

Bénard-von Kármán Vortex Street in a Bose-Einstein Condensate

Univ. of Electro-Communications

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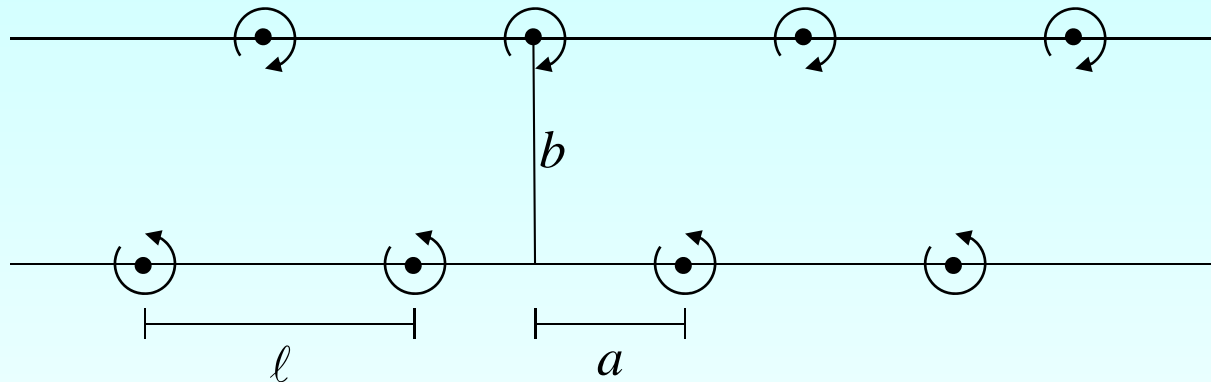
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The University of Electro-Communications

Stability of vortex street

Stability of point vortices (inviscid and incompressible)



This configuration is stable only for
$$\left\{ \begin{array}{l} a = \ell / 2 \\ \cosh \frac{\pi b}{\ell} = \sqrt{2} \end{array} \right.$$

T. von Kármán, Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl., 509 (1911)

Viscous fluids

Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \underbrace{\nu \nabla^2 \mathbf{u}}$$

viscosity

Reynolds number

ν : kinematic viscosity

$$\text{Re} \equiv \frac{\text{typical length} \times \text{typical velocity}}{\nu}$$

Motivation of this study

Viscosity is important for vortex street generation.

How about superfluids ?

- Viscosity is absent and the Reynolds number cannot be defined.
- Vortices are quantized.

Formalism

2D Gross-Pitaevskii (GP) equation

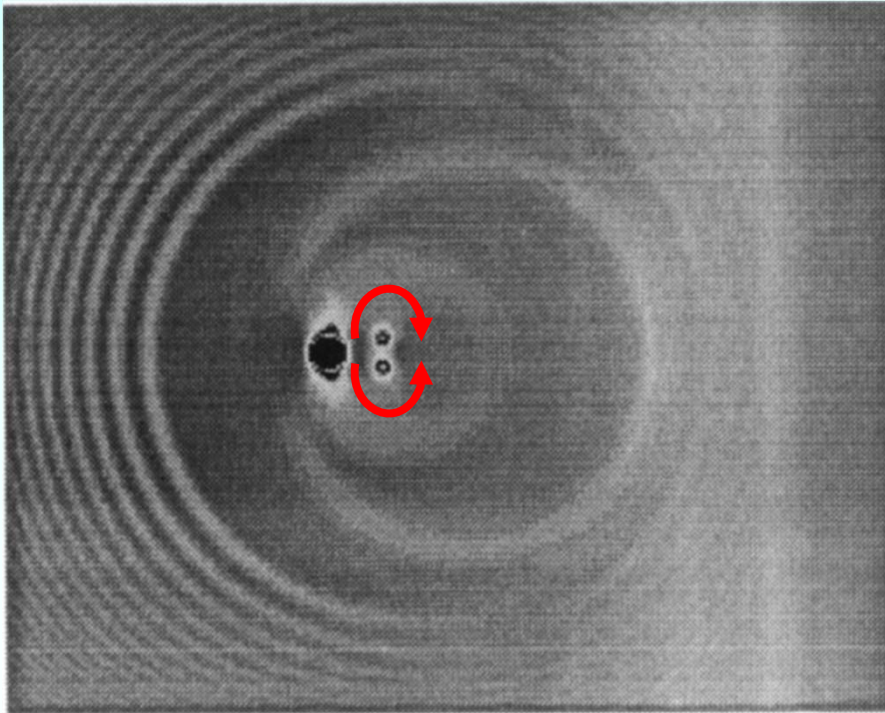
$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(x, y, t) + g |\psi|^2 \right] \psi$$

Gaussian moving potential

$$V(x, y, t) = V_0 \exp \left[-\frac{(x + vt)^2 + y^2}{d^2} \right]$$

Previous work

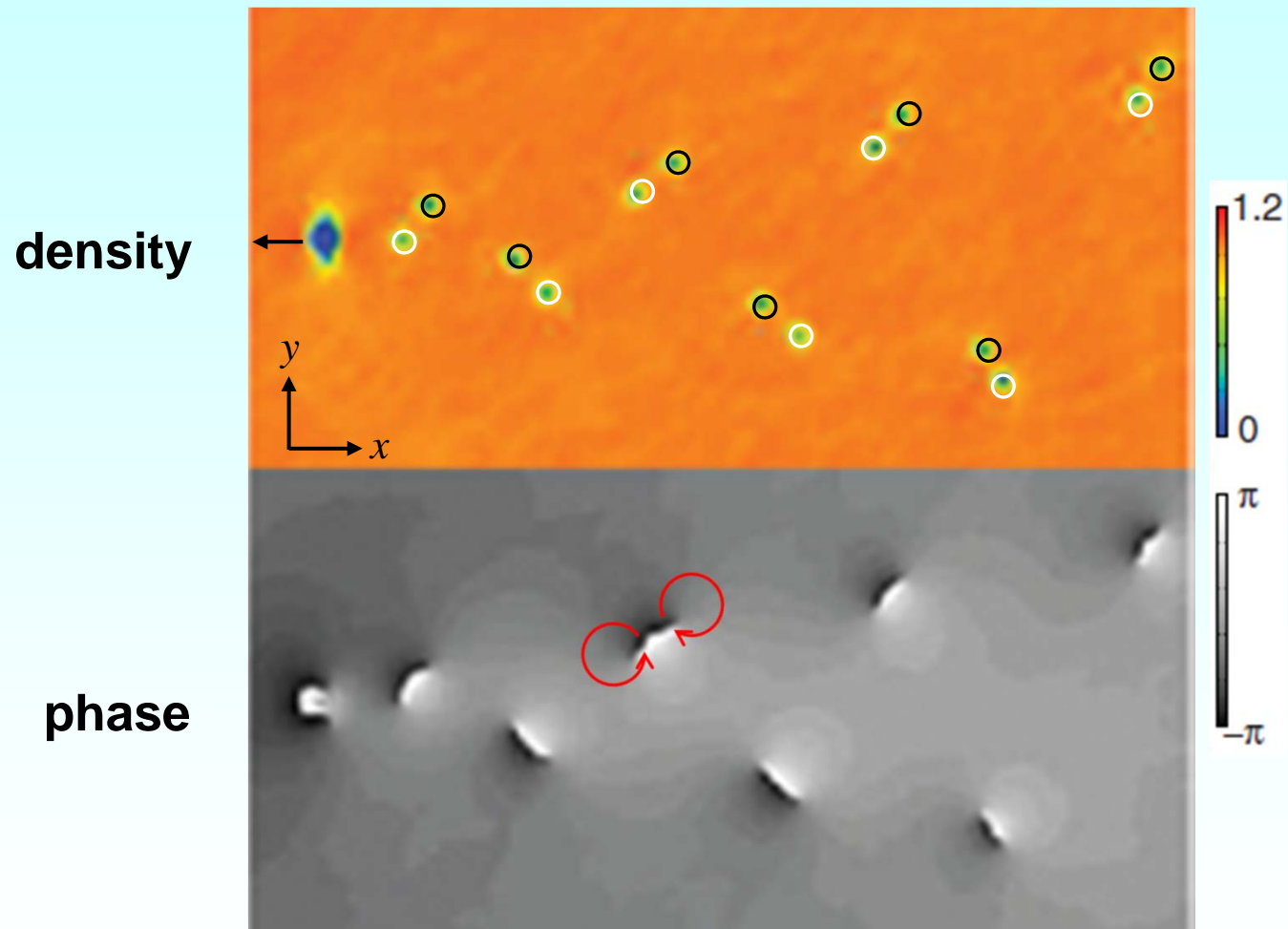
Numerical simulation of the GP eq.



**Vortex-antivortex
pairs are released,
above a critical velocity.**

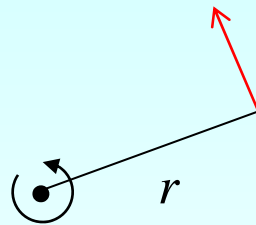
T. Frisch, Y. Pomeau, and S. Rica, *Phys. Rev. Lett.* **69**, 1644 (1992)

Vortex pairs in the wake



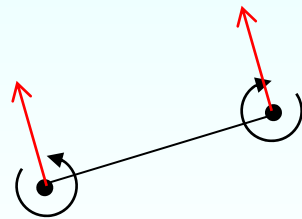
Motion of vortex pairs

Point vortex in inviscid and incompressible fluid

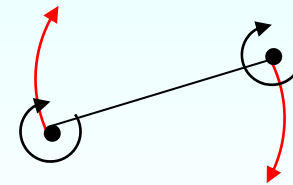


$$v = \frac{\Gamma}{2\pi r} \quad \Gamma : \text{vorticity}$$

Vortex pairs



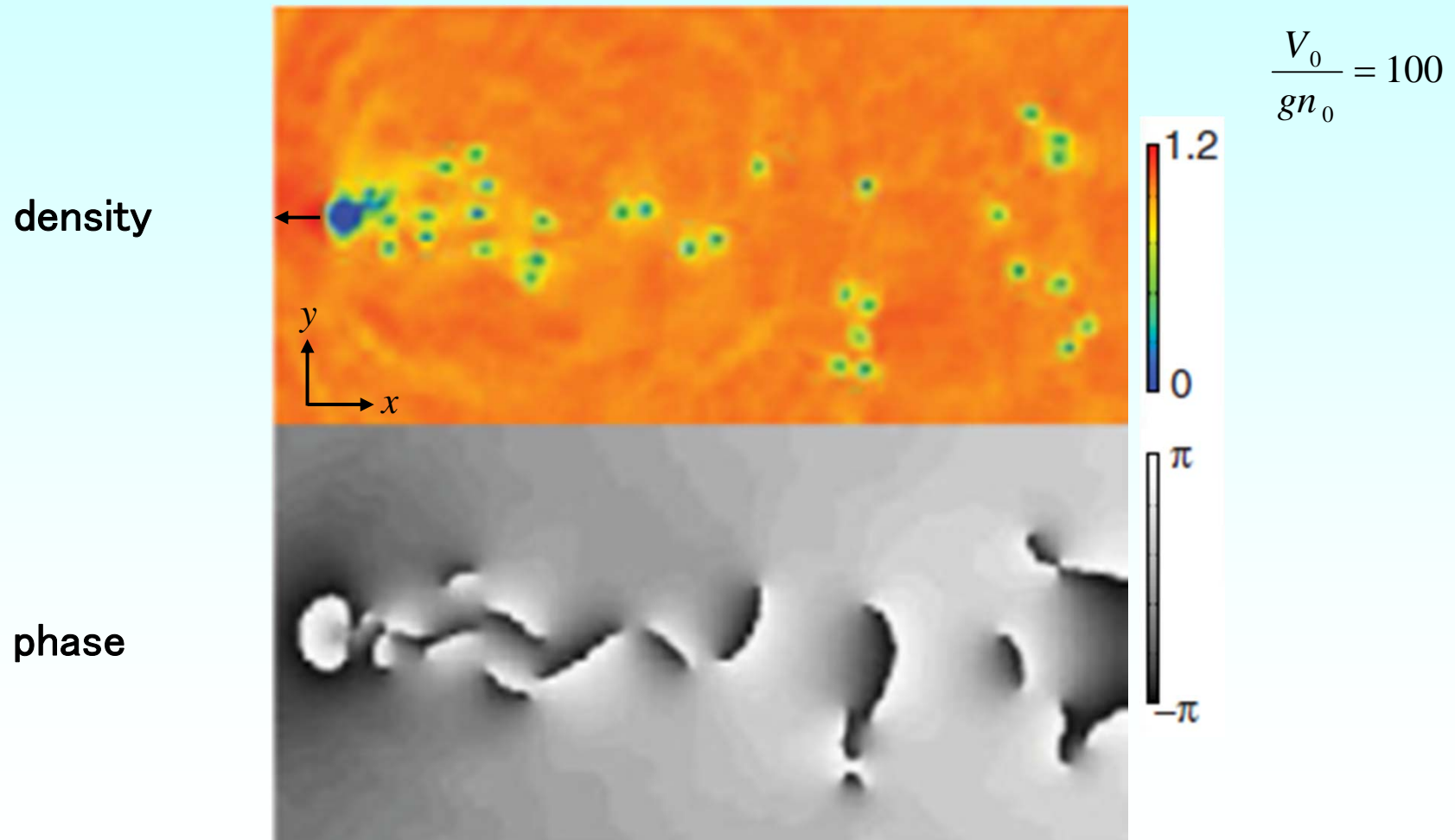
Propagate with $v = \frac{\Gamma}{2\pi r}$



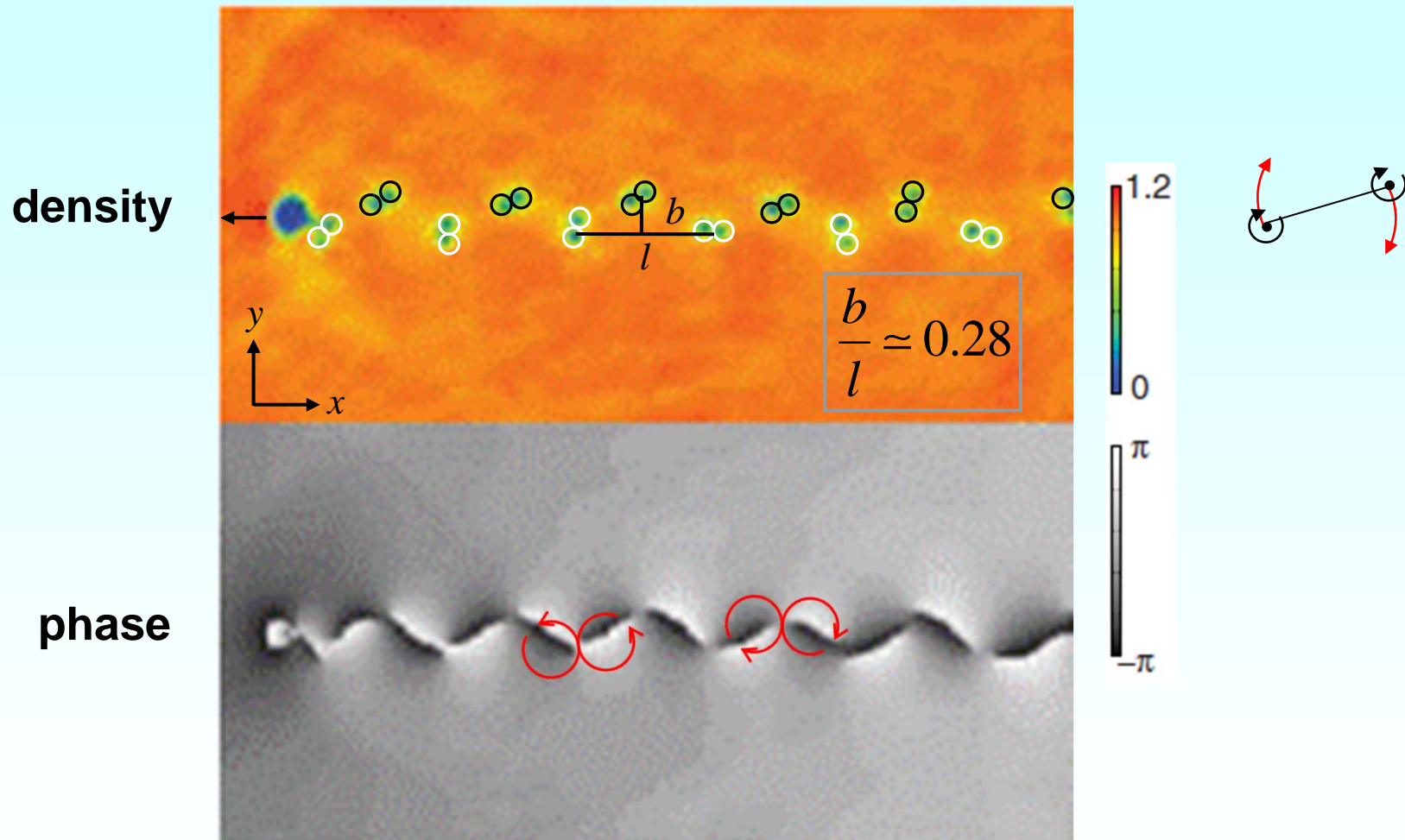
Rotate at $\omega = \frac{\Gamma}{\pi r^2}$

Irregular pattern for large velocity

$$v/v_s = 0.60, d/\xi = 1.58$$

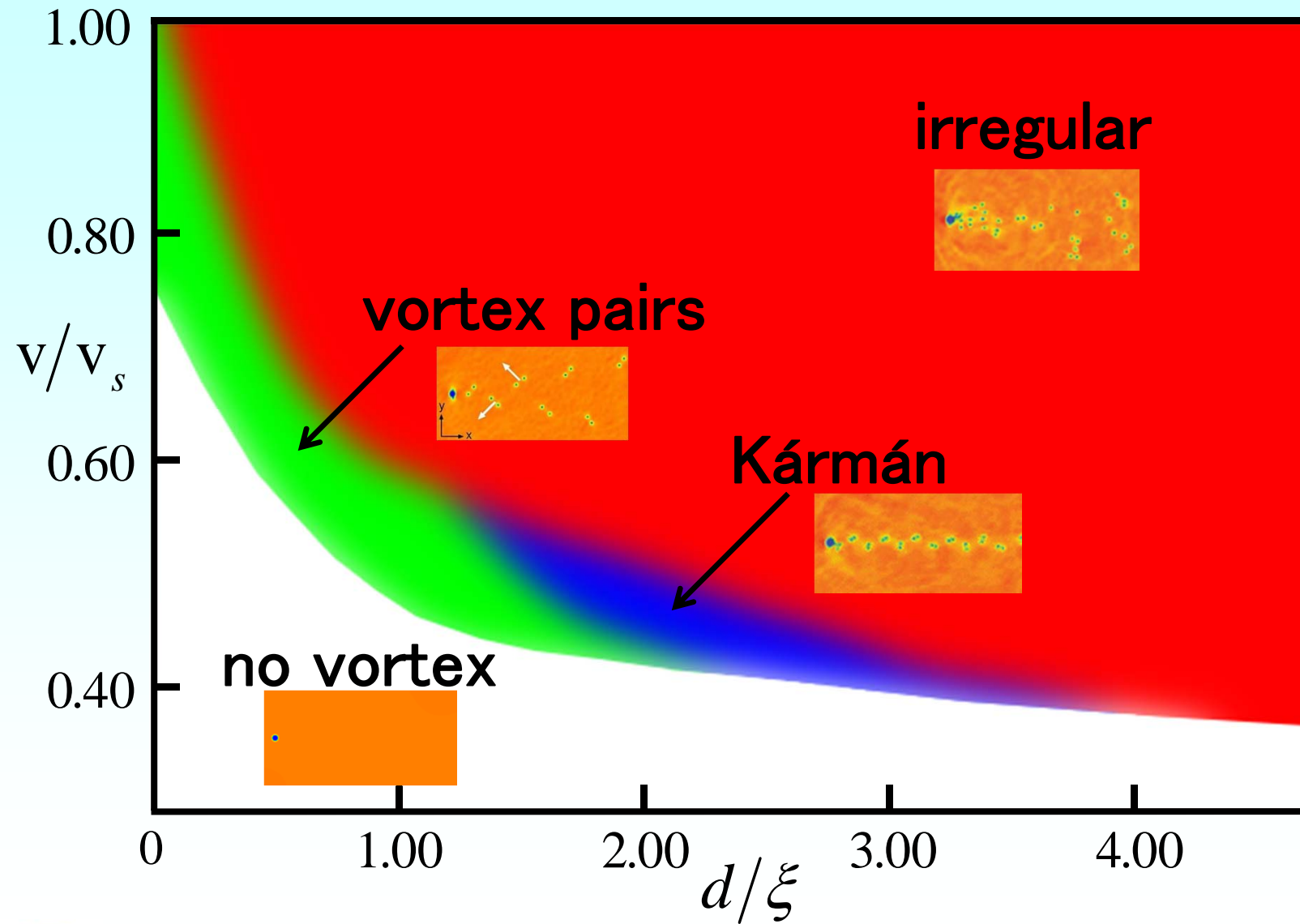


Kármán vortex street

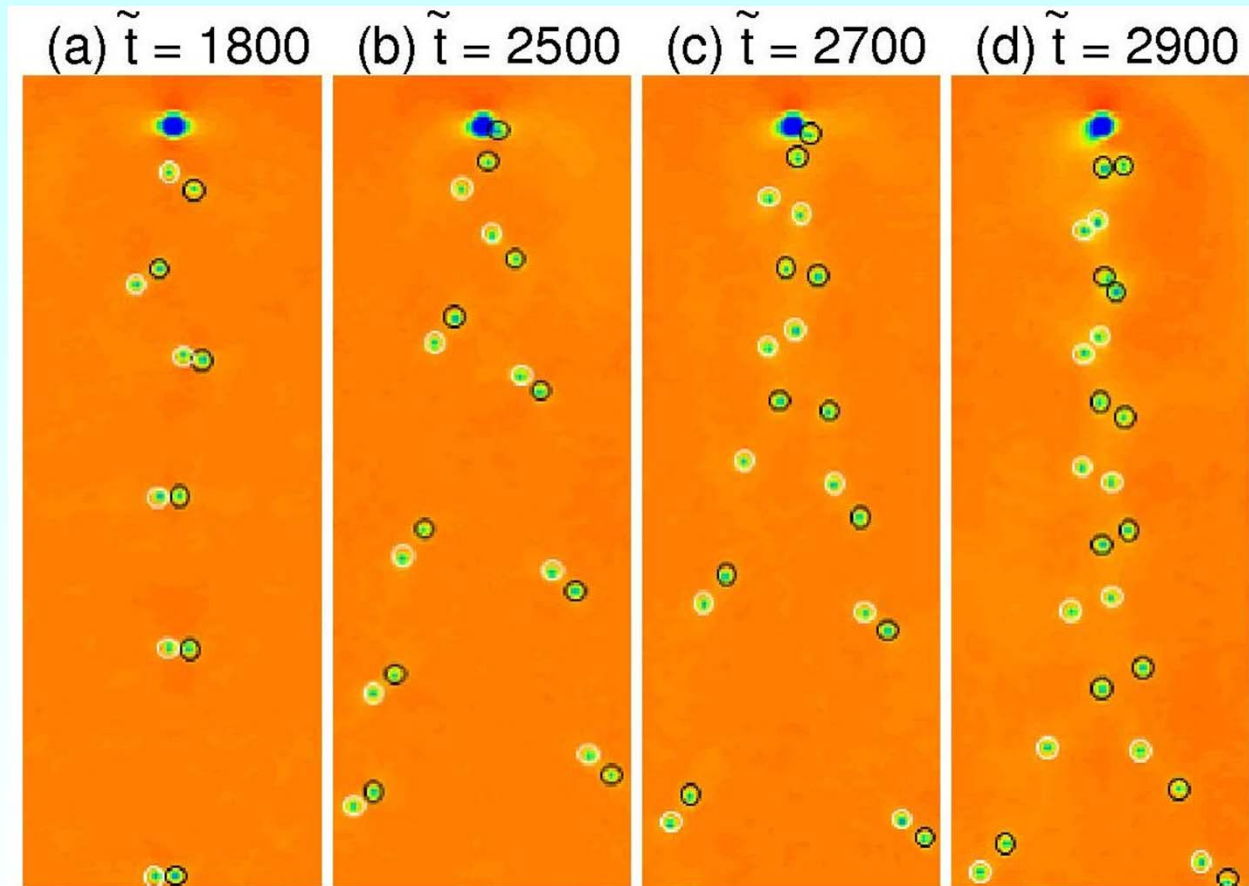


von Kármán's stability condition $\cosh \frac{\pi b}{l} = \sqrt{2} \Rightarrow \frac{b}{l} \approx 0.28$

Parameter dependence



Increasing velocity

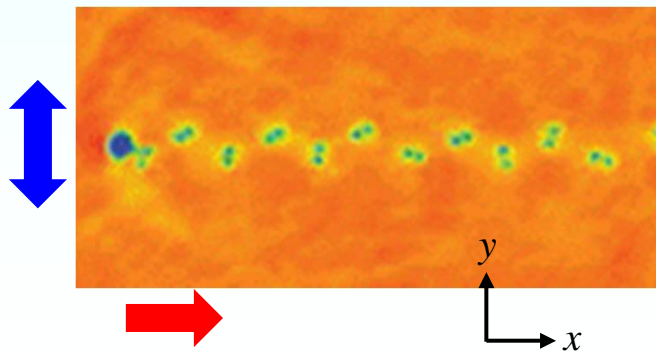
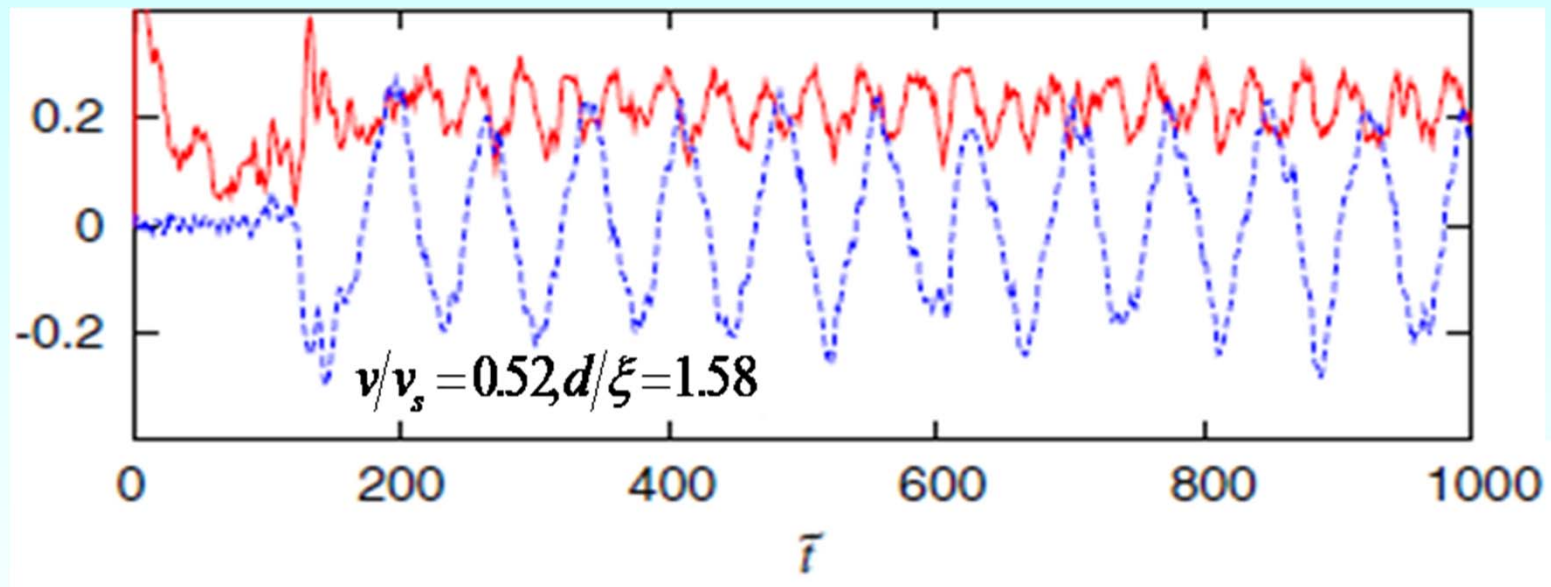


**Gradual change from vortex-antivortex pairs
to Kármán vortex street**

Drag force

$$\mathbf{F} = \partial_t \int d\mathbf{r} \psi^* (i\hbar \nabla) \psi$$

$$\tilde{\mathbf{F}} = \frac{2\mathbf{F}}{A m n_0 v^2} \quad \text{area } A$$



Trapped system (3D)

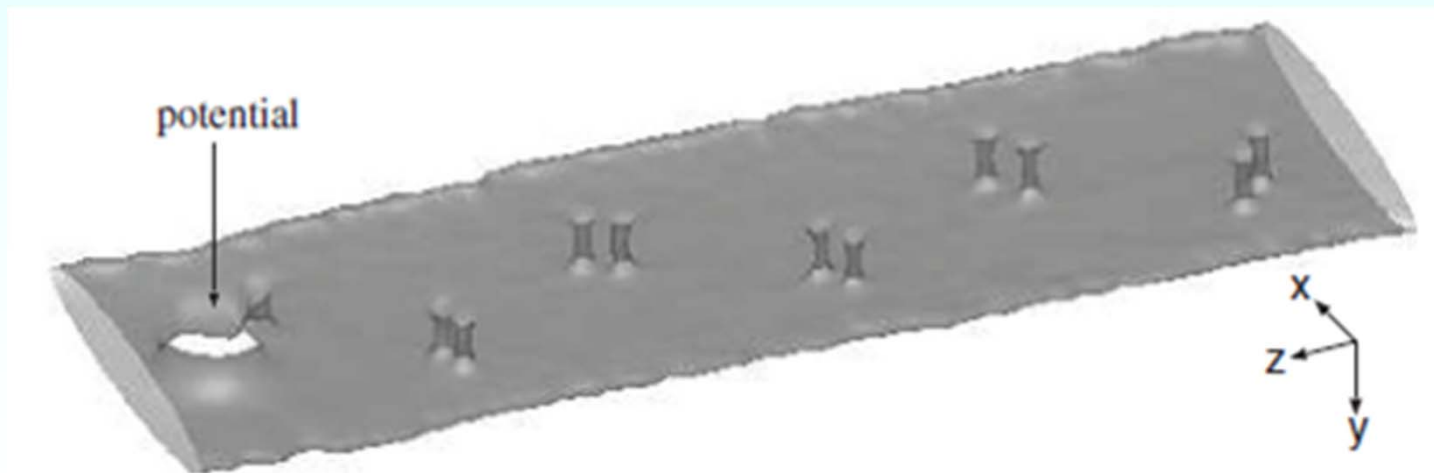
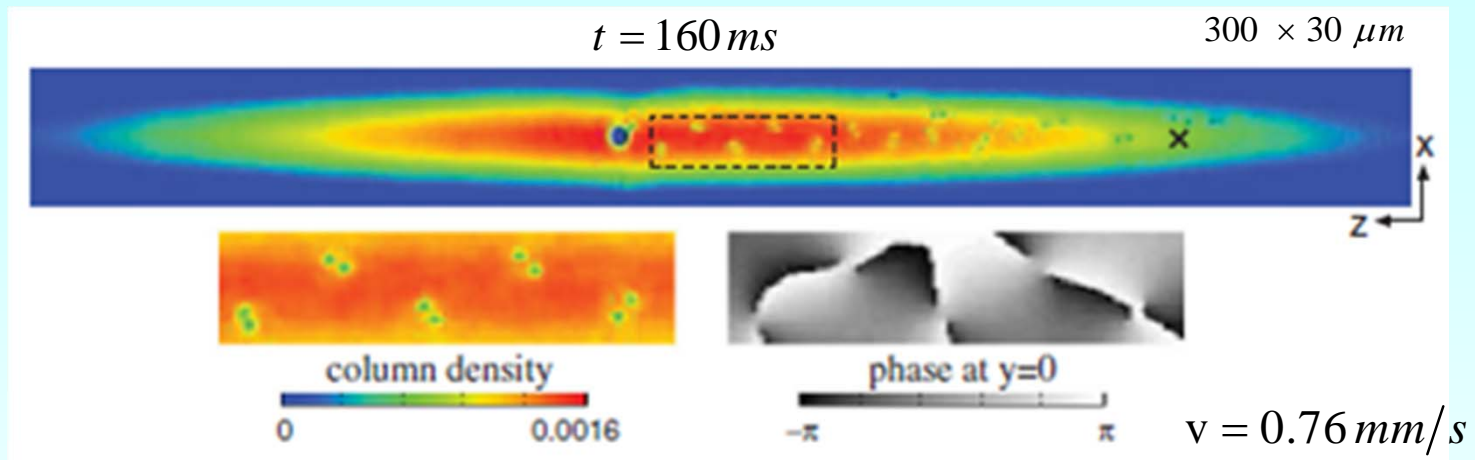
$$V = \frac{1}{2} m (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + V_0 \exp \left\{ - \frac{[x^2 + (z - vt)^2]}{d^2} \right\}$$

$$^{87}\text{Rb} \quad N = 2.1 \times 10^6$$

$$(\omega_x, \omega_y, \omega_z) = 2\pi \times (56, 350, 4.3) \text{ Hz} \quad \text{Surfboard shaped}$$

The same condition as in Sadler *et al.*, Nature 443, 312 (2006)

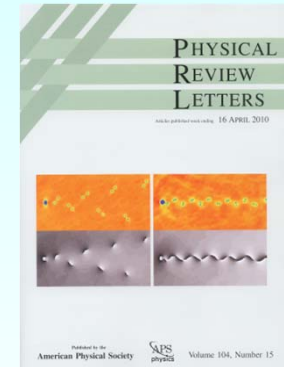
Trapped system (3D)



Summary

- Bénard-von Kármán vortex street emerges in superfluids.
- The phenomenon can be realized in an experiment.

K. Sasaki, N. Suzuki, and HS, *Phys. Rev. Lett.* 104, 150404 (2010)



Future work

- Mechanism of vortex street generation.