Bénard-von Kármán Vortex Street in a Bose-Einstein Condensate

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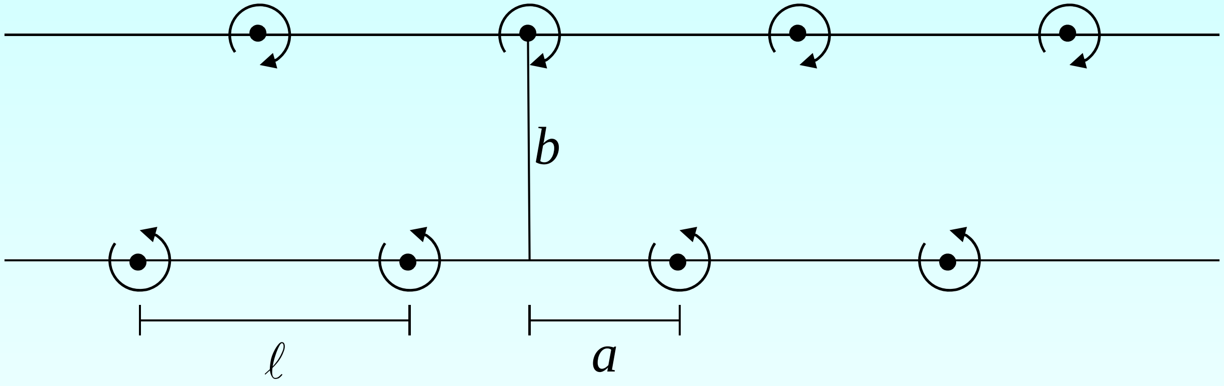
Stability of vortex street

Stability of point vortices (inviscid and incompressible)



This configuration is stable only for $\begin{cases} a = \ell / 2 \\ \cosh \dfrac{\pi b}{\ell} = \sqrt{2} \end{cases}$

T. von Kármán, Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl., 509 (1911)

Viscous fluids

Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \underbrace{\nu \nabla^2 \mathbf{u}}_{\textbf{viscosity}}$$

Reynolds number

ν : kinematic viscosity

$$\mathrm{Re} \equiv \frac{\text{typical length} \times \text{typical velocity}}{\nu}$$

Motivation of this study

Viscosity is important for vortex street generation.

How about superfluids ?

Viscosity is absent and the Reynolds number cannot be defined.

Vortices are quantized.

Formalism

2D Gross-Pitaevskii (GP) equation

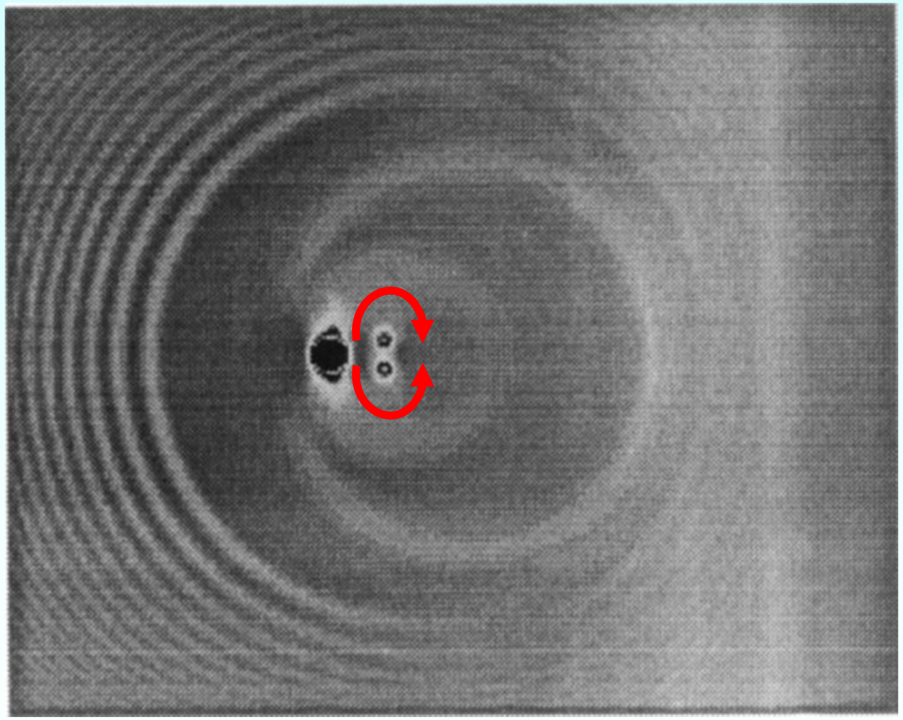
$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(x, y, t) + g \left| \psi \right|^2 \right] \psi$$

Gaussian moving potential

$$V(x, y, t) = V_0 \exp\left[-\frac{\left(x + \mathrm{v}t\right)^2 + y^2}{d^2} \right]$$

Previous work

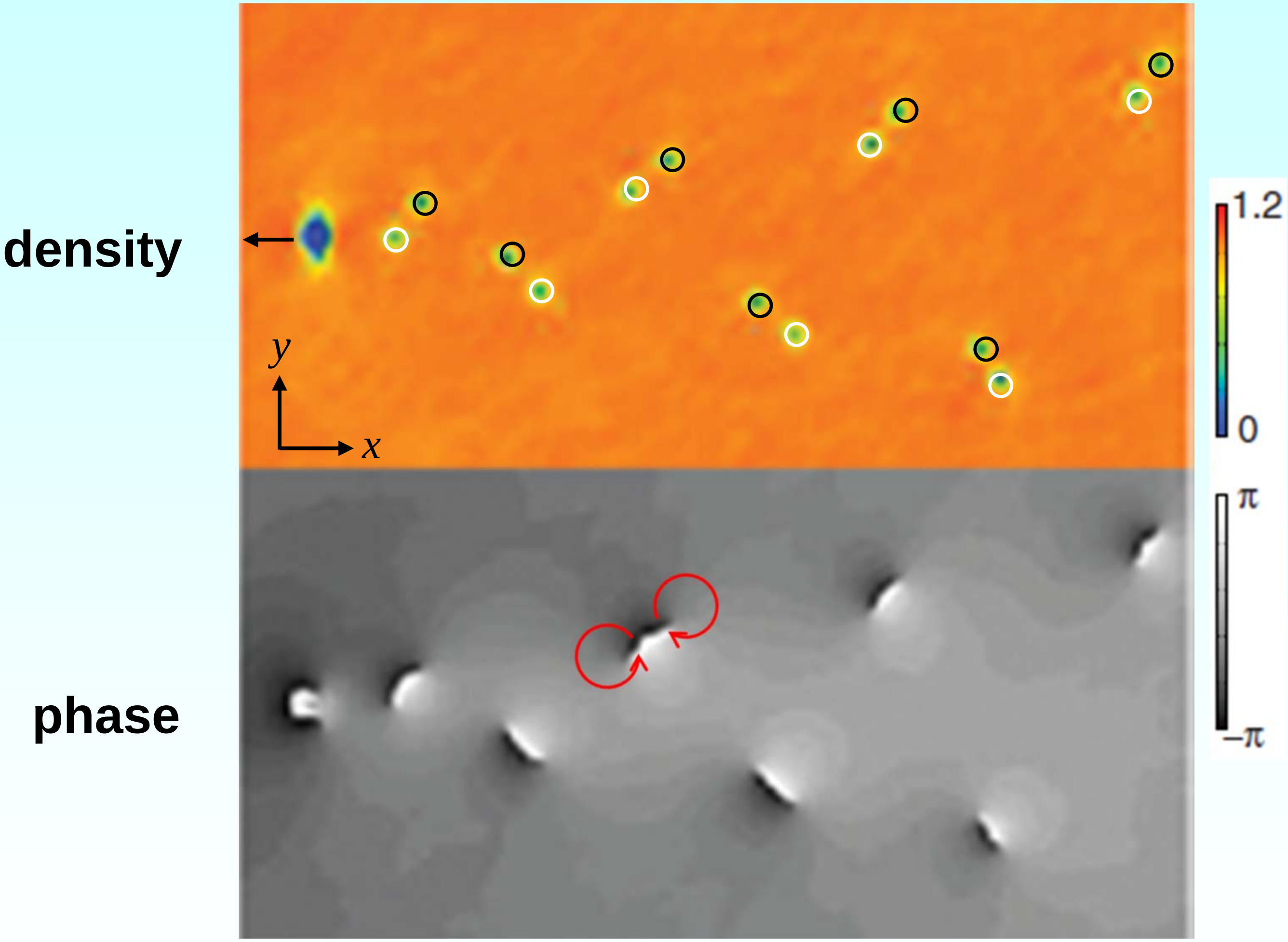
Numerical simulation of the GP eq.



Vortex-antivortex pairs are released, above a critical velocity.

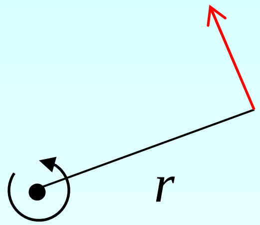
T. Frisch, Y. Pomeau, and S. Rica, *Phys. Rev. Lett.* **69**, 1644 (1992)

Vortex pairs in the wake



Motion of vortex pairs

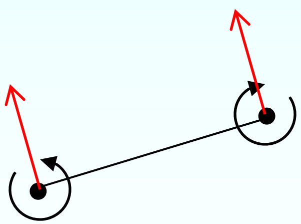
Point vortex in inviscid and incompressible fluid

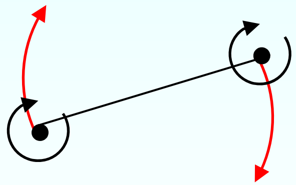


$$v = \frac{\Gamma}{2\pi r}$$

Γ : vorticity

Vortex pairs





Propagate with $v = \dfrac{\Gamma}{2\pi r}$

Rotate at $\omega = \dfrac{\Gamma}{\pi r^2}$

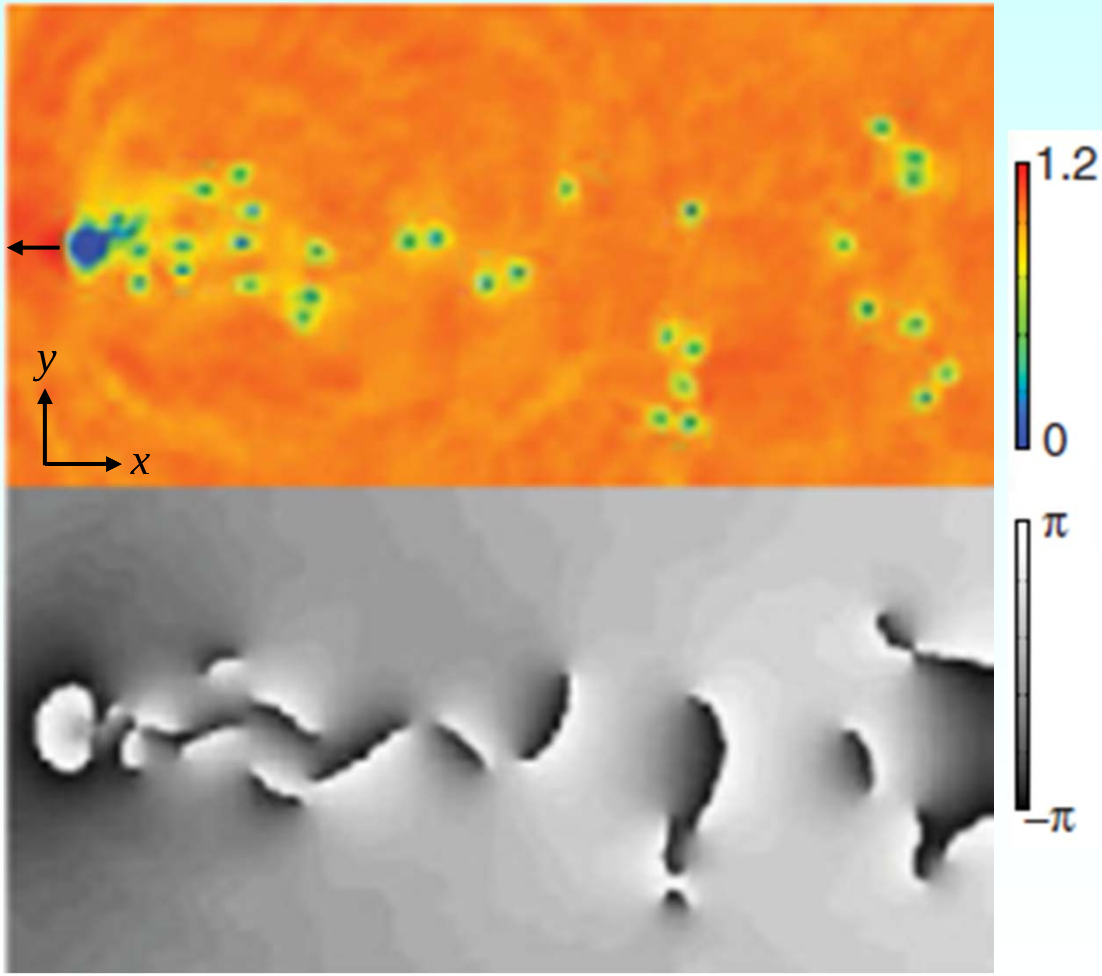
Irregular pattern for large velocity

$v/v_s = 0.60, d/\xi = 1.58$

$$\frac{V_0}{gn_0} = 100$$

density

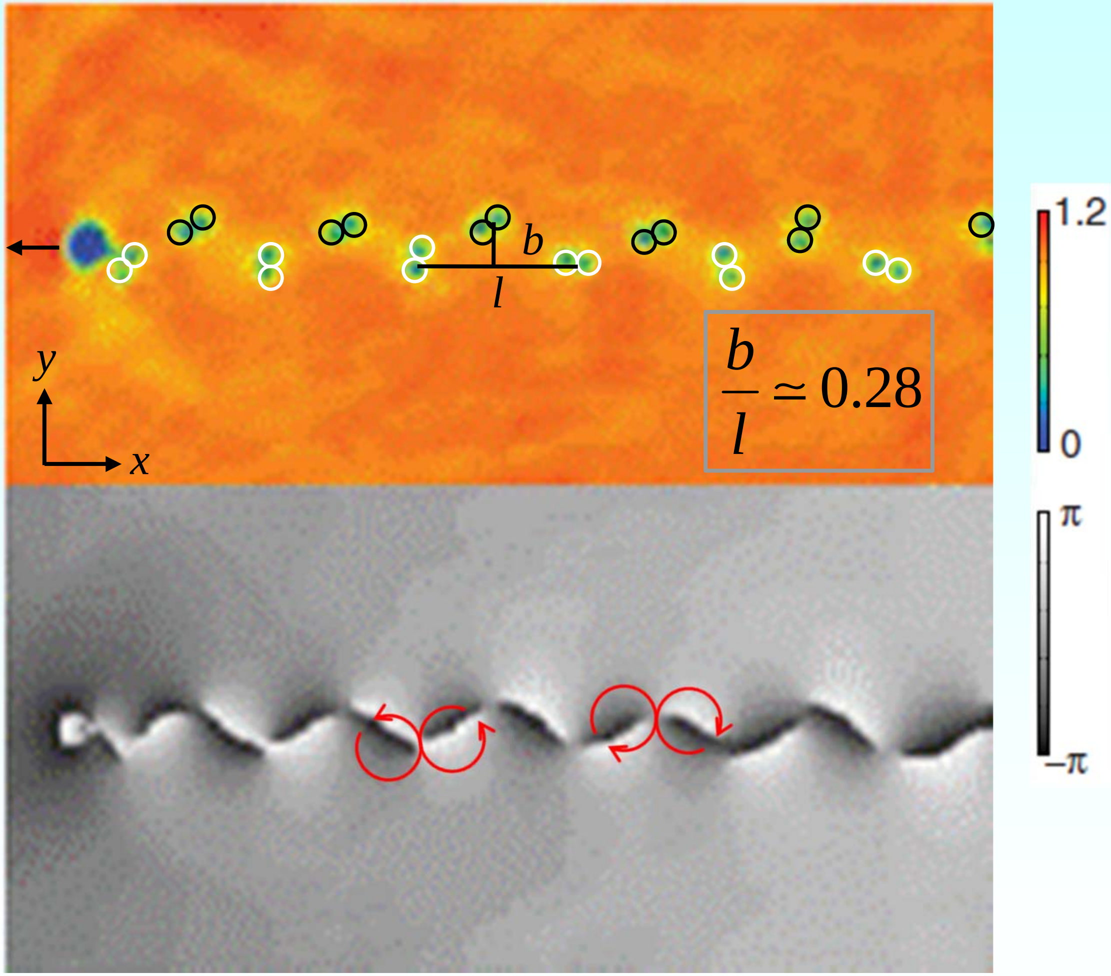
phase

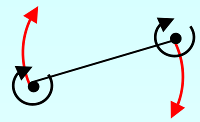


Kármán vortex street

density

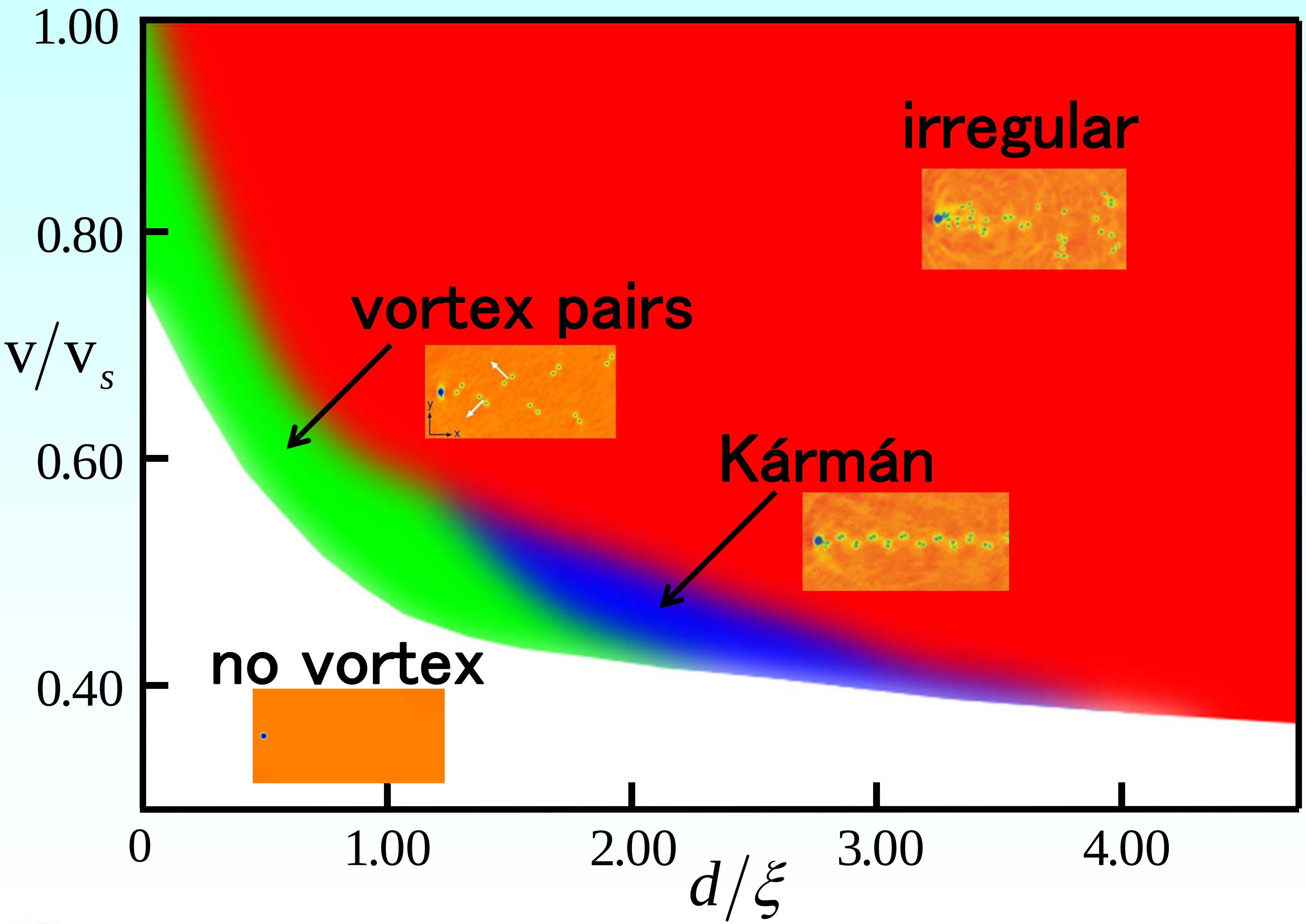
phase



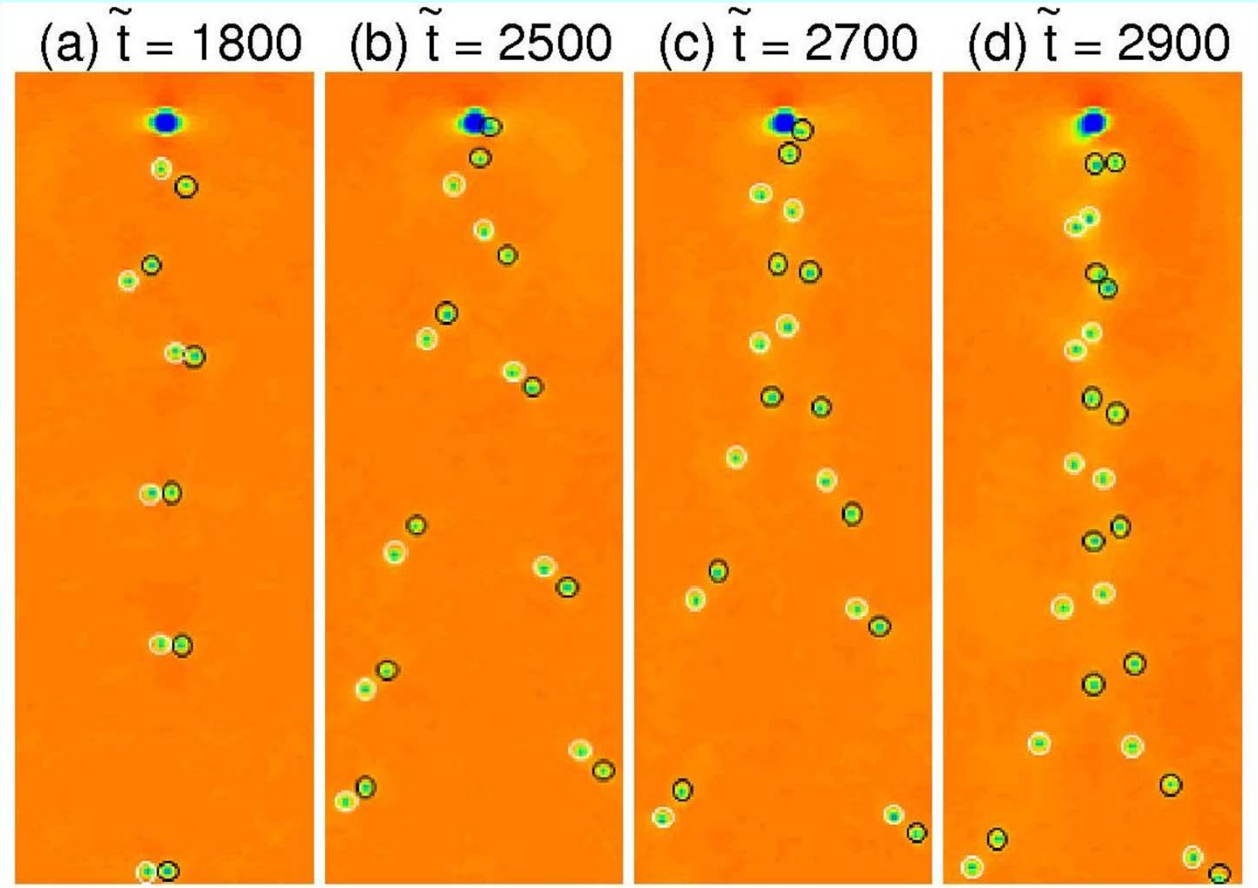


von Kármán's stability condition $\cosh \frac{\pi b}{\ell} = \sqrt{2} \Rightarrow \frac{b}{l} \simeq 0.28$

Parameter dependence



Increasing velocity

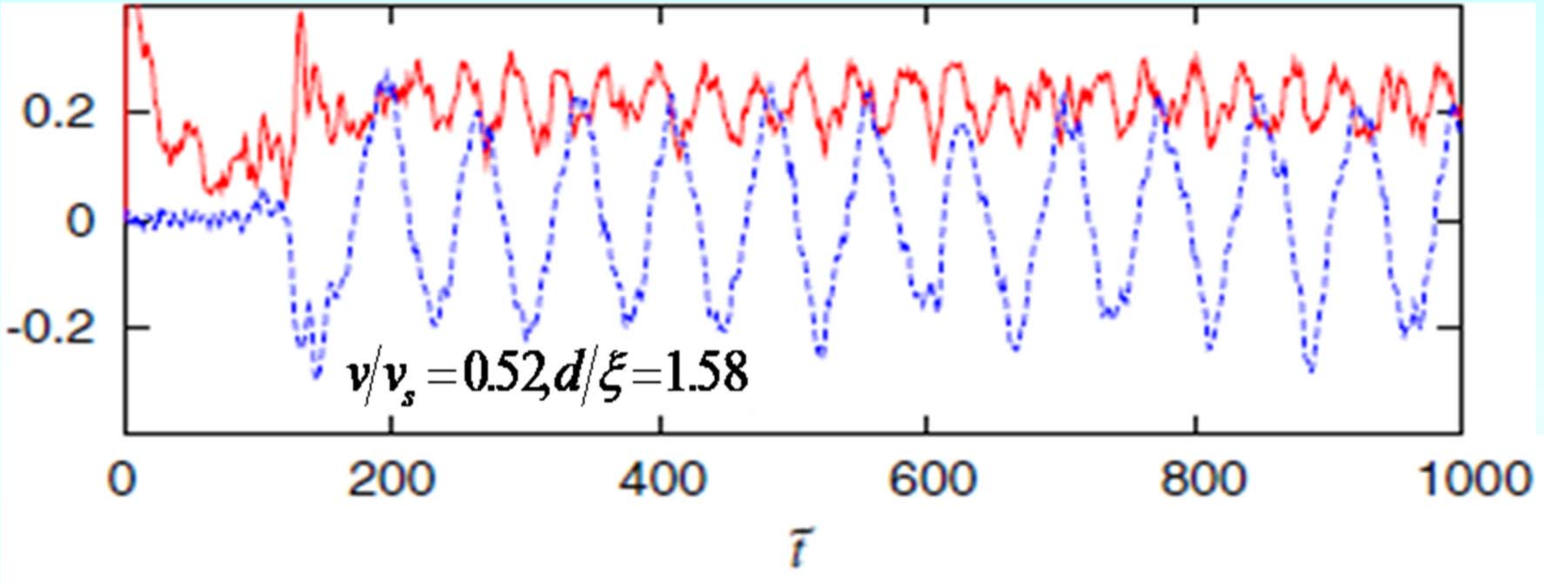


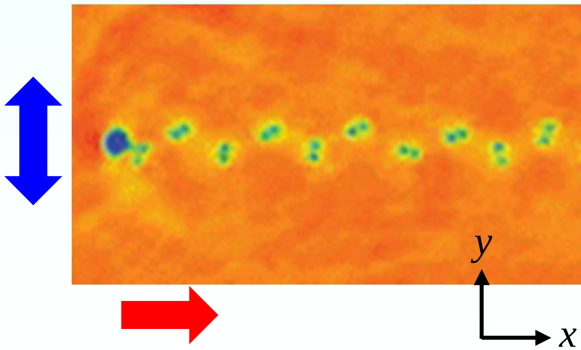
Gradual change from vortex-antivortex pairs to Kármán vortex street

Drag force

$$\mathbf{F} = \partial_t \int d\mathbf{r}\, \psi^* (i\hbar\nabla)\psi$$

$$\tilde{\mathbf{F}} = \frac{2\mathbf{F}}{Amn_0 v^2}$$ area A





$\tilde{\mathbf{F}}_x$

$\tilde{\mathbf{F}}_y$

Trapped system (3D)

$$V = \frac{1}{2} m\left(\omega^2{}_x x^2 + \omega^2{}_y y^2 + \omega^2{}_z z^2\right) + V_0 \exp\left\{-\frac{\left[x^2 + (z - \mathrm{v}t)^2\right]}{d^2}\right\}$$

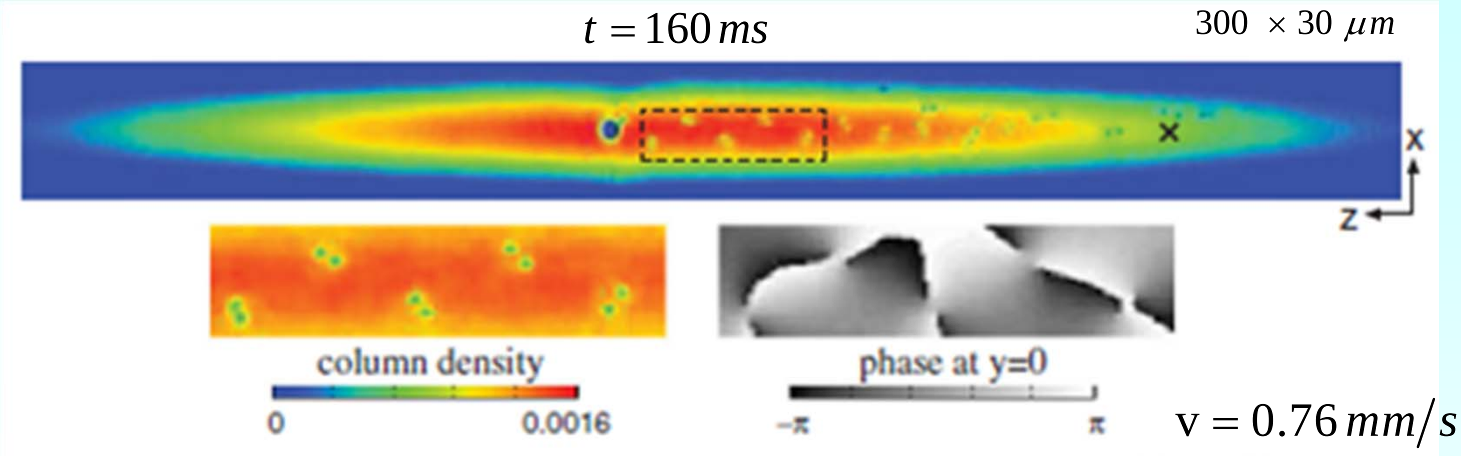
$$^{87}\mathrm{Rb} \quad N = 2.1 \times 10^6$$

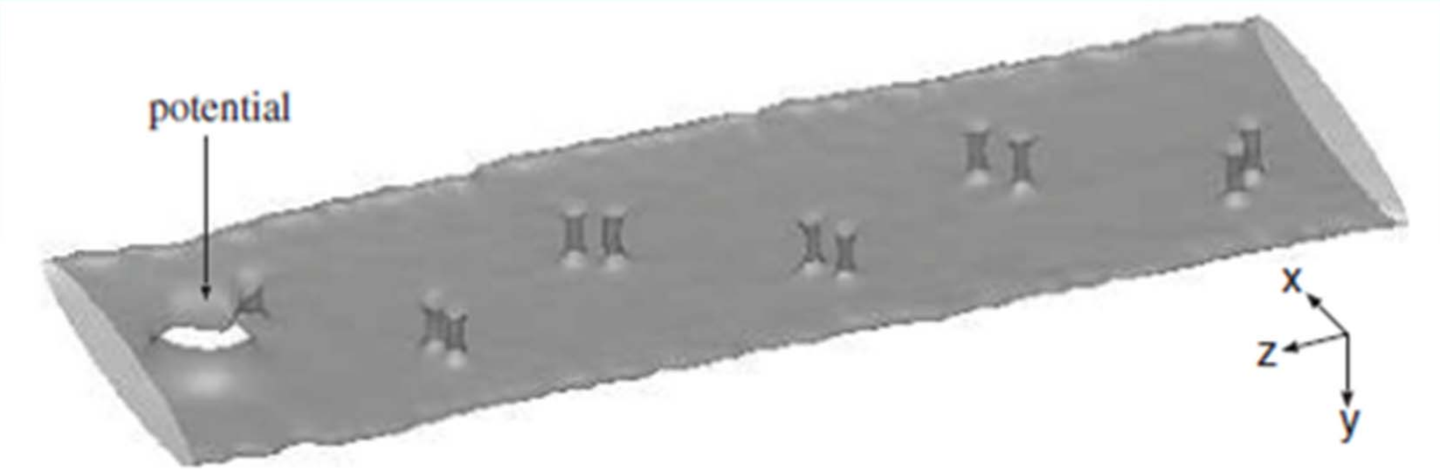
$$\left(\omega_x, \omega_y, \omega_z\right) = 2\pi \times \left(56, 350, 4.3\right) Hz$$

Surfboard shaped

The same condition as in Sadler *et al*., Nature 443, 312 (2006)

Trapped system (3D)





Summary

- Bénard-von Kármán vortex street emerges in superfluids.
- The phenomenon can be realized in an experiment.

K. Sasaki, N. Suzuki, and HS, *Phys. Rev. Lett.* 104, 150404 (2010)

Future work

- Mechanism of vortex street generation.