Novel Topological Effects in Dense QCD in a Magnetic Field*

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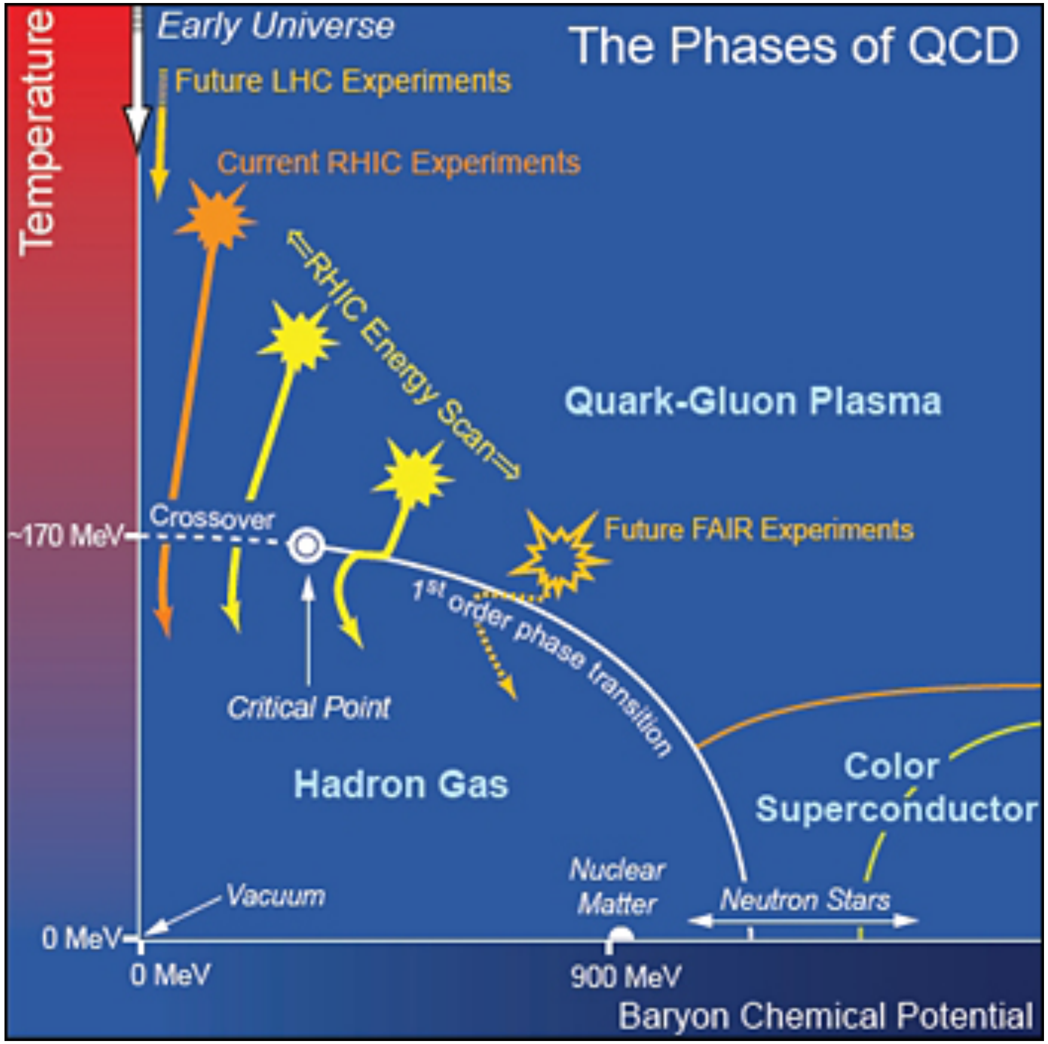
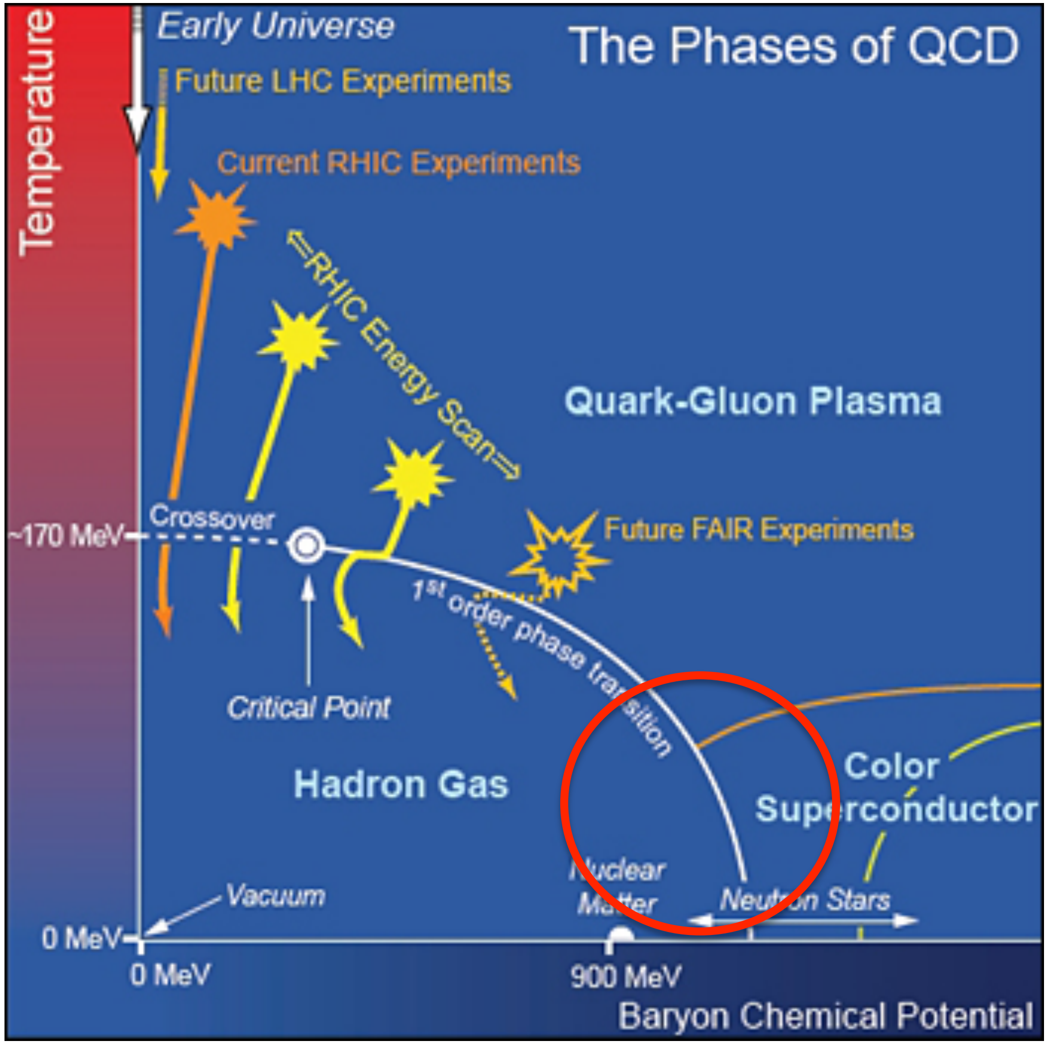
Outline

- Motivation
- DCDW in a Magnetic Field
- Spectral asymmetry, anomalous quark number density
- Axion electrodynamics in QCD at high density: anomalous electric charge and anomalous Hall Current
- Role of B at finite density: no B no topology
- (Observable) Topological effects
- Strange bedfellows at high density: Topology and IR physics
- Outlook

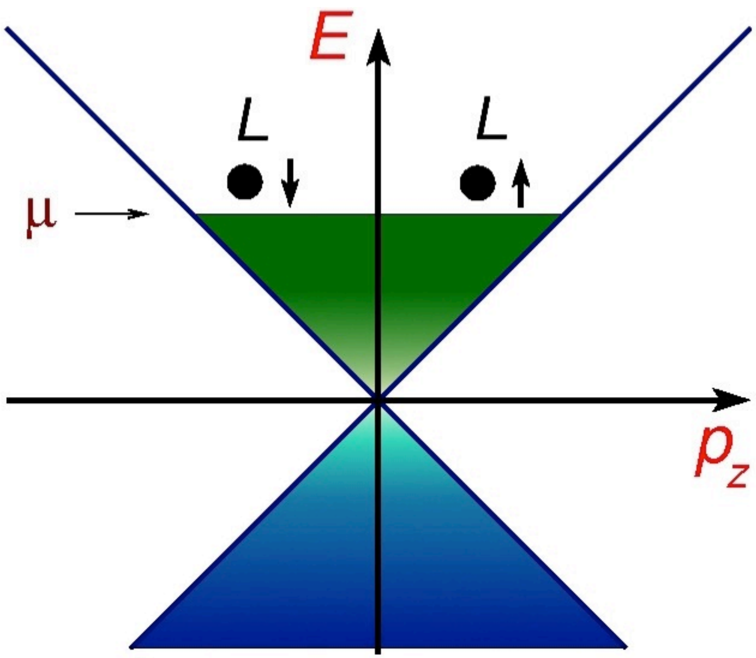
RHIC Energy Scans & Future Experiments

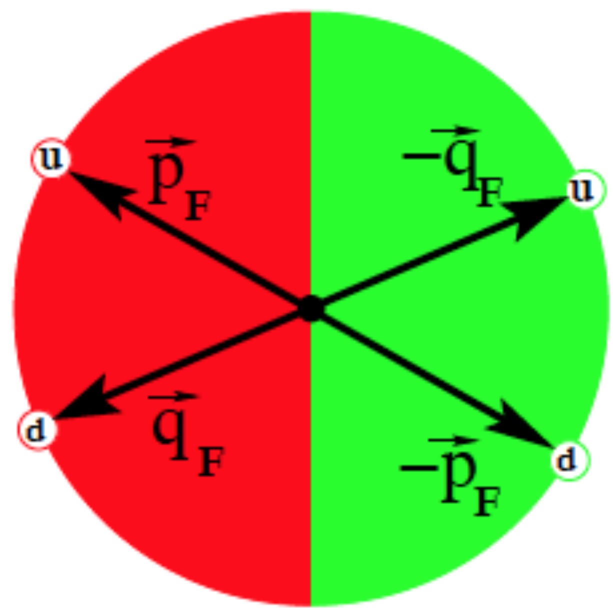


RHIC Energy Scans & Future Experiments



What do We Expect at Very Large Density?

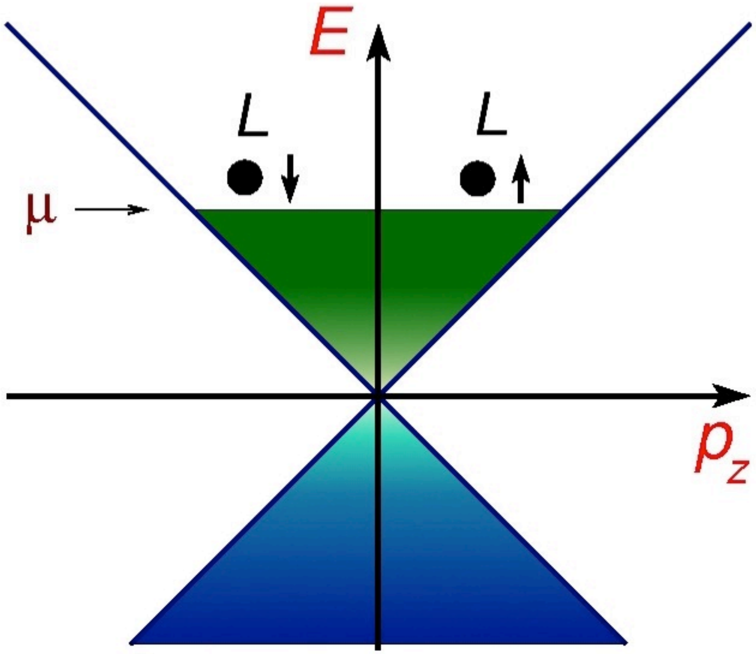


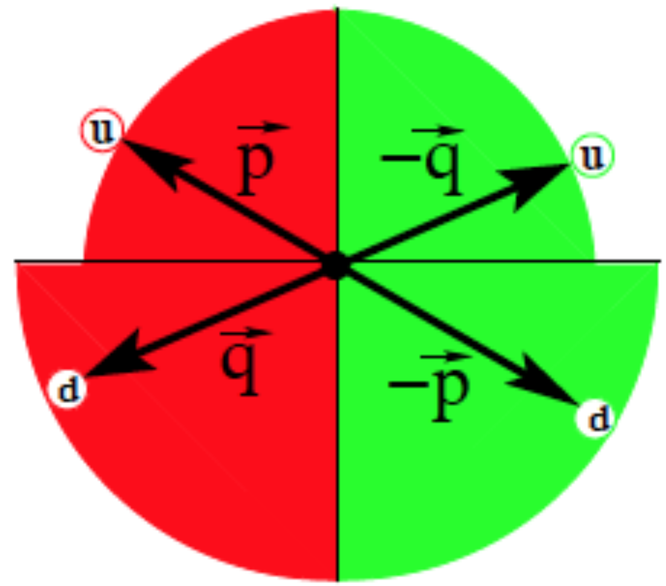


Cooper Pairing via the color anti-triplet channel

Color Superconductivity:
Favored at asymptotically large densities

Intermediate Densities:
No crystal-clear picture

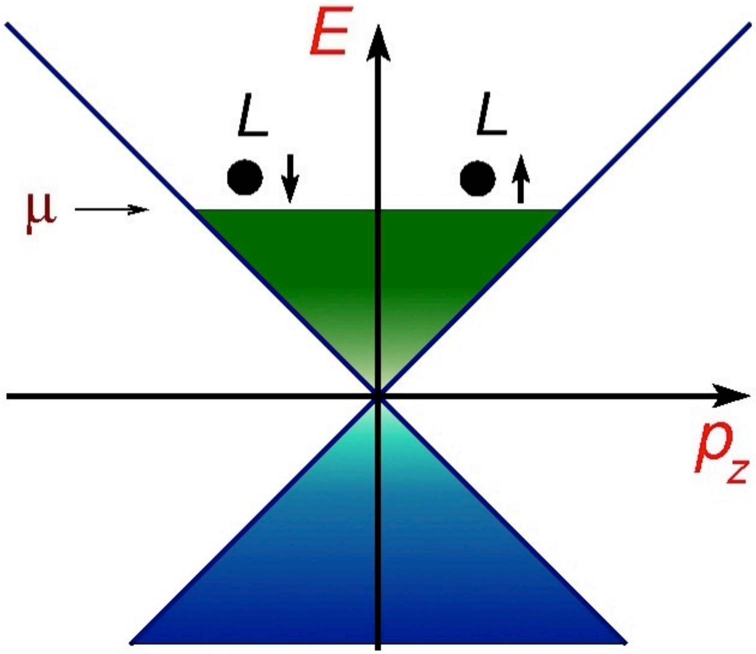




Cooper pairing is distorted by the mismatch of the flavors' Fermi surfaces. It leads to gapless CS and chromomagnetic instabilities.

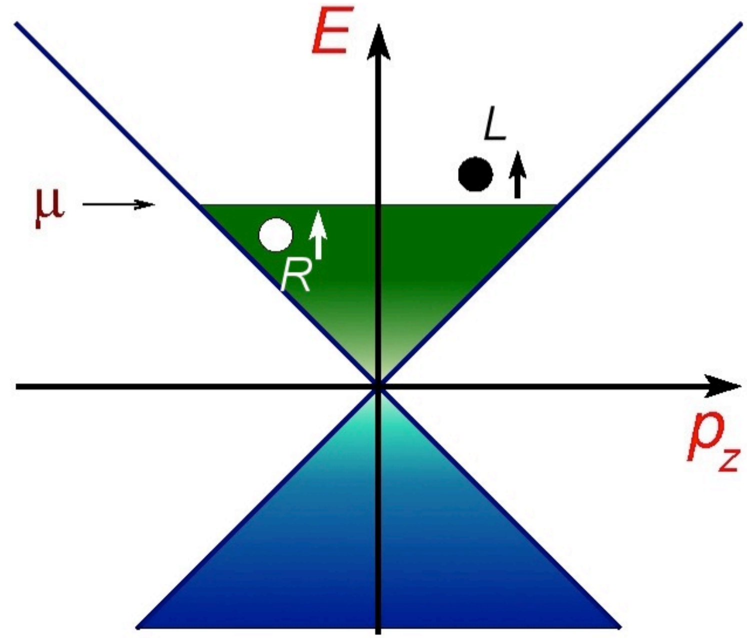
Possible solution: inhomogeneous CS phases

Competing Pairings at Intermediate Density



Cooper Pairing

Becomes less favored with decreasing densities. Color nonsinglet, hence not favored at large Nc.



Particle-Hole Pairing

No Fermi surface mismatch issue because it pairs fermions with holes. Favored over CS at large Nc.

Dual Chiral Density Wave

Nakano & Tatsumi, PRD71, 2005

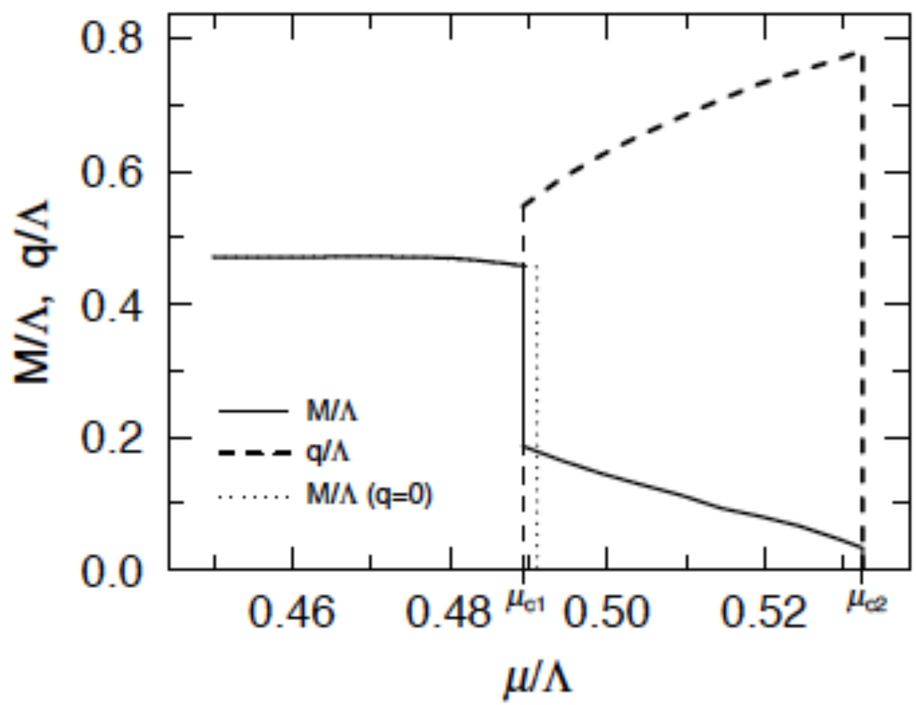
NJL model

$$\mathcal{L} = \bar{\psi}\left(\gamma^{\mu}\left(i\partial_{\mu} + \mu\delta_{\mu 0}\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]\right.$$

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \Delta\cos(\mathbf{q}\cdot\mathbf{r})\\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle &= \Delta\sin(\mathbf{q}\cdot\mathbf{r}),\end{aligned}$$

Not favored over Nickel's real kink solution

Order Parameters at T=0



DCDW in a Magnetic Field (MDCDW)

Frolov, Zhukovsky, & Klimenko PRD82, 2010
Tatsumi, Nishiyama, & Karasawa, PLB 743, 2015

$$H_D = \boldsymbol{\alpha} \cdot \mathbf{P} + \gamma^0 \left[\frac{1+\gamma_5\tau_3}{2} M(\mathbf{x}) + \frac{1-\gamma_5\tau_3}{2} M^*(\mathbf{x}) \right]$$

$$\mathbf{P} = -i\nabla + Q\mathbf{A}, \qquad M(\mathbf{x}) = m(\mathbf{x}) \exp(i\theta(\mathbf{x}))$$

Spectral Asymmetry of the LLL

$$\lambda_{n,p,\zeta,\epsilon} = \epsilon\sqrt{\left(\zeta\sqrt{m^2+k_3^2}+q/2\right)^2+2eBn}, n=1,2,...,$$

$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2+k_3^2}+q/2$$

Anomalous Quark Number Density in MDCDW

Tatsumi, Nishiyama, & Karasawa, PLB 743, 2015

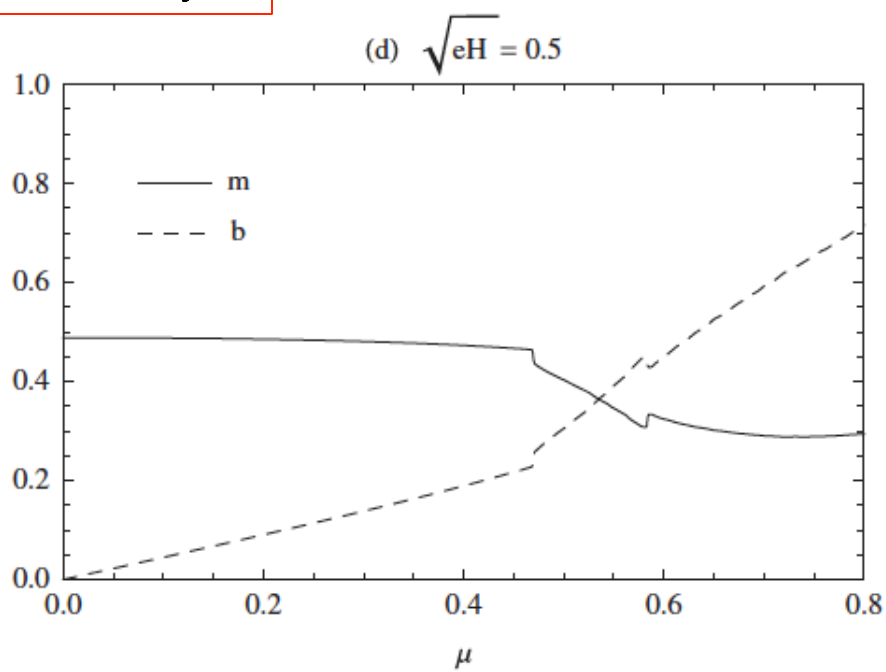
Topology enters through a parameter that measures the LLL spectral asymmetry

$$\eta_H = \lim_{s\to 0}\sum_k sgn(\lambda_k)|\lambda_k|^{-s}$$

$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2+k_3^2}+q/2$$

Anomalous quark number density

$$\rho_B^{anom} = -\frac{\partial\Omega_{anom}}{\partial\mu} = N_c\sum_f \frac{|e_f|}{4\pi^2}\vec{\nabla}\theta\cdot\vec{B} = 3\frac{|e|}{4\pi^2}qB$$



inhomogeneous solution is favored due to the anomaly

Frolov, Zhukovsky, & Klimenko PRD82, 2010

Anomalous Electric Charge in MDCDW

Ferrer & VI, 1512.03972 [nucl-th]

$$\rho_B^{anom} = -\frac{\partial \Omega_{anom}}{\partial \mu} = N_c \sum_f \frac{|e_f|}{4\pi^2} \vec{\nabla}\theta \cdot \vec{B} = 3\frac{|e|}{4\pi^2} qB$$

$$\downarrow$$

$$Q_B^{anom} = N_c \sum_f \frac{e_f |e_f|}{4\pi^2} \vec{\nabla}\theta \cdot \vec{B} = \frac{e^2}{4\pi^2} qB$$

Axion Term in MDCDW

Ferrer & VI, 1512.03972 [nucl-th]

Same anomalous electric charge can be obtained from the mean-field partition function following Fujikawa's method

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi - m\bar{\psi}e^{i\tau_3\gamma_5\mathbf{q}\cdot\mathbf{x}}\psi - \frac{m^2}{4G}$$

The effective Lagrangian has an axion term

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu}} - \frac{m^2}{4G}$$

$$\kappa = \frac{N_c}{2\pi^2}[e_d^2 - e_u^2] = -\frac{e^2}{2\pi^2} \qquad \theta = qz/2$$

Axion Electrodynamics in MDCDW

Ferrer & VI, 1512.03972 [nucl-th]

Same anomalous E.C. as from the spectral asymmetry

$$\nabla \cdot \mathbf{E} = J_0 - \boxed{\kappa \nabla\theta \cdot \mathbf{B}}$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_V + \kappa\left(\frac{\partial \theta}{dt}\mathbf{B} + \boxed{\nabla\theta \times \mathbf{E})}\right.$$

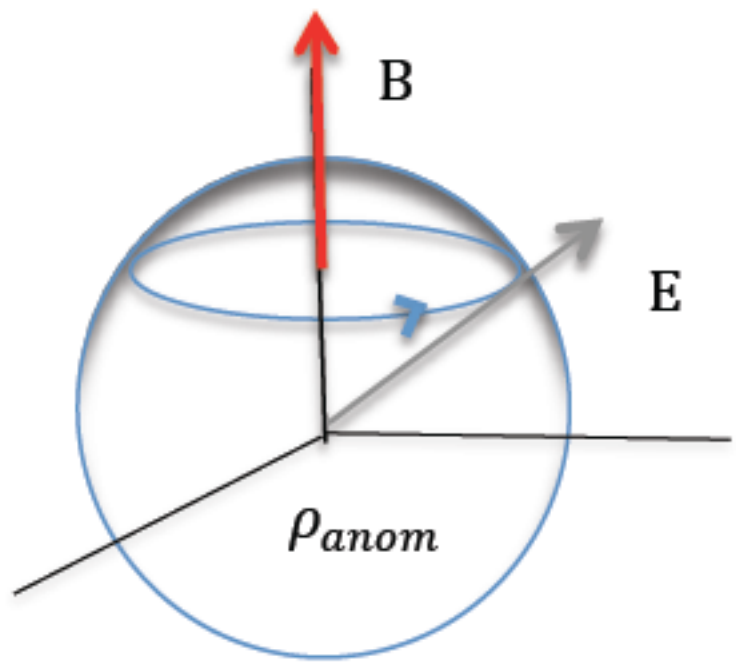
$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0,$$

Anomalous Hall current: Non-dissipative, $\perp$ to both B and E

$$\kappa = -\frac{e^2}{2\pi^2}$$

$$\theta = qz/2$$

Circular non-dissipative Hall Currents are formed in a sphere filled with anomalous charge density

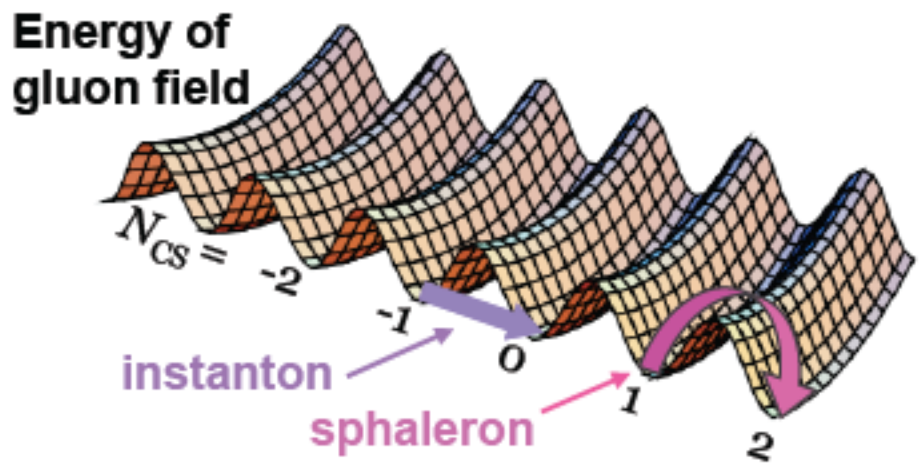


- Observable effects in future high baryon HIC experiments?

- Neutron stars?

Different Roles of B at High T and High Density

At high T: B is a probe for the CME, but the topology is produced by the sphaleron transitions



$$\lambda_{n=0,p,\epsilon} = \epsilon\sqrt{m^2 + k_3^2} + q/2,$$

$$L_{ax} \sim qzF^{\mu\nu}\tilde{F}_{\mu\nu}$$

At high density: B is needed together with the DCDW, to produce the LLL spectral asymmetry and hence the nontrivial topology

Linear Magnetoelectric Effect

$$\nabla \cdot \mathbf{D} = J_0 \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_V$$

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\mathbf{H} = \mathbf{B} - \kappa\theta\mathbf{E} \qquad \mathbf{D} = \mathbf{E} + \kappa\theta\mathbf{B}$$

Axion Polariton in Magnetic Topological Insulators

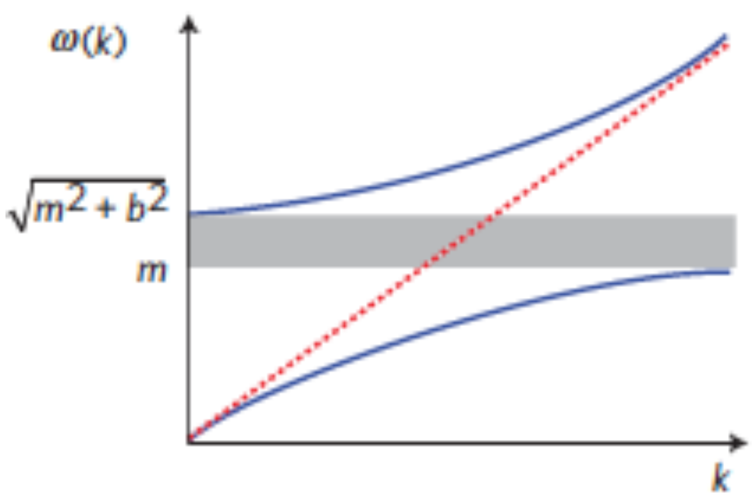
Li, Wang, Qi & Zhang, Nature Physics 2010

It has been discussed in a TI with long-range antiferromagnetic order where TR symmetry is spontaneously broken and the axion field can take continuous values from 0 to π

$$\frac{\partial^2}{\partial t^2}\mathbf{E} - c'^2\nabla^2\mathbf{E} + \frac{\alpha \mathbf{B}_0}{\pi\epsilon}\frac{\partial^2}{\partial t^2}\delta\theta = 0$$

$$\frac{\partial^2}{\partial t^2}\delta\theta - v^2\nabla^2\delta\theta + m_0^2\delta\theta - \frac{\alpha \mathbf{B}_0}{8\pi^2 g^2 J}\mathbf{E} = 0$$

Axion Polariton: coupled mode of light and axion field

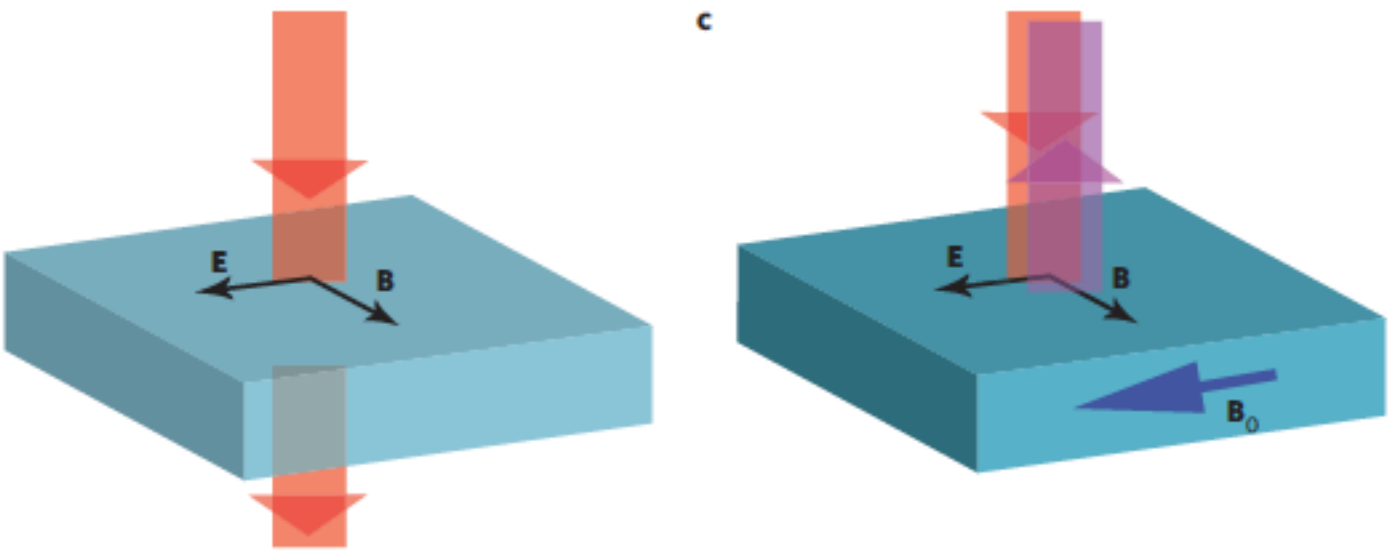


$$\omega_\pm^2(k) = \frac{1}{2}\Big[(c'^2k^2 + m^2 + b^2) \pm \sqrt{(c'^2k^2 + m^2 + b^2)^2 - 4c'^2k^2m^2}\Big]$$

$$b^2 = \alpha^2 \mathbf{B}_0{}^2/8\pi^3\epsilon g^2 J$$

If the electric field of incident light is parallel to the background magnetic field, some light frequencies will be attenuated

Li, Wang, Qi & Zhang, Nature Physics 2010



Same thing should happen in the MDCDW!

Potential for observable signature of MDCDW in future experiments

Topology Influences IR Physics!

Topology comes from spectral asymmetry of the LLL. It is an UV phenomenon

$$\eta_H = \lim_{s \to 0} \sum_k sgn(\lambda_k) |\lambda_k|^{-s}$$

$$Q_B^{anom} = \frac{e^2}{4\pi^2} qB$$

The topology affects the gap equation through the anomalous term, so it influences the value of q, which comes from the pairing dynamics, hence IR

$$\frac{\partial \Omega}{\partial \mathrm{q}} = \boxed{\frac{\partial \Omega_{anom}}{\partial \mathrm{q}}} + \frac{\partial \Omega_{vac}}{\partial \mathrm{q}} + \frac{\partial \Omega_{med}}{\partial \mathrm{q}} = 0$$

Outlook

- Quantitative studies are needed to produce detailed measurable observables that will allow to probe the presence of the MDCDW phase at high baryon density at NICA
- Since we are still trying to figure out how to verify effects at much lower baryon densities, there is plenty of time to figure this out…