter out worlds

- $[\![\textit{Bill is tall}]\!](C) = \{w_1, w_2, w_3\}$

Barker's 'Dynamics of Vagueness'

Example Context

Example context C

w	$\delta(w)({}^{\uparrow}tall)$	tall(w)(b)	$\delta(w)({}^{\uparrow}clever)$	clever(w)(b)
w_1	170	175	100	105
w_2	160	170	120	125
w_3	170	180	100	95
w_4	180	175	105	100
w_5	170	165	110	115

context update: filter out worlds

- $[\![\textit{Bill is tall}]\!](C) = \{w_1, w_2, w_3\}$
- $[\![\textit{Bill is clever}]\!](C) = \{w_1, w_2, w_5\}$

Barker's 'Dynamics of Vagueness'

Example Context

Example context C

w	$\delta(w)(^{\uparrow}tall)$	tall(w)(b)	$\delta(w)(^{\uparrow}clever)$	clever(w)(b)
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w_4	180	175	105	100
w_5	170	165	110	115

context update: filter out worlds

- $[\![\textit{Bill is tall}]\!](C) = \{w_1, w_2, w_3\}$
- $[\![\textit{Bill is clever}]\!](C) = \{w_1, w_2, w_5\}$
- $[\![\textit{Bill is tall and Bill is clever}]\!](C) = \{w_1, w_2\}$

Barker: several issues and shortcomings

- simplify notation (linear scales)
- compose predicate modifiers:
  definitely very tall, *very very tall*
- introduce logical operators
  - disjunction, conjunction, negation via
    union, intersection, and complement of contexts.
  - conditional via material implication

Find connections to fuzzy logics

- determine proportion of worlds filtered out

Example context C

w	$\delta(w)(^{\uparrow}tall)$	tall(w)(b)	$\delta(w)(^{\uparrow}clever)$	clever(w)(b)
w_1	170	175	100	105
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w_4	180	175	105	100
w_5	170	165	110	115

- $\|Bill\ is\ tall\|_C = 3/5$

Barker's 'Dynamics of Vagueness'

Measuring contexts

Find connections to fuzzy logics

- determine proportion of worlds filtered out

Example context C

w	$\delta(w)(^{\uparrow}tall)$	tall(w)(b)	$\delta(w)(^{\uparrow}clever)$	clever(w)(b)
w_1	170	175	100	105
w_2	160	170	120	125
w_3	170	180	100	95
w_4	180	175	105	100
w_5	170	165	110	115

- $\|\textit{Bill is tall}\,\|_C = 3/5$
- $\|\textit{Bill is tall and Bill is clever}\|_C = 2/5$

- not truth-functional

Theorem

The following boundaries are tight:

- $*_{\text{Ł}}(\|\phi\|_C, \|\psi\|_C) \leq \|\phi \wedge \psi\|_C \leq *_{\text{G}}(\|\phi\|_C, \|\psi\|_C)$,
- $\bar{*}_{\text{G}}(\|\phi\|_C, \|\psi\|_C) \leq \|\phi \vee \psi\|_C \leq \bar{*}_{\text{Ł}}(\|\phi\|_C, \|\psi\|_C)$, and
- $I_G(\|\phi\|_C, \|\psi\|_C) \leq \|\phi \rightarrow \psi\|_C \leq \|\phi\|_C \Rightarrow_{\text{Ł}} \|\psi\|_C$.

- t-norms, co-t-norms, etc., occur as bounds

Barker's 'Dynamics of Vagueness'

Saturated contexts

Saturated context

- predicates and individuals are independent of each other
- contexts are 'dense'

- contexts determined by lower and upper bounds for values
- fuzzy truth values for atomic propositions directly computable

For ϕ and ψ with disjoint predicate symbols:

$$\|\phi \wedge \psi\|_C = \|\phi\|_C *_{\Pi} \|\psi\|_C$$
$$\|\phi \vee \psi\|_C = \|\phi\|_C \bar{*}_{\Pi} \|\psi\|_C$$

Shapiro's 'Vagueness in Context'

Motivation

- philosophical contextual, delineation-based approach

'Vagueness in Context'

- contexts are partial valuations
- fixed context space: all admissible contexts
- define local and global connectives and quantifiers

- local connectives: strong Kleene tables

Shapiro's 'Vagueness in Context'

Contexts

$$
\begin{tikzcd}
 & & N_0 : \{\} \arrow[dll] \arrow[d] \arrow[dr] & \\
N_1 : \{\mathit{tall}(a)\} \arrow[dr] & & N_2 : \{\mathit{heavy}(a)\} \arrow[dl] & N_3 : \{\neg \mathit{tall}(a)\} \\
 & N_5 : \{\mathit{tall}(a), \mathit{heavy}(a)\} & &
\end{tikzcd}
$$

- directed acyclic structures with root N_0
- ordered by $\succeq$ 'sharpens'
- truth defined in terms of forcing

Shapiro's 'Vagueness in Context'

Evaluation games

a Hintikka-style evaluation game for Shapiro's logic:

- players refer to truth values

a Giles-style evaluation game:

- three-player game between you, me, and nature
- assertions relative to sharpenings
- modifiers 'maybe' and 'surely'

Shapiro's 'Vagueness in Context'

Evaluation games

a Hintikka-style evaluation game for Shapiro's logic:

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a Giles-style evaluation game:

- three-player game between you, me, and nature
- assertions relative to sharpenings
- modifiers 'maybe' and 'surely'

Theorem

For the game $\mathcal{G} = [\ \mid F_P]$ starting with my assertion of F at P

(i) I have a winning strategy for $\mathcal{G}$ iff F is true,

(ii) you have a winning strategy for $\mathcal{G}$ iff F is false, and

(iii) neither of us has a winning strategy for $\mathcal{G}$ iff F is indefinite

at the world P.

Relating Barker's and Shapiro's approaches

Motivation

- different motivations and formalisms

Questions

- compare possible inferences using Barker / Shapiro
- compare expressiveness of approaches

- Shapiro: handle conflicting information
- Barker: all worlds are classical
  - require completability for Shapiro's contexts

Relating Barker's and Shapiro's approaches

Partitioning the context space - Shapiro

$$
\begin{tikzcd}
 & & & N_0 : \{\} \arrow[dlll] \arrow[dl] \arrow[dr] \arrow[drrr] & & & \\
N_1 : \{tall(a)\} \arrow[dr] & & N_2 : \{heavy(a)\} \arrow[dl] & & N_3 : \{\neg tall(a)\} \arrow[dr] & & N_4 : \{\neg heavy(a)\} \arrow[dl] \\
 & N_5 : \{tall(a), heavy(a)\} & & & & N_6 : \{\neg tall(a), \neg heavy(a)\} &
\end{tikzcd}
$$

- partition possible worlds by their completions
- same propositions are forced within a partition block

$$ {}^{W}T_{sh}(N) =_{df} \{N' \in W : N' \succeq N, N' \text{ is complete}\} $$

Relating Barker's and Shapiro's approaches

Partitioning the context space - Barker

w	$\delta(w)(^{\uparrow}tall)$	tall(w)(a)	$\delta(w)(^{\uparrow}heavy)$	heavy(w)(a)
w_1	170	175	100	105
w_2	165	170	90	125
w_3	170	177	95	100
w_4	180	175	90	80
w_5	170	165	105	100

- partition sets of possible worlds
- three partitions:
  - $C \subseteq \{w_1, w_2, w_3\}$
  - $C \subseteq \{w_4, w_5\}$
  - $C \cap \{w_1, w_2, w_3\} \neq \emptyset$ and $C \cap \{w_4, w_5\} \neq \emptyset$

Relating Barker's and Shapiro's approaches

Partitioning the context space - Barker

w	$\delta(w)(^{\uparrow}tall)$	tall(w)(a)	$\delta(w)(^{\uparrow}heavy)$	heavy(w)(a)
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$$T_b(C) =_{df} \{s(w) : w \in C\} \text{ with } s(w) \text{ classical s.t.}$$
$$R(u) \text{ is true at } s(w) \text{ iff } w \in [\![R]\!](u)(C) \text{ and}$$
$$R(u) \text{ is false at } s(w) \text{ iff } w \notin [\![R]\!](u)(C)$$

Relating Barker's and Shapiro's approaches

Corresponding models

Definition

$\langle W, N_0 \rangle_{Sh}$ ... Shapiro model
$\langle \mathcal{P}(C_0), C_0 \rangle_B$ ... Barker model
Corresponding models iff:

- agree on relevant predicates and individuals
- $N \in W$: there exists $C \in \mathcal{P}(C_0)$ such that ${}^W T_{sh}(N) = T_b(C)$
- $C \in \mathcal{P}(C_0)$: there exists $N \in W$ such that $T_b(C) = {}^W T_{sh}(N)$

Relating Barker's and Shapiro's approaches

Corresponding models

Definition

$\langle W, N_0 \rangle_{Sh}$ ...Shapiro model
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Corresponding models iff:

- agree on relevant predicates and individuals
- $N \in W$: there exists $C \in \mathcal{P}(C_0)$ such that ${}^{W}T_{sh}(N) = T_b(C)$
- $C \in \mathcal{P}(C_0)$: there exists $N \in W$ such that $T_b(C) = {}^{W}T_{sh}(N)$

Proposition

$\langle W, N_0 \rangle_{Sh}$ and $\langle \mathcal{P}(C_0), C_0 \rangle_B$ corresponding models:
There exists a bijection between $[\,\cdot\,]^{W}_{Sh}$ and $[\,\cdot\,]^{C_0}_{B}$ induced by

$${}^{C_0}T_b^{-1} \circ {}^{W}T_{sh} \qquad \text{and} \qquad {}^{W}T_{sh}^{-1} \circ T_b$$

Theorem

$\langle W, N_0 \rangle_{Sh}$ and $\langle \mathcal{P}(C_0), C_0 \rangle_B$ corresponding models,
ϕ unsettled in N_0 and in C_0

- perform context update with ϕ
- resulting contexts: N_1 and C_1
- $\left[N_1\right]^W_{Sh} \simeq \left[C_1\right]^{C_0}_B$ holds

- N_1 and C_1: satisfy same first-order formulas
- can be iterated

Conclusion

Summary

- generalization of Giles's game to a more abstract framework
- random witness selection for semi-fuzzy quantification
- recover (co-)t-norms within Barker's approach to vagueness
- present evaluation games for Shapiro's logic
- find connections between Barker's and Shapiro's approaches

Outlook

- logically model conclusions drawn due to scale structure
- combine Kyburg and Moreau's contextual approach with degree-based decision making

Publications

C.G. Fermüller and C. Roschger.
From games to truth functions: A generalization of Giles's game.
Studia Logica, Special issue on Logic and Games, 2014.

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Logic Journal of the IGPL, 2014.

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Evaluation games for Shapiro's logic of vagueness in context.
The Logica Yearbook 2009, 2010.

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Comparing context updates in delineation and scale based models of vagueness.
Understanding Vagueness - Logical, philosophical, and linguistic perspectives, 2011.