Comparison of shot encoding functions for reverse-time migration

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SUMMARY

Reverse-time migration (RTM) represents one of the most accurate, but also one of the costliest algorithms available for imaging complex media. The RTM computational cost can be reduced by shot encoding, i.e. by imaging simultaneously a large number of shots. Different encoding strategies are possible, including linear and random phase encoding, which represent end members of a more general family of encodings. For a fixed maximum delay (i.e. computational cost), we can trade spatial bandwidth for crosstalk noise. Linear phase encoding is characterized by low bandwidth and high SNR, while random phase encoding by high bandwidth and low SNR. Mixed encodings allow us calibrating the amount of resolution desired in the migrated image, given an acceptable level of noise.

WAVE-EQUATION MIGRATION

Wave-Equation migration consists of two steps:

- wavefield reconstruction
- Imaging condition

The wavefield reconstruction extrapolates the source and receiver wavefields, respectively, forward and backward in time in the velocity model:

- source wavefield: $s_k = s_k(\mathbf{x},t)$
- receiver wavefield: $r_k = r_k(\mathbf{x},t)$

The index k indicates the shot number.

The imaging condition is a nonlinear operation: zero-time crosscorrelation of source and receiver wavefield

$$I(\mathbf{x}) = \sum_k \int s_k(\mathbf{x},t) r_k(\mathbf{x},t)\, dt$$

The summation is over all the shots/experiments.

SHOT ENCODING MIGRATION

Synthetic source and receiver wavefields:

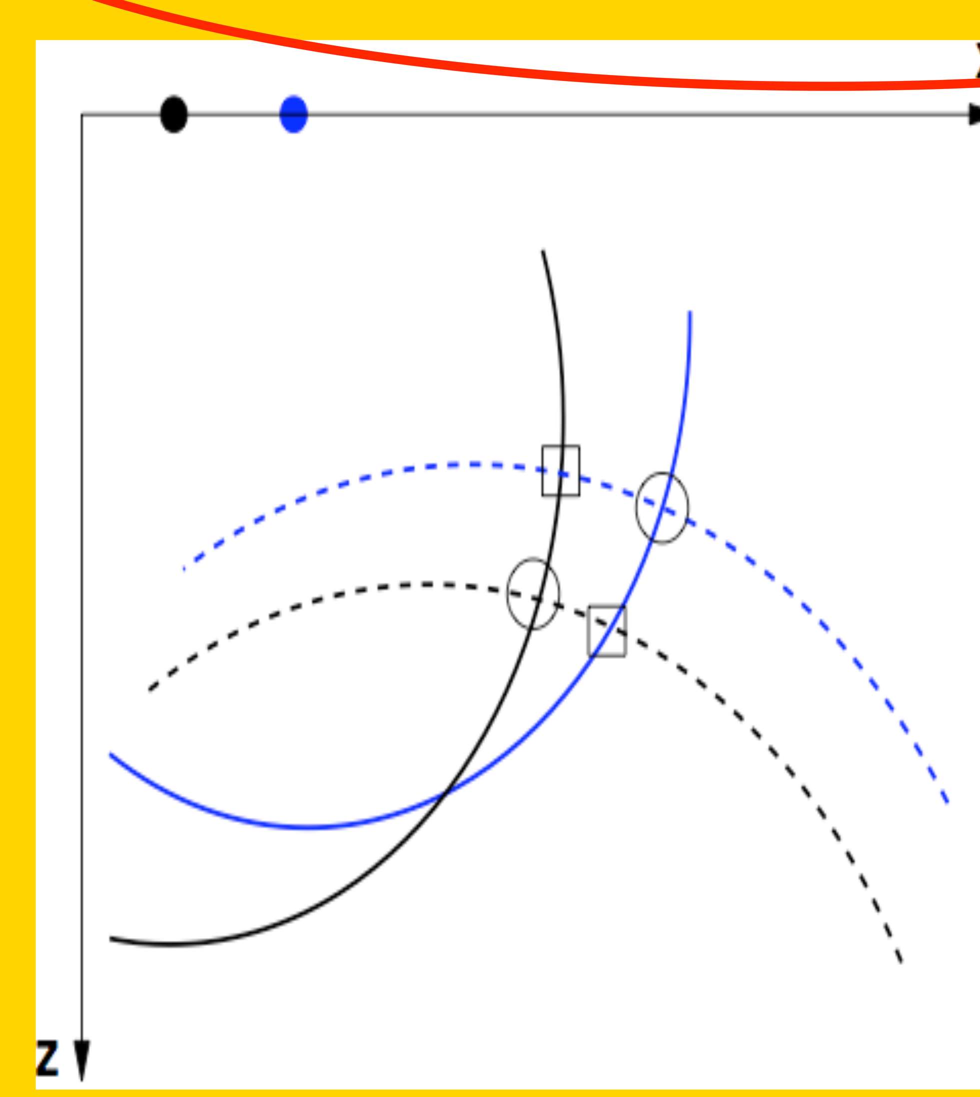
$$S(\mathbf{x},t,\theta) = \sum_k s_k(\mathbf{x},t) * \delta(t - f(\mathbf{x}_k,\theta))$$

$$R(\mathbf{x},t,\theta) = \sum_l r_l(\mathbf{x},t) * \delta(t - f(\mathbf{x}_l,\theta))$$

<u>Imaging condition</u>

$$I(\mathbf{x}) = \sum_k I_k + \sum_{k \neq l} \int s_k(\mathbf{x},\omega) r_l^*(\mathbf{x},\omega) \sum_\theta e^{i\omega(f(\mathbf{x}_l,\theta) - f(\mathbf{x}_k,\theta))}\, d\omega$$

CROSSTALK



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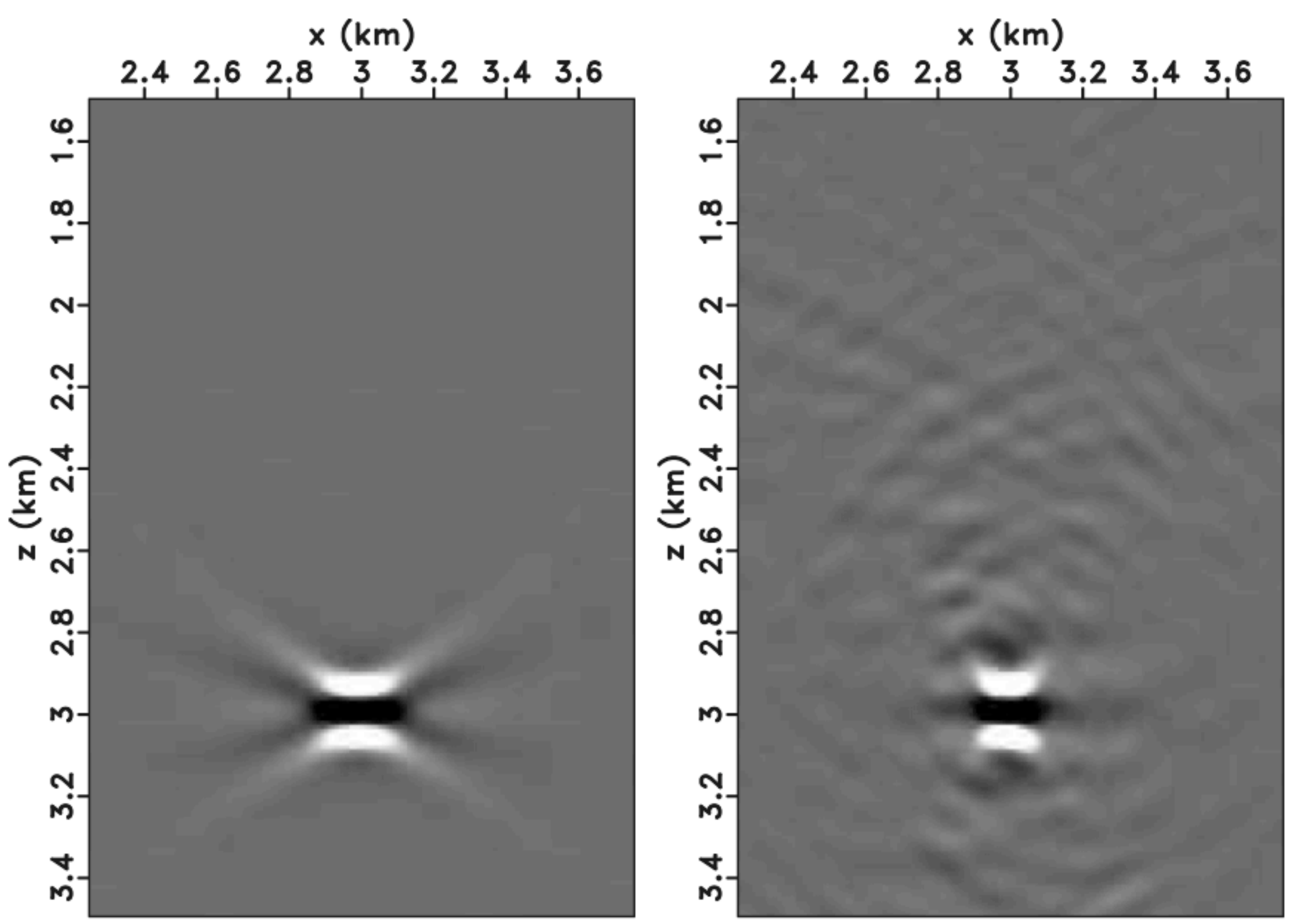
ENCODING EFFECTS

Point scatterer in constant velocity



LINEAR **RANDOM**

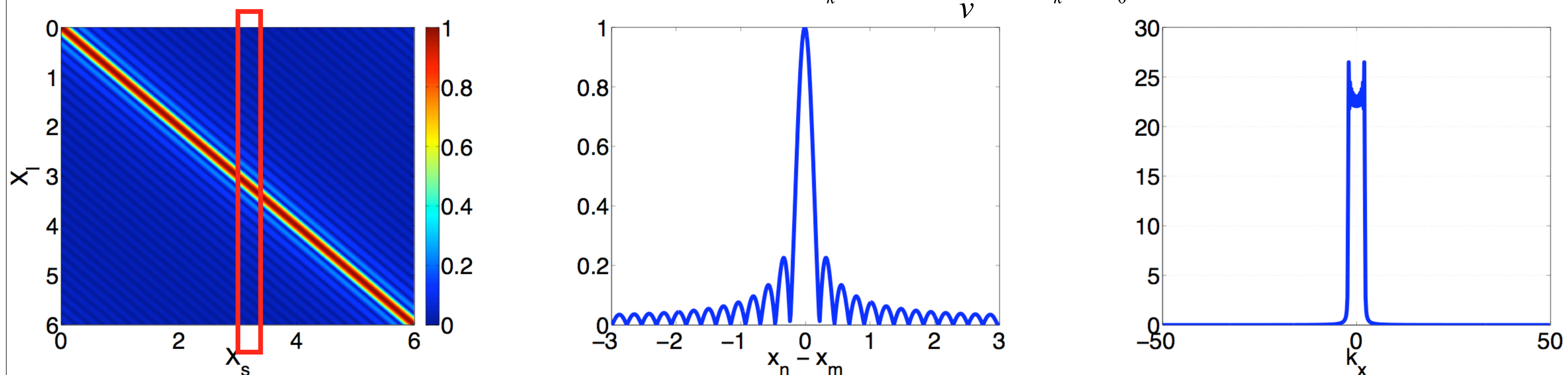
Linear-shot encoding migration (L-SEM) produces low artifacts but recovers a limited number of components of the spatial spectrum of the image.

Random-shot encoding migration (R-SEM) recovers more spatial components but the crossterms in the imaging condition spread over the entire image.

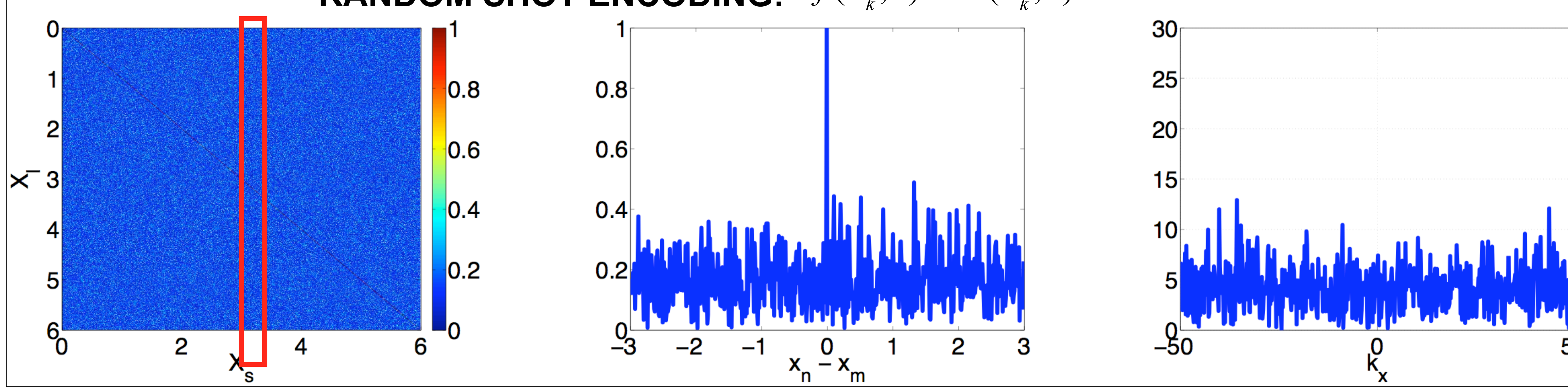
APPROXIMATION TO THE IDENTITY

CROSSTALK MATRIX: $$\sum_{\theta} e^{i\omega(f(\mathbf{x}_l,\theta)-f(\mathbf{x}_k,\theta))}$$

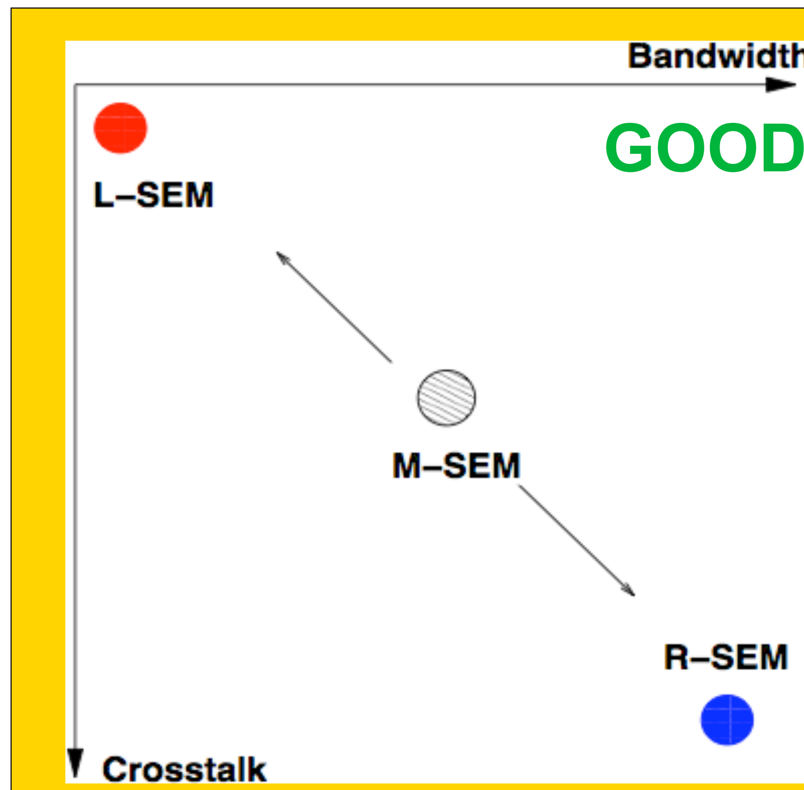
LINEAR SHOT ENCODING: $f(\mathbf{x}_k,\theta)=\frac{\sin(\alpha\theta)}{v}(\mathbf{x}_k-\mathbf{x}_0)$



RANDOM SHOT ENCODING: $f(\mathbf{x}_k,\theta)=\mathcal{U}(\mathbf{x}_k,\theta)$



OBSERVATIONS



- Trade off crosstalk VS spatial bandwidth.
- The equivalence between shot-profile migration and encoding migration can be achieved only as a limit.
- We can combine these two extrema, sharing advantages and drawbacks.

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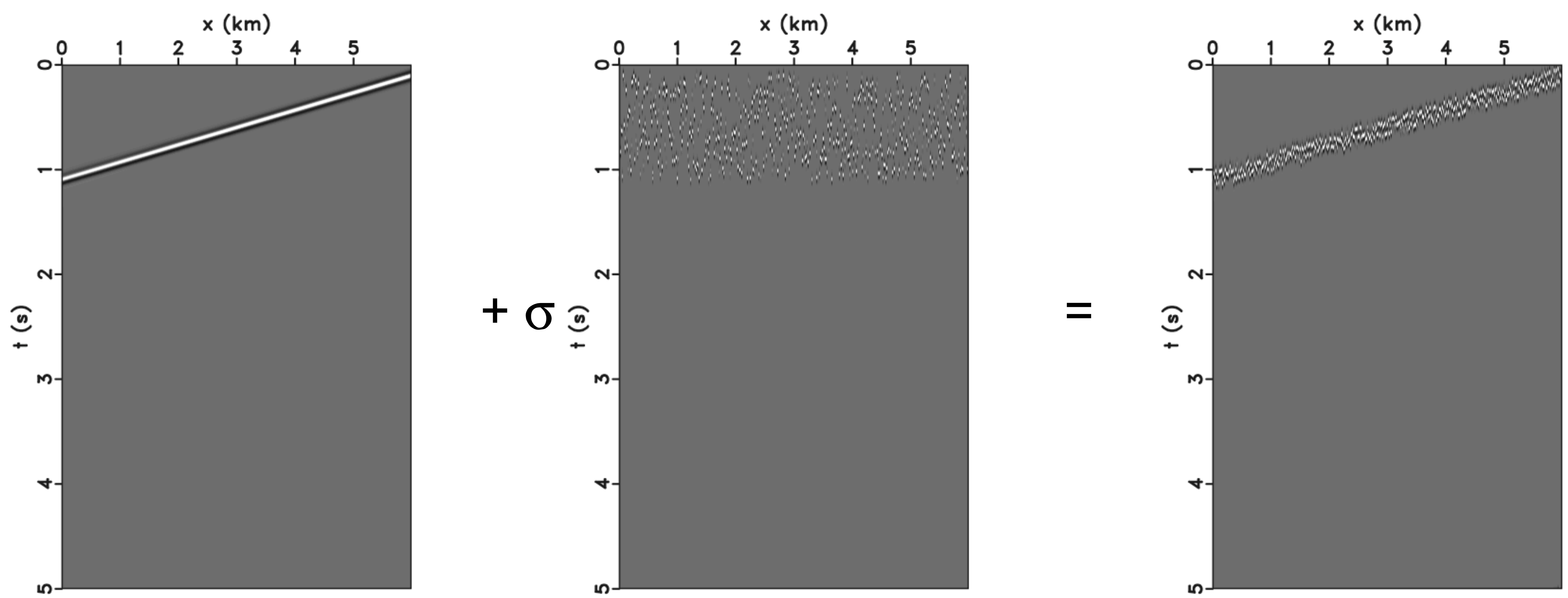
MIXED SHOT ENCODING



LINEAR RANDOM MIXED

MIXED SHOT ENCODING
Plane wave dithering

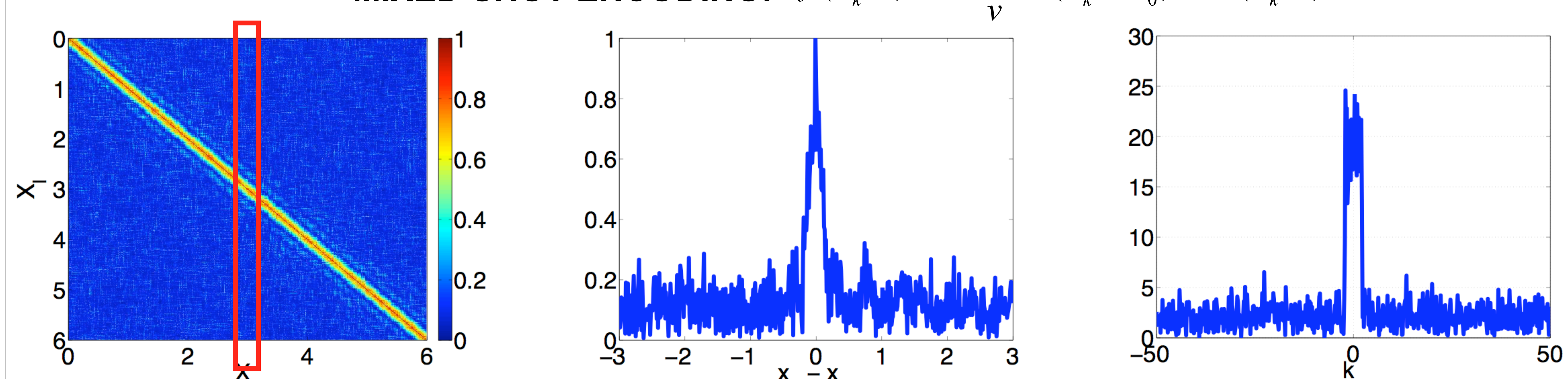
Combining the small crosstalk noise of Linear shot encoding and the high spatial bandwidth of Random shot encoding we can move toward an intermediate scheme that increases the spatial bandwidth without introducing an overwhelming level of crosstalk.

APPROXIMATION TO THE IDENTITY

- A bigger number of plane-wave components is considered with respect to standard L-SEM, even though with smaller weight.
- Reduced crosstalk from different experiments.
- The random-like nature of crosstalk is preserved.

CROSSTALK MATRIX: $$\sum_{\theta} e^{i\omega(f(\mathbf{x}_l,\theta)-f(\mathbf{x}_k,\theta))}$$

MIXED SHOT ENCODING: $$f(\mathbf{x}_k,\theta)=\frac{\sin(\alpha\theta)}{v}(\mathbf{x}_k-\mathbf{x}_0)+\mathcal{U}(\mathbf{x}_k,\theta)$$



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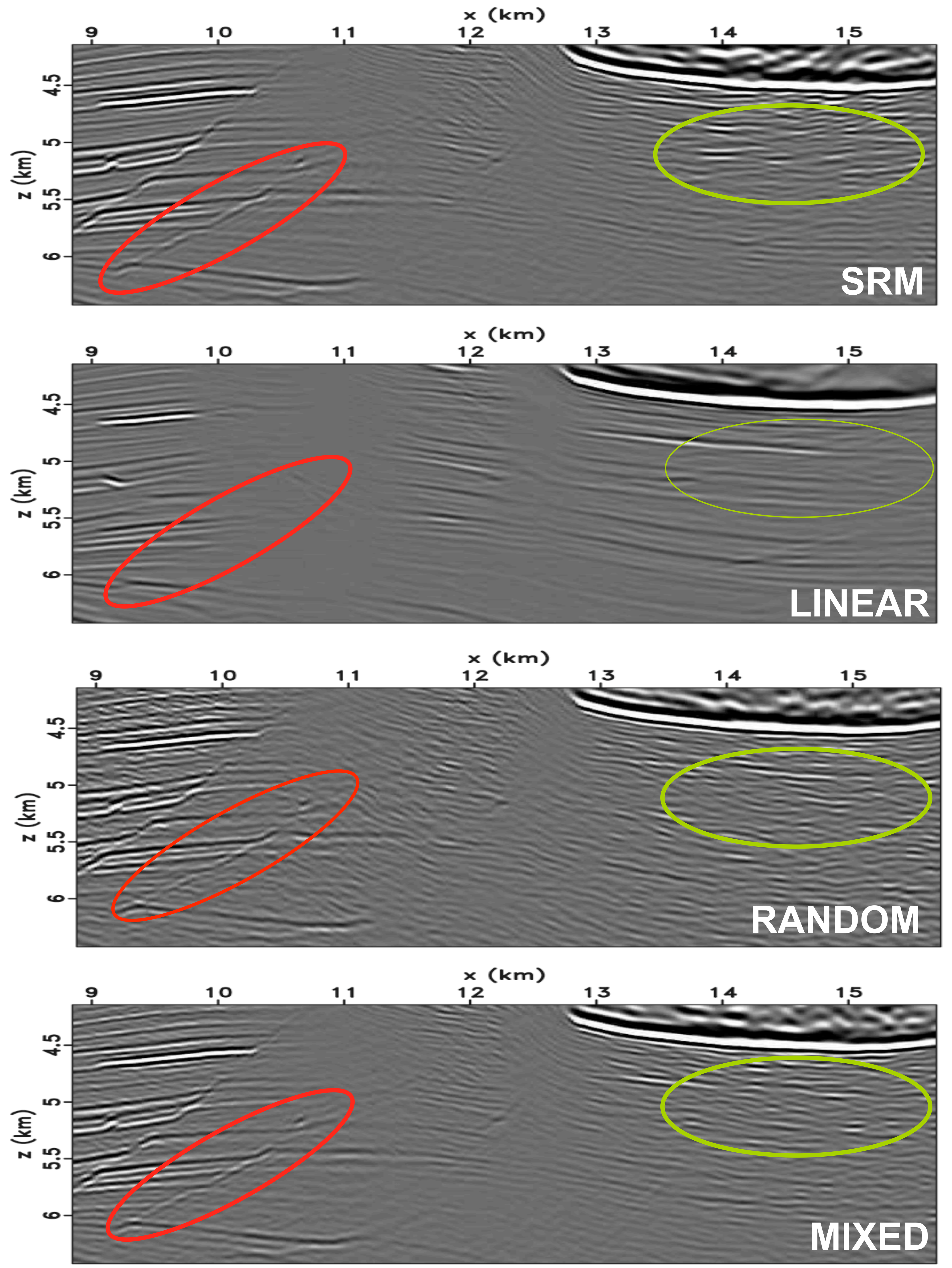
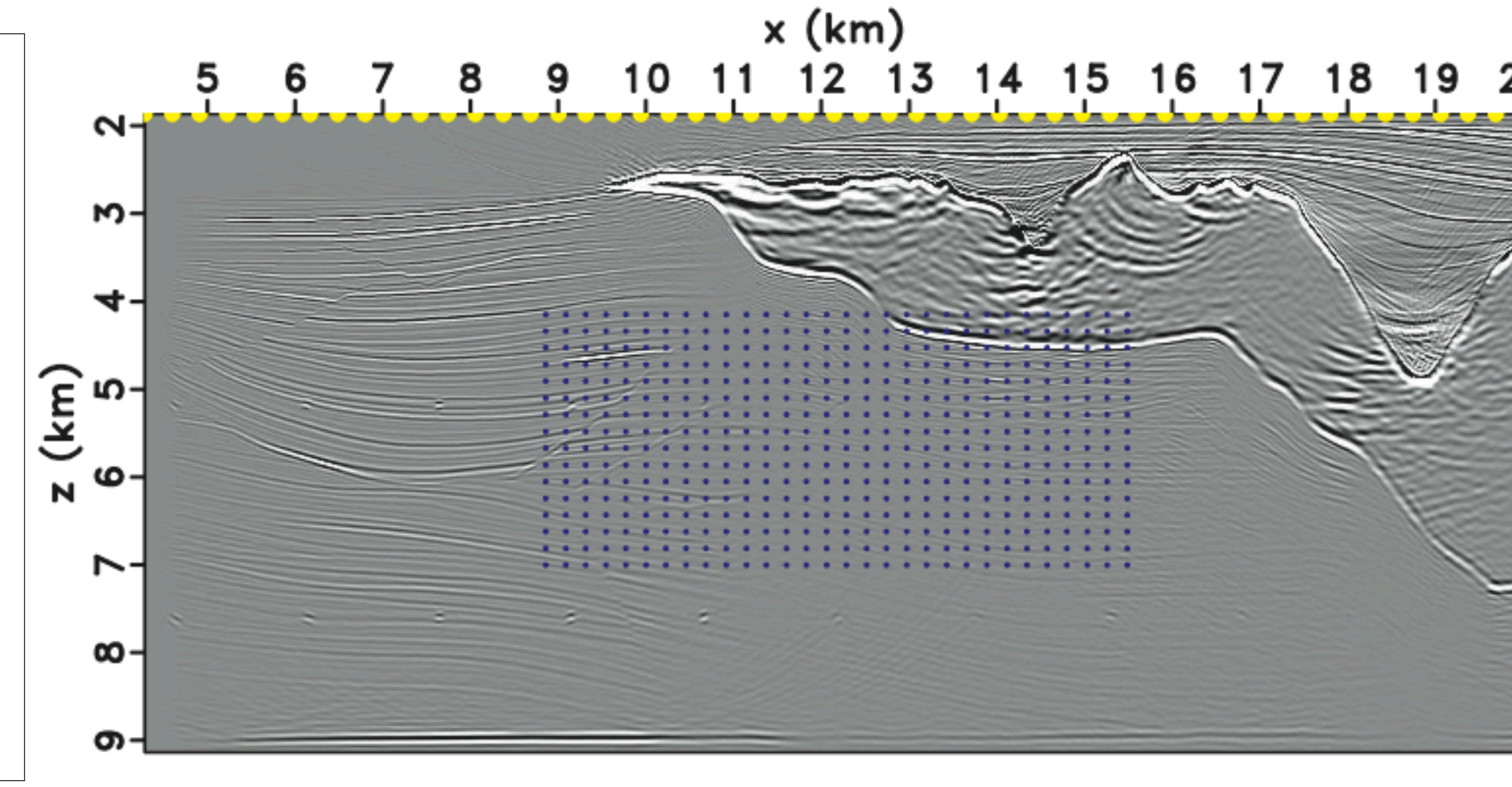
NUMERICAL RESULTS



- SRM: 50 shots
- L-SEM: 50 plane-waves
- R-SEM: 50 random delays realizations
- M-SEM: 50 dithered plane-waves



CONCLUSIONS

- Comparison of Linear and Random Shot Encoding and analysis with respect to crosstalk and spatial bandwidth in the image.
- Combination of L-SEM and R-SEM is effective at increasing spatial bandwidth and controlling crosstalk.
- M-SEM reduces the computational cost for RTM for specified bandwidth and SNR.
- M-SEM achieves spatial resolution comparable to SRM without suffering from undersampling problems.

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