Twin Paradox Revisited

Relativity and Astrophysics
Lecture 19
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Outline

- Simultaneity Again
  - Sample Problem L-2
- Twin Paradox Revisited
  - Time dilation viewpoint
  - Length contraction viewpoint
  - Paradox & why it's not!
- Problem L-13, page 117 (due Friday/Monday)
  - Will hand back on Monday if you hand it in on Friday

Revisiting Simultaneity and Paradoxes

- We will look one more time at
  - Simultaneity
  - Time dilation / twin paradox

<u>As we showed previously</u>

- If two events occur in our lab frame separated by distance x and time t, we can use the Lorentz transformation to find a “rocket” frame in which –
  - Events occur at same location (for $x < t$)
  - Events occur at same time (for $x > t$)

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Timelike Intervals

- In lab frame we see two events (event A and event B) separated by a distance x (= $x_B - x_A$) and time t (= $t_B - t_A$)
  - Can we now find a rocket frame in which the **events occur at the same location**?
- Let the coordinates for event A be zero in both frames. Then we have

$x_A = x'_A = 0$	x_B = known	$x'_B = 0$
$t_A = t'_A = 0$	t_B = known	t'_B = unknown

- We want the speed v_{rel} of the rocket such that $x'_B = 0$. We use the Lorentz transformation equations

$$x'_B = \gamma x_B - v_{rel}\gamma t_B \underset{x'_B = 0}{\Rightarrow} x_B = v_{rel} t_B \Rightarrow v_{rel} = \frac{x_B}{t_B}$$

- Again v_{rel} is the required speed of the rocket so that $x'_B = 0$.
  - Note that since $v_{rel} < 1$, we must have $x_B < t$
  - Thus $t^2_B - x^2_B > 0$ (the spacetime interval is **timelike**)

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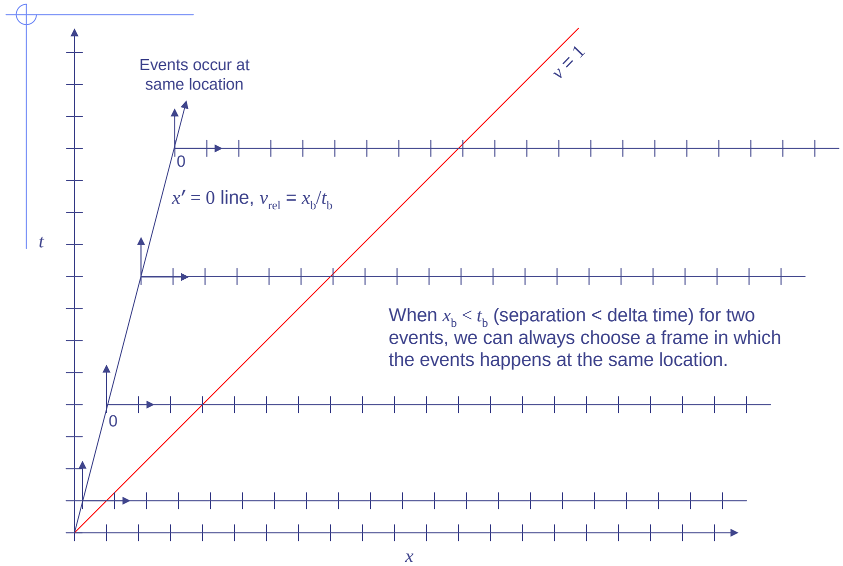
Events at the same location



Spacelike Intervals

- In lab frame we see two events (event A and event B) separated by a distance x (= $x_B - x_A$) and time t (= $t_B - t_A$)
  - Can we find a rocket frame in which the **events are simultaneous**?
- Let the coordinates for event A be zero in both frames. Then we have

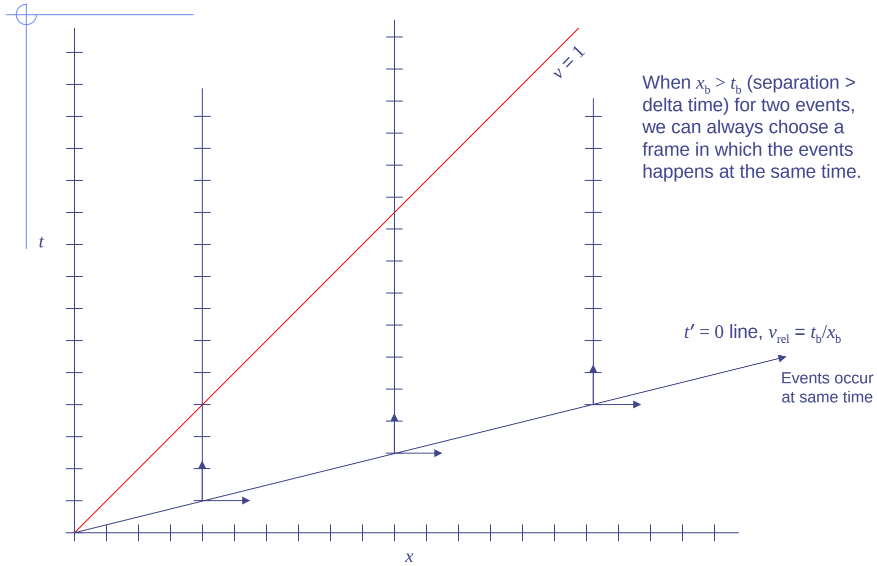
$x_A = x'_A = 0$	x_B = known	x'_B = unknown
$t_A = t'_A = 0$	t_B = known	$t'_B = 0$

- We want the speed v_{rel} of the rocket such that $t'_B = 0$. We use the Lorentz transformation equations

$$t'_B = \gamma t_B - v_{rel}\gamma x_B \quad \underset{t'_B = 0}{\Rightarrow} \quad t_B = v_{rel} x_B \quad \Rightarrow \quad v_{rel} = \frac{t_B}{x_B}$$

- v_{rel} is the required speed of the rocket so that $t'_B = 0$.
  - Note that since $v_{rel} < 1$, we must have $x_B > t$
  - Thus $t^2_B - x^2_B < 0$ (the spacetime interval is **spacelike**)

Events at the same time



When $x_b > t_b$ (separation > delta time) for two events, we can always choose a frame in which the events happens at the same time.

Simultaneity Revisited: Sample Prob. L-2

- Problem Setup
  - Julius Caesar was murdered approximate 2000 years ago. Is there a way using the laws of physics to save his life?
- Let Caesar's death be the reference event with coordinates, $x_o = 0$, $t_o = 0$
- Event A is the present time which is $x_A = 0$ lyr, $t_A \sim$ 2000 yr
- Simultaneous with event A, the Starship Enterprise sets off a fire cracker in the Andromeda galaxy
  - Event B is the firecracker at $x_B = 2\times10^6$ lyr, $t_B = 2000$ yr.
  - Let $x'_o = 0$, $t'_o = 0$ for the Enterprise (Caesar's murder is the reference event)

Sample Problem L-2 (cont'd)

- How fast must the Enterprise be going in the Earth frame in order that Caesar's murder is happening NOW, that is, $t'_B = 0$?
- In lab frame we see two events (event A and event B) separated by a distance x ($= x_B - x_A$) and time t ($= t_B - t_A$)
  - Can we find an Enterprise frame in which the **events are simultaneous**?
- Let the coordinates for event A be zero in both frames. Then we have
  - $x_A = x'_A = 0$ $\quad x_B = 2\text{x}10^6$ yr $\quad x'_B$ = unknown
  - $t_A = t'_A = 0$ $\quad t_B = 2\text{x}10^3$ yr $\quad t'_B = 0$
- We want the speed v_{rel} of the rocket such that $t'_B = 0$. We use the Lorentz transformation equations

$$t'_B = \gamma t_B - v_{rel}\gamma x_B \quad \underset{t'_B = 0}{\Rightarrow} \quad t_B = v_{rel} x_B \quad \Rightarrow \quad v_{rel} = \frac{t_B}{x_B} = \frac{2\times10^3\ \text{yr}}{2\times10^6\ \text{yr}}$$

- Thus $v_{rel} = 10^{-3} = 0.001$ we have $t'_B = 0$, that is, Caesar's murder happens NOW in the Enterprise frame,

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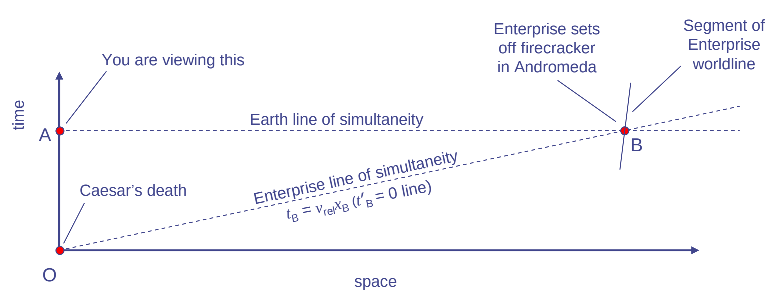
Sample Problem L-2 (cont'd)

- Draw a spacetime diagram which contains:
  - Event O (Caesar's death), Event A (you here), event B (firecracker in Andromeda), your line of NOW simultaneity, the position of the Enterprise, the worldline of the Enterprise, and the Enterprise NOW line of simultaneity.



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Sample Problem L-2 (cont'd)

- In the Enterprise frame what are the x and t coordinates of the firecracker.
  - Note that we choose the relative velocity of the Enterprise so that $t'_B = 0$, which gives $v_{rel} = 10^{-3}$.
- Compute γ and use the inverse Lorentz transformation.

$$\gamma = \frac{1}{\sqrt{1-v_{rel}^2}} \sim 1 + \frac{v_{rel}^2}{2} = 1 + \frac{10^{-6}}{2}$$

$$t'_B = -v_{rel}\gamma x_B + \gamma t_B = \gamma\left(-10^{-3}\cdot 2\times 10^6 + 2\times 10^3\right) = 0 \text{ yr}$$

$$\begin{aligned} x'_B = \gamma x_B - v_{rel}\gamma t_B &= \gamma\left(2\times 10^6 - 10^{-3}\cdot 2\times 10^3\right) \\ &= \gamma\left(1-10^6\right)2\times 10^6 \\ &= \left(1+10^{-6}/2\right)\left(1-10^{-6}\right)2\times 10^6 \\ &= \left(1-10^{-6}/2-10^{-12}/2\right)2\times 10^6 \\ &= 1.999999\times 10^6 \text{ lyr} \end{aligned}$$

Coordinate is not much different in the two frames because the velocity is small

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Sample Problem L-2 (cont'd)

- Can the Enterprise firecracker explosion warn Caesar, thus changing the course of history?
- There exists a frame (the rest frame of the Enterprise) in which Caesar's death and the firecracker explosion occur at the same time.
  - In this frame a signal connecting the two events would have to travel faster than light.
- Enterprise can't warn Caesar.
  - The spatial coordinate is hugely different
- Events separated in this fashion are called spacelike
  - Spacelike events cannot be causally linked, that is, they cannot have a cause and effect relationship.

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Twin Paradox

From: Lecture 2, slide 8, using spacetime interval

- A starship leaves Earth (Event 1) and travel at 95% light speed, later arriving at Proxima Centauri (Event 2), which lies 4.3 light-years from Earth.
  1. What are the space and time separation between the two events as measured in the Earth frame, in years?
  2. What are the space and time separations in the frame of the starship?
- **Part 1:** The distance separation is 4.3 light-years, as stated in the problem. The time separation is:

$$t_{earth} = \frac{4.3\,\text{lyr}}{0.95\,(\text{lyr/yr})} = 4.526\text{ years}$$

- **Part 2:** Both events occur at the position of the starship
  => space separation is 0 m in starship frame.
- Once more, we use invariance of spacetime interval to get time separation in the proton frame: $t_{ship}^2 - d_{ship}^2 = t_{earth}^2 - d_{earth}^2$

$$\Rightarrow\ t_{ship}^2 = (4.526\text{ yr})^2 - (4.3\text{ yr})^2 = (20.48 - 18.49)\text{ yr}^2 = 1.99\text{ yr}^2$$

$$\Rightarrow\ t_{ship} = 1.41\text{ years}$$

This is the famous "twin paradox" – time goes by more slowly on the starship than Earth.

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The Twin Paradox - revisited

- One twin travels away from the
- Earth at high speed for a long time.
  - Returning to Earth she finds herself much younger than her brother!
- How?
  - Both twins think each other's clock slows down, so what is going on?
- Suppose one twin is on a rocket ship traveling at v = 0.95 to a star 4.3 lyr away.
  - She sees the distance contracted to:

No "warp" drive

Analyze using Lorentz contraction

$$L = 4.3\times\sqrt{1-0.95^2}\text{ lyr} = 1.34\text{ lyr}$$

- She computes her time to get there as

$$t' = (1.34\text{ lyr}/0.95) = 1.41\text{ yr}$$

- The twin on Earth sees the distance as still 4.3 lyr, so he computes her time to go out as:

$$t = (4.3\text{ lyr}/0.95) = 4.526\text{ yr}$$

- So he ages 4.53 years, while his twin sister ages only 1.41 years.

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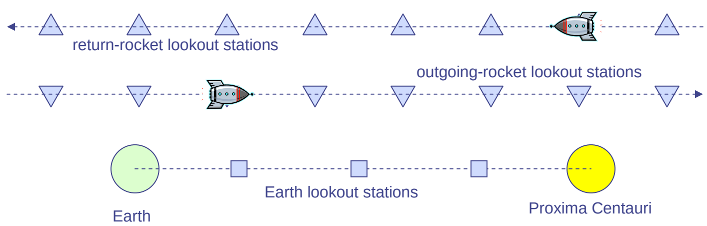
Relativity of Simultaneity (chpt. 4.9)

- How do we "explain" the twin paradox
  - Both twins see each other aging less but the one on the Earth ages more than the one in the starship
  - We will consider a round trip to Proxima Centauri
- Let us set up a set of observing (lookout) stations that are associated with the Earth and with the rocket ship
  - We want to see what they measure?
- Note that all three frames are NOT the same
- The person in the rocket has to turn around to return!
  - Thus the outgoing and return frames are completely different
  - Thus we can think of these as two different rockets



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Outgoing Rocket

- So it takes 1.41 years by outgoing-rocket time to reach Proxima Centauri (PC).
  - What time does the outgoing rocket lookout read on the Earth clock as she passes it (when the rocket reaches PC?
  - Time dialation must be the same => $1.41\times(1.41/4.53) = 0.44$ years!
- Note what we have –
  - This is what the Earth clock reads *at the same time* as the outgoing rocket arrives at PC *as measured in the outgoing rocket frame*.
  - But *at the same time* as the outgoing rocket arrives at PC, the Earth clock reads 4.53 years *as measured in the Earthbound frame*.
- Observers in different reference frames do not agree on what events occur *at the same time* when these events occur far apart along the line of motion.
  - Called Relativity of Simultaneity

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Return Rocket

- When return rocket arrives at Earth the clocks read
  - Rocket clock = 2.82 years, Earth clock = 9.06 years
  - The readings are occurring at the same location so we don't need to worry about the relativity of simultaneity
- According to return rocket frame observations the Earth clock again records an elapsed time of $1.41\times(1.41/4.53) = 0.44$ years on the return trip.
  - Therefore at the turn around in the return rocket frame the Earth clock read 9.06 – 0.44 = 8.62 years
- However, in the outgoing rocket frame the Earth clock read 0.44 years when the rocked reached PC
- Which is right? – both are!
  - The two observations are from different frames
  - The "jump" is the result of the traveler changing frames at Proxima Centauri.

Paradox Summary

- The experiences of the rocket traveler and the Earth dweller are not symmetric.
  - The traveler has to change directions to return (or stop) => acceleration
  - Thus the traveler changes reference frames
- All frames are consistent and non-paradoxical

Event	Time measured in Earth-linked frame	Time measured by traveler	Earth-clock reading observed by: outgoing-rocket lookout station passing Earth	Earth-clock reading observed by: return-rocket lookout station passing Earth
Depart Earth	0 yr	0 yr	0 yr	
Arrive P. Cent	4.53 yr	1.41 yr	0.44 yr	
Depart P. Cent	4.53 yr	1.41 yr	0.44 yr	8.62 yr
Arrive Earth	9.06 yr	2.82 yr		9.06 yr

Travel Time – Twin Paradox

- In Euclidean geometry the *shortest path length* between two points is by a travel moving in a straight line
  - Or the traveler who changes direction the least!
- In Spacetime, the *greatest aging* between two events is experience by the traveler who does not change direction.

$$d\tau^2 = dt'^2 - dx'^2 = dt^2 - dx^2$$

- The minus sign means that dt has to be larger than in a frame which moves between events ($dx' = 0$).

See Chapter 4.6 of Spacetime Physics