Geometric asian option pricing in general affine stochastic volatility models with jumps

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Main contribution

direct contribution

- explicit evaluation formulae for geometric asian options, both average price and average stike options, in a general affine framework;

indirect contribution

- general financial framework where almost any financial model fits in;
- general evaluation scheme for the fair price of financial options.

Framework

- evaluate the probability that a given option is exercised leads to the fair price of the same option;
- fundamental to know the characteristic function of the undelying;
- let X_t the log-price process and V_t the volatily process, the couple (X_t, V_t) is an affine process if

$$(1) \quad log\mathbb{E}\left[e^{uX_t + wV_t} \middle| X_0, V_0 \right] = \phi(t; u, w) + V_0\psi(t; u, w) + X_0 u \, ;$$

- any affine process is completely determined by two functions, known as functional characteristic

$$F(u, w) := \frac{\partial \phi}{\partial t}(t, u, w)\bigg|_{t=0^+} \, , \quad R(u, w) := \frac{\partial \psi}{\partial t}(t, u, w)\bigg|_{t=0^+} \, ;$$

Framework

- the functions ϕ and ψ in (1) satisfy the following generalized Riccati equations

$$(2) \qquad \begin{cases} \partial_t \phi = F(u, \psi)\,, \\ \phi(0, u, w) = 0\,, \end{cases} \quad \begin{cases} \partial_t \psi = R(u, \psi)\,, \\ \phi(0, u, w) = w\,, \end{cases}$$

- if we know $F(u, w)$ and $R(u, w)$ of the joint process (X_t, V_t), then solving the Riccati equations (2) we have the functions ϕ and ψ and thus the characterstic function of the couple (X_t, V_t);
- the fair price P of an option is reduced to an inverse Laplace transform

$$P = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} F(u) e^{\kappa(u;\phi,\psi)} du\,.$$

Main results

Exploting previous methods they were able to retrieve closed or semi-closed formulae for ϕ and ψ for many stochastic volatily models (possibily with jumps) such as:

- Heston model;
- Bates model;
- Turbo-Bates model;
- Barndorff-Nielson-Shepard model;
- OU time-changed Lévy process;
- CIR time-changed Lévy process.

Comments

- the theory is well motivated by applications;
- interesting setting that allows for good analytical results;
- wide number of models fit into the present setting;
- simple yet non trivial idea;
- the evaluation scheme introduced allows to deal non only with european option but also with more complicated exotic/path-dependent options.

Questions

- functional characteristic;
- numerical evaluation of inverse Laplace transform;