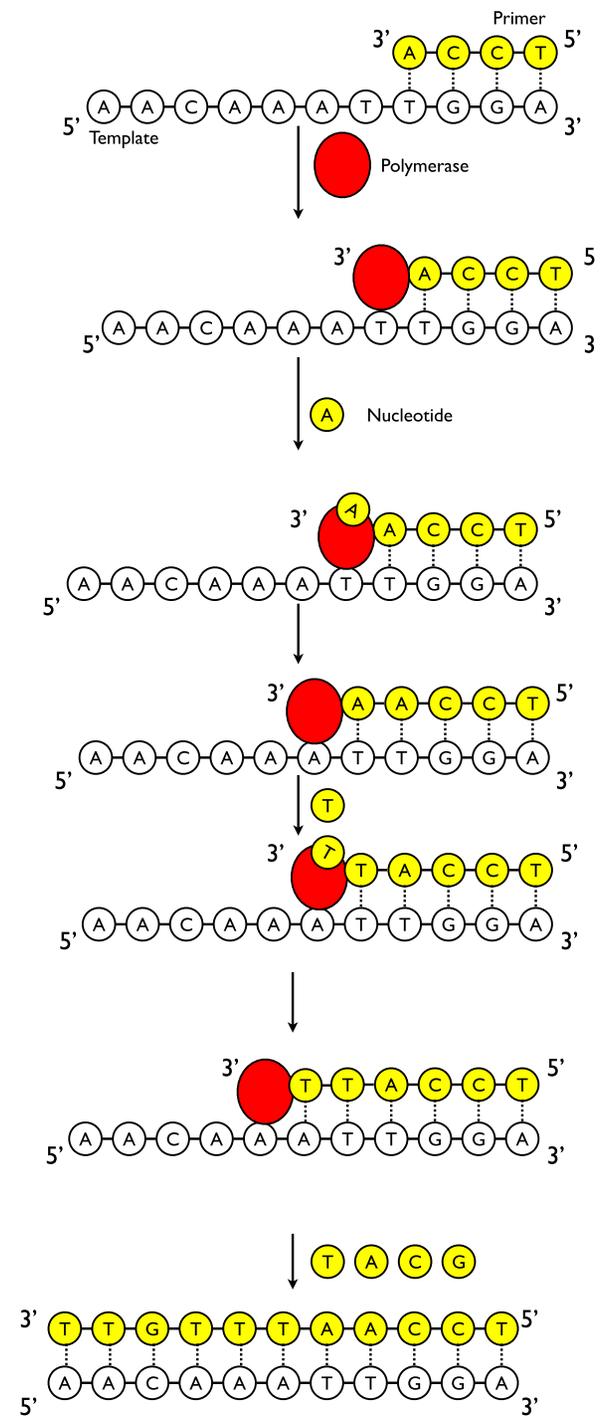




# Polymerization Reaction

## Primer Extension via Polymerase



extension of primer strand bound to the template by DNA polymerase

# PCR

**Polymerase Chain Reaction (PCR)** is a protocol used to amplify a template strand.

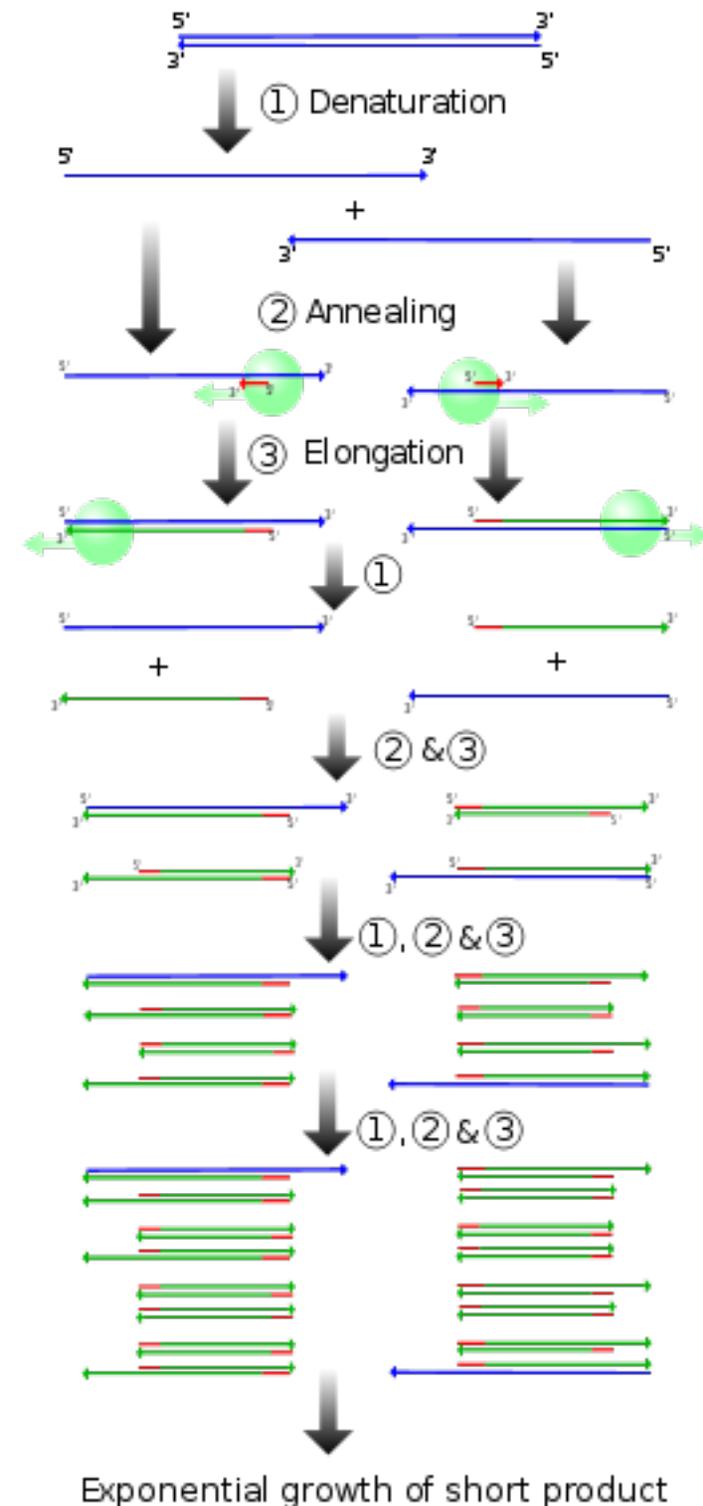
It uses repeated stages of thermal cycling between two temperatures  $t_1 < t_2$

**At temperature  $t_1$ :**

- a primer hybridizes to a segment of the template sequence and
- polymerase enzyme extends the primer sequence to form a complementary copy of the template sequence

**At temperature  $t_2$ :**

- the copied sequence melts off so both the original template sequence and the complementary copy can be used for further PCR cycles.



Exponential growth of short product

# Whiplash PCR

## Whiplash PCR

### History:

- Invented by Hagiya et al 1997]
- Improved by Erik Winfree 1998
- Made Isothermal by John Reif and Urmi Majumder 2008

# Whiplash PCR (WPCR)

**Whiplash PCR** is a protocol used to do computation using a single strand  $s$  of single stranded DNA consisting of  $n$  pairs of a primer sequence and an extension sequence, followed by a stop sequence (that stops the polymerization on each stage).

**Note:** multiple identical primer sequences may be paired with distinct extension sequences to allow for nondeterministic operation.

It uses repeated stages of thermal cycling between two temperatures  $t_1 < t_2$

**At temperature  $t_1$ :**

- The 3' end of  $s$  hybridizes to a primer segment of  $s$  and
- polymerase enzyme extends the 3' end of  $s$  to form a complementary copy of the corresponding extension sequence.

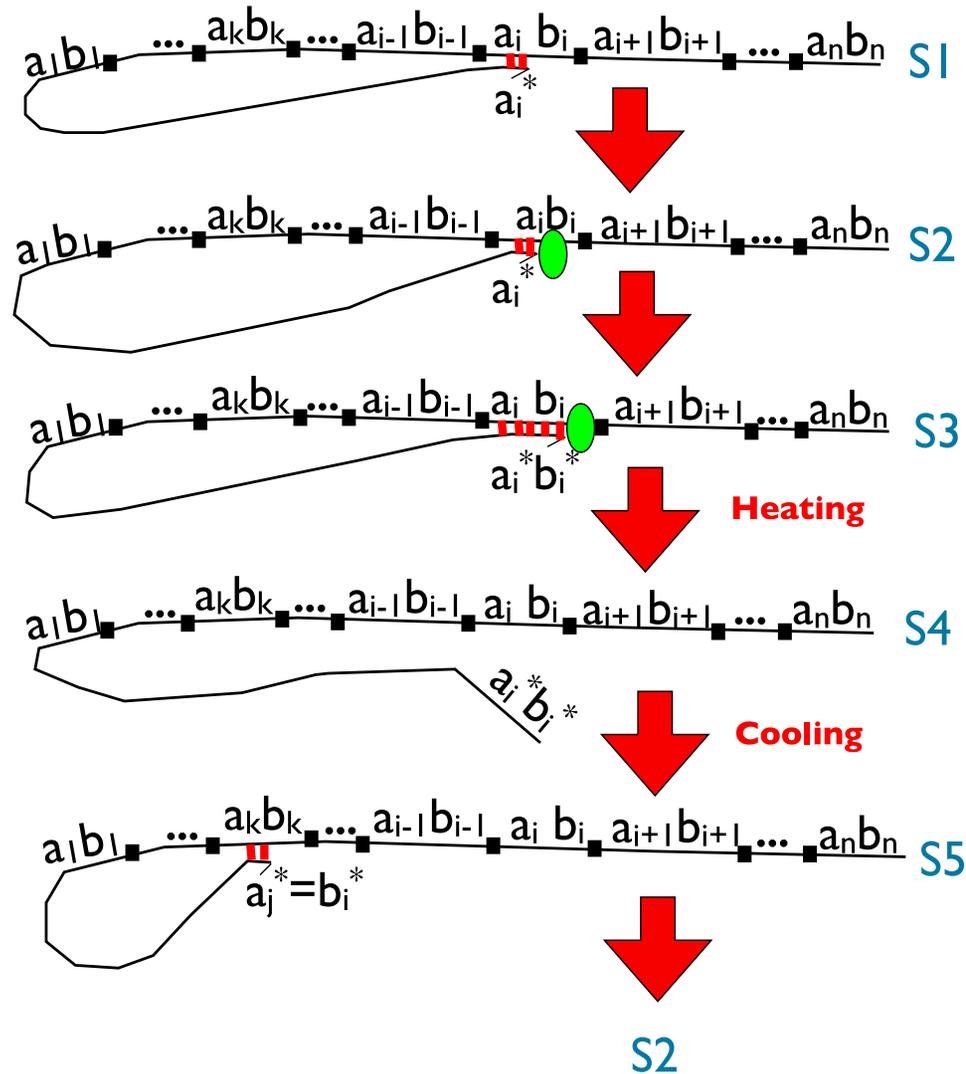
**At temperature  $t_2$ :**

- the copied sequence melts off the 3' end of  $s$  so a further stage of Whiplash PCR can be performed.

.

# Original Whiplash PCR Machine

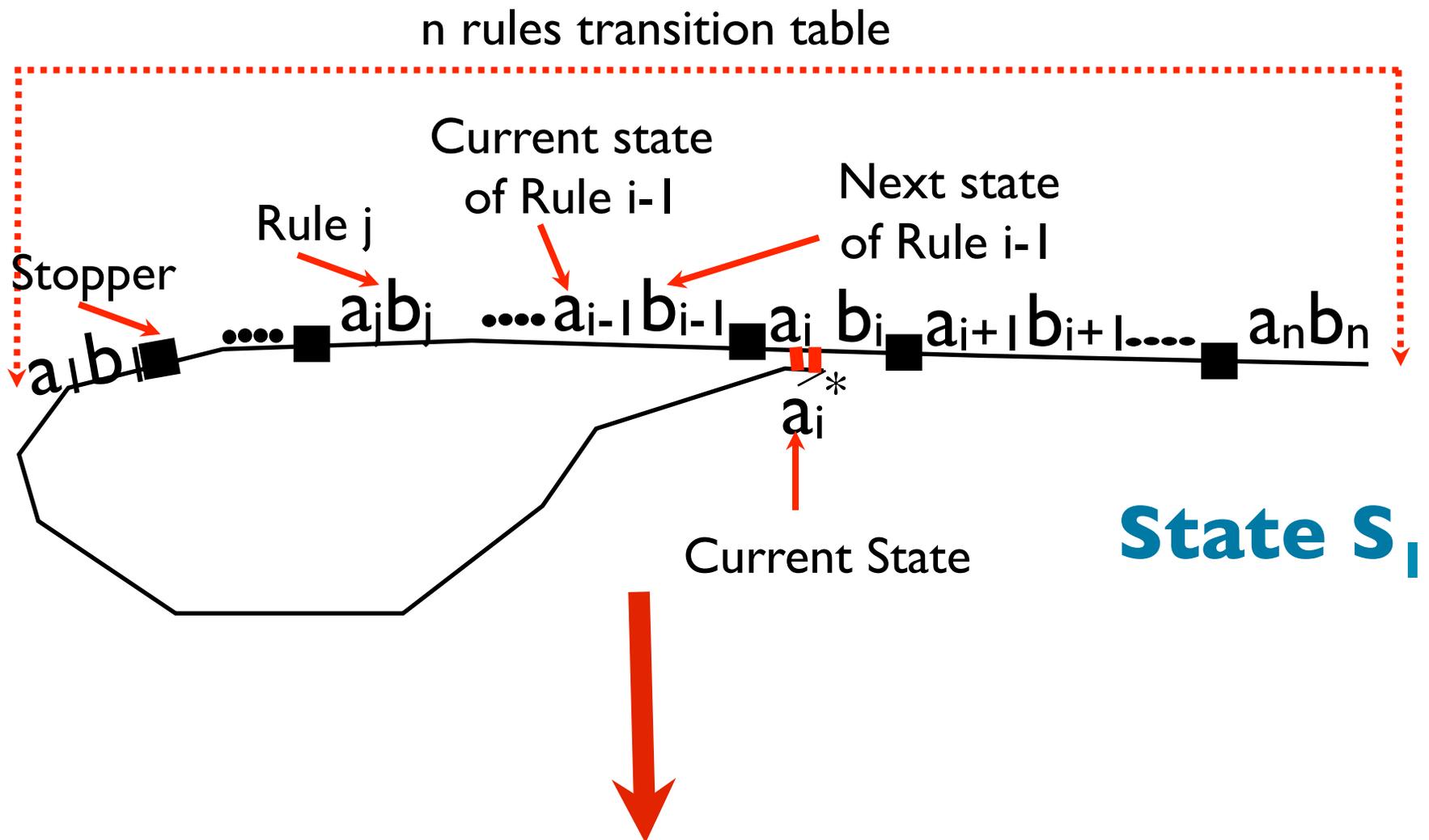
Reference: M Hagiya, M Arita, D Kiga, K Sakamoto and SYokomaya,  
DNA Based Computers III, pp:55-72, American Mathematical Society, 1999



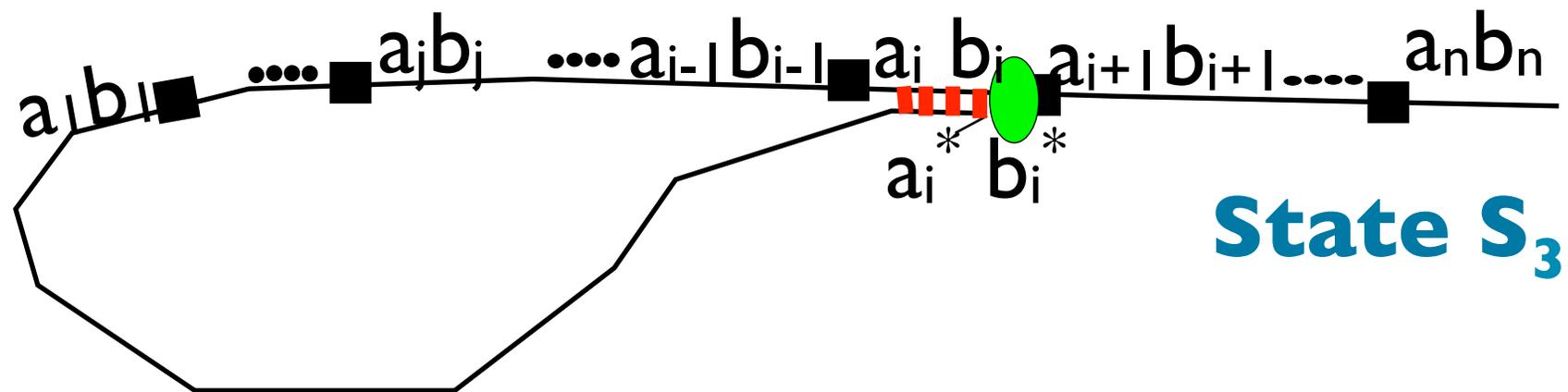
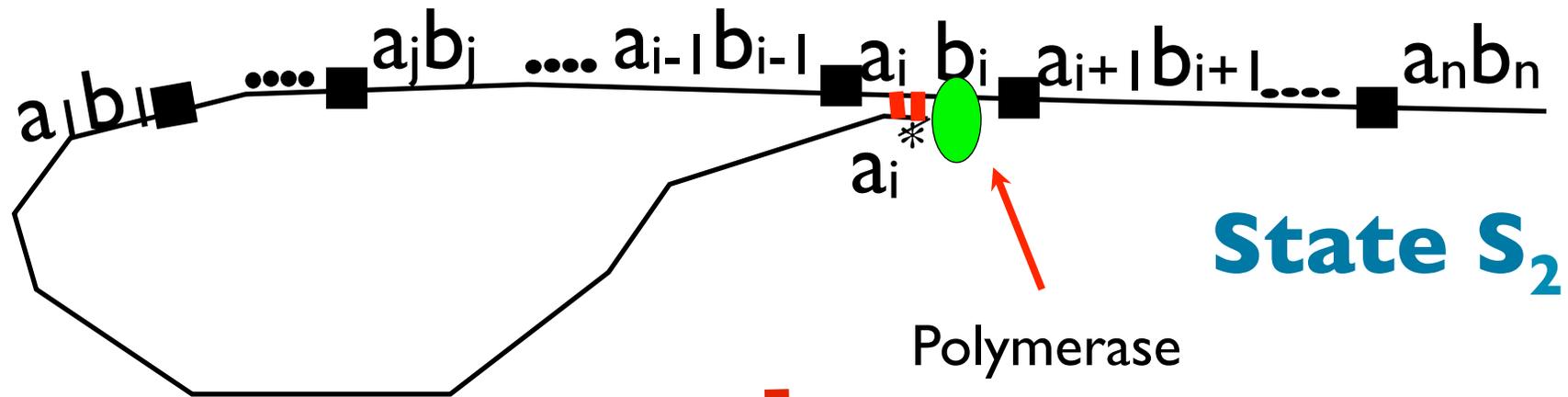
**Fig. 1** Schematic of the protocol for the original Whiplash PCR machine:  $S1$ : initial state of the WPCR strand  $W$  with current state being  $a_i^*$ .  $S2$ : polymerase binds to the 3' end of  $W$  (bearing the current state).  $S3$ : next state  $b_i^*$  is copied at the head of  $W$  by primer extension.  $S4$ : the mixture is heated so that  $W$  loses its hairpin structure.  $S5$ : the solution is cooled so that the head of  $W$  can bind to the new current state  $b_i^* = a_j^*$  encoded at the 3' end of the strand and the whole state transition repeats again beginning with State  $S2$

# Original Whiplash PCR Machine

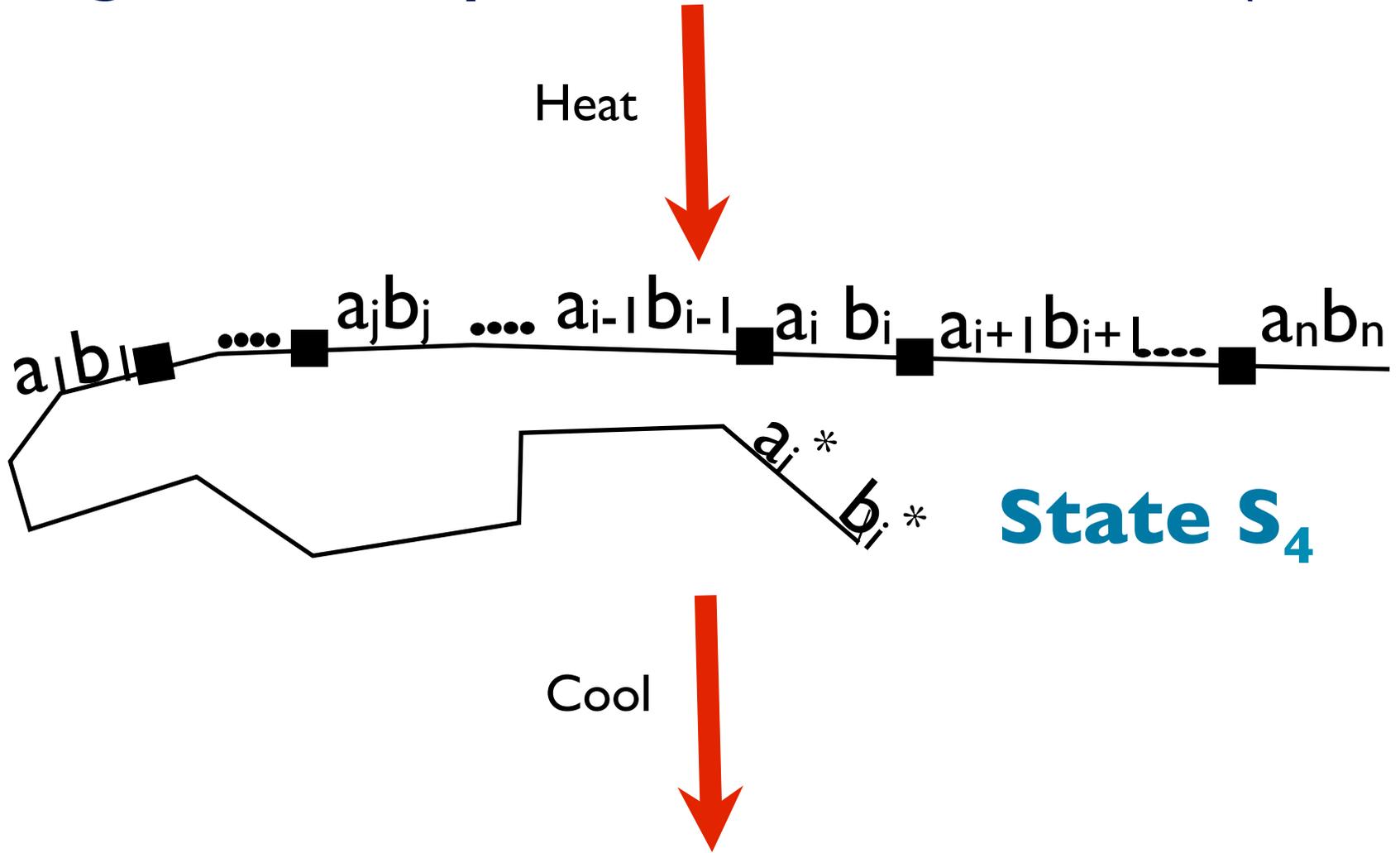
Reference: M Hagiya, M Arita, D Kiga, K Sakamoto and S Yokomaya,  
DNA Based Computers III, pp:55-72, American Mathematical Society, 1999



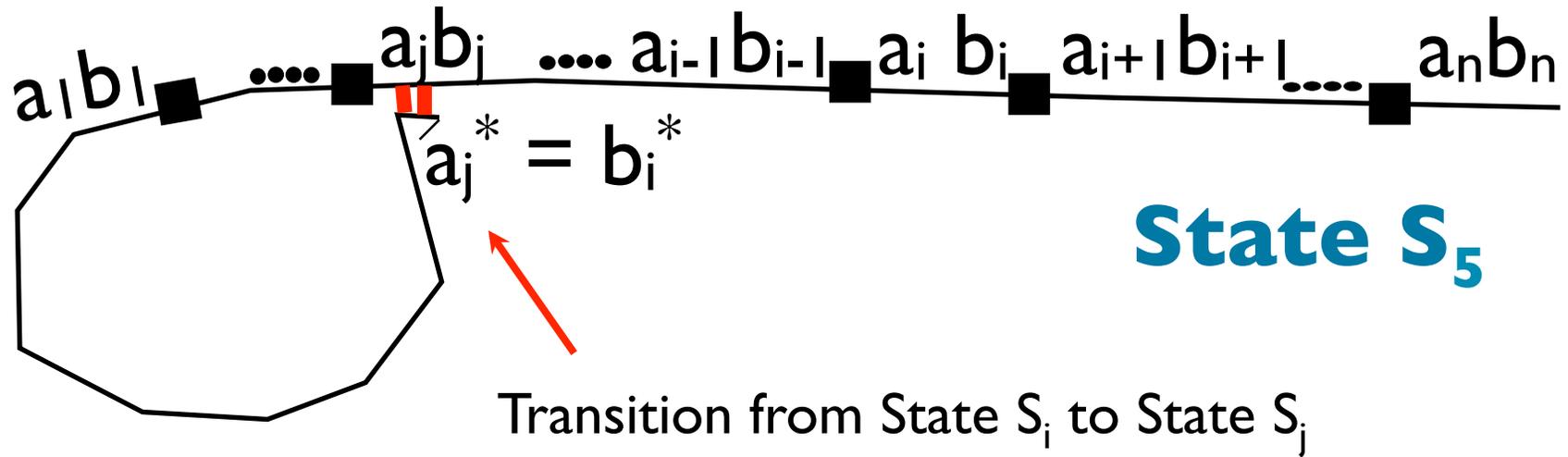
# Original Whiplash PCR Machine (Contd)



# Original Whiplash PCR Machine(Contd)



# Original Whiplash PCR Machine(Contd)



**State  $S_2$**

# Importance

- ◆ *Allows sequential molecular computations*
- ◆ *Also allows parallel execution of distinct programs*

*unlike other forms of molecular computation (e.g. tiling assembly):*

- ◆ *Each WPCR machine holds its own program*
- ◆ *Operation on local rules rather than global rules*

Note: Tiling assembly can be made to do multiple programs in parallel if we start with a universal cellular automata tile set with different seed rows. However, it is not very practical to generate such a large til

# Whiplash PCR

- **Applied to solve NP search Problems  
by Erik Winfree 1998**

# *Limitations of WPCR*

- ◆ Requires thermal cycling and hence its computing is **not isothermal**
- ◆ *Need a controlled laboratory environment*
- ◆ *No flexibility of application*
- ◆ **Back-hybridization**
- ◆ Program execution is limited to only a few steps

# *Previous techniques to address back-hybridization*

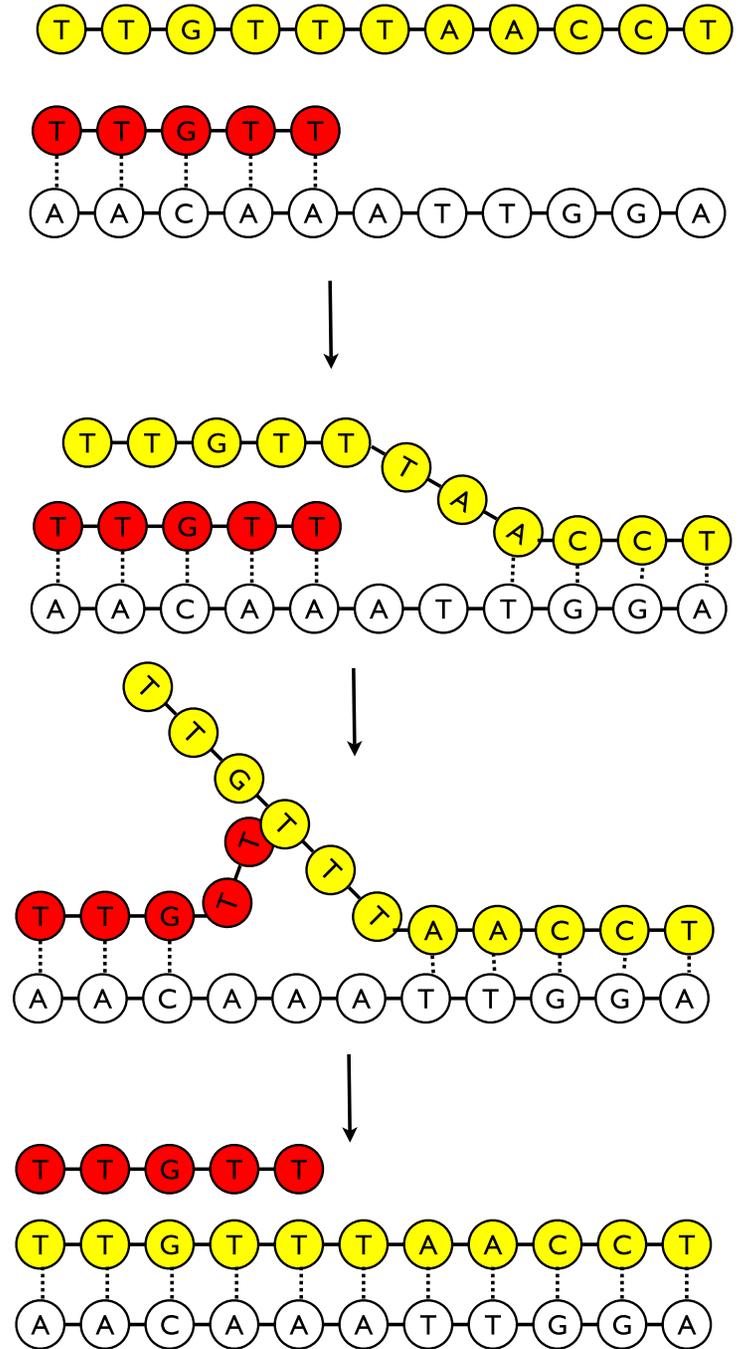
- ◆ Protocol with successive transitions in one step (Sakamoto et al., 1999):
  - ◆ did not significantly increase number of steps of program execution
- ◆ PNA Mediated WPCR (Rose et al., 2001):
  - ◆ not autocatalytic
- ◆ Displacement Whiplash PCR (Rose et al., 2006):
  - ◆ not autocatalytic

# *Need for isothermal & autocatalytic WPCR machine*

- ◆ Elimination of thermal cycles will allow more flexibility of applications
- ◆ Improve the yield of the system by minimizing back-hybridization



**Key technique  
to get system  
Isothermal:  
Strand Displacement**

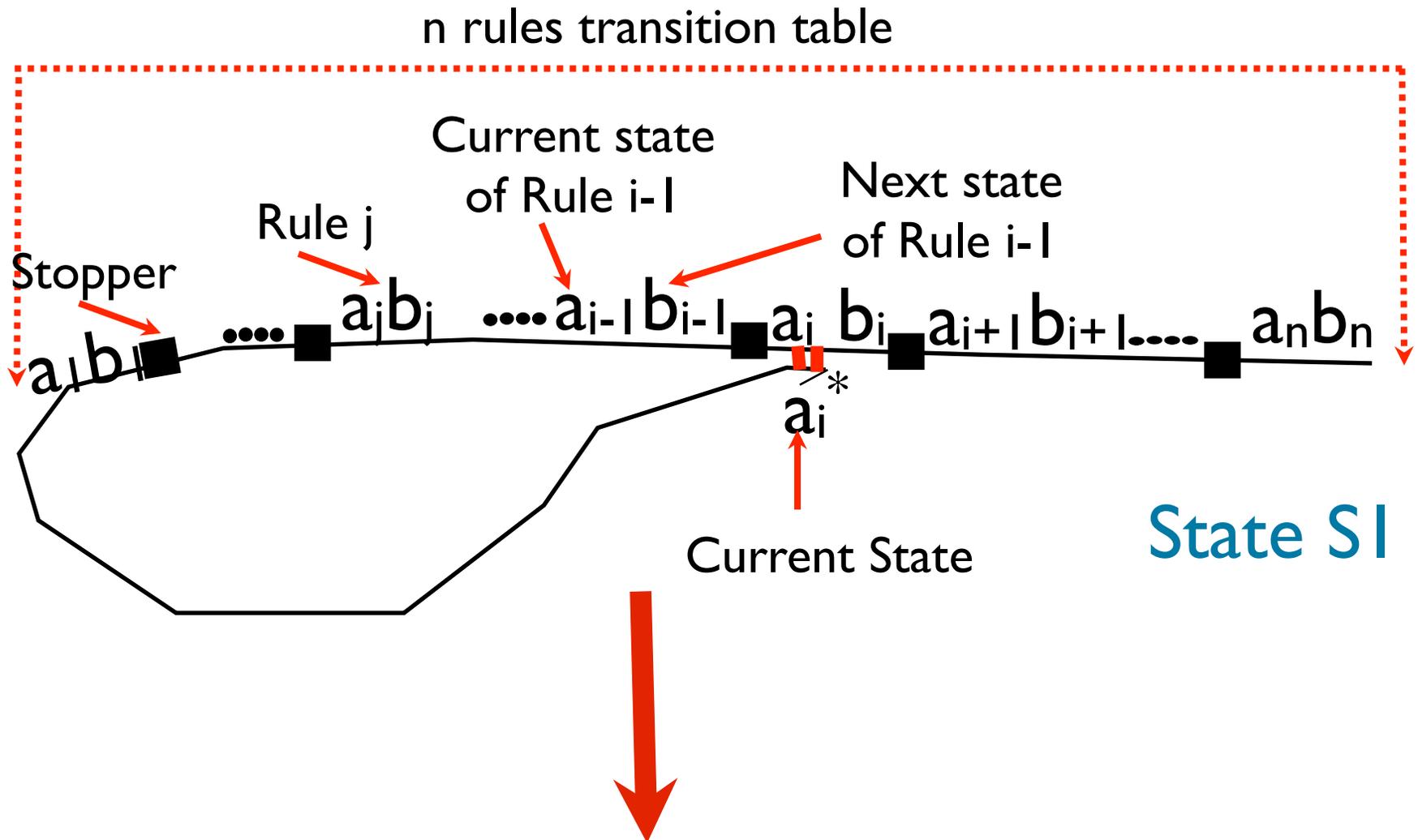


# Outline

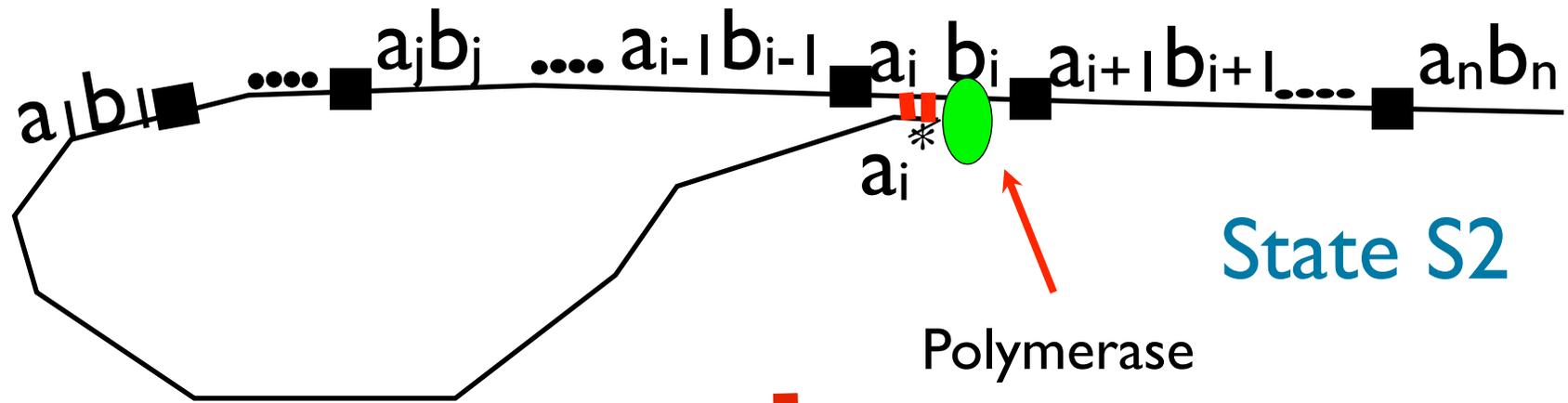
- ◆ Original Whiplash PCR (WPCR) Machine
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- ◆ Our Contribution: Isothermal and Reactivating WPCR (IR-WPCR) machine
  - ◆ IR-WPCR machine with non-reusable rules
  - ◆ IR-WPCR machine with reusable rules
  - ◆ Preparation Stage
- ◆ Proof of correctness of the system
- ◆ Experimental Verification Plan
- ◆ Conclusion

# Original Whiplash PCR Machine

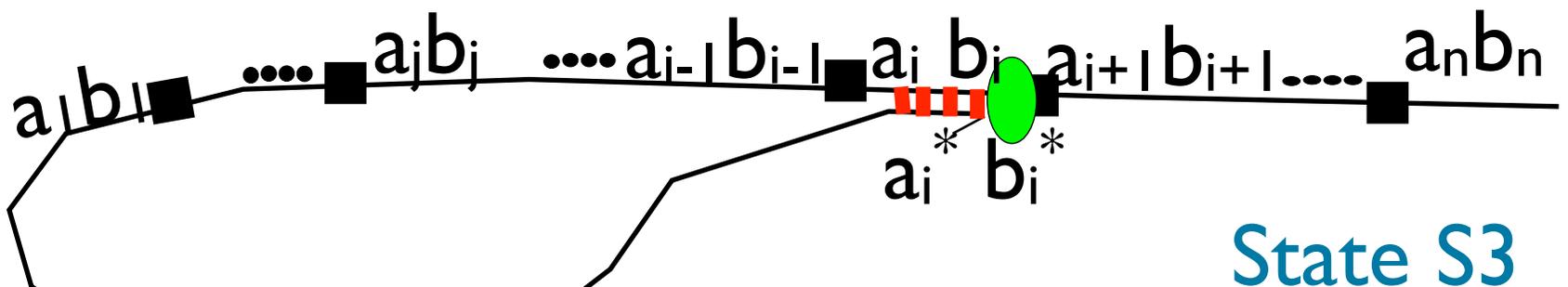
Reference: M Hagiya, M Arita, D Kiga, K Sakamoto and S Yokomaya,  
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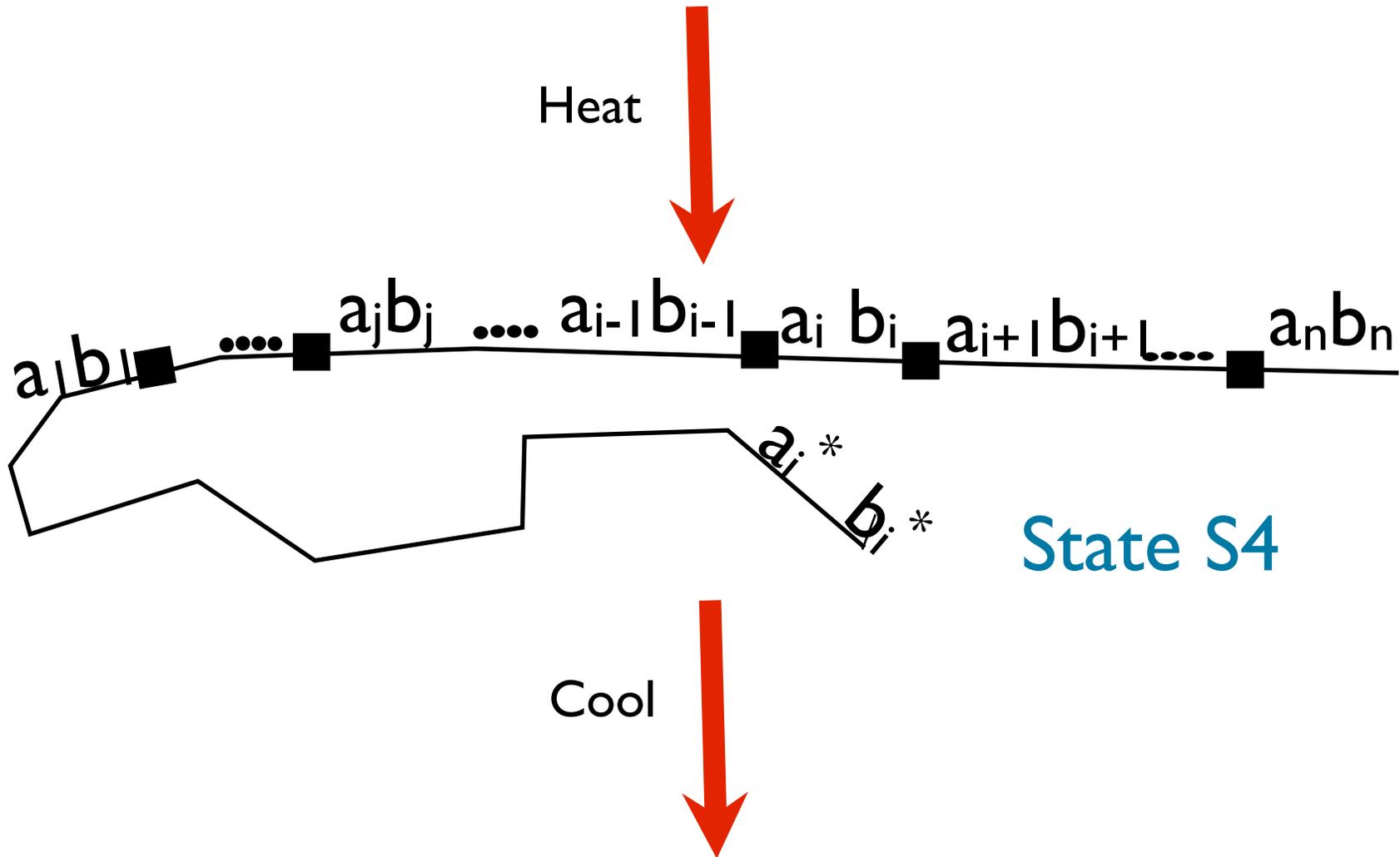
# Original Whiplash PCR Machine (Contd)



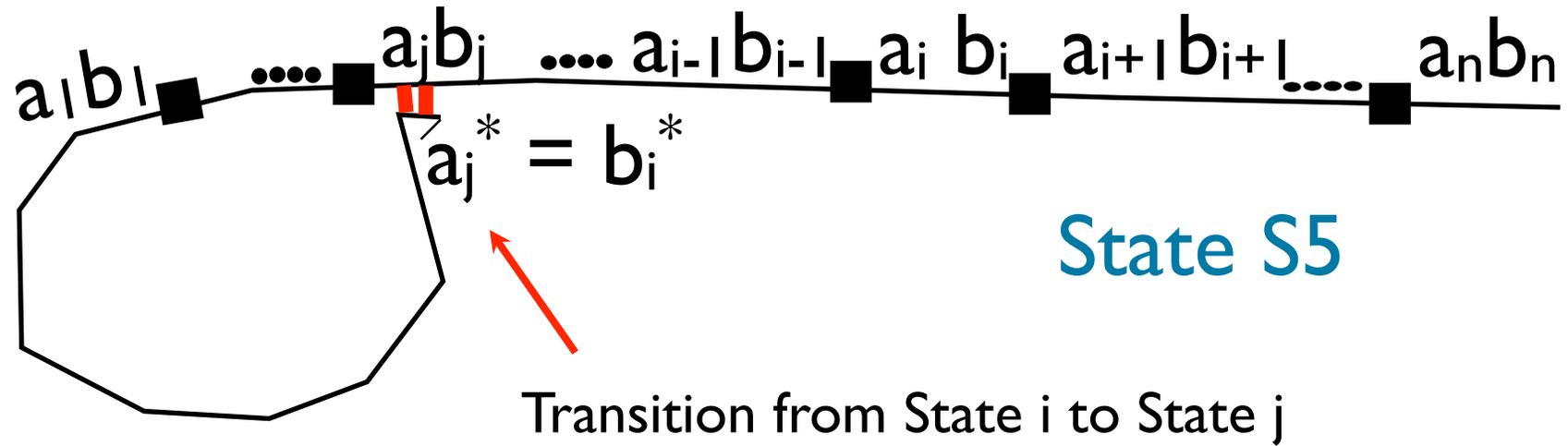
Next state copied



# Original Whiplash PCR Machine(Contd)



# Original Whiplash PCR Machine(Contd)



State S2

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# *Importance*

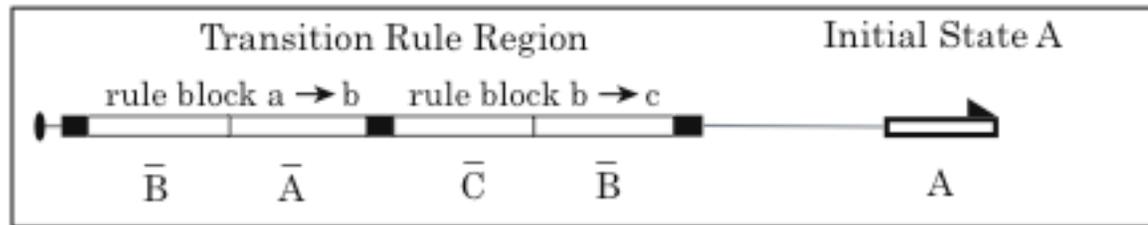
- ◆ *Allows autonomous molecular computation*
- ◆ *Allows parallel execution of distinct programs unlike other forms of molecular computation (e.g. tiling assembly)*
  - ◆ *Each WPCR machine holds its own program*
  - ◆ *Operation on local rules rather than global rules*

Note: Tiling assembly can be made to do multiple programs in parallel if we start with a universal cellular automata tile set with different seed rows. However, it is not very practical to generate such a large til

# Limitations

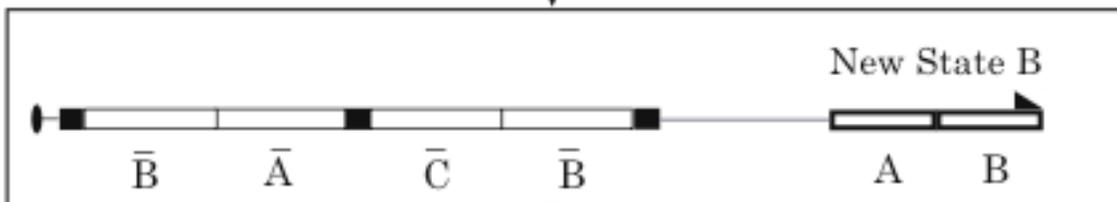
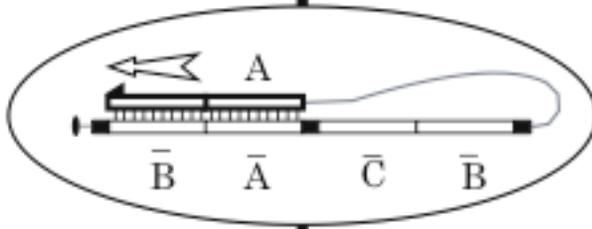
- ◆ Requires thermal cycling and hence its computing is **not isothermal**
- ◆ *Need a controlled laboratory environment*
- ◆ *No flexibility of application*
- ◆ **Back-hybridization**
- ◆ Program execution is limited to only a few steps

# Back-hybridization



Transition 1:  
(a → b)

(1)

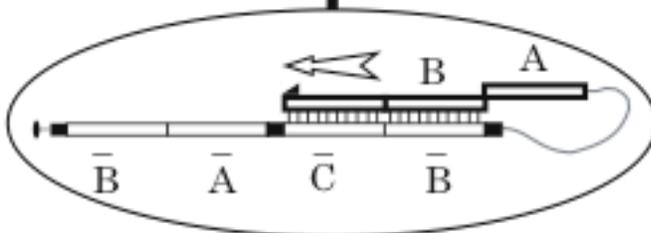


Backhybridization ( $p_2(bh) \cong 1$ )

Transition 2:  
(b → c)

$\epsilon_2 \cong 10^{-5}$

(2)



**Back-hybridization** is a phenomenon where a hairpin with a longer double stranded (ds) DNA region is preferentially formed over one with a shorter ds-DNA region.

# *Previous techniques to address back-hybridization*

- ◆ Protocol with successive transitions in one step (Sakamoto et al., 1999):
  - ◆ did not significantly increase number of steps of program execution
- ◆ PNA Mediated WPCR (Rose et al., 2001):
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# *Need for isothermal & autocatalytic WPCR machine*

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- ◆ Improve the yield of the system by minimizing back-hybridization

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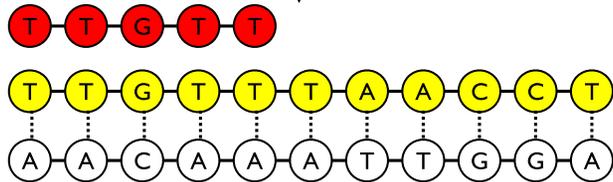
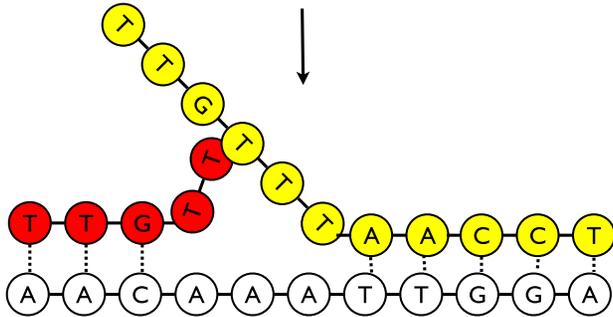
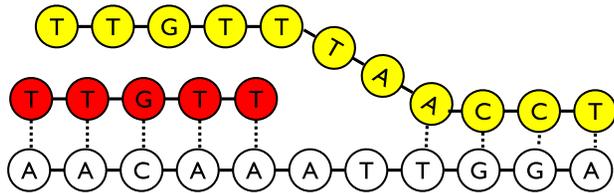
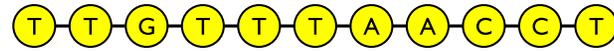
# *Isothermal Reactivating WPCR Machine*

- ◆ Addresses all the cons of a WPCR machine
- ◆ **Key concept:** use extension of a secondary primer by a DNA polymerase with good strand displacement capability to trigger state transition
- ◆ A non-isothermal preparation stage precedes the computation stage
- ◆ Two types:
  - ◆ IR-WPCR machine with non-reusable states
    - ◆ Prevents **back-hybridization**
  - ◆ IR-WPCR machine with reusable states
    - ◆ Original WPCR machine but isothermal

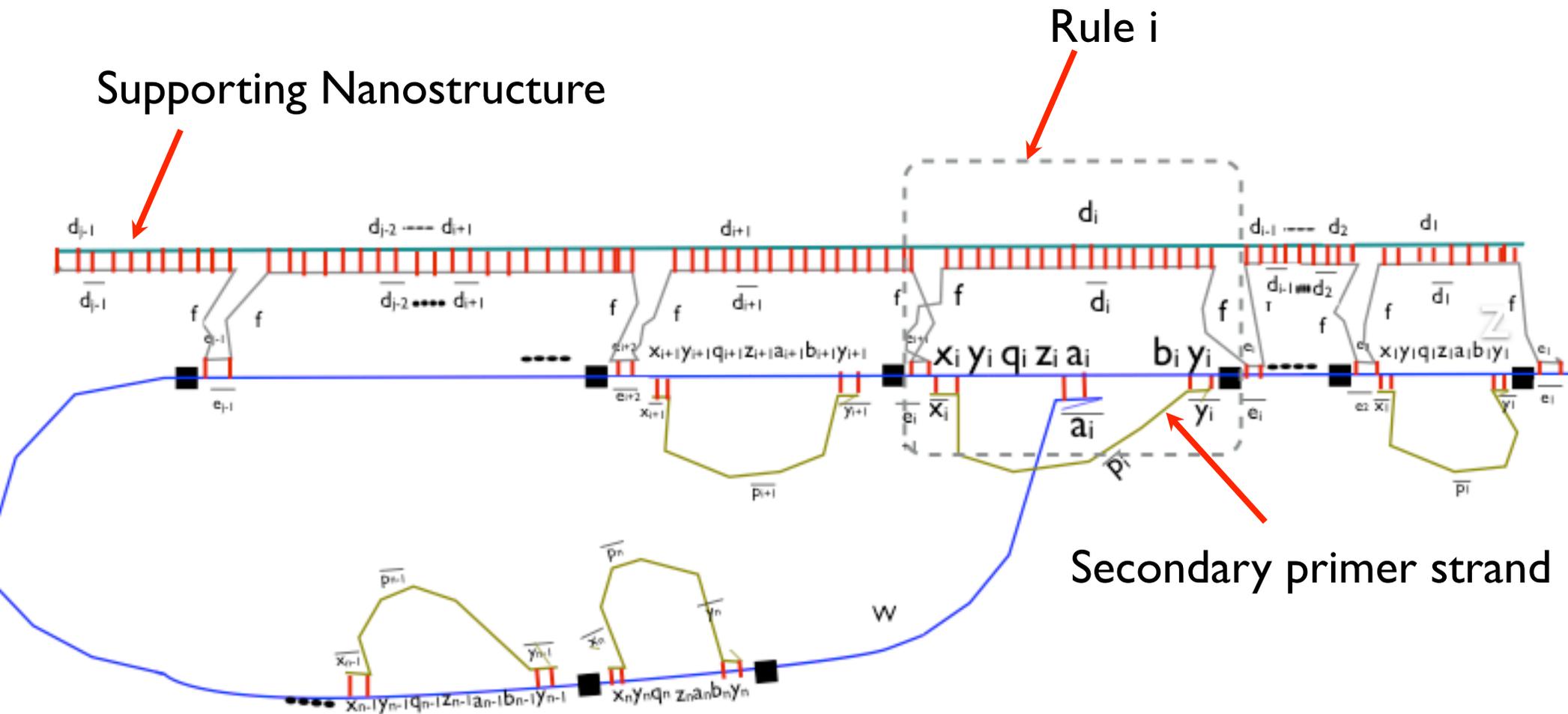
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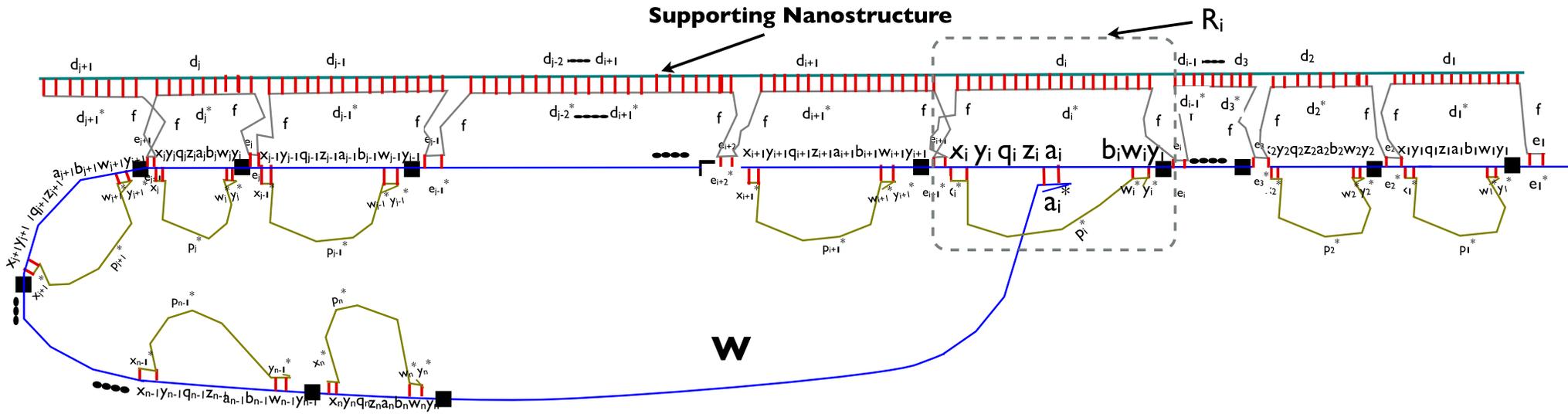
# Branch Migration



# IR-WPCR Strand after preparation stage

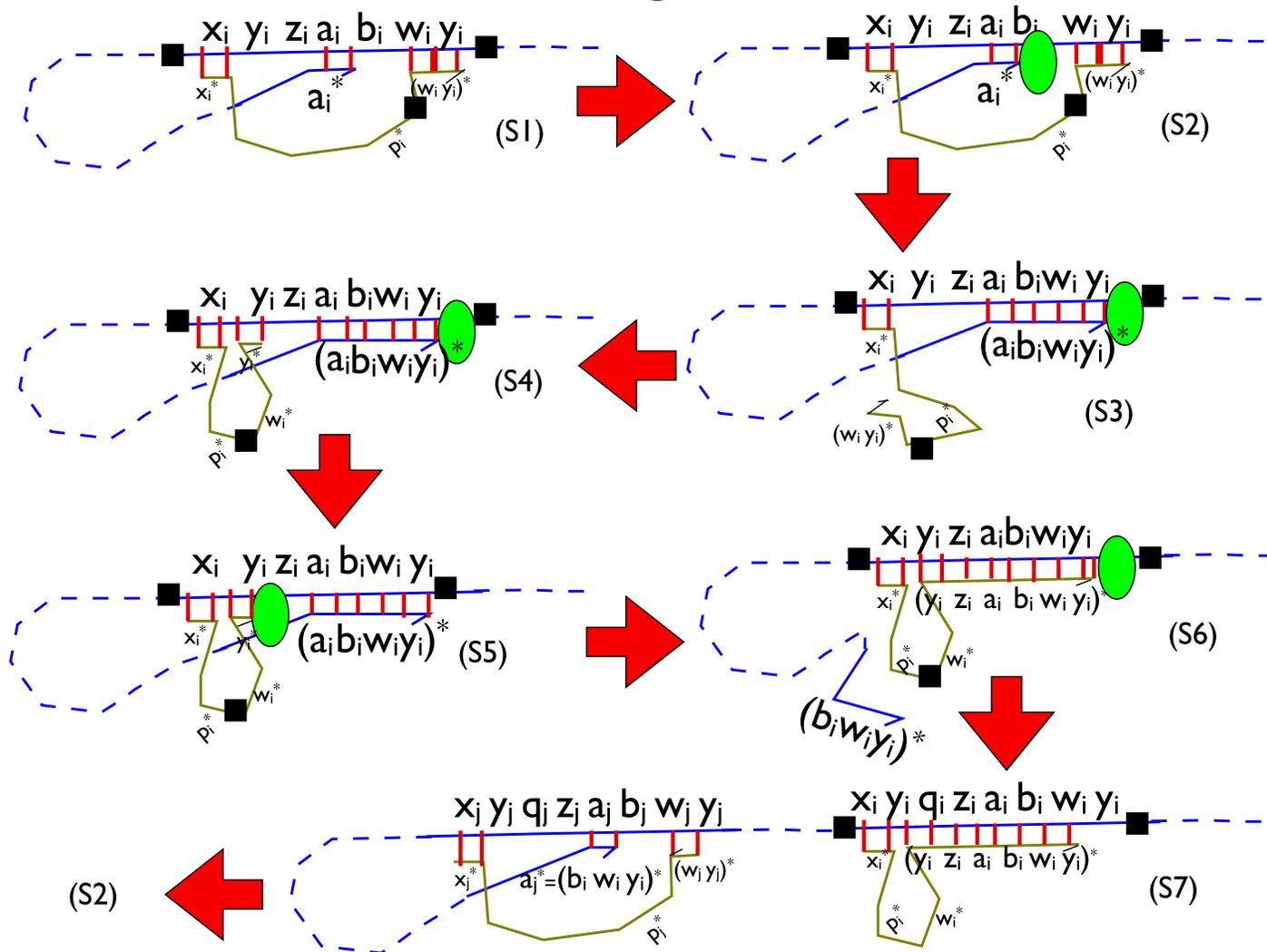


# Details of WPCR Strand for Isothermal execution



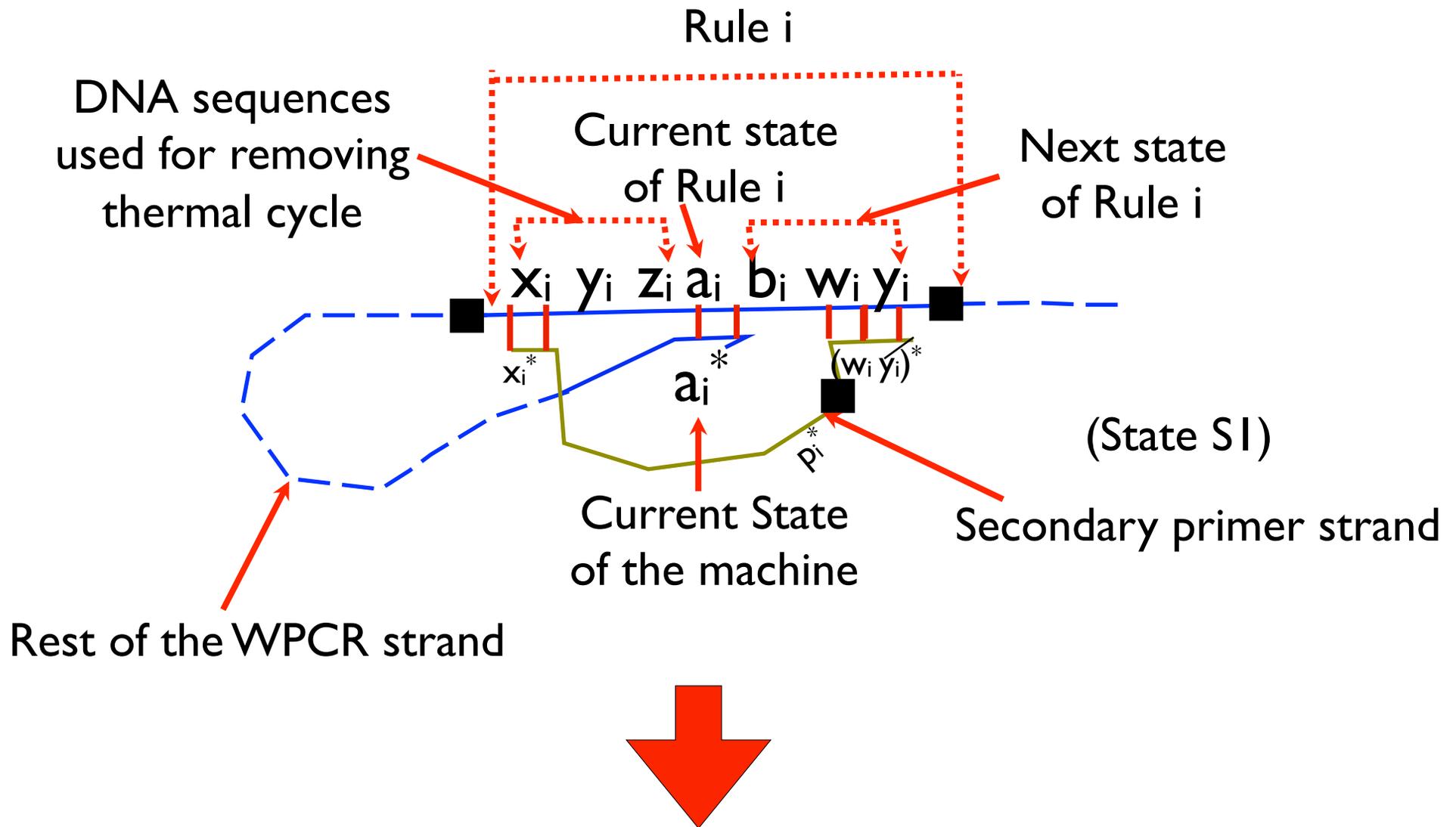
**Fig. 3** Complete WPCR Strand for isothermal and autocatalytic program execution (Rule  $R_i$  on focus). Although details are provided in this figure, the emphasis is on the layout of the overall strand. In particular, note that most of the strand representing the transition rules is stabilized using a supporting DNA nanostructure and only the current state of the machine is allowed to freely bind to an appropriate rewrite rule using a lag region  $W$

# Evaluation Stage for Non-Reusable Rules

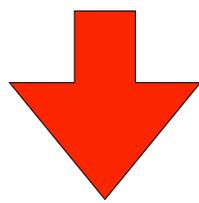
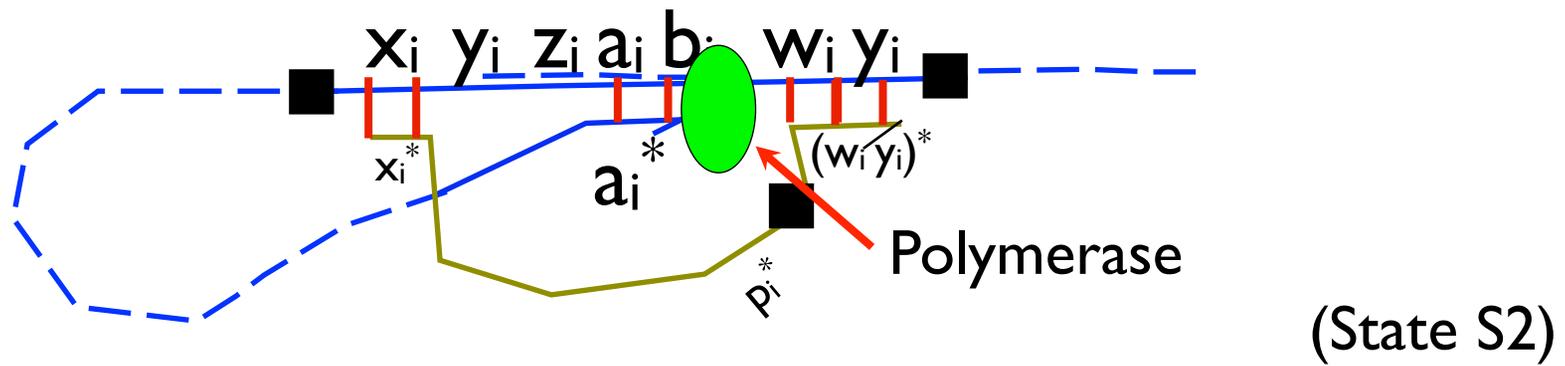


**Fig. 5** Evaluation stage for non-reusable rules IR-WPCR protocol with the focus being only on the transition rule  $R_i$  to which the current state is hybridized:  $S1$  WPCR strand  $W$  with protection strand  $P_i$  encoded as  $(x_i p_i y_i)^*$  partially hybridized with rule  $R_i$ . Also the 3' end of  $W$ , bearing the current state  $a_i^*$  is hybridized to  $a_i$  of  $R_i$ .  $S2$ : polymerase binds to the 3' end of  $W$ .  $S3$ : polymerase extends  $a_i^*$  to copy  $b_i w_i y_i$ , thus displacing  $w_i^* y_i^*$  of  $P_i$  from  $w_i y_i$  of rule  $R_i$  located further away from  $x_i$  in  $R_i$ .  $S4$ :  $y_i^*$  of  $P_i$  binds to  $y_i$  located next to  $x_i$  in  $R_i$ .  $S5$ : polymerase binds with the 3' end of  $P_i$ .  $S6$ : 3' end of  $P_i$  is extended by the polymerase to copy  $z_i a_i b_i w_i y_i$ , thus displacing 3' end of  $W$  which has the new current state  $a_j = b_i w_i y_i$ .  $S7$ : 3' end of  $W$  bearing  $a_j^*$  binds to the  $a_j$  in rule  $R_j$  and the process repeats starting with the polymerase binding to the 3' end of  $W$  as shown in State S2

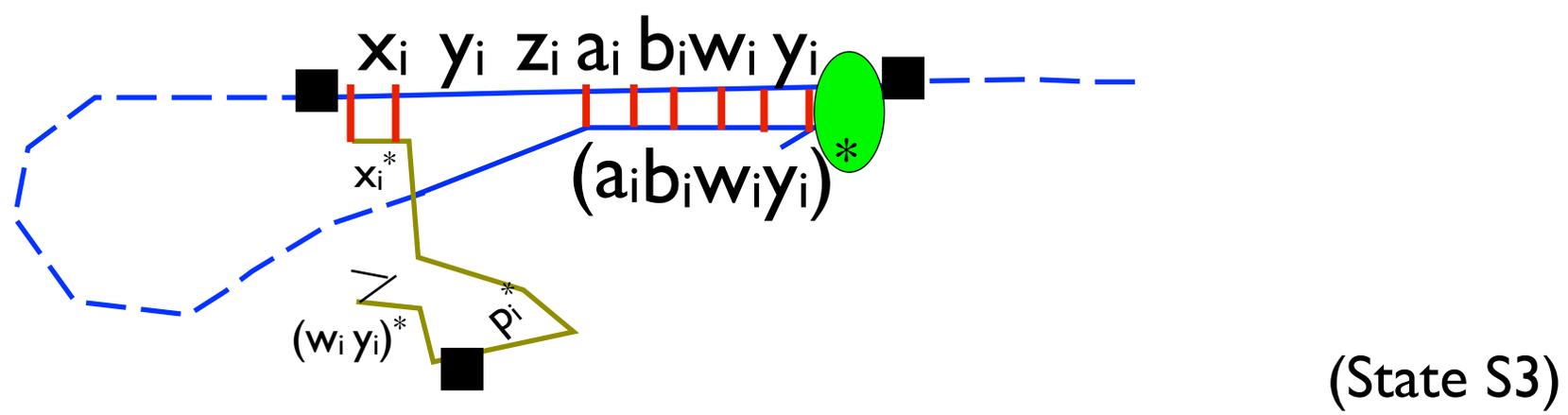
# IR-WPCR machine with non-reusable states



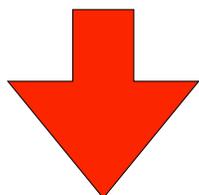
# IR-WPCR machine with non-reusable states



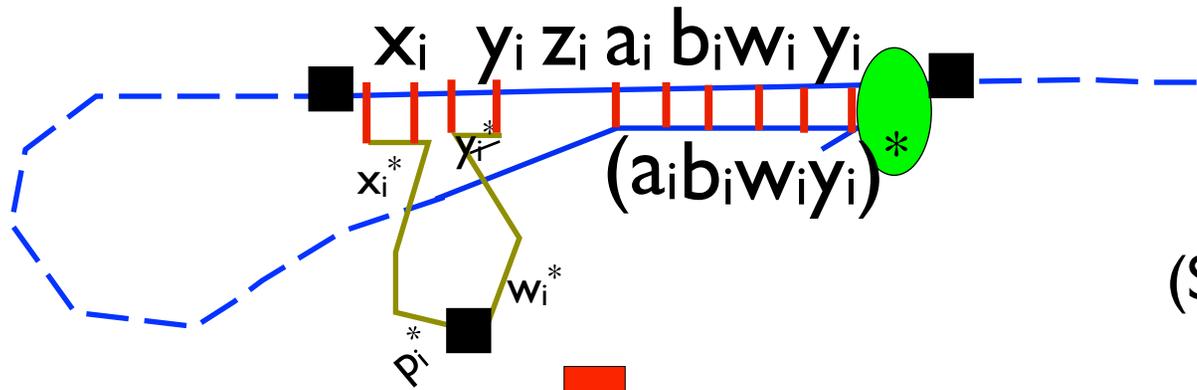
Next state copied while displacing the secondary primer



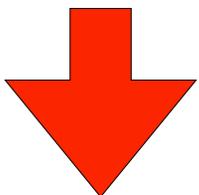
# IR-WPCR machine with non-reusable states



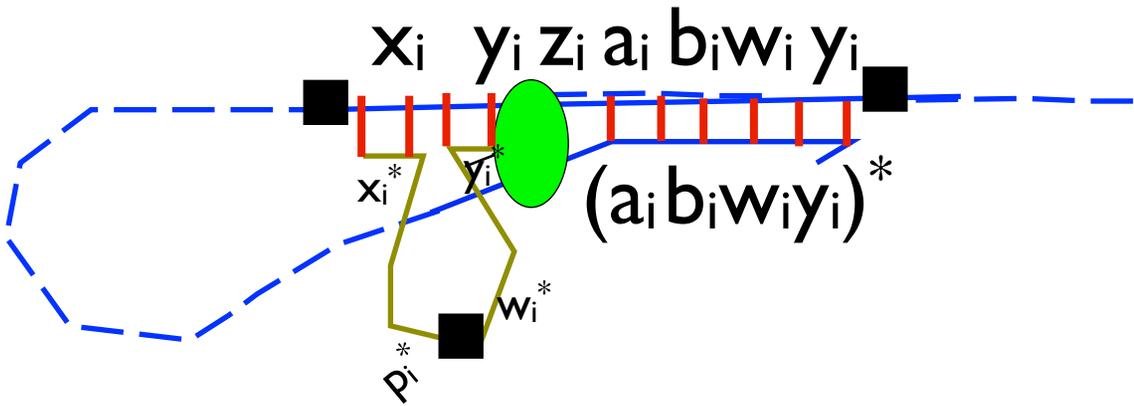
Secondary primer binds to the second best location in the neighborhood



(State S4)

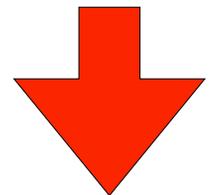


(Polymerase binds to the 3' end of bound secondary primer)

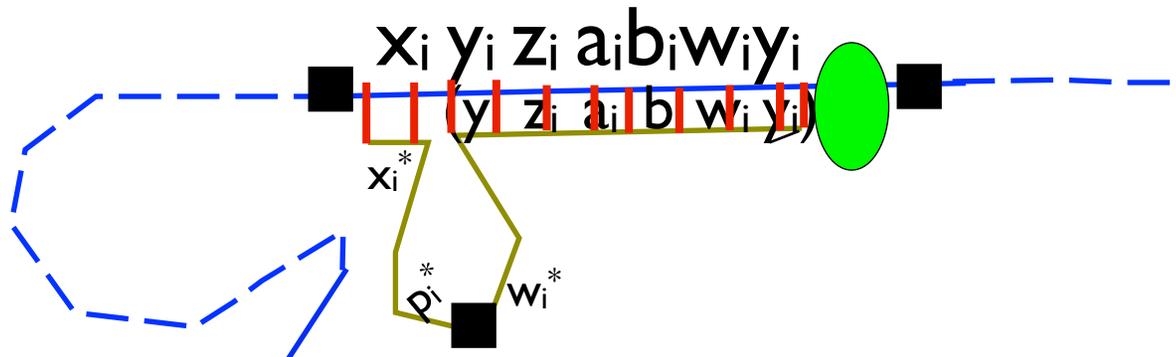


(State S5)

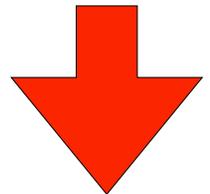
# IR-WPCR machine with non-reusable states



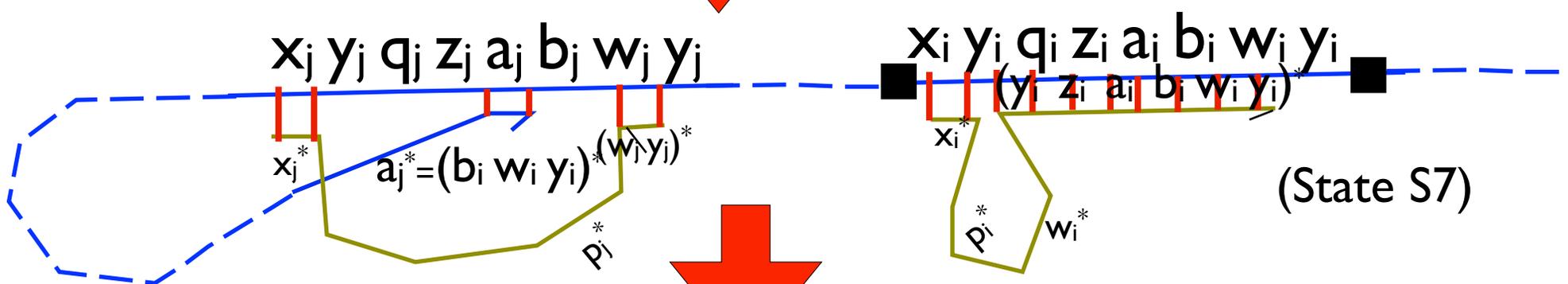
Secondary primer extended to stopper  
displacing 3' end of WPCR strand



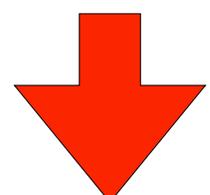
(State S6)



3' end of WPCR strand binds to  
appropriate rule (state transition)



(State S7)



(State S2)

$(b_i w_i y_i)^*$

$x_j y_j q_j z_j a_j b_j w_j y_j$

$x_j^*$   $a_j^* = (b_i w_i y_i)^* (w_i y_i)^*$

$x_i y_i q_i z_i a_i b_i w_i y_i$

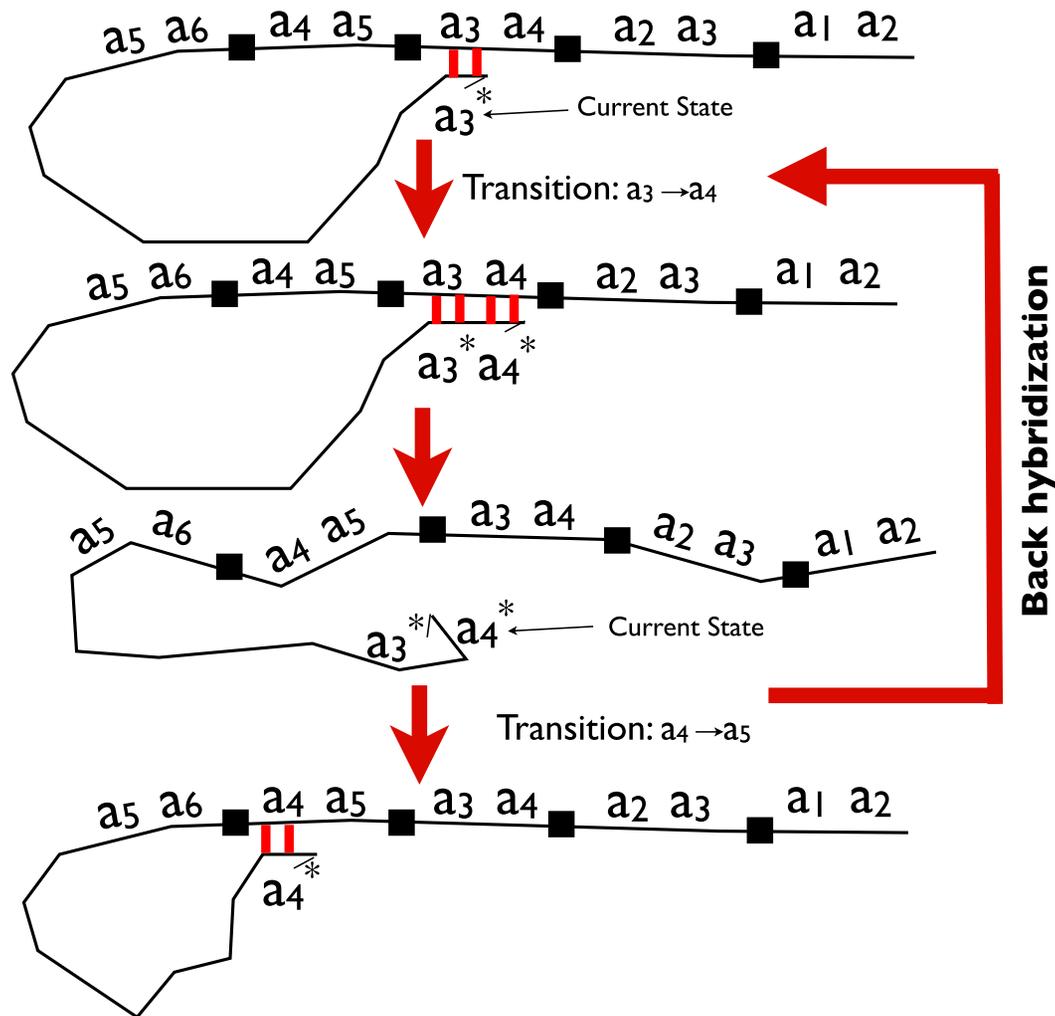
$(y_i z_i a_i b_i w_i y_i)^*$

# IR-WPCR machine with non-reusable states

## Pros & Cons

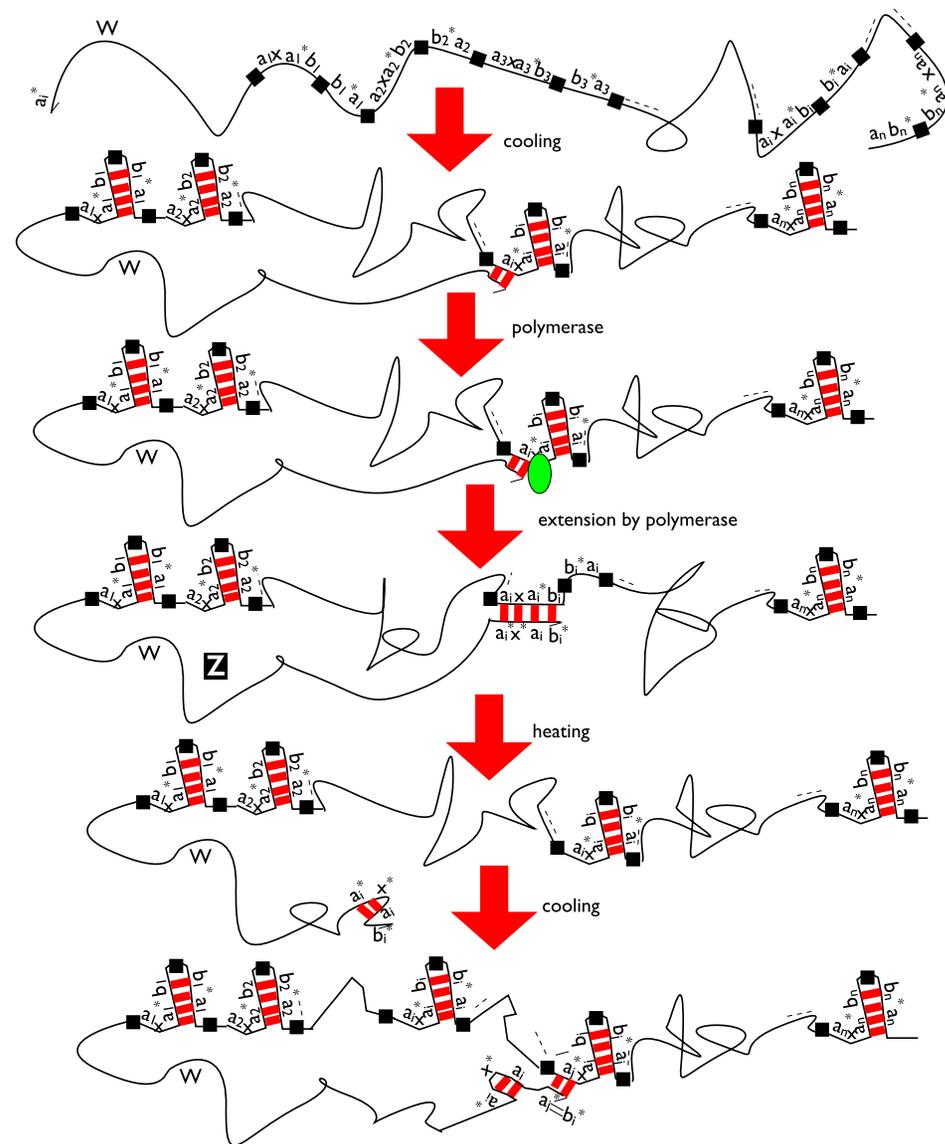
- ◆ **Pros of IR-WPCR with non-reusable states:**
  - ◆ Prevents Back-hybridization since rule once used is not available any more
    - ◆ Isothermal
- ◆ **Cons of IR-WPCR with non-reusable states:**
  - ◆ Rule cannot be reused
  - ◆ Needs redundant encodings of a rule for complex finite state machine
- ◆ IR-WPCR Machine with reusable states has all the power of the original WPCR machine and yet operates **isothermally**

# Back-Hybridization



**Fig. 4** *Back-hybridization*: transition from state  $a_3$  to state  $a_4$  happens as usual but for the next transition  $a_4$  to  $a_5$ , the 3' end of the machine preferentially binds with the old transition rule. This is because  $a_3^*$  along with  $a_4^*$  at the 3' end of the machine has a longer hybridization region when bound with rewrite rule  $a_3 \rightarrow a_4$  compared to when only  $a_4^*$  binds with the current state of the rewrite rule  $a_4 \rightarrow a_5$ . Consequently, the machine is stuck in state  $a_4$

# Protocol for Folding Whiplash PCR to avoid back-hybridization

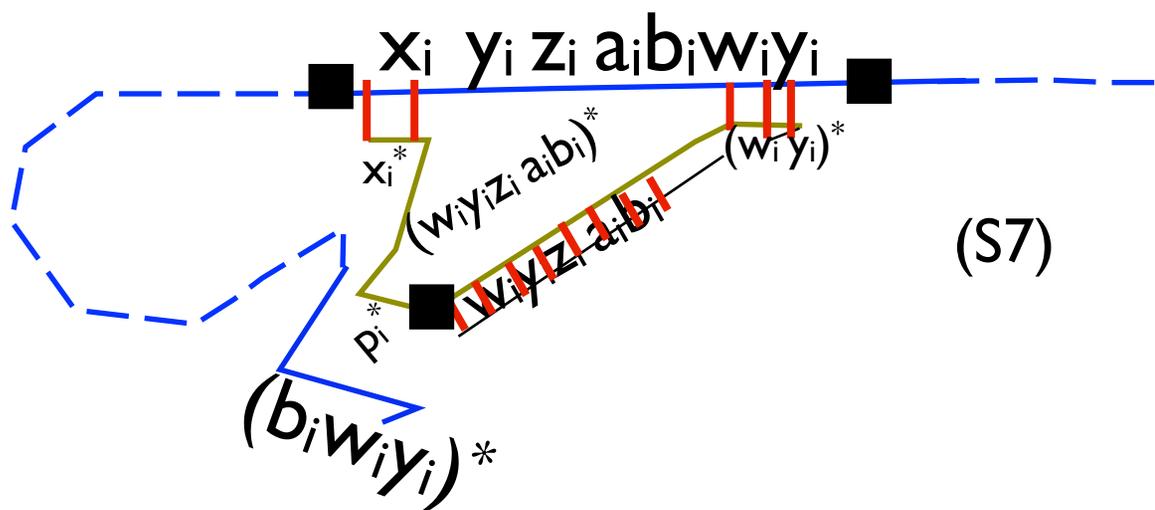
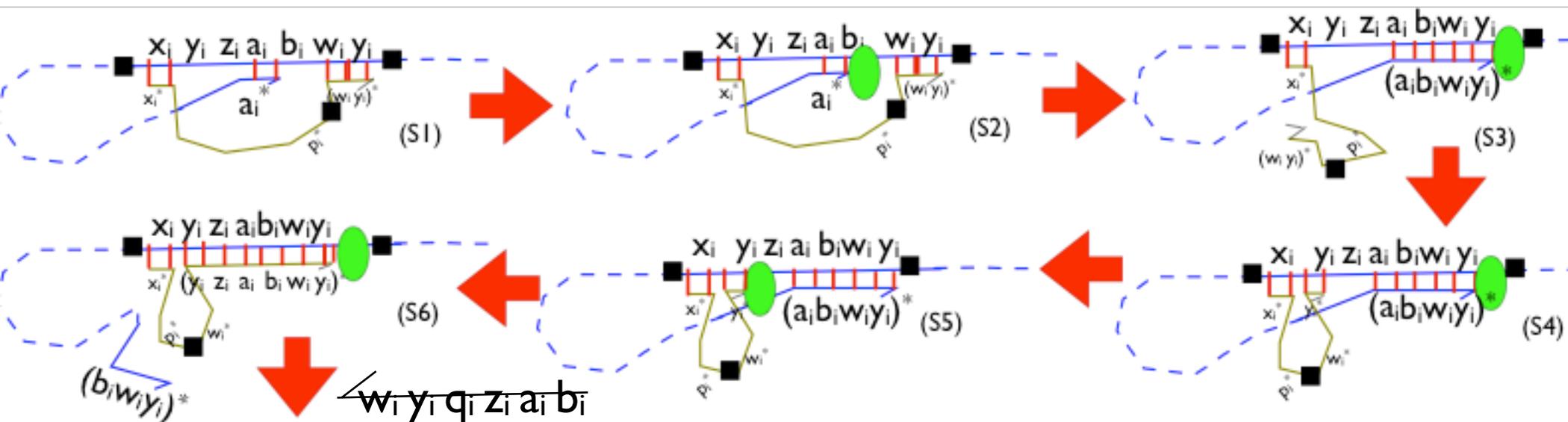


**Fig. 8** Schematic of the protocol for the folding Whiplash PCR machine: *S1*: initial state of the WPCR strand  $W$ . *S2*: the solution is heated such that the next state in each rule hidden in a hairpin loop with current state of the machine being  $a_i^*$ . *S3*: polymerase binds to the 3' end of  $W$  (bearing the current state). *S4*: next state  $b_i^*$  is copied at the head of  $W$  by primer extension and hairpin loop is opened. *S5*: the mixture is heated so that  $W$  loses its hairpin structure (It may even open up the individual hairpin loops in each rule, not shown here). *S6*: the solution is cooled so that the head of  $W$  can bind to the new current state  $b_i^* = a_j^*$  encoded at the 3' end of the strand and the whole state transition repeats again beginning with State S2. Note that the next state in each rule is hidden in a stem loop as is the old current state encoded at the 3' end of the WPCR strand. These two stem loop formations are key to preventing back-hybridization in this protocol

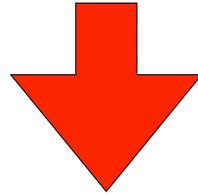
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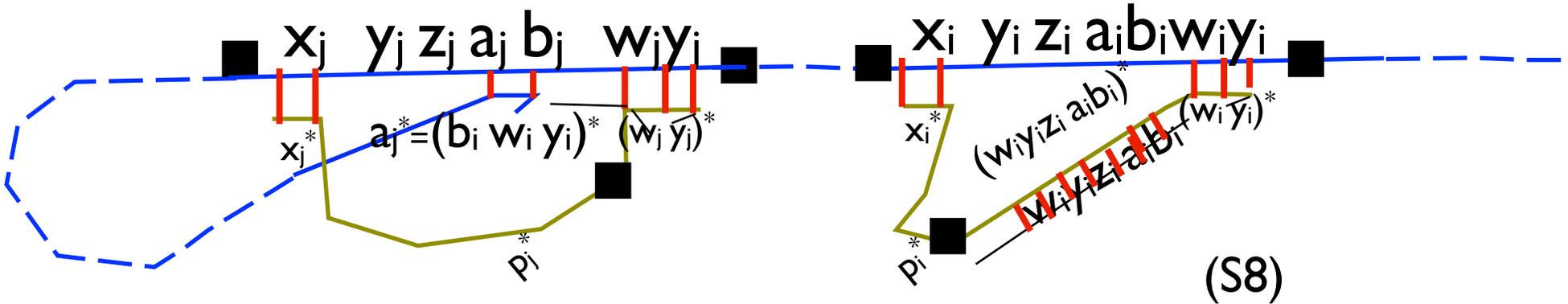
# IR-WPCR machine with reusable states



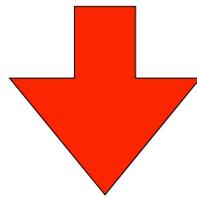
# IR-WPCR machine with reusable states



Transition from state i to state j

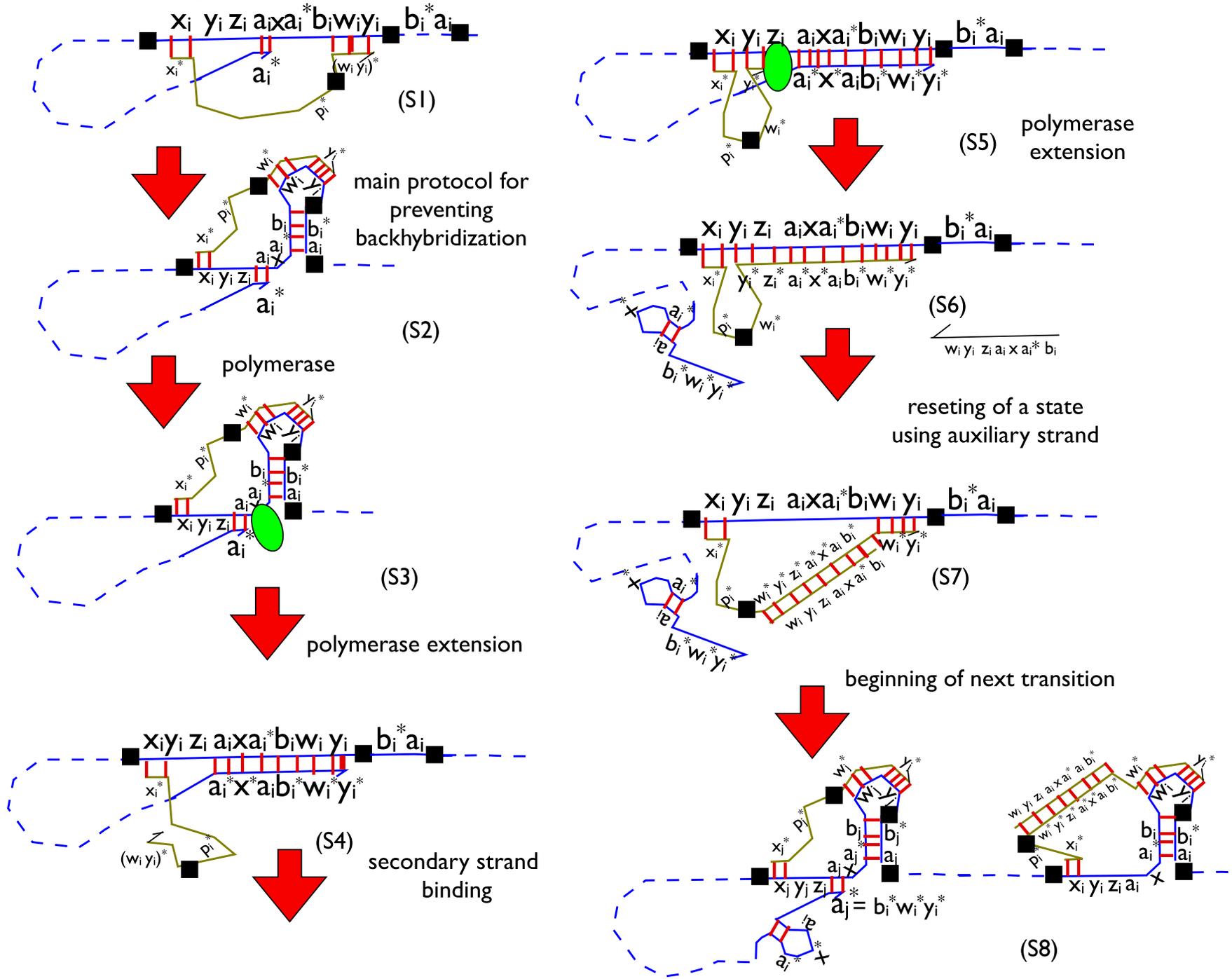


(S8)



(S2)

# Summary IR-WPCR machine with reusable states



# *IR-WPCR machine with reusable states*

## *Pros & Cons*

- ◆ **Pros of IR-WPCR with non-reusable states:**

- ◆ Isothermal

- ◆ States reusable allowing us to build complex finite state machines

- ◆ **Cons of IR-WPCR with non-reusable states:**

- ◆ Back-hybridization

# *Handling of inputs in IR-WPCR machine*

- ◆ Each input can be encoded between current and next state
- ◆ Symbols in input encoded uniquely to maintain sequentiality
- ◆ External input ligated at the 3' end of WPCR strand at the start of the corresponding state transition

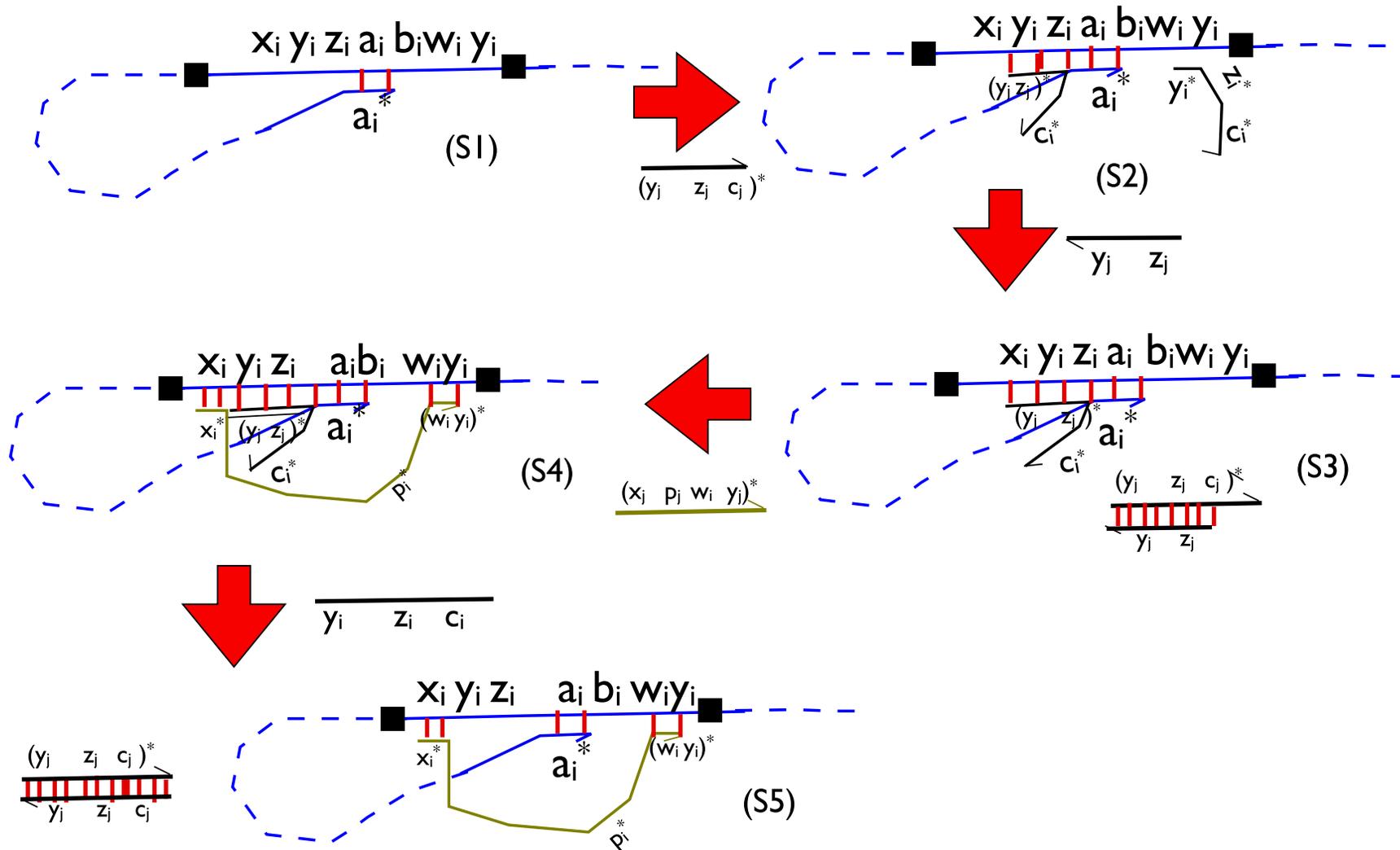
# Outline

- ◆ Original Whiplash PCR (WPCR) Machine
- ◆ Pros and Cons of the original WPCR Machine
- ◆ Our Contribution: Isothermal and Reactivating WPCR (IR-WPCR) machine
  - ◆ IR-WPCR machine with non-reusable rules
  - ◆ IR-WPCR machine with reusable rules
  - ◆ **Preparation Stage**
- ◆ Proof of correctness of the system
- ◆ Experimental Verification Plan
- ◆ Conclusion

# *Preparation Protocol*

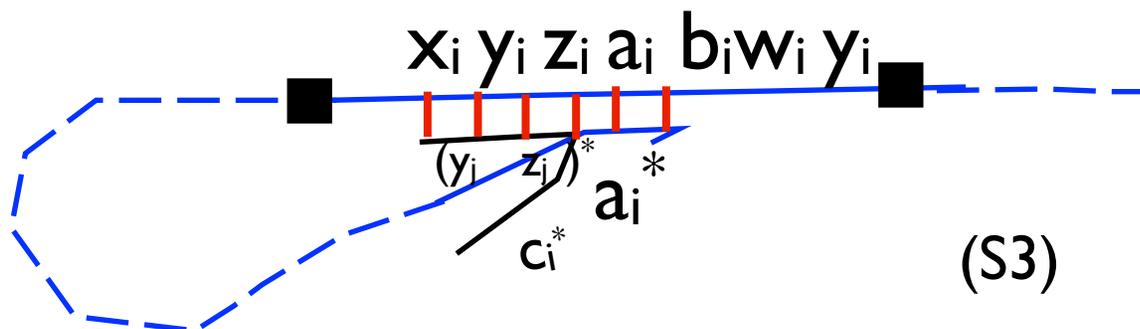
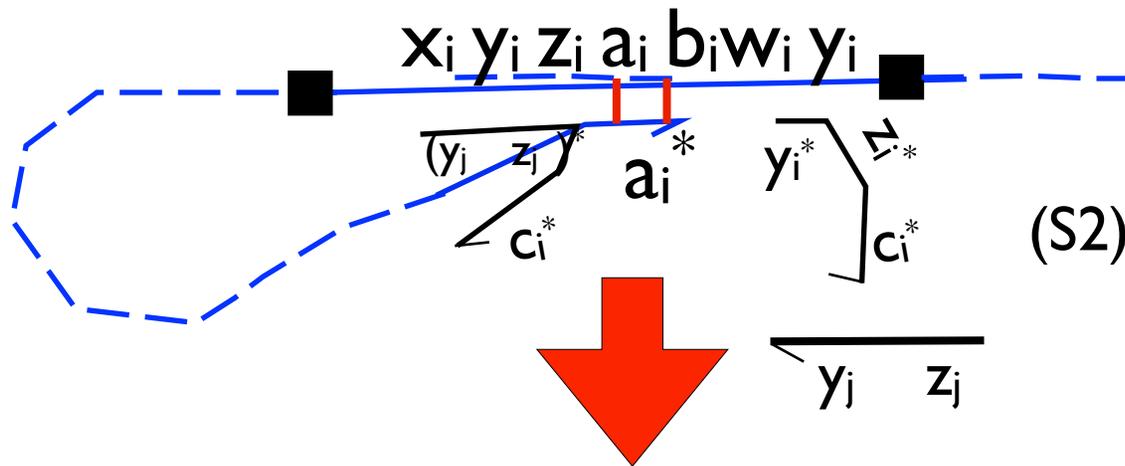
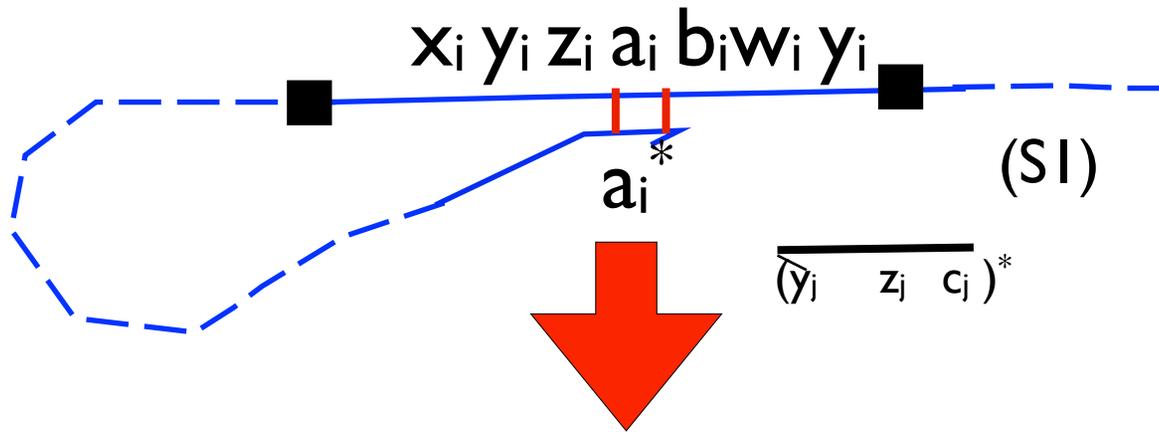
- ◆ Simple Preparation Protocol
  - ◆ Secondary primer hybridizes as desired since  $w$  is longer than just  $y$  on the rule encoding
- ◆ Complex Preparation Protocol
  - ◆ Elaborate protocol to increase the probability of desired secondary structures of WPCR strand before computation starts

# DNA Complex Preparation Protocol

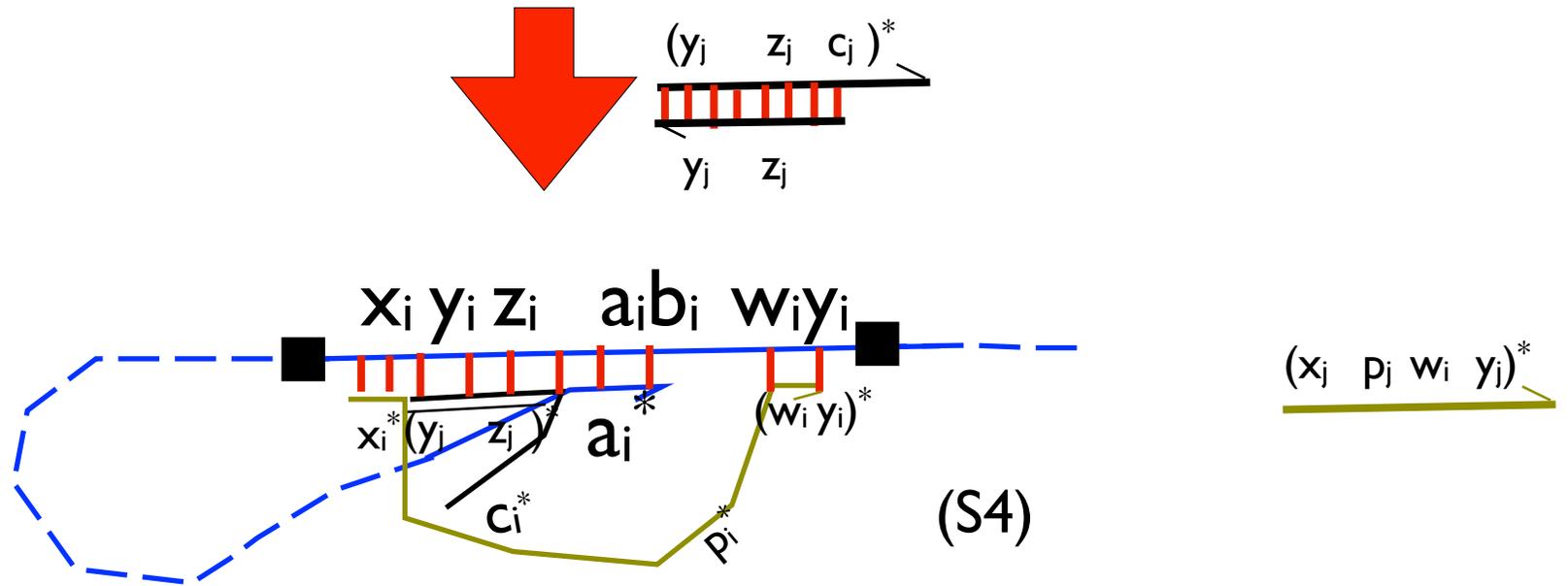


**Fig. 6** Complex preparation protocol with respect to only rule  $R_i$ :  $S1$  WPCR strand  $W$  tethered to support (not shown in the Figure).  $S2$ :  $(y_i z_i c_i)^*$  is added to the solution. One copy binds to the  $y_i$  near  $x_i$  and another binds to  $y_i$  further away from it.  $S3$ : the copy of  $(y_i z_i c_i)^*$  that binds to the  $y_i$  in  $R_i$  further away from  $x_i$  is removed by the addition of  $y_i z_i$ . The duplex thus formed is then removed from the solution using magnetic beads (not shown here).  $S4$ : Protection strand  $P_i$  encoded as  $(x_i p_i w_i y_i)^*$  is introduced and it hybridizes with the  $x_i$  and free  $w_i y_i$  of rule  $R_i$ .  $S5$ : the copy of  $(y_i z_i c_i)^*$  that is bound to the  $y_i$  in  $R_i$  nearer to  $x_i$  is removed by the addition of  $y_i q_i z_i$ . Here too, the duplex is later removed using magnetic beads

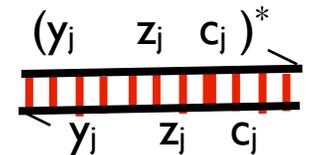
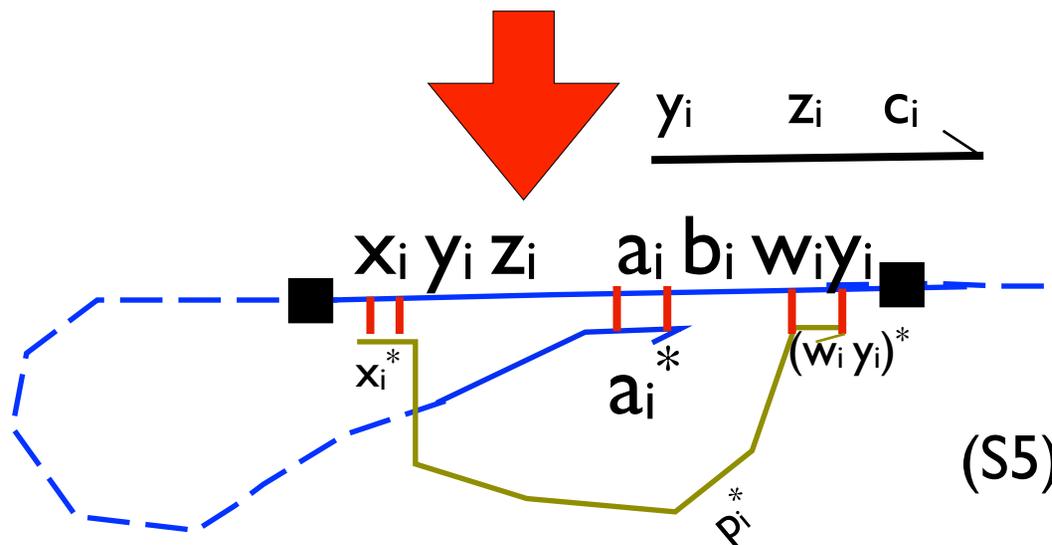
# Complex Preparation Protocol



# Complex Preparation Protocol (Contd)



WPCR strand ready to compute isothermally!

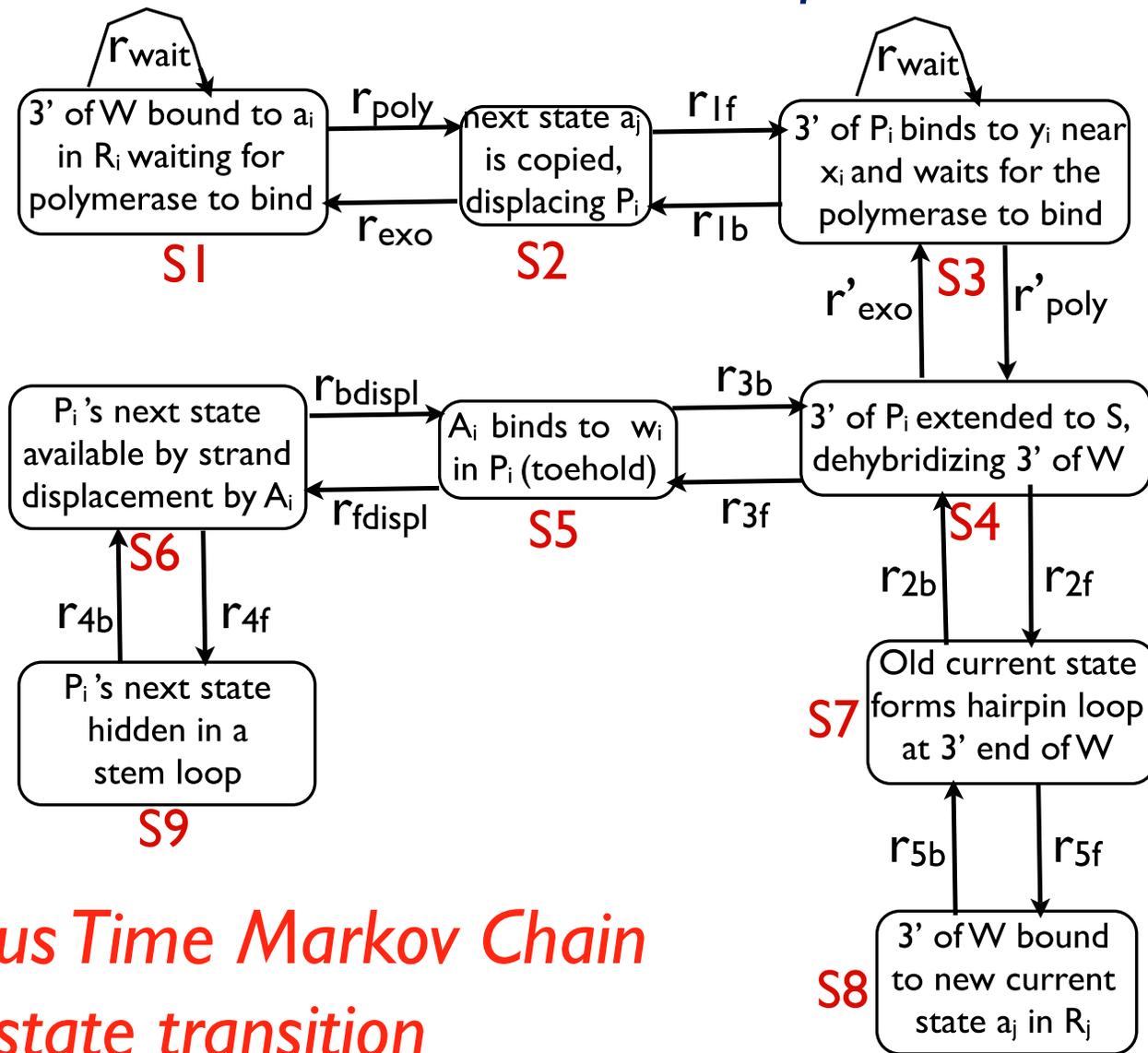


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# Proof of Correctness of IR-WPCR Machine

Continuous Time Markov Chain for reusable rule  $R_i$



*A Continuous Time Markov Chain  
for state transition*

**Fig. 10** Continuous time Markov Chain for rule  $R_i$  in the reusable rules IR-WPCR protocol that prevents back-hybridization using folding WPCR

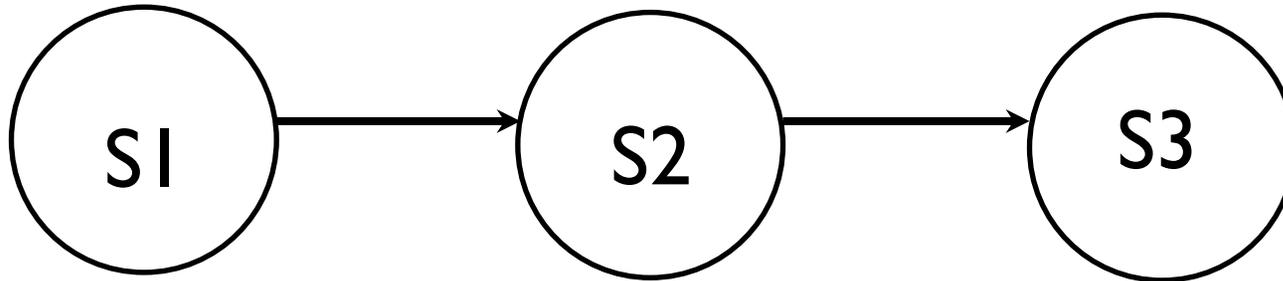
# Proof of Correctness of IR-WPCR Machine

- ◆ Assume proof of correctness of the original WPCR machine
- ◆ Stochastic system: Likelihood and rate of a state transition
- ◆ **Rate of Polymerization**
  - ◆ Rate formulation [Rose et al, 2001]
  - ◆  $\Phi$ -29 Rates [Saturno et al, 1995]
- ◆ **Rate of hybridization [Winfrey, 1998]**
- ◆ **Rate of dehybridization [Winfrey, 1998]**
- ◆ **Rate of strand displacement**
  - ◆ 1D random walk
  - ◆ Mean time for single base migration [Thompson 1976]

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# Experimental Verification Plan



Encode a 3 state machine in an IR-WPCR strand

$x_1 - y_1 - z_1 - a_1 - b_1 - w_1 - y_1 - S - x_2 - y_2 - z_2 - a_2 - b_2 - w_2 - y_2 - S - S' - a_1^*$

**Two experiments:** to verify both transitions happen using FRET  
(molecular beacon technique)

# *Validate first transition*

- ◆ Encode only first rule in the WPCR strand
- ◆ Encode a molecular beacon as  $h(b|w|y|h^*)$  with a fluorophore and quencher at the two ends (hybridized to WPCR strand and emitting signal)
- ◆ When next state is copied molecular beacon is released and forms a hairpin, thus quenching the fluorescence
- ◆ Other transition can be validated similarly

# Summary

- ◆ Isothermal Reactivating WPCR machine
  - ◆ uses extension of a secondary primer by a DNA polymerase with good strand displacement capability to trigger state transition
- ◆ IR-WPCR machine with non-reusable states
  - ◆ prevents back-hybridization
- ◆ IR-WPCR with reusable states
  - ◆ similar to original WPCR machine but **isothermal**
- ◆ Proof of correctness of IR-WPCR machine
- ◆ Experimental verification plan using molecular beacons and polymerase  $\Phi$ -29

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Urmi Majumder



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