Information Globalization, Risk Sharing, and International Trade

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Abstract

Information frictions are often invoked to explain low levels of international trade beyond those that measured trade frictions (tariffs, transportation costs, etc.) can explain. But to explain why international trade is lower then domestic trade, home firms have to know something that foreigners do not. Without information asymmetry, domestic trade and foreign trade would be inhibited equally. This paper incorporates a simple information asymmetry in a standard, two-country Armington trade model and studies its effect on international risk sharing and trade flows. We find that ameliorating information asymmetry – information globalization – reduces trade and international risk sharing. In other words, asymmetric information frictions behave in the opposite manner as a standard trade cost.

Many researchers have explored the possibility that information frictions account for the low volumes of cross-border trade. Portes and Rey (2005) show that the volume of phone calls between two countries predicts how much they trade. Gould (1994) and Rauch and Trindade (2002) argue that immigrants trade more with their home countries. The argument that information frictions create uncertainty about foreign economies, and that this uncertainty deters risk-averse potential exporters is compelling. In many settings, the effect of an increase in uncertainty is to reduce the certainty-equivalent expected return, which acts like a tax on transactions.

But to explain why firms trade less across borders than within borders, it is essential to consider not just imperfect information, but asymmetric information. If an information friction inhibits foreign, but not domestic trade, then domestic firms must know something about each other that foreign firms do not know. We take the most standard, simple trade model – a two-good, two-country Armington model – and add information asymmetry in the most obvious way: Each country experiences a random endowment shock that its home firms observe perfectly, but foreigners observe imperfectly, with a noisy signal. Each firm

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chooses what fraction of their endowment to export to an international market. The international relative price clears that market, goods are immediately shipped to their destination country, and agents consume. Although the argument that a lack of information deters trade is compelling, in this setting, that argument turns out to be wrong. When home and foreign firms have less precise information about each other, they trade more.

One way to understand how information can inhibit trade is to think about how information undermines the ability to share risk. Suppose two people each hold a ticket to an identical, independent lottery. As long as neither knows the lottery outcome, they can trade half their ticket for half of the other ticket and share risk. But once a piece of information reveals the lottery outcomes, the only possible price for half a ticket is half the realized value. The post-information sale no longer shares risk. Reducing information asymmetry (information globalization) is like revealing the outcome of the lottery. It causes international relative prices to adjust in a way that lowers risk-sharing. Since symmetric, full risk-sharing entails each country exporting half of its endowment, full risk-sharing achieves maximum trade volume. When information globalization inhibits risk-sharing, it also moves the economy away from maximum trade.

So contrary to folk wisdom, countries that know more about each other should trade less. The results highlight the difference between models with a lack of information, which affects home and foreign firms alike, and trade under asymmetric information. More importantly, our findings point to a new research agenda: If a standard model with asymmetric information cannot explain the lack of cross-border trade, are there other frictions that interact with asymmetric information that can? Can richer models reinstate the folk wisdom? While answering that question is beyond the scope of this paper, we lay out the challenges and offer a foundation on top of which those other frictions might be added.

Section 1 sets up our two-country, two-good endowment economy with asymmetric information. Agents value the consumption of a composite good that aggregates the consumption of the two goods with a constant elasticity of substitution. Each country has an aggregate endowment of one of the goods and must trade to obtain some of the other good. The size of a country's aggregate endowment is known to residents of the country, but not to foreigners. Agents have prior beliefs about the aggregate endowment in the other country, observe a noisy signal about the endowment, and update with Bayes' Law. Because they don't know how much the other country has, they are uncertain about the terms of trade for their export good. That causes export goods to have an uncertain return. Agents form beliefs about the terms of trade and then choose a quantity of their good to export. All goods arrive in an international market. The price clears that market. Then each country gets some of the other country's goods in return for their exports. Finally, all agents consume. In short, this is a standard bilateral trade model where the only novel piece is the imperfect information that each country has about the other's endowment.

Incorporating information asymmetry allows us to analyze another realistic feature of international trade: real exchange rate risk. Not knowing what others know means not knowing at what price they will trade. Because home information is symmetric, the return to home sales is known. From an exporter's perspective, the source of the export return risk is irrelevant. The key issue is that the relative price of home and foreign goods is unknown when exports are chosen.

To analyze the effect of smaller or larger information asymmetries, we use a calibrated model to study how varying the precision of the foreign endowment signal affects three objects of interest: trade volume, international risk sharing and expected utility. Section 2 describes our parameter calibration and numerical simulation method.

To illustrate the effect of asymmetric information frictions, section 3 shows results for three economies. In the first economy, called "no information" agents have prior beliefs about the productivity in the other country and receive no additional information about that productivity level before they choose how much to export. In the second economy, called "noisy signals," agents get a signal that conveys some information about what the other country's aggregate productivity will be. This signal allows them to choose exports that covary with foreign exports. In the third economy, called "full information," each country knows exactly what the aggregate productivity in the other country will be. The results show that exports rise by 4% of GDP when the economy moves from full information to no information. Thus information globalization represents a moderate trade friction.

Although the intuition for the result that information inhibits trade can be conveyed simply, the mechanism is subtle and relies crucially on higher-order beliefs (e.g., home agents' beliefs about what foreigners believe about the home economy).[1] To isolate the second-order belief channel, we decompose the first-order condition into a term that depends on second-order beliefs and a residual covariance term. We show numerically that information asymmetry has little effect on the covariance term, but a large effect on the second-order belief term. In addition, we shut off the second-order belief effect by solving a model where only the home country gets an increase in signal precision. In this setting, with second-order beliefs held fixed, an increase in signal precision has almost no effect on the average trade share. We learn that home trade

[1] This feature is similar to other models with asymmetric information, such as Morris and Shin (1998) or Angeletos and La'O (2013).

falls, not because home knows more about the foreign economy, but because home knows that foreigners know more about the home economy. It is really *knowing that others know more* that lowers the fraction of goods exported by both countries.

What we take away from this investigation is not that information globalization is our preferred explanation for any particular puzzle or observed phenomenon in trade. To the contrary, we argue that missing trade is still a puzzle. To explain low observed levels of trade, we may need to look to other frictions, besides asymmetric information about foreign economic conditions.

Related papers There is a spate of recent papers modeling and measuring information frictions in trade. The most closely related is Steinwender (2014), where exporters in one country learn about exogenous market prices in another country. More precise information decreases uncertainty and increases the expected profits, trade volume and welfare. Our paper is similar because agents learn about aggregate economic conditions in another country and then choose exports. But instead of one trader facing an exogenous price, our model features equilibrium two-country trade. The fact that both parties know something the other does not creates non-trivial higher-order beliefs that are essential for our surprising results.

Other papers look at different types of information frictions. In Allen (2013), Petropoulou (2011), Rauch and Watson (2004) and Eaton, Eslava, Krizan, Kugler, and Tybout (2011), producers are uncertain about firm- or match-specific variables such as the location of the best trading partner, the quality of their match or local demand for their specific product. These are undoubtedly important information frictions. But if these frictions inhibit foreign trade more than domestic trade, there must be some country component to them that is known at home, but not abroad. As such, our model complements these theories by filling in that missing piece, the role of uncertainty about a foreign economy.

The effect of information globalization on risk sharing is similar to the effect of allowing international borrowing (Brunnermeier and Sannikov (2014)). Both mechanisms undermine risk-sharing, but ours has the opposite predictions for trade volumes.

In financial markets, information also frequently inhibits risk-sharing. The Hirshleifer (1971) effect arises when information precludes trade in assets whose payoffs are contingent on an outcome revealed by the information. Our effect is distinct because 1) our signals are private to each country, not public and 2) it works through an effect of the quantity exported on the international relative price. Our effect does not change the set of securities traded because no financial securities are traded in this model.

1 Benchmark Model

In order to understand how an information asymmetry affects trade, we write down a simple model with two countries, an endowment economy and an information asymmetry. The first two ingredients constitute a standard equilibrium model of trade. The information asymmetry is that agents in each country know their own country's aggregate endowment, but have imperfect information about the other country's endowment. The key feature of the model is that the relative price of goods is not known at the time when exports are chosen. It could be that there is shipping delay. This could be a demand shock. It could be that some fixed costs must be incurred to export before export contracts are written. But what this captures is the idea that exporting is risky. It is the essence of an information asymmetry: Something is known when one sells domestically that is unknown when selling abroad. Our question is what happens when the unknowns in exporting become more known.

This is a repeated static model with the following economic environment.

Preferences: There are 2 countries and a continuum of agents within each country. We denote individual variables with lower case and aggregates with upper case. The problems are symmetric across countries, so we only describe the problem for the domestic country. Agents like to consume two goods, x and y and their utility flow each period is $\mathbb{E}[c]$, where the consumption composite good is $c = (c_x^\theta + c_y^\theta)^{1/\theta}$, $\theta \in (0, 1)$. The elasticity of substitution across goods is $\nu = \frac{1}{1-\theta}$.

Endowments: Each agent in the domestic country has an idiosyncratic endowment of z_x units of good x, where $\ln z_x \sim \mathcal{N}(\mu_x, \sigma_x^2)$. Each agent in the foreign country has an idiosyncratic endowment z_y units of good y, where $\ln z_y \sim \mathcal{N}(\mu_y, \sigma_y^2)$. The means of these distributions are themselves independent random variables: $\mu_x \sim \mathcal{N}(m_x, s_x^2)$ and $\mu_y \sim \mathcal{N}(m_y, s_y^2)$. Because they are average endowment around which all individual endowments are distributed, μ_x and μ_y are aggregate shocks. If we let Φ denote the cumulative density (cdf) of a standard normal variable, then the conditional cdf's of z_x and z_y are $F(ln(z_x)|\mu_x) = \Phi((ln(z_x) - \mu_x)/\sigma_x)$ and $F(ln(z_y)|\mu_y) = \Phi((ln(z_y) - \mu_y)/\sigma_y)$.

Notice that there are no financial markets open here that agents could use to hedge their productivity shocks or those of the foreign country. The equilibrium movements in international prices are an important source of risk-sharing Cole and Obstfeld (1991). One of the key insights of this model will be that the information countries have about each others' productivity changes how they share risk. If we introduce another source of risk sharing, we would obscure that effect.

Information: At the beginning of the period, firms in country x observe their own endowment z_x and the mean of their country's endowment μ_x. Likewise, agents in country y observe z_y and μ_y. Furthermore, we assume that both countries know the distribution from which mean productivities are drawn and the cross-sectional variance of firm outcomes. In other words, $m_x, m_y, s_x, s_y, \sigma_x$ and σ_y are common knowledge.

In addition, agents in country x observe a signal about the y-endowment $\tilde{m}_y = \mu_y + \eta_y$ where $\eta_y \sim N(0, \tilde{s}_y^2)$. Similarly, agents in country y observe a signal about the x-endowment $\tilde{m}_x = \mu_x + \eta_x$ where $\eta_x \sim N(0, \tilde{s}_x^2)$. Let $\mathcal{I}_x$ denote the information set of an agent in the home country and let $\mathcal{I}_y$ denote the information set of a foreign agent. All home country choices will be a function of the three random variables in the home agents' information set: $\mathcal{I}_x = \{z_x, \mu_x, \tilde{m}_y\}$. Likewise, foreign choices depend on $\mathcal{I}_y = \{z_y, \mu_y, \tilde{m}_x\}$.

Bayesian updating Agents in each country combine their prior knowledge of the distribution of the others' productivity and their signal to form posterior beliefs. By Bayes' law, the posterior probability distribution is normal with mean $\hat{m}$ and variance $\hat{s}^2$ given by

$$F(\mu_y|\mathcal{I}_x) = \Phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right) \quad \text{where} \quad \hat{m}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2} \tilde{m}_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{s}_y^2 = \frac{1}{s_y^{-2} + \tilde{s}_y^{-2}} \tag{1}$$

$$F(\mu_x|\mathcal{I}_y) = \Phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \quad \text{where} \quad \hat{m}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \tilde{m}_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{s}_x^2 = \frac{1}{s_x^{-2} + \tilde{s}_x^{-2}} \tag{2}$$

In order to forecast prices, agents will need to forecast the other country's export choices. Since others' export choices depend on their forecasts of one's own endowment, actions will also depend on beliefs about the beliefs of others. According to Bayes' law, these second-order beliefs are

$$F(\hat{m}_x|\mathcal{I}_x) = \Phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) \quad \text{where} \quad \hat{\hat{m}}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \mu_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{\hat{s}}_x^2 = \frac{\tilde{s}_x^{-2}}{(s_x^{-2} + \tilde{s}_x^{-2})^2} \tag{3}$$

$$F(\hat{m}_y|\mathcal{I}_y) = \Phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) \quad \text{where} \quad \hat{\hat{m}}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2} \mu_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{\hat{s}}_y^2 = \frac{\tilde{s}_y^{-2}}{(s_y^{-2} + \tilde{s}_y^{-2})^2} \tag{4}$$

Note that there is a one-to-one mapping between signals $\tilde{m}$ and posterior beliefs $\hat{m}$. Instead of using signals as a state variable, we will use posterior beliefs, for simplicity and without loss of generality. Thus, we write $\mathcal{I}_x = \{z_x, \mu_x, \hat{m}_y\}$ and $\mathcal{I}_y = \{z_y, \mu_y, \hat{m}_x\}$.[2]

Price and budget set: Each agent chooses how much to export, t_x or t_y. The relative price q is the number of units of x good required to purchase one unit of y good on the international market. In

[2]In fact, all higher orders of beliefs can matter for export choices. But, because there are only two shocks observed by each country, the first two orders of beliefs are sufficient to characterize the entire hierarchy.

equilibrium, this price clears the market. An agent who exports t_x units of x goods receives $\frac{t_x}{q}$ units of y, for immediate consumption (there is no secondary resale market). We restrict exports and consumption to be non-negative. Therefore the country x budget set is:

$$c_x \quad \epsilon \quad [0, z_x - t_x] \tag{5}$$

$$c_y \quad \epsilon \quad \left[0, \frac{t_x}{q}\right] \tag{6}$$

and the country y budget set is:

$$c_x \quad \epsilon \quad [0, t_y q] \tag{7}$$

$$c_y \quad \epsilon \quad [0, z_y - t_y] \tag{8}$$

When the export decision is made, firms do not know the price q. It is a random variable whose realization depends on their own (known) aggregate state, on the foreign (unknown) aggregate state, on home beliefs about the foreign state and on foreign beliefs about the home state.

Equilibrium An equilibrium is given by export policy functions for domestic $t_x(z_x, \mu_x, \hat{m}_y)$ and foreign $t_y(z_y, \mu_y, \hat{m}_x)$ countries, aggregate exports $T_x(\mu_x, \hat{m}_y), T_y(\mu_y, \hat{m}_x)$, perceived price functions $\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ for each country and an actual price function $q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ such that:

1. Given perceived price functions $\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$, export policies maximize expected consumption of every firm in each country. Substituting the budget sets (5) to (8) into $E[(c_x^\theta + c_y^\theta)^{1/\theta}]$, we can write this problem as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max E\left[\left((z_x - t_x)^\theta + \left(\frac{t_x}{\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}\right)^\theta\right)^{1/\theta} \middle| \mathcal{I}_x\right] \tag{9}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max E\left[\left((t_y \tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y))^\theta + (z_y - t_y)^\theta\right)^{1/\theta} \middle| \mathcal{I}_y\right] \tag{10}$$

Using the conditional densities (1), (2), (4) and (3), we can compute expectations as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \int\int \left((z_x - t_x)^\theta + \left(\frac{t_x}{\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}\right)^\theta\right)^{1/\theta} dF(\mu_y|\mathcal{I}_x) dF(\hat{m}_x|\mathcal{I}_x) \tag{11}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \int\int \Big((t_y \tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y))^\theta + (z_y - t_y)^\theta \Big)^{1/\theta} dF(\mu_x|\mathcal{I}_y) dF(\hat{m}_y|\mathcal{I}_y) \quad (12)$$

2. The relative price q clears the international market. Since every unit of x-good exported must be sold and paid for with y exports, and conversely, every unit of y exports must be sold and paid for with x exports, the only price that clears the international market is the ratio of aggregate exports:

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{T_x(\mu_x, \hat{m}_y)}{T_y(\mu_y, \hat{m}_x)} \quad (13)$$

where aggregate exports are

$$T_x(\mu_x, \hat{m}_y) = \int t_x(z_x, \mu_x, \hat{m}_y) dF(z_x|\mu_x) \quad (14)$$

$$T_y(\mu_y, \hat{m}_x) = \int t_y(z_y, \mu_y, \hat{m}_x) dF(z_y|\mu_y). \quad (15)$$

3. The perceived and actual price functions coincide:

$$\tilde{q}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) \qquad \forall (\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$$

1.1 Characterization of Equilibrium

Optimal exports The next result shows that within each country, every firm exports the same fraction of their endowment. But the size of that fraction depends on the average aggregate endowment at home, their beliefs about the average endowment abroad and their beliefs about what the other country knows.

Lemma 1 *Export policies that are proportional to firm productivity, $t(z, \mu_x, \hat{m}_y) = z\Psi(\mu_x, \hat{m}_y)$ for home firms and $t(z, \mu_x, \hat{m}_y) = z\Gamma(\mu_x, \hat{m}_y)$ for foreign firms, are solutions to the problems in (11) and (12).*

Thus, aggregate domestic and foreign exports are

$$T_x(\mu_x, \hat{m}_y) = \int z_x \Psi(\mu_x, \hat{m}_y) dF(z_x|\mu_x) = e^{\mu_x + \frac{1}{2}\sigma_x^2} \Psi(\mu_x, \hat{m}_y) \quad (16)$$

$$T_y(\mu_y, \hat{m}_x) = \int z_y \Gamma(\mu_y, \hat{m}_x) dF(z_y|\mu_y) = e^{\mu_y + \frac{1}{2}\sigma_y^2} \Gamma(\mu_y, \hat{m}_x), \quad (17)$$

where the last equality holds because z_x and z_y are lognormal.

Equilibrium relative price Equation (13) tells us that the world market clearing price q is ratio of aggregate x exports to aggregate y exports. Since this ratio arises often, it is useful to define the ratio of the average firm fundamentals $f(\mu_x, \mu_y) \equiv E[z_x]/E[z_y] = \exp[(\mu_x - \mu_y) + 1/2(\sigma_x^2 - \sigma_y^2)]$. For example, if countries endowment distributions were symmetric, then $m_x = m_y$ and $\sigma_x = \sigma_y$, which would imply that f is a lognormal variable, with median value of 1. Regardless of symmetry, realized equilibrium price is $T_x(\mu_x, \hat{m}_y)/T_y(\mu_y, \hat{m}_x)$, which can be rewritten as

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = f(\mu_x, \mu_y)\frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)} \tag{18}$$

Fixed point problems. Equilibrium condition 3 imposes rational expectations, which requires that the agents' beliefs about relative prices coincide with their probability distributions under the true law of motion of the economy. This consistency takes the form of a fixed point problem. Substituting (18) in for q in the expressions for Ψ and Γ reveals that the expressions that describe the fraction of output that firms choose to export take the form $\Psi = g_1(\mathcal{I}_x, \Psi, \Gamma)$ and $\Gamma = g_2(\mathcal{I}_y, \Psi, \Gamma)$. (See appendix for derivation.) In other words, the optimal fraction of exports depends on what firms believe will be the fraction of output that others will export. When the optimal export policy and beliefs about others' export policies coincide, that is an equilibrium. Thus, the equilibrium is described by functions Ψ and Γ that solve equations g_1 and g_2.

2 Simulation strategy and parameter choice

We solve and simulate different versions of the model: (i) full information, where both countries know their own productivity and the other country's productivity exactly ($\hat{m}_x = \mu_x$ and $\hat{m}_y = \mu_y$); (ii) no information, where each country knows its own productivity and since neither country gets any signal, their beliefs about the other country's productivity are given by the unconditional distribution ($\hat{m}_x = m_x$ and $\hat{m}_y = m_y$); and finally (iii) noisy signals, where each country knows its own productivity and receives signals about the other country's productivity. In this last case we will solve and simulate the model for various levels of signal precision, always keeping the precision symmetric across countries.

For each information model, we solve the fixed point problem by iterating on the export policy functions Ψ and Γ which are approximated using linear splines. For each country we define grids for their two states: aggregate productivity and posterior mean of foreign productivity. We also define grids for foreign productivity and second order beliefs that countries use to evaluate their perceived price function. Expectations with respect to foreign productivity and second order beliefs are computed using Gaussian quadrature.

Once we have solved the fixed point problem, we simulate the repeated economy for T=100,000 periods and compute average statistics across simulations. We first draw a series of aggregate productivities and fix it across information models. Then for each information model we generate posterior means of foreign productivity by drawing signals centered at the realized aggregate productivities and with precision determined by the model at hand. See Appendix B for more details.

Parameters Table 1 describes the parameters we use to simulate the models and how they are chosen or calibrated. The unconditional mean of aggregate productivity m and the dispersion of firm productivity σ only appear in the solution as differences between home and foreign values. Only the differences $m_x - m_y$ or $\sigma_x - \sigma_y$ affect the export shares or the relative prices. The absolute quantities produced and traded do depend on these parameters, but not any relative quantities or prices. Since the question of fundamental asymmetry between two countries and its effect on trade is not the focus of this paper, we assume that there is no difference in the distribution of fundamentals by imposing symmetry: $m_x = m_y$ and $\sigma_x = \sigma_y$. We normalize $m = 0$ and $\sigma = \sqrt{2}$ so that the mean of aggregate (log) endowment is unity: $\mathbb{E}[\log f_x] = m_x + \frac{1}{2}\sigma_x^2 = 1$. To set volatility of the aggregate productivity shocks, we note that it is the ratio of signal to aggregate dispersion what matters for outcomes $\frac{\tilde{s}}{s}$. Since we will vary signal precision across simulations, we assume symmetry and normalize the volatility of aggregate shocks to unity ($s_x = s_y = 1$). We set θ equal to 0.75 that implies an elasticity of substitution across the two consumption goods of $\nu = \frac{1}{1-\theta} = 4$, which is a standard value in the trade literature. Finally, the precision of the signals that each country observes about the other's productivity $\tilde{s}^{-2}$ are varied across simulations in the range $[0, \infty]$.

Table 1: **Summary of Model Parameters**

Parameter	$m_x = m_y$	$\sigma_x = \sigma_y$	$s_x = s_y$	θ
Value	0	$\sqrt{2}$	1	0.75

3 Results

Our main findings are that information globalization, meaning more precise signals about foreign productivity, can reduce trade, increase utility, and increase risk. To explain the mechanics of how asymmetric information plays out in a two-country equilibrium model, we first need to explore how information improves utility by increasing trade coordination. Next, we explain how information globalization and the resulting trade coordination make the terms-of-trade a less effective hedge against productivity risk. Finally, we come back to the reason why information globalization reduces average trade volume. The decline in trade arises because of how additional information changes second-order beliefs, which are reflected in the terms-of-trade. Knowing that foreigners are better informed about the home economy causes home firms to slash exports in low-productivity states and increase exports slightly in high-productivity states. On average, knowing that others know more lowers the fraction of goods exported by both countries.

In the subsections that follow, we illustrate each of these results in our calibrated model, propose explanations for why each effect arises, and look for additional evidence in the model and simulations to support our explanation.

3.1 Export Coordination

Greater precision of information about foreign economic conditions (information globalization) enbales countries to achieve more coordinated trade. The motive for coordination comes from movements in relative prices. If the foreign country exports a lot, the relative price of the foreign goods must be low to clear the international market. That means that the relative price of home goods is high. High returns to exporting make the home country want to export more in these times when foreign exports are also high. When each country knows more about the other's productivity, they can anticipate better when the other country will export more. Thus, Result 1 is that information enables better coordination.

Result 1 ***More Precise Information Enables Better Trade Coordination.*** *In a neighborhood around perfect information, increasing the precision of both countries' signals* $\tilde{s}^{-2}$ *reduces var*$(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x))$.

To understand Result 1 and the origin of the coordination motive, it is helpful to examine the export first order condition. By replacing the export volume with the endowment times export share, $t_x = z_x\Psi$, we can re-express the home export choice problem (9) as

$$\max_{\Psi} z_x \mathbb{E}\left[\left((1-\Psi)^{\theta} + \left(\frac{\Psi}{q}\right)^{\theta}\right)^{1/\theta} \Big| (\mu_x, \hat{m}_y)\right]$$

Taking a first-order condition with respect to Ψ yields the following solution

$$\mathbb{E}\left[\left(\frac{C}{1-\Psi(\mu_x,\hat{m}_y)}\right)^{1-\theta}\Big|(\mu_x,\hat{m}_y)\right] = \mathbb{E}\left[\left(\frac{C}{\Psi(\mu_x,\hat{m}_y)}\right)^{1-\theta}\frac{1}{q^{\theta}}\Big|(\mu_x,\hat{m}_y)\right]$$

which can be re-arranged as

$$\underbrace{\left(\frac{\Psi(\mu_x,\hat{m}_y)}{1-\Psi(\mu_x,\hat{m}_y)}\right)^{1-\theta}}_{(\text{Trade Share})^{1-\theta}} = \underbrace{\mathbb{E}\left[q^{-\theta}\Big|(\mu_x,\hat{m}_y)\right]}_{I} + \underbrace{\frac{\mathbb{C}\left[C^{1-\theta},q^{-\theta}\Big|(\mu_x,\hat{m}_y)\right]}{\mathbb{E}\left[C^{1-\theta}\Big|(\mu_x,\hat{m}_y)\right]}}_{\frac{II}{III}} \tag{19}$$

Since we assumed that $\theta < 1$, the left side of the equation is increasing in the export share Ψ. Thus, anything that increases the right side will increase the equilibrium trade share. The right side of the equation is the sum of two terms, labeled I and II/III. The first term is the expected relative price, raised to the power $-\theta$. The second term (II/III) captures the covariance between consumption and the relative price.

The expected level of foreign exports enters this condition through the expected relative price q. Recall that the relative price is the ratio of home to foreign exports: $q = T_x(\mu_x,\hat{m}_y)/T_y(\mu_y,\hat{m}_x)$. Consider an experiment where we increase the mean of foreign exports T_y, without changing its variance or covariance properties. Since $\partial q/\partial T_y < 0$, the relative price falls, but $q^{-\theta}$ rises. So, term I increases. Since we held covariances fixed, term II does not change. Expected home consumption C may change. But for a sufficiently small term II, this effect will be small, and the optimal home trade share will increase in foreign trade. That positive partial derivative is the coordination motive in trade.

Figure 1 shows that the effect on export correlation can be large. Raising signal precision – information globalization – increases export correlation from zero to almost 1 for perfect information.

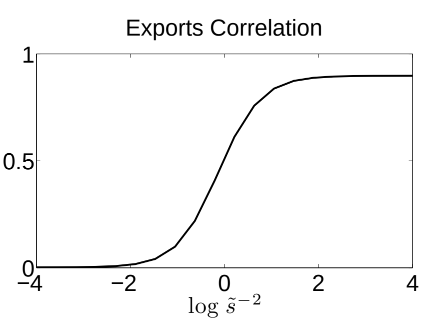

Figure 1: **Information globalization increases export correlation.** Horizontal axis is log signal precision. Vertical axis is $corr(T_x, T_y)$.

3.2 Risk-Sharing

As noted by Cole and Obstfeld (1991), equilibrium movements in the terms of trade are a good hedge for a country's production risk. When a country's productivity is high, the country will export lots, but the benefits of high productivity will be partially offset by the fact that the price of those abundant exports will be low. Conversely, when productivity is low and exports are low, the high relative price of the scarce export good will help to increase utility. The risk-sharing mechanism only works if prices are highly flexible. If the relative price doesn't vary much, countries cannot share risk. Result 2 shows that information globalization and the resulting trade coordination undermines this risk-sharing mechanism by making relative price less flexible.

Result 2 ***Coordinated Trade Reduces Price Variance.*** *Take the home country's export policy as given by fixing the distribution of* T_x*. If foreign firms coordinate more (*$var(T_y \perp T_x)$ *falls), then* $var(1/q)$ *decreases.*

To understand the logic of Result 2, recall that the relative price is the ratio of home to foreign exports: $q = T_x(\mu_x, \hat{m}_y)/T_y(\mu_y, \hat{m}_x)$. If exports are uncoordinated, then the ratio of home exports to foreign exports fluctuates greatly and the relative price is volatile. When information globalization makes exports coordinated, as explained in the previous section, $T_x(\mu_x, \hat{m}_y) \approx T_y(\mu_y, \hat{m}_x)$, the ratio of home to foreign exports is more stable, and the relative price fluctuates less. If the relative price does not move, it cannot share risk.

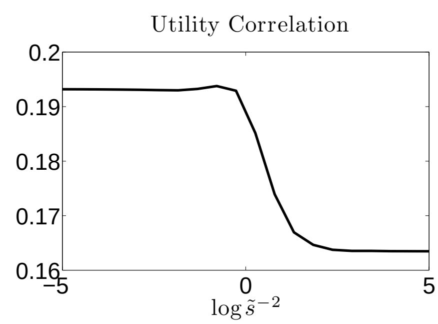

Figure 2: Information globalization inhibits risk-sharing. The horizonal axis is the log precision $\log(\tilde{s}^{-2})$ of the signal that each country gets about the foreign aggregate endowment. Vertical axis is $corr(E[c|\mathcal{I}_x], E[c^*|\mathcal{I}_y])$.

One metric of risk-sharing in a model is the correlation of utilities of each country's representative agent. When risk is shared, a productivity shock that hits one country is transmitted to the other country, resulting in utility correlation. Figure 2 shows that when two countries have more precise information about each others' output, they share less risk. As signal precision increases, utility correlation decreases (in most of the space). In other words, information globalization inhibits risk sharing.

The mechanism by which information worsens risk-sharing is that it makes the relative international price less flexible. Define an international price index as $Q \equiv \left(1 + q^{\theta/(1-\theta)}\right)^{(1-\theta)/\theta}$. This is the standard CES price aggregator. Since x is a numeraire good with price 1, the relative price of the home good to the composite consumption good is $1/Q$. Figure 3 shows that when information is noisy, the relative price looks like a very flexible price, it absorbs much of the risk of changes in productivity, and therefore is quite volatile. As information becomes more precise, the resulting prices look more like sticky prices, in that they move less, absorb less risk and cause real quantities to fluctuate more. Again, the effect of information globalization is the opposite of a decline in trade costs.

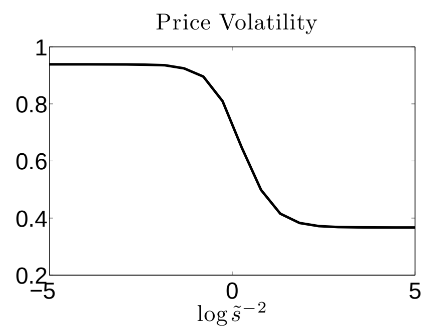

Figure 3: Information globalization makes international prices less flexible.

If foreign firms anticipate that home productivity is high, they expect a higher relative price for their goods and export more. By exporting more, the foreign country mitigates the decline in the home good's price, which encourages the home country to export even more and further increases home utility. Conversely, when foreign firms anticipate low home exports, they expect a low price for foreign goods and thus export less, which offsets some of the rise in the relative price of home goods. Because better information allows each country to better anticipate the other's exports (coordinate trade), it makes the relative price less sensitive to each country's productivity. That insensitivity makes the price a less effect risk-sharing device.

A finance comparison The idea that information inhibits risk-sharing is similar in spirit to the Hirshleifer (1971) effect in asset pricing. In that setting, when traders get information about the payoff of a risky asset before they trade, it reduces expected utility because it reduces the set of outcomes agents can contract on. Gamblers cannot bet on known outcomes. Our trade model differs in a few respects. First, the nature of the information is different. Our information is about the relative abundance of two goods, not the true value of a risky asset. Second, this model has Pareto gains from trade. Financial markets are typically zero-sum games. Third, we don't let firms write risk-sharing contracts. The relative price is the only mechanism for sharing risk. The key similarity to Hirschleifer is that when the price incorporates more information, it shares less risk between the trading counter-parties.

3.3 Trade Share and Volume

The question posed at the start of the paper is whether asymmetric information frictions impede international trade. In this subsection, we demonstrate that information globalization – a reduction in asymmetric information – impedes trade. We use the previous results on trade coordination and risk sharing to explain this result. Finally, we demonstrate the key role of second-order beliefs. We conclude that what deters exporting is not better information for one's own country. Rather, it is knowing that others know more that is a barrier to trade.

We begin by describing our measure of trade volume. To normalize the volume of trade, we consider the ratio of exports to domestic consumption of the home good as our measure of the volume of trade. We call this trade share:

$$\text{Trade Share}_x \equiv \frac{t_x}{c_x}$$

It is defined analogously for country y. The previous section demonstrated why more information increases trade coordination and lowers price variance. Result 3 shows that this lower price variance reduces the average trade share.

Result 3 ***Lower Price Variance Reduces Trade*** *Holding fixed* $cov(C^{1-\theta}, q^{\theta})/E[C^{1-\theta}]$, *a mean-preserving decrease in* $var(1/q)$ *increases* $\Psi/(1-\Psi)$.

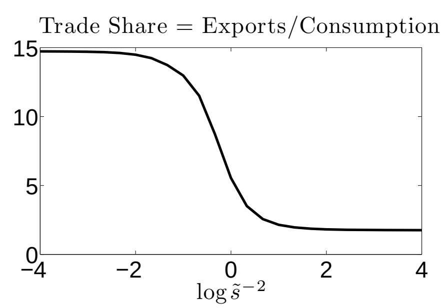

Figure 4: Information globalization reduces the average trade share.

Putting these three pieces of the argument together, our main result is that information globalization decreases the trade share. Put differently, trade share is decreasing in signal precision, for a large range of parameters.[3] Figure 4 plots the average trade share, as signal precision increases. The same moments for the foreign country exhibit the same patterns.

Recall that the optimal export choice is given by the first order condition (20), which is the sum of two terms. The first term (I) is the expected relative price, raised to the power $-\theta$. The second term (II/III) captures the covariance between consumption and the relative price. Figure 5 shows that most of the effect on export shares comes from the first, relative price term. The left and middle panels plot terms I and II

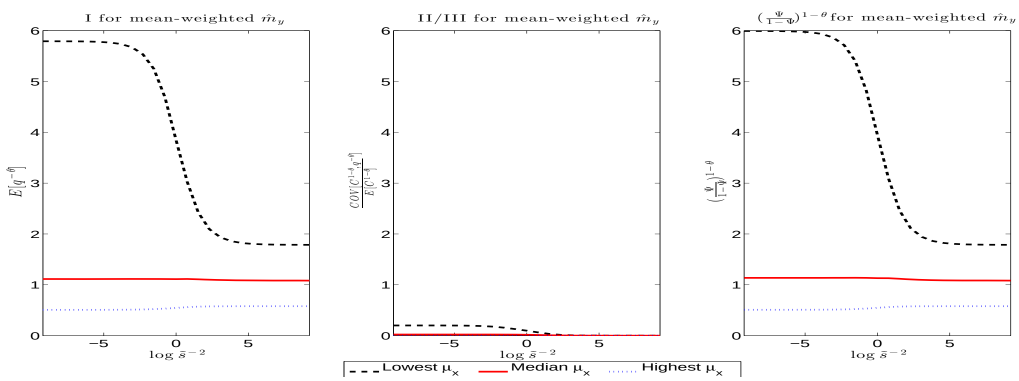

Figure 5: Terms from the first-order condition for different levels of information precision $\tilde{s}^{-2}$. Most of the decline in trade share comes from term I.

separately, as signal precision for both countries rises. The right panel plots their sum. We see that when signal precision increases (information globalization rises), trade share falls, almost entirely because of term I ($E[q^{-\theta}]$), and only in low endowment (low μ_x) states.

These results tell us where to look for the source of our low-trade result: Information globalization lowers $E[q^{-\theta}]$ in states where the home endowment is low. Raising the relative price q to a negative power is a convex function of q. It makes $q^{-\theta}$ high when the relative price of home goods is high. Raising the price to an exponent between $(-1, 1)$ weights low q's (high home good prices) more than high q's (low home good prices) in the expectation. In other words, this makes the trade share Ψ more sensitive to changes in relative price when the home relative price is high than when it is low. Figure 6 illustrates this effect.

The left panel illustrates the price rigidity effect that inhibited risk sharing in Section 3.2. As information becomes more precise, the relative price responds less to endowment shocks. This is represented by a flatter price function in the high-information regime. The right panel plots the same two curves from the left panel,

[3] If we instead define the trade share as exports over endowment, the results do not change.

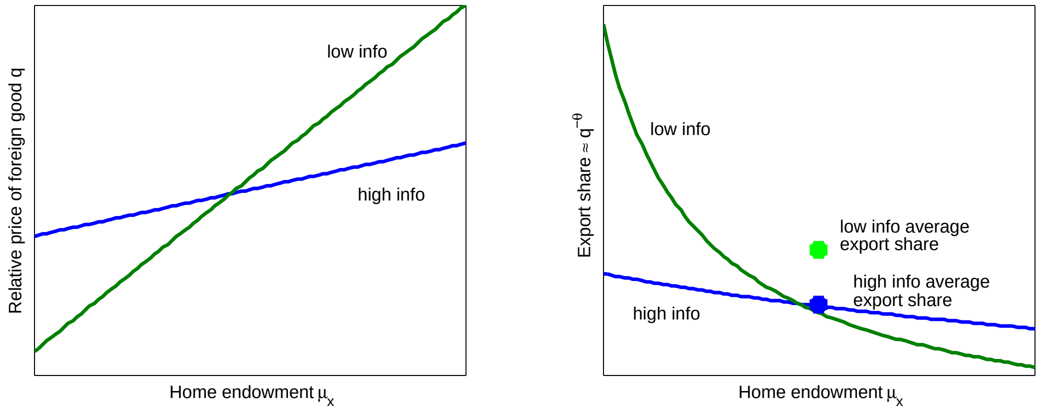

Figure 6: **Why does information inhibit trade?** Average export share falls because information flattens the price function (risk-sharing effect) and exports are convex function of that relative price. Relative price (left panel, plotted as linear for illustration) and export share (right panel, a convex function of relative price). Dots indicate the average value of each curve.

but raised to the power $-\theta$. The lines become downward sloping, because a high q represents a lower relative price for home goods, which induces home to export a smaller share of their endowment. Crucially, these curves are convex. In the high-information regime, a flatter relative price curve, raised to the power $-\theta$, results in a convex function that is much lower in low states and slightly higher in high states. It's average level (the large dot) is lower than for the convex function of the more steeply sloped line (the low-information relative price). These convex functions are the export share in various home endowment states. The lower average level in the high-information economy represents a lower average share of exports.

When the consumption aggregator is Cobb-Douglas ($\theta \to 0$), this mechanism breaks down. The export share becomes constant because $q^{-\theta} = 1$. Thus, trade does not change with information precision.

The economics behind the result: second-order beliefs When global information is precise, export shares fall because of how information changes one country's beliefs about what the other country believes (second-order beliefs). When home output is low, the home good is scarce and thus the relative price of home goods is high, making the relative price of foreign goods low. If the foreign country anticipates this low relative price, they'll export less. This reduction in foreign exports makes home goods relatively more abundant and thus offsets some of the increase in the relative price of home goods. If the home firms anticipate that foreigners will forsee low home output, export less and mitigate the increase in home relative price, the home firms will choose a lower export share. This logic is represented in right panel of Figure 6. Notice the large difference between the export share with low information and with high information in low

μ_x states.

Of course, this mechanism should produce the opposite effect when home output is high. Home relative prices will be low, foreign high. Anticipating this, foreigners export more, which offsets the decline in home relative prices. Thus, when home firms know that foreigners are well-informed about their economic state, they export more in booms then they would without that precise foreign information. This logic is also represented in right panel of Figure 6. Notice that the export share is higher with high information than with low information in high-μ_x states. But export share is more sensitive to relative price in low-endowment states: The curves in the right panel of Figure 6 are more steeply sloped for low μ_x. Therefore, the decline in trade in low states is larger than the increase in trade in the high states, resulting in a net decrease in the export share.

To see that it is really second-order beliefs that are at the heart of the negative relationship between information and average trade shares, we turn off movements in second-order beliefs. Then we check if the trade share still decreases in information precision, holding second order beliefs fixed. To do this experiment, we vary the precision of both countries' information, but we hold fixed the home country beliefs about foreign signal precision. This fixes second-order beliefs: home beliefs about what foreigners believe about home country productivity. But it allows all first-order beliefs to vary: Each country knows that the precision of their beliefs about the other country is rising. This is violating the assumption of rational expectations. That does not make this a desirable benchmark model. Instead, it is simply an experiment that isolates one single channel to see if that channel is responsible for the decline of trade share in signal precision.

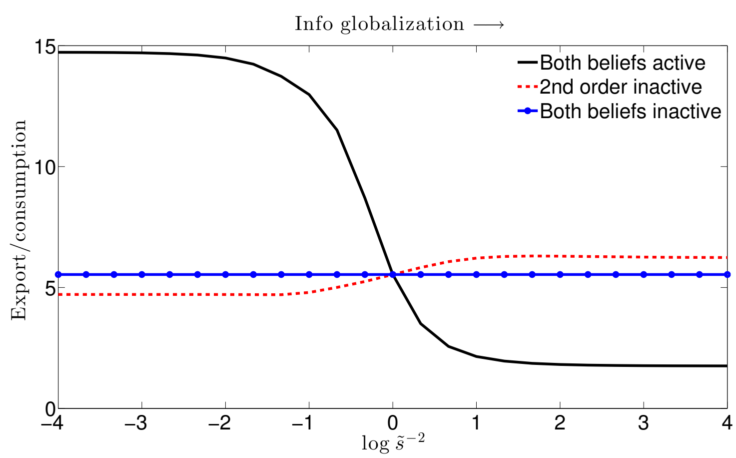

Figure 7: Turning off the change in second order beliefs. Increases in signal precision no longer reduce the trade share.

Figure 7 reveals that when the precision of home second-order beliefs is held fixed, the average trade share no longer declines, even if true signal precision increases for both countries. It is not an increase in the precision of one's own signal that makes trade share decline. Rather, it is the increase in one beliefs about the other country's signal precision that makes a country reduce the fraction of output it exports on average, and particularly in low-productivity states. The message from these results is that more precise information reduces the average export share because of what one country anticipates that a well-informed foreign trade partner will do to worsen their terms of trade, in times when the home country productivity is low.

How can the trade share fall while total exports rise? We have shown that as information globalization increases, the trade share falls. In contrast, Figure 8 shows that the total units exported rise in signal precision initially and then fall off as signal precision increases further, on average.

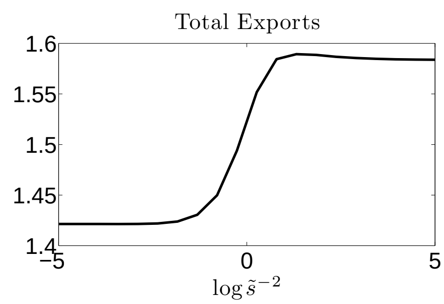

Figure 8: Reducing information asymmetry increase trade volume for small values of precision.

These results appear contradictory. But in fact, it comes from a change in covariance. We can write the amount of x goods exported as the fraction of the endowment exported, times the endowment: $T_x = f_x \Psi(\mu_x, \hat{m}_y)$. Taking an expectation yields

$$E[T_x] = E[f_x]E[\Psi(\mu_x, \hat{m}_y)] + Cov\left(f_x, \Psi(\mu_x, \hat{m}_y)\right)$$

When noisy signals are introduced, $E[T_x]$ rises, $E[f_x]$ is constant and $E[\Psi(\mu_x, \hat{m}_y)]$ falls. That is only possible if $Cov\left(f_x, \Psi(\mu_x, \hat{m}_y)\right)$ rises. The simulation results confirm that this covariance rises in signal precision.

This covariance is at the heart of most of the results in the paper. It is the covariance between home aggregate productivity and the share of home goods exported. When signal precision rises, the fact that the other country anticipates periods of high home output and high foreign relative price makes the foreign country export more. When foreigners export more, it increases the relative price of the home good and induces the home country to export even more. That is why $Cov\left(f_x, \Psi(\mu_x, \hat{m}_y)\right)$ rises in signal precision. It is the same reason why information increases export volatility and consumption risk.

3.4 Welfare Analysis

Countries' expected utility is higher with greater information precision because both countries achieve more balanced consumption bundles by coordinating their exports. We have established that when each country knows more about the other's productivity, they can anticipate better when the other country will export a lot. If exports are coordinated, then times when the home good is abundant, home agents will also get to consume lots of foreign goods. When home goods are scarce, foreign exports will be lower and consumption of both goods at home will be low. This balance in consumption bundles makes trade volumes and utility more volatile, but also achieves higher average utility.

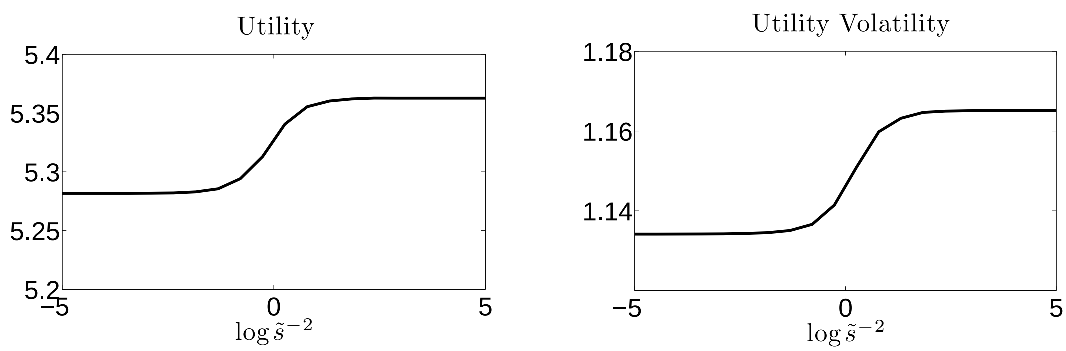

Figure 9: Information globalization increases average utility, but also increases utility volatility.

Figure 9 documents that higher signal precision – information globalization – increases expected utility. But instead of making utility less variable, like a trade cost would, information globalization makes utility more variable. That is a consequence of the fall in risk sharing that we explain in the next section.

To understand why coordination increases utility, we can look at a second order Taylor approximation to the utility function around the point $(\mathbb{E}[c_x], \mathbb{E}[c_y])$:

$$\mathbb{E}[c] \approx \bar{c}\left\{1 + \frac{(1-\theta)}{2}\left(\frac{\mathbb{E}[c_x]}{\bar{c}}\right)^{\theta}\left(\frac{\mathbb{E}[c_y]}{\bar{c}}\right)^{\theta}\left[2\frac{\mathbb{C}[c_x, c_y]}{\mathbb{E}[c_x]\mathbb{E}[c_y]} - \mathbb{CV}[c_x]^2 - \mathbb{CV}[c_y]^2\right]\right\}$$

where $\bar{c} \equiv c(\mathbb{E}[c_x], \mathbb{E}[c_y])$ is the certainty equivalent. This expression shows that utility is increasing in the covariance between the consumption goods, $\mathbb{C}[c_x, c_y]$, and this covariance increases when trade is more coordinated (high $\text{corr}(T_x, T_y)$). More precise information about foreign productivity leads countries to choose export quantities that are more positively correlated. This trade coordination is what explains the higher average level of composite consumption.

3.5 Sensitivity Analysis

Since the elasticity of substitution is a key parameter in many theories of trade, we explore its effect on our results here (Figure 10). Also, since the degree of aggregate volatility is also the prior uncertainty of the key random variable, we explore increasing or decreasing its value as well (Figure 10).

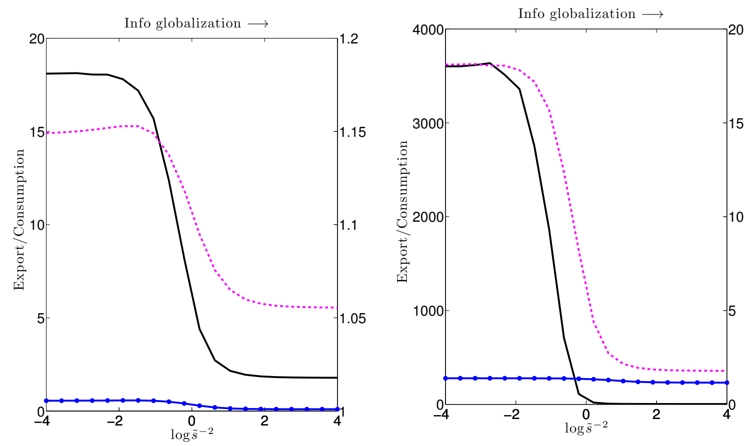

Figure 10: **Sensitivity Analysis** The elasticity of substitution governs the magnitude of the trade effect, but not its direction (left panel). The effect of information (right panel) on the trade share for small and medium aggregate volatility (right scale) and high aggregate volatility (left scale). The effect increases with aggregate volatility.

For the parameter θ that governs intertemporal elasticity, we explored 0.01, 0.2 and 0.75. These impliy intertemporal elasticities $(1/1-\theta)$ of 1.01, which is almost Cobb-Douglas utility, 1.25 and 4, which is a value commonly used in the macro trade literature. We see that while the effect on trade share is quite large for the elasticity of 4, it falls dramatically as preferences become less elastic and approach Cobb-Douglas. We know that in the Cobb-Douglas case, countries always export a fixed fraction of their endowment and our effect disappears completely. Our work in progress explores elasticities below 1.

For the parameters s_x and s_y that govern the standard deviation of the aggregate endowment shocks, we explore 0.5, 1 (our benchmark case) and 1.5. For low volatility, relative prices are very predictable, information has little effect because there is little uncertainty to resolve anyway, and the effect of information on trade share vanishes. This is reflected as a flat trade share. The effect of information increases as we increase the volatility of the aggregate shocks. In sum, we find that neither changes in the elasticity of substitution nor changes in the volatility of aggregate shocks reverses the effect of information globalization on the trade share. But both can have large consequences for the magnitude of the effect. Next, we explore a limiting case where the effect is not reversed, but it does disappear.

Solution with Cobb-Douglas Utility Clearly, the form of the utility function and the preference parameters matter here. As an example suppose that instead of CES preferences, we have Cobb-Douglas preferences, which are the limit of the CES preferences[4] as $\theta \to 0$. In this limit, preferences are

$$\mathbb{E}[c] = \mathbb{E}\left[c_x^{1-\gamma} c_y^{\gamma}\right].$$

Substituting in the budget constraint and taking first order conditions for home and foreign exports yields a simple optimal export policy:

$$t_x = \gamma z_x \qquad t_y = \gamma z_y \qquad c_x = (1-\gamma) z_x \qquad t_y = (1-\gamma) z_y$$

The policy is to have a fixed proportion of endowment for consumption of x and the rest to export, regardless of the prices and regardless of information sets. Since each firm's exports are a constant fraction of their output, aggregate exports are a constant fraction of aggregate output. Since this policy is independent of information sets, information globalization has no effect on exports, and subsequently, no effect on international relative prices or risk-sharing.

3.6 Introducing instruments that share international risk

So far, we have assumed away all instruments that agents might use to share international risk. Exchange rate futures, international equity holdings, profit-sharing contracts, secondary goods markets, all could help to share international risk and undermine some of the effects we have identified in this paper. Does that make the paper's results less relevant?

We started out wanting to understand what the effect of reductions in international information asymmetry might be on trade. If we start from a setting where perfect risk sharing makes information unnecessary, then of course, we cannot say much about the effect of information frictions. Instead, we give information its best shot to facilitate trade, by considering a world where the inability to share risk makes learning about foreign shocks very valuable. And yet, we still find that information globalization reduces trade and

[4] We consider a more general specification of the aggregator before taking the limit: $\lim_{\theta \to 0} \left((1-\gamma)c_x^{\theta} + \gamma c_y^{\theta}\right)^{\frac{1}{\theta}} = c_x^{1-\gamma} c_y^{\gamma}$

undermines risk-sharing.

If we did include a complete set of risk-sharing instruments and allowed agents to contract before signals were observed, we could achieve perfect risk sharing for any information structure. But even in this environment, our main point is still true, that the effects of information frictions (here no effect) is quite different from the effect of a proportional trade cost, which would still affect trade volume in this setting.

Finally, our results in progress quantify the utility in a constrained planner's problem where the planner can choose each country's exports, subject to the information that country has available. This socially optimal policy and its associated utility will give us some idea of what can be achieved with a limited set of financial instruments.

4 Conclusions

If information frictions are responsible for lower levels of foreign trade than domestic trade, the the information friction must take the form of an information asymmetry: home firms must know something about other home firms that foreigners do not. Otherwise, the information friction would inhibit home and domestic trade equally. This paper articulates a simple benchmark model of international trade with asymmetric information.

This benchmark model tells us that asymmetric information is not a barrier to trade. Instead, pairs of countries with more asymmetric information should trade more. The results quantify and explain the following logical steps. If another country exports more, my country would like to also export more because when others' goods are abundant, my goods fetch a higher price. Better information about the foreign endowment facilitates this export coordination. Better coordinated trade means that the ratio of home exports to foreign exports fluctuates less. But this ratio of home to foreign exports is the relative price of foreign goods. Movements in the relative price are the key instrument for sharing international endowment risk. When that relative price is less flexible, risk-sharing breaks down. Finally, since optimal exports are a convex function of the expected relative price, when the relative price varies less, the positive Jensen inequality term gets smaller and the average amount of trade falls.

These results do not imply that information frictions cannot rationalize low trade. It teaches us that the relationship between asymmetric information and trade share is not as simple as previously thought. The results should prompt researchers to look for ways to interact asymmetric information with other frictions in a way that might reduce trade. This model provides the foundation for such further exploration.

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A Proofs

A.1 Preliminaries

Notation: For each firm, we define the utility-and-trade-friction adjusted price aggregator as:

$$Q(z,\mu,\hat{m};p,s) \equiv \left[\mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[(\lambda(z,\mu,\hat{m})sp)^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}$$

where $(z,\mu,\hat{m})$ is the state of the firm, $\mathcal{I}$ is her information set, $p>0$ are the country's terms of trade, $s\in[0,1]$ is a trade cost and $\lambda(z,\mu,\hat{m}) \equiv c(z,\mu,\hat{m})^{\frac{1-\theta}{\theta}}$ is a measure of firm's utility. Also denote $\bar{\lambda}(z,\mu,\hat{m}) \equiv \mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}}$. Each firm will aggregate prices using this function evaluated at the corresponding terms of trade and trade costs faced by its country. Note that this price index is firm specific.

Optimal exports of home country The maximization problem for the x country and its FOC are:

$$V(z_x,\mu_x,\hat{m}_y) = \max_{t_x}\mathbb{E}\left[\left((z_x-t_x)^{\theta} + \left(\frac{t_x}{q}\right)^{\theta}\right)^{1/\theta}\middle|\mathcal{I}_x\right]$$

$$E\left[\frac{1}{\theta}\left((z_x-t_x)^{\theta} + \left(\frac{t_x}{q}\right)^{\theta}\right)^{(1-\theta)/\theta}\left(-\theta(z_x-t_x)^{\theta-1} + \theta\frac{t_x^{\theta-1}}{q^{\theta}}\right)\middle|\mathcal{I}_x\right] = 0$$

Let $\lambda(z_x,\mu_x,\hat{m}_y) \equiv c(z_x,\mu_x,\hat{m}_y)^{\frac{1-\theta}{\theta}}$, write first term as $\lambda(z_x,\mu_x,\hat{m}_y)^{\theta} = c(z_x,\mu_x,\hat{m}_y)^{1-\theta}$ and break the expectation:

$$E\left[\frac{t_x^{\theta-1}}{q^{\theta}}\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right] = (z_x-t_x)^{\theta-1}E\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]$$

Pulling out the non random term t_x we get:

$$\left(\frac{t_x}{z_x-t_x}\right)^{\theta-1} = \frac{E\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]}{E\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_y)}{q}\right)^{\theta}\middle|\mathcal{I}_x\right]}$$

Rearranging and using the definition of the price aggregator, we get an implicit expression for optimal exports t_x:

$$\begin{aligned}
t_x(z_x,\mu_x,\hat{m}_y) &= \frac{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}}{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_x)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_x)}{q}\right)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}} \\
&= \left(\frac{\left[\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_x)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\left(\frac{\lambda(z_x,\mu_x,\hat{m}_x)}{q}\right)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\lambda(z_x,\mu_x,\hat{m}_y)^{\theta}\middle|\mathcal{I}_x\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}} \\
&= z_x\left(\frac{Q\left(z_x,\mu_x,\hat{m}_x;\frac{1}{q}\right)}{\bar{\lambda}(z_x,\mu_x,\hat{m}_x)}\right)^{\frac{\theta}{1-\theta}}
\end{aligned}$$

Note that with perfect information: $Q\left(z_x,\mu_x,\hat{m}_x;\frac{1}{q}\right) = \lambda(z_x,\mu_x,\hat{m}_x)\left(1+\left(\frac{1}{q}\right)^{\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}$ and $\bar{\lambda}(z_x,\mu_x,\hat{m}_x) = \lambda(z_x,\mu_x,\hat{m}_x)$, then the export expression simplifies to a linear function of z_x:

$$t_x = z_x\left(\frac{1}{1+(q)^{\frac{\theta}{1-\theta}}}\right)$$

Analogously, one can show that firm optimal exports in the foreign country are given by:

$$t_y(z_y,\mu_y,\hat{m}_x) = z_y\left(\frac{Q\left(z_y,\mu_y,\hat{m}_x;q\right)}{\bar{\lambda}(z_y,\mu_y,\hat{m}_x)}\right)^{\frac{\theta}{1-\theta}}$$

and with perfect information they reduce to:

$$t_y = z_y\left(\frac{1}{1+\left(\frac{1}{q}\right)^{\frac{\theta}{1-\theta}}}\right)$$

A.2 Lemma 1: Exports are proportional to firm productivity

Proof. Guess a solution $t(z,\mu,\hat{m}) = z\Psi(\mu,\hat{m})$. First we show that the composite good is also proportional to z:

$$\begin{aligned}
c(z,\mu,\hat{m}) &= \left((z-t(z,\mu,\hat{m}))^{\theta} + (t(z,\mu,\hat{m})sp))^{\theta}\right)^{1/\theta} \\
&= \left(z^{\theta}(1-\Psi(\mu,\hat{m}))^{\theta} + z^{\theta}\left(\Psi(\mu,\hat{m})sp\right)^{\theta}\right)^{1/\theta} \\
&= z\left((1-\Psi(\mu,\hat{m}))^{\theta} + (\Psi(\mu,\hat{m})sp)^{\theta}\right)^{1/\theta} \\
&= z\Psi_2(\mu,\hat{m})
\end{aligned}$$

where $\Psi_2 \equiv \left((1-\Psi)^{\theta} + (\Psi sp)^{\theta}\right)^{1/\theta}$.

Second, we substitute the composite consumption in $\bar{\lambda}$ and we obtain a separable function between idiosyncratic and aggregate variables:

$$\begin{aligned}
\bar{\lambda}(z,\mu,\hat{m}) &\equiv \mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\
&= \mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\
&= \mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}} \\
&= z^{\frac{1-\theta}{\theta}}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\middle|\mathcal{I}\right]^{\frac{1}{\theta}}
\end{aligned}$$

Third, substitute λ in Q and again we obtain a separable function:

$$\begin{aligned}
Q(z,\mu,\hat{m};p,s) &\equiv \left[\mathbb{E}\left[\lambda(z,\mu,\hat{m})^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[(\lambda(z,\mu,\hat{m})sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}\\
&= \left[\mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[c(z,\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}\\
&= \left[\mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[z^{1-\theta}\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}\\
&= \left[z^{-1}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+z^{-1}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}\\
&= z^{\frac{1-\theta}{\theta}}\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}
\end{aligned}$$

Finally, substituting all the elements above in the implicit function that defines the export policy we get that terms with z inside Q and $\bar{\lambda}$ cancel out:

$$\begin{aligned}
t(z,\mu,\hat{m}) &= z\left(\frac{Q(z,\mu,\hat{m};p,s)}{\bar{\lambda}(z,\mu,\hat{m})}\right)^{\frac{\theta}{1-\theta}}\\
&= z\left(\frac{z^{\frac{1-\theta}{\theta}}\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{z^{\frac{1-\theta}{\theta}}\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}}\\
&= z\left(\frac{\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}}\\
&= z\Psi(\mu,\hat{m})
\end{aligned}$$

Therefore, we have verified that the export policy is indeed linear in idiosyncratic productivity z.

Furthermore, the component that depends on aggregate shocks is given by:

$$\Psi(\mu,\hat{m})=\left(\frac{\left[\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}+\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}(sp)^{\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[\Psi_2(\mu,\hat{m})^{1-\theta}\Big|\mathcal{I}\right]^{\frac{1}{\theta}}}\right)^{\frac{\theta}{1-\theta}}$$

where

$$\Psi_2(\mu,\hat{m})=\Big((1-\Psi(\mu,\hat{m}))^{\theta}+(\Psi(\mu,\hat{m})sp)^{\theta}\Big)^{1/\theta}$$

■

A.3 Derivation of fixed point problems

Using Lemma 1, we can aggregate the exports of domestic and foreign firms:

$$T_x(\mu_x, \hat{m}_y) = \int z_x \Psi(\mu_x, \hat{m}_y) dF(z_x|\mu_x) = e^{\mu_x + \frac{1}{2}\sigma_x^2} \Psi(\mu_x, \hat{m}_y)$$

$$T_y(\mu_y, \hat{m}_x) = \int z_y \Gamma(\mu_y, \hat{m}_x) dF(z_y|\mu_y) = e^{\mu_y + \frac{1}{2}\sigma_y^2} \Gamma(\mu_y, \hat{m}_x)$$

where the last equality holds because z_x and z_y are lognormal. The realized equilibrium price will be given by:

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{T_x(\mu_x, \hat{m}_y)}{T_y(\mu_y, \hat{m}_x)} = \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2} \Psi(\mu_x, \hat{m}_y)}{e^{\mu_y + \frac{1}{2}\sigma_y^2} \Gamma(\mu_y, \hat{m}_x)} = f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}$$

where $f \equiv e^{(\mu_x - \mu_y)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)}$ are the relative fundamentals.

Rearranging Ψ and Γ and impose consistency of beliefs (the actual price function is used by agents to form their beliefs), we get that equilibrium is given by three functions Ψ, Γ and q such that they solve the following fixed point problems:

$$\Psi(\mu_x, \hat{m}_y) = \frac{1}{1 + \left(\frac{\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{1-\theta} \middle| \mathcal{I}_x \right]}{\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{1-\theta} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{-\theta} \middle| \mathcal{I}_x \right]} \right)^{\frac{1}{1-\theta}}}$$

$$\Gamma(\mu_y, \hat{m}_x) = \frac{1}{1 + \left(\frac{\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \mu_x, \hat{m}_y, \hat{m}_x)^{1-\theta} \middle| \mathcal{I}_y \right]}{\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \mu_x, \hat{m}_y, \hat{m}_x)^{1-\theta} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\theta} \middle| \mathcal{I}_y \right]} \right)^{\frac{1}{1-\theta}}}$$

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}$$

where the auxiliary functions are:

$$\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \left((1 - \Psi(\mu_x, \hat{m}_y))^\theta + \left(\frac{\Psi(\mu_x, \hat{m}_y)}{q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \right)^\theta \right)^{1/\theta}$$

$$\Gamma_2(\mu_y, \mu_x, \hat{m}_y, \hat{m}_x) = \left((1 - \Gamma(\mu_y, \hat{m}_x))^\theta + (\Gamma(\mu_y, \hat{m}_x) q(\mu_y, \mu_x, \hat{m}_x, \hat{m}_y))^\theta \right)^{1/\theta}$$

This system can be expressed compactly as:

$$\begin{aligned} \Psi &= g_1(\mathcal{I}_x, \Psi, \Gamma) \\ \Gamma &= g_2(\mathcal{I}_y, \Psi, \Gamma) \\ q &= f g_3(\Psi, \Gamma) \end{aligned}$$

B Algorithm

B.1 Polynomial approximation to policy functions

Functional Basis Let $\{\Phi_k\}_{k=1}^{M}$ be a basis of polynomials with support $x \in [a, b]$. We use linear splines and uniform nodes for the 2 states of each country.

i) **Grid for state 1: Own productivity**:

- In x country it is distributed $\mu_x \sim \mathcal{N}(m_x, s_x^2)$, where m_x, s_x are parameters. We construct uniform nodes $\{\mu_x^i\}_{i=1}^N$ in the support $[m_x - 4s_x, m_x + 4s_x]$.

- In y country it is distributed $\mu_y \sim \mathcal{N}(m_y, s_y^2)$, where m_y, s_y are parameters. We construct uniform nodes $\{\mu_y^i\}_{i=1}^N$ in the support $[m_y - 4s_y, m_y + 4s_y]$.

ii) **Grid for state 2: Posterior mean of foreign productivity**:

- In x country, the posterior mean of foreign productivity is $\hat{m}_y \sim \mathcal{N}(m_y, \bar{s}_y^2)$ where $\bar{s}_y^2 = \frac{s_y^4}{s_y^2 + \tilde{s}_y^2}$. However, to use a fixed grid that does not change with the precision of information, we construct the nodes $\{\hat{\mu}_y^j\}_{j=1}^N$ over the support $[m_y - 4s_y, m_y + 4s_y]$.

- In y country, the posterior mean of foreign productivity is $\hat{m}_x \sim \mathcal{N}(m_x, \bar{s}_x^2)$ where $\bar{s}_x^2 = \frac{s_x^4}{s_x^2 + \tilde{s}_x^2}$. Analogously, we construct the nodes $\{\hat{m}_x^j\}_{j=1}^N$ over the support $[m_x - 4s_x, m_x + 4s_x]$.

Approximating functions We approximate four conditional expectations with polynomials:

$$
\begin{aligned}
\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta} \middle| \mathcal{I}_x\right] &\approx g^1(\mu_x, \hat{m}_y) \\
\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{1-\theta} q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{-\theta} \middle| \mathcal{I}_x\right] &\approx g^2(\mu_x, \hat{m}_y) \\
\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta} \middle| \mathcal{I}_y\right] &\approx h^1(\mu_y, \hat{m}_x) \\
\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)^{1-\theta} q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)^{\theta} \middle| \mathcal{I}_y\right] &\approx h^2(\mu_y, \hat{m}_x)
\end{aligned}
$$

where the polynomials are constructed using the basis for each dimension evaluated at the nodes described above:

$$
\begin{aligned}
g^1(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^1_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
g^2(\mu_x^i, \hat{m}_y^j) &\equiv \sum_{k,k' \in K \times K'} g^2_{k,k',i,j} \Phi_k(\mu_x^i) \Phi_{k'}(\hat{m}_y^j) \\
h^1(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^1_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j), \\
h^2(\mu_y^i, \hat{m}_x^j) &\equiv \sum_{k,k' \in K \times K'} h^2_{k,k',i,j} \Phi_k(\mu_y^i) \Phi_{k'}(\hat{m}_x^j)
\end{aligned}
$$

B.2 Computing expectations

For each country, we have two random variables, foreign productivity and second order beliefs, for which we will evaluate expectations using Gaussian Quadrature method. For this, we must define a set of nodes $\{x_a\}_{a=1}^{N_q}$ and weights $\{w_a\}_{a=1}^{N_q}$ such that

$$
\mathbb{E}[f(X)] = \sum_{a=1}^{N_q} w_a f(x_a)
$$

and further moments conditions are satisfied.

- **Grid for random variable 1: foreign productivity**: The distribution of foreign aggregate productivity depends on the second state, the posterior mean $\hat{m}$.

  - In x country, for each value of the second state (the posterior mean) we have that foreign productivity is Normal with mean equal to the posterior mean $\hat{m}_y^j$ and variance equal to the posterior variance $\hat{s}_y^2 = \left(s_y^{-2} + \tilde{s}_y^{-2}\right)^{-1} = \frac{1}{\frac{1}{s_y^2}+\frac{1}{\tilde{s}_y^2}}$

  $$\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, \hat{s}_y^2)$$

  Then for each $j = 1, ..., N$, Gaussian Quadrature procedure constructs nodes of foreign productivity $\{\mu_y^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$. Note that the weights do not depend on j.

  - In y country, for each value of the second state (the posterior mean $\hat{m}_x^j$) we have that foreign productivity is Normal with mean equal to the posterior mean $\hat{m}_x^j$ and variance equal to the posterior variance $\hat{s}_x^2 = \left(s_x^{-2} + \tilde{s}_x^{-2}\right)^{-1} = \frac{1}{\frac{1}{s_x^2}+\frac{1}{\tilde{s}_x^2}}$

  $$\mu_x^j \sim \mathcal{N}(\hat{m}_x^j, \hat{s}_x^2)$$

  Then for each $j = 1, ..., N$, Gaussian Quadrature procedure constructs nodes of foreign productivity $\{\mu_x^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$.

  Extreme cases

  - Perfect Info: As $\tilde{s}_y \to 0$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, 0) = \mathcal{N}(\mu_y^j, 0)$. The grid degenerates to a single point for each j: $\mu_y^{j,b} = \mu_y^j$.
  - No Info: As $\tilde{s}_y \to \infty$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, s_y^2) = \mathcal{N}(m_y, \bar{s}_y^2)$ which is equal the distribution of the posterior mean (the second state). Clearly, $\bar{s}_y^2 = \frac{s_y^4}{s_y^2+\tilde{s}_y^2} \to s_y^2$ as well, which makes the distribution of foreign productivity equal to the prior. However, in the code we have fixed grids for the states so that they do not depend on signal precision. Therefore, as we reduce signal precision, the grid will not converge to the prior. However, the simulations take care of it.

- **Grid for random variable 2: second order beliefs**: From the perspective of the domestic country, the second order beliefs about the posterior mean (this is, what the domestic country thinks the posterior mean of the foreign country is) is a Normal random variable that depends on the first state, the domestic aggregate productivity μ.

  - In the x country, for each value of the first state (aggregate productivity μ_x^i), we have that the second order belief is Normal with mean and variance as follows:

  $$\hat{m}_x^i \sim \mathcal{N}(\hat{\hat{m}}_x^i, \hat{\hat{s}}_x^2) \qquad \text{with} \qquad \hat{\hat{m}}_x^i \equiv \frac{s_x^{-2}m_x + \tilde{s}_{p_x}^{-2}\mu_x^i}{s_x^{-2} + \tilde{s}_{p_x}^{-2}}, \qquad \hat{\hat{s}}_x^2 \equiv \tilde{s}_{p_x}^{-2}(s_x^{-2} + \tilde{s}_{p_x}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{p_x}}{s_x^2} + \frac{1}{\tilde{s}_{p_x}})^2}$$

  where $\tilde{s}_{p_x}$ is the foreign signal noise as perceived by the domestic country. With known information structures $\tilde{s}_{p_x} = \tilde{s}_x$, but with unknown information structures $\tilde{s}_{p_x} \neq \tilde{s}_x$.

  Then for each $i = 1, ..., N$, Gaussian Quadrature procedure constructs nodes for second order beliefs $\{\mu_x^{i,a}\}_a = 1, ..., N_q$ and corresponding weights $\{\gamma^a\}_a = 1, ..., N_q$.

  - In the y country, we have that for each value of the first state μ_y^i the second order belief is distributed as:

  $$\hat{m}_y^i \sim \mathcal{N}(\hat{\hat{m}}_y^i, \hat{\hat{s}}_y^2) \qquad \text{with} \qquad \hat{\hat{m}}_y^i \equiv \frac{s_y^{-2}m_y + \tilde{s}_{p_y}^{-2}\mu_y^i}{s_y^{-2} + \tilde{s}_{p_y}^{-2}}, \qquad \hat{\hat{s}}_y^2 \equiv \tilde{s}_{p_y}^{-2}(s_y^{-2} + \tilde{s}_{p_y}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{p_y}}{s_y^2} + \frac{1}{\tilde{s}_{p_y}})^2}$$

Extreme cases

– Perfect Info: As $\tilde{s}_{p_x} \to 0$, then the distribution becomes degenerate at the true realizations: $\hat{m}_x^i \sim \mathcal{N}(\mu_x^i, 0) \quad \forall i$ and the grid becomes: $\hat{m}_x^{i,a} = \mu_x^i,\ a = 1, ..., N_q$

– Imperfect Info: As $\tilde{s}_{p_x} \to \infty$, then the distribution becomes degenerate at the prior means $\hat{m}_x^i \sim \mathcal{N}(m_x, 0) \quad \forall i$ and the grid becomes $\hat{m}_x^{i,a} = m_x,\ a = 1, ..., N$

B.3 Finding the fixed point

1. For reference, we organize the states as follows. For x - country: $(\mu_x, \hat{m}_y)$ and for y - country: $(\mu_y, \hat{m}_x)$. For the price and other economy wide variables, we make the following convention: $q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)$.
2. Guess initial set of coefficients for polynomials $\{g^1_{k,k',i,j}, g^2_{k,k',i,j}, h^1_{k,k',i,j}, h^2_{k,k',i,j}\}$.
   - We start by solving the perfect information case and approximate the policies with the polynomials to get the first set of coefficients. Since with perfect information $\hat{m}_y = \mu_y$ and $\hat{m}_x = \mu_x$, we have the following system of equations:

$$\begin{aligned}
\Psi^{PI}(\mu_x, \mu_y) &= \frac{1}{1 + [q^{PI}(\mu_x, \mu_y)]^{\frac{\theta}{1-\theta}}} \\
\Gamma^{PI}(\mu_y, \mu_x) &= \frac{1}{1 + \left[\frac{1}{q^{PI}(\mu_x, \mu_y)}\right]^{\frac{\theta}{1-\theta}}} \\
q^{PI}(\mu_x, \mu_y) &= f\frac{\Psi^{PI}(\mu_x, \mu_y)}{\Gamma(\mu_y, \mu_x)}
\end{aligned}$$

   Thus q^{PI} is

$$q^{PI}(\mu_x, \mu_y) = f^{1-\theta} \quad \forall(\mu_y, \mu_x)$$

   Once we have the price, we recover the policies and construct the first guess of coefficients and approximating functions.
3. For the X - country:
   - For each state $(\mu_x^i, \hat{m}_y^j)$, approximate Ψ using the polynomials g^1 and g^2 evaluated at the state:

$$\Psi(\mu_x^i, \hat{m}_y^j) \approx \frac{1}{1 + \left(\frac{g^1(\mu_x^i, \hat{m}_y^j)}{g^2(\mu_x^i, \hat{m}_y^j)}\right)^{\frac{1}{1-\theta}}}$$

   - For each quadrature node $(\mu_y^a, \hat{m}_x^b)$ approximate Γ using the polynomials h^1 and h^2 evaluate at the nodes $\{\mu_y^a\}_{a=1}^{N_q}, \{\hat{m}_x^b\}_{b=1}^{N_q}$

$$\Gamma(\mu_y^a, \hat{m}_x^b) \approx \frac{1}{1 + \left(\frac{h^1(\mu_y^a, \hat{m}_x^b)}{h^2(\mu_y^a, \hat{m}_x^b)}\right)^{\frac{1}{1-\theta}}}$$

   - Construct q and Ψ_2 in 4 dimensions using $\Psi(\mu_x^i, \hat{m}_y^j)$ and $\Gamma(\mu_y^a, \hat{m}_x^b)$:

$$q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) \approx e^{(\mu_x^i - \mu_y^a)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)} \frac{\Psi(\mu_x^i, \hat{m}_y^j)}{\Gamma(\mu_y^a, \hat{m}_x^b)}$$

$$\Psi_2(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) = \left((1 - \Psi(\mu_x^i, \hat{m}_y^j))^\theta + \left(\frac{\Psi(\mu_x^i, \hat{m}_y^j)}{q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)}\right)^\theta\right)^{\frac{1}{\theta}}$$

- Compute the conditional expectations of $\Psi_2^{1-\theta}$ and $\Psi_2^{1-\theta}q^{-\theta}$ that integrate out the two random variables $(\mu_y, \hat{m}_x)$ as the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
&\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\Big|\mathcal{I}_x\right] \\
=& \int_{\mu_y}\int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \quad \phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right)\phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) d\mu_y d\hat{m}_x \\
\approx& \sum_{a=1}^{N_q}\sum_{b=1}^{N_q} \omega^a\gamma^b\Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\Big|\mathcal{I}_x\right] \\
=& \int_{\mu_y}\int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right)\phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) d\mu_y d\hat{m}_x \\
\approx& \sum_{a=1}^{N_q}\sum_{b=1}^{N_q} \omega^a\gamma^b\Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)q^{-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

4. For the Y- country, we do analogous calculations.

   - For each state $(\mu_y^i, \hat{m}_x^j)$, approximate Γ using the polynomials h^1 and h^2 evaluated at the state:

$$\Gamma(\mu_y^i, \hat{m}_x^j) \approx \frac{1}{1 + \left(\frac{h^1(\mu_y^i, \hat{m}_x^j)}{h^2(\mu_y^i, \hat{m}_x^j)}\right)^{\frac{1}{1-\theta}}}$$

   - For each quadrature node $(\mu_x^a, \hat{m}_y^b)$ approximate Ψ using the polynomials g^1 and g^2 evaluate at the nodes $\{\mu_x^a\}_{a=1}^{N_q}, \{\hat{m}_y^b\}_{b=1}^{N_q}$

$$\Psi(\mu_x^a, \hat{m}_y^b) \approx \frac{1}{1 + \left(\frac{g^1(\mu_x^a, \hat{m}_y^b)}{g^2(\mu_x^a, \hat{m}_y^b)}\right)^{\frac{1}{1-\theta}}}$$

   - Construct q and Γ_2 in 4 dimensions using $\Gamma(\mu_y^i, \hat{m}_x^j)$ and $\Psi(\mu_x^a, \hat{m}_y^b)$ (note that the state for the price is in the same order as for the X-country):

$$q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j) \approx e^{(\mu_x^a - \mu_y^i)}e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)}\frac{\Psi(\mu_x^a, \hat{m}_y^b)}{\Gamma(\mu_y^i, \hat{m}_x^j)}$$

$$\Gamma_2(\mu_y^i, \hat{m}_x^j, \mu_x^a, \hat{m}_y^b) = \left((1 - \Gamma(\mu_y^i, \hat{m}_x^j))^\theta + \left(\Gamma(\mu_y^i, \hat{m}_x^j)q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j)\right)^\theta\right)^{\frac{1}{\theta}}$$

   - Compute the conditional expectations of $\Gamma_2^{1-\theta}$ and $\Gamma_2^{1-\theta}q^\theta$ that integrate out the two random variables $(\mu_x, \hat{m}_y)$. This is just the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
& \mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)\Big|\mathcal{I}_y\right] \\
= & \int_{\mu_y}\int_{\hat{m}_x}\Gamma_2^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) \quad \phi\left(\frac{\mu_x-\hat{m}_x}{\hat{s}_x}\right)\phi\left(\frac{\hat{m}_y-\hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right)d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\gamma^b\Gamma_2^{1-\theta}(\mu_y^i,\hat{m}_x^j,\mu_x^a,\hat{m}_y^b)
\end{aligned}$$

$$\begin{aligned}
& \mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)q^\theta(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\Big|\mathcal{I}_y\right] \\
= & \int_{\mu_y}\int_{\hat{m}_x}\Gamma_2^{1-\theta}(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)q^\theta(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)\phi\left(\frac{\mu_x-\hat{m}_x}{\hat{s}_x}\right)\phi\left(\frac{\hat{m}_y-\hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right)d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\gamma^b\Gamma_2^{1-\theta}(\mu_x^i,\hat{m}_y^j,\mu_y^a,\hat{m}_x^b)q^\theta(\mu_x^a,\hat{m}_y^b,\mu_y^i,\hat{m}_x^j)
\end{aligned}$$

5. Update coefficients by i) fitting polynomials to approximate the conditional expectations and ii) using a linear combination of the new coefficients with the previous guess.
6. Repeat steps until convergence of coefficients.
7. Once convergence is achieved, recover all variables at the firm level and at the aggregate level.

   Recall the definitions of domestic, foreign and relative fundamentals:

$$f_x \equiv e^{\mu_x+\frac{1}{2}\sigma_x^2}, \qquad f_y \equiv e^{\mu_y+\frac{1}{2}\sigma_y^2} \qquad f \equiv e^{(\mu_x-\mu_y)}e^{\frac{1}{2}(\sigma_x^2-\sigma_y^2)}$$

   (a) Price function:

$$q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) = f\frac{\Psi(\mu_x,\hat{m}_y)}{\Gamma(\mu_y,\hat{m}_x)}$$

   (b) Firms' export policy and consumptions in x country:

$$\begin{aligned}
t_x(z_x,\mu_x,\hat{m}_y) &= z_x\Psi(\mu_x,\hat{m}_y) \\
c_x(z_x,\mu_x,\hat{m}_y) &= z_x(1-\Psi(\mu_x,\hat{m}_y)) \\
c_y(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= \frac{t_x(z_x,\mu_x,\hat{m}_y)}{q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)} \\
c(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_x\Psi_2(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)
\end{aligned}$$

   (c) Firms' export policy and consumptions in y country:

$$\begin{aligned}
t_y(z_y,\mu_y,\hat{m}_x) &= z_y\Gamma(\mu_y,\hat{m}_x) \\
c_y^*(z_y,\mu_y,\hat{m}_x) &= z_y(1-\Gamma(\mu_y,\hat{m}_x)) \\
c_x^*(z_y,\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) &= t_y(z_y,\mu_y,\hat{m}_x)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y) \\
c^*(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_y\Gamma_2(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)
\end{aligned}$$

(d) Aggregate variables in x country:

$$\begin{aligned}
T_x(\mu_x, \hat{m}_y) &= f_x \Psi(\mu_x, \hat{m}_y) \\
C_x(\mu_x, \hat{m}_y) &= f_x(1 - \Psi(\mu_x, \hat{m}_y)) \\
C_y(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= \frac{T_x(\mu_x, \hat{m}_y)}{q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \\
C(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) &= f_x \Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)
\end{aligned}$$

(e) Aggregate variables in y country:

$$\begin{aligned}
T_y(\mu_y, \hat{m}_x) &= f_y \Gamma(\mu_y, \hat{m}_x) \\
C_y^*(\mu_y, \hat{m}_x) &= f_y(1 - \Gamma(\mu_y, \hat{m}_x)) \\
C_x^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= T_y(\mu_y, \hat{m}_x) q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) \\
C^*(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) &= f_y \Gamma_2(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y)
\end{aligned}$$

C Proofs of Analytical Results

C.1 Proof of Result 1: Information Enables Trade Correlation

In a neighborhood around perfect information, decreasing the precision of both countries' signals increases var$(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x))$.

Let $T_y \perp T_x$ denote the part of T_y that is orthogonal to T_x. In other words, if we estimate linear projection (an OLS regression) of T_y on T_x, then $T_y \perp T_x$ is the projection (regression) residual.

Starting from the fixed point problem in Appendix A.3, we have

$$\begin{aligned}
\Psi(\mu_x, \hat{m}_y) &= \frac{1}{1 + \left(\frac{\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{1-\theta} \middle| \mathcal{I}_x \right]}{\mathbb{E}_{\mu_y, \hat{m}_x}\left[\Psi_2(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{1-\theta} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{-\theta} \middle| \mathcal{I}_x \right]} \right)^{\frac{1}{1-\theta}}} \\
\Gamma(\mu_y, \hat{m}_x) &= \frac{1}{1 + \left(\frac{\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \mu_x, \hat{m}_y, \hat{m}_x)^{1-\theta} \middle| \mathcal{I}_y \right]}{\mathbb{E}_{\mu_x, \hat{m}_y}\left[\Gamma_2(\mu_y, \mu_x, \hat{m}_y, \hat{m}_x)^{1-\theta} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\theta} \middle| \mathcal{I}_y \right]} \right)^{\frac{1}{1-\theta}}} \\
q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) &= f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}
\end{aligned}$$

When information is perfect, we can remove the expectation operators and cancel the ψ and Γ terms in the denominator of each expression. We find

$$\begin{aligned}
\Psi(\mu_x, \hat{m}_y) &= \frac{1}{1 + q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\frac{\theta}{1-\theta}}} \\
\Gamma(\mu_y, \hat{m}_x) &= \frac{1}{1 + q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\frac{-\theta}{1-\theta}}}
\end{aligned}$$

Rearranging the expression for Γ yields

$$\Gamma(\mu_y, \hat{m}_x) = \frac{q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\frac{\theta}{1-\theta}}}{1 + q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\frac{\theta}{1-\theta}}} = 1 - \frac{1}{1 + q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\frac{\theta}{1-\theta}}}$$

Thus, $\Gamma = 1 - \Psi$. Since the two export shares are perfectly negative correlated, $\Gamma \perp \Psi = 0$.

For any level of uncertainty about μ_x and μ_y, the expectations do no cancel, $(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x))$ depends on μ_x and μ_y. It is therefore is a random variable with positive variance. Since $\text{var}(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x)) = 0$ for perfect information and $\text{var}(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x)) > 0$ for imperfect information, there must exist a neighborhood around perfect information such that $\text{var}(\Psi(\mu_x, \hat{m}_y) \perp \Gamma(\mu_y, \hat{m}_x))$ is increasing in signal noise.

C.2 Proof of Result 2: Coordinated Trade Reduces Price Variances

Prove: Take the home country's export policy as given by fixing the distribution of T_x. If foreign firms coordinate more ($var(T_y \perp T_x)$ falls), then $var(1/q)$ decreases.

We can rewrite T_y as the sum of its linear projection on T_x and the projection residual:

$$T_y = \alpha + \beta T_x + e_{xy}$$

where $E[e_{xy}] = 0$ and $E[e_{xy} T_x] = 0$.

Recall that the market clearing condition dictates that $1/q = T_y/T_x$. Thus

$$\begin{aligned} var(1/q) &= var\left(\frac{\alpha + \beta T_x + e_{xy}}{T_x}\right) \\ &= var\left(\beta + \frac{\alpha + e_{xy}}{T_x}\right) \\ &= var\left(\frac{1}{T_x}\right)\alpha^2 + var(e_{xy}) E\left[\frac{1}{T_x}\right]^2 \end{aligned}$$

Since we assumed that the distribution of T_x is fixed, $var\left(\frac{1}{T_x}\right)$ is fixed as well. If $var(e_{xy})$ increases and $var\left(\frac{1}{T_x}\right)$ does not change, then $var(1/q)$ increases.

Note that e_{xy} is a mean-zero variable and changes in its variance do not affect $E[1/q]$. Thus, changes in $var(e_{xy})$ change the variance of $1/q$ in a mean-preserving way.

C.3 Proof of Result 3: Lower Price Variance Reduces Average Trade Share

Prove: Holding fixed $cov(C^{1-\theta}, q^{\theta})/E[C^{1-\theta}]$, a mean-preserving decrease in $var(1/q)$ increases $\Psi/(1-\Psi)$.

From equation (19) in the main text, we have

$$\underbrace{\left(\frac{\Psi(\mu_x, \hat{m}_y)}{1 - \Psi(\mu_x, \hat{m}_y)}\right)^{1-\theta}}_{(\text{Trade Share})^{1-\theta}} = \underbrace{\mathbb{E}\left[q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]}_{I} + \underbrace{\frac{\mathbb{C}\left[C^{1-\theta}, q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]}{\mathbb{E}\left[C^{1-\theta}\middle|(\mu_x, \hat{m}_y)\right]}}_{\frac{II}{III}} \tag{20}$$

If the last term is constant by assumption, then anything that increases $E\left[q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]$ increases the left side, which is a monotonic increasing function of the trade share. Thus, if $E\left[q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]$ increases, the trade share increases.

Note that $q^{-\theta} = (1/q)^{\theta}$. Since $\theta \in (0, 1)$, this is a concave function of $1/q$. Consider first the effect on the expected value of $q^{-\theta}$ from increasing the variance of $1/q$. For any random variable x, we can represent an increase in variances as a mean-preserving spread, constructed as a compound lottery where one takes a draw from the distribution of the original variable x with probability π and a draw x' from another distribution with the same mean, with probability $1 - \pi$. Then, by the definition of concavity, the function x^{θ} of the higher-variance variable has a lower mean: $E[\pi x^{\theta} + (1 - \pi)(x')^{\theta}] < \pi E[x]^{\theta} + (1 - \pi)E[x']^{\theta} = E[x]^{\theta}$. Thus, a mean-preserving increase in variance of $1/q$ lowers $E\left[q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]$. By the same logic, a mean-preserving decrease in variance of $1/q$ increases $E\left[q^{-\theta}\middle|(\mu_x, \hat{m}_y)\right]$ and therefore increases $\Psi/(1 - \Psi)$.