Fisher Information Determines Capacity of ε-secure Steganography

Tomáš Filler and Jessica Fridrich

Dept. of Electrical and Computer Engineering
SUNY Binghamton, New York

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BINGHAMTON UNIVERSITY
State University of New York

Perfect vs. Imperfect Steganography

Stegosystems can be divided into two classes:

Perfectly secure stegosyst.

KL diverg. cover & stego
$D_{KL}(P||Q) = 0$

Detector does NOT exist.

Secure capacity is linear.

Communication rate is POSITIVE.

Imperfect stegosyst.

$D_{KL}(P||Q) = \varepsilon$

Detector DOES exist.

Sec. capacity is SUBlinear.

Communication rate is ZERO.

- perfectly secure stegosystems exist for artificial cover sources
- all known stegosystems for digital media are imperfect

Secure Capacity of ε-secure Stegosystems

From Taylor expansion ... ($n\beta = \#$ of changes)

$$D_{KL}\left(P^{(n)} || Q_\beta^{(n)}\right) = \frac{1}{2} n\beta^2 \mathtt{I} + O(\beta^3) = \varepsilon$$

$$\frac{1}{2} n\beta^2 \mathtt{I} \approx \varepsilon \quad \Rightarrow \quad n\beta \approx \sqrt{\frac{1}{\mathtt{I}}}\sqrt{2\varepsilon n}$$

$\mathtt{I}$ **... Fisher information (rate) at $\beta = 0$.**

Capacity of ε-secure stegosystems scales as $r\sqrt{n}$.

Root rate: (more refined measure of capacity)

$$r \approx \frac{1}{\sqrt{\mathtt{I}}}$$

Is Root Rate Useful?

Root rate: (more refined measure of capacity)

$$r \approx \frac{1}{\sqrt{\mathrm{I}}}$$

Fisher information rate can be expressed in a closed-form.

Applications:

- BENCHMARKING - compare stegosystems by their root rates.
- STEGANOGRAPHY DESIGN - maximize root rate w.r.t. embedding operation for fixed cover source.

Presentation Outline

1. ASSUMPTIONS
   model of cover objects, model of embedding impact
2. STEGANOGRAPHIC CAPACITY & ROOT RATE
   formal connection between detectability and root rate
3. APPLICATION
   comparison of LSB and ± 1 embedding in spatial domain
4. CONCLUSION

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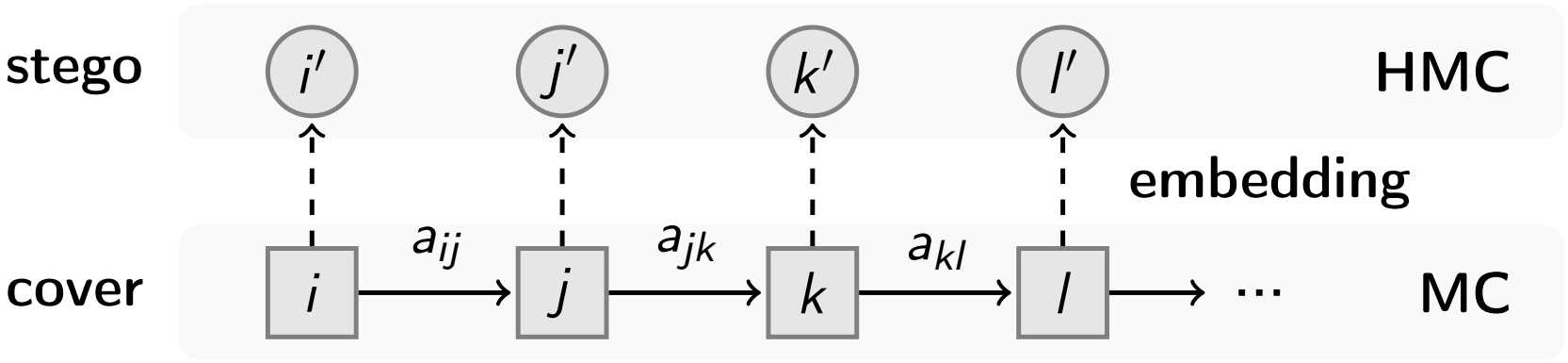
ASSUMPTIONS

Assumptions Summary

1. COVER SOURCE - Markov Chain
   first order Markov Chain with tran. prob. mat. $\mathbb{A} = (a_{ij})$

2. EMBEDDING ALGORITHM - MI embedding
   independent substitution of states (next slide)

3. STEGOSYSTEM - ε-secure (imperfect)
   stegosystem is ε-SECURE



Mutually Independent Embedding Operation

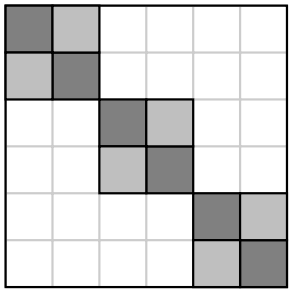
Impact of embedding is modeled as probabilistic mapping acting on each cover element independently - MI embedding.

$$Pr(Y_l = j | X_l = i) = b_{ij}(\beta)$$

X_l **... l-th cover element**
Y_l **... l-th stego element**
β **... change rate (rel. payload)**

Matrix $\mathbb{B} = (b_{ij})$ is "transition probability matrix" for $\beta \geq 0$.

LSB embedding:

$\mathbb{B} =$ 

$= 1 - \beta$ $= \beta$

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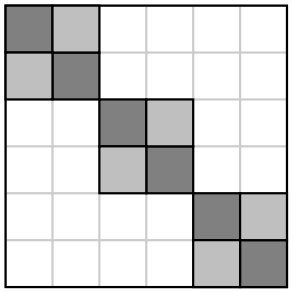
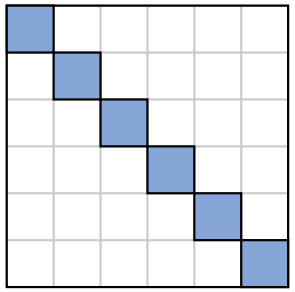
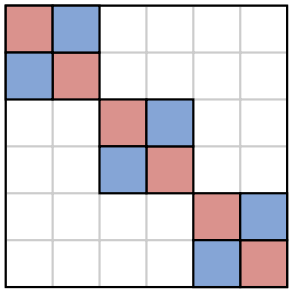
$\mathbb{B} =$  $= \mathbb{I} + \beta\mathbb{C} =$  $+ \beta \cdot$ 

$= 1 - \beta$ $= \beta$ $= 1$ $= -1$

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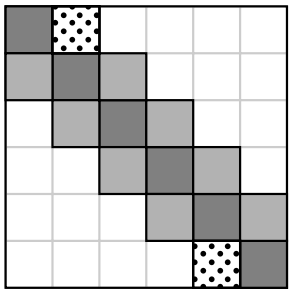
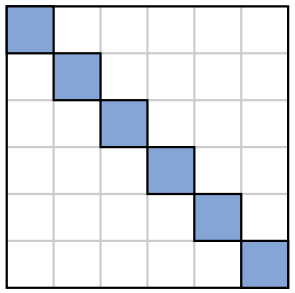
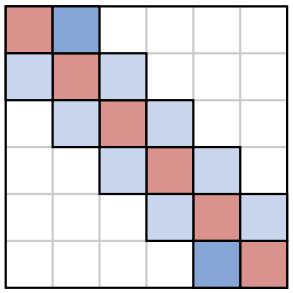
± 1 embedding:

$\mathbb{B} =$  $= \mathbb{I} + \beta \mathbb{C} =$  $+ \beta \cdot$ 

$= 1 - \beta$ $= \frac{\beta}{2}$ $= \beta$ $= 1$ $= 0.5$ $= -1$

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$$Pr(Y_l = j | X_l = i) = b_{ij}(\beta)$$

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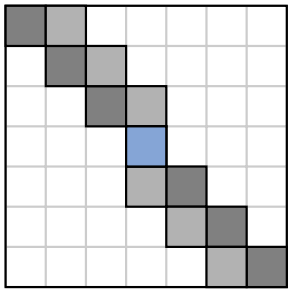
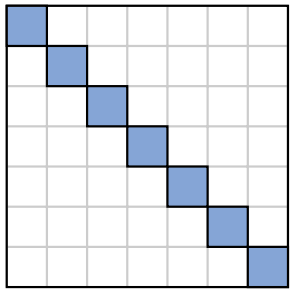
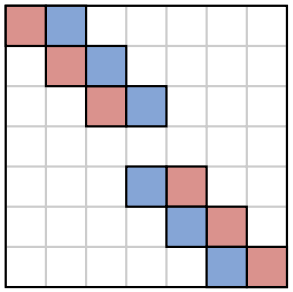
F5 embedding:

$\mathbb{B} =$  $= \mathbb{I} + \beta\mathbb{C} =$  $+ \beta \cdot$ 

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Assumption:

$$\mathbb{B}(\beta) = \mathbb{I} + \beta \mathbb{C}$$

$\mathbb{C}$ describes the inner workings of the embedding algorithm.

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STEGANOGRAPHIC CAPACITY & $\sqrt{RATE}$

Square Root Law

1. If $\frac{n\beta_n}{\sqrt{n}} \to 0$ then the stegosyst. are asymptotically secure.
2. If $\frac{n\beta_n}{\sqrt{n}} \to +\infty$ then arbitrarily accurate stego detectors exist.

For fixed level of security,

$$\frac{n\beta_n}{\sqrt{n}} < C \quad \Rightarrow \quad \beta_n \to 0 \text{ as } n \to \infty.$$

[Filler, Ker, Fridrich, "The Square Root Law of Steganographic Capacity for Markov Covers", Proc. SPIE, 2009]

The Best Possible Steganalyzer

Hypothesis test:

$$\begin{array}{lcll} \mathrm{H}_0 & : & \beta = 0 & \text{decide cover} \\ \mathrm{H}_1 & : & \beta > 0 \text{ (known)} & \text{decide stego.} \end{array}$$

Detection statistics (log-likelihood ratio):

$$L_\beta^{(n)}(X) = \frac{1}{\sqrt{n}} \ln \frac{Q_\beta^{(n)}(X)}{P^{(n)}(X)}$$

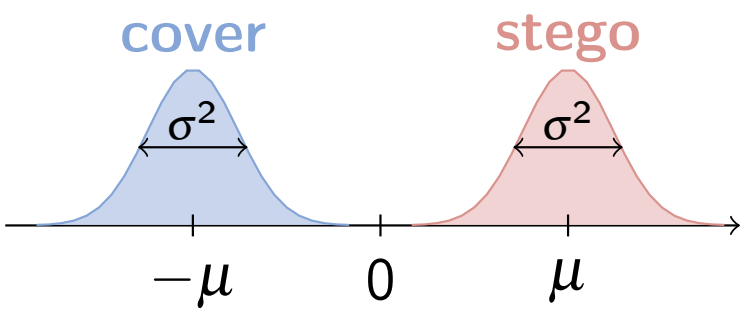
stego distribution with n elements

cover distribution with n elements

change rate

The Root Rate

For small β, $L_\beta^{(n)}(X)$ is Gaussian (Local Asymptotic Normality).



$$\mu = \sqrt{n}\beta^2 \mathtt{I}/2$$
$$\sigma^2 = \beta^2 \mathtt{I}$$

Fisher information rate $\mathtt{I} = \lim_{n\to\infty} \frac{1}{n}\frac{d^2}{d\beta^2} D_{KL}\left(P^{(n)} || Q_\beta^{(n)}\right)\Big|_{\beta=0}$

Deflection coefficient: $d^2 = (-\mu-\mu)^2/\sigma^2 = n\beta^2\mathtt{I} < \varepsilon$

$$n\beta < r\sqrt{\varepsilon n} \qquad r = \sqrt{\frac{1}{\mathtt{I}}} \text{ ROOT RATE}$$

Closed Form for Fisher Information Rate

Theorem (Fisher Information Rate)

Let $\mathbb{A} = (a_{ij})$ define the MC cover model and $\mathbb{B} = \mathbb{I} + \beta\mathbb{C}$ represent MI embedding. Then, the Fisher information rate $\mathtt{I}$ exists and can be written as

$$\mathtt{I} = \mathbf{c}^T \mathbb{F} \mathbf{c}.$$

- $\mathbf{c} = (c_{11}, \ldots, c_{NN})^T$ *is column vector obtained directly from* $\mathbb{C}$.
- *Matrix* $\mathbb{F} \in \mathscr{R}^{N^2 \times N^2}$, $\mathbb{F} = f(\mathbb{A})$ *and does not depend on* $\mathbb{B}$.

Maximizing root rate $\Rightarrow$ minimizing $\mathtt{I}$ w.r.t. $\mathbb{C}$.

Closed form for $\mathtt{I}$ enables us to

- **compare stegosystems**
- **maximize capacity w.r.t. embedding operation $\mathbb{C}$.**

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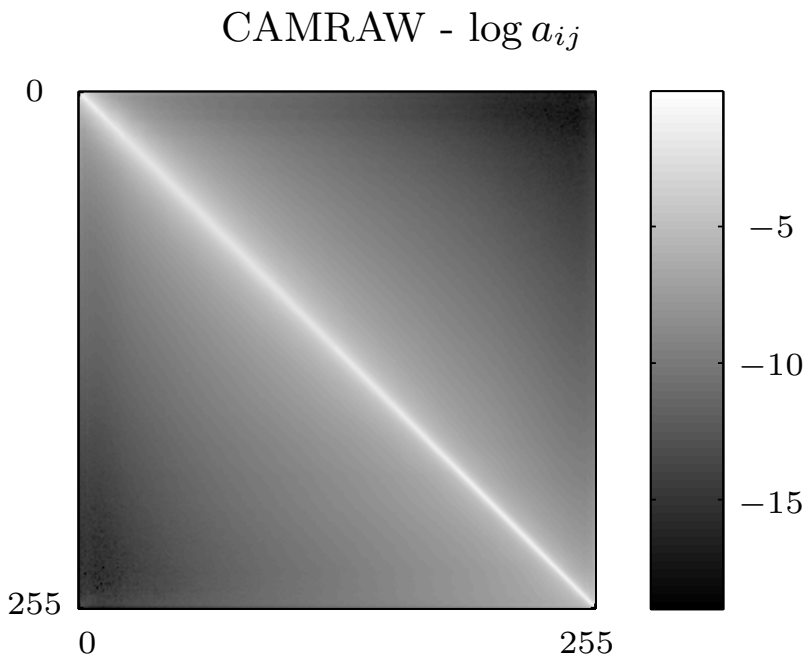
APPLICATIONS

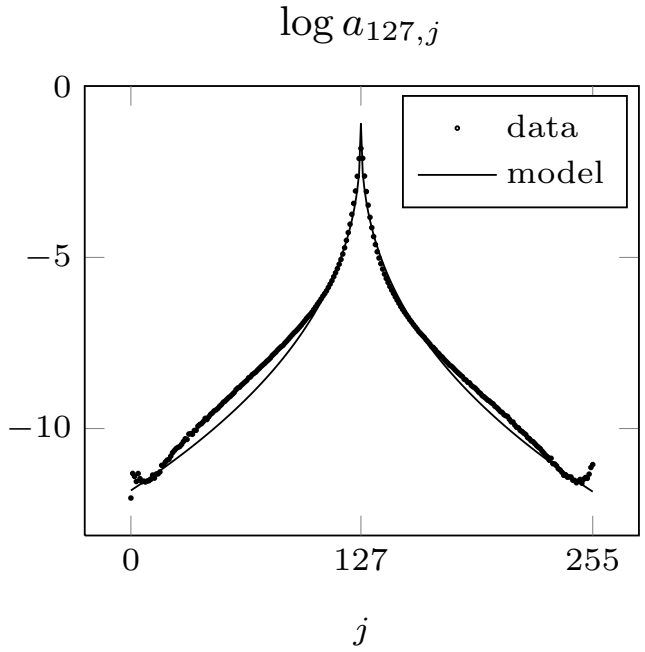
Markov Model of Spatial Domain Images

Image databases: CAMRAW, NRCS, NRCS-JPEG70.

Spatial domain 8-bit grayscale images: $\mathbb{A} = (a_{ij}) \in \mathscr{R}^{256\times 256}$

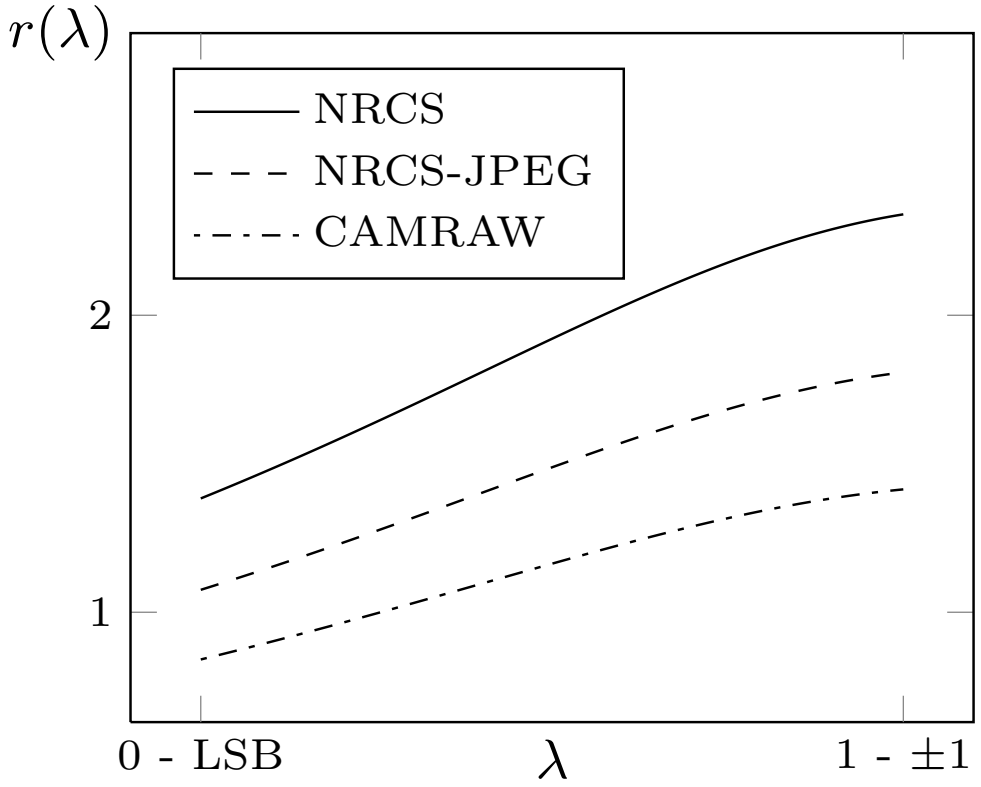
$$a_{ij} = \frac{1}{Z_i} e^{-(|i-j|/\tau)^\gamma}$$





Convex Combination of LSB and ± 1 Embedding

Use ± 1 embedding in first λn pixels and LSB embedding in the rest.



Root rate $r(\lambda)$:

$$\mathbf{c}_\lambda = \lambda \mathbf{c}_{\pm 1} + (1-\lambda)\mathbf{c}_{\mathrm{LSB}}$$

$$r(\lambda) = \sqrt{\frac{1}{\mathbf{c}_\lambda^T \mathbb{F} \mathbf{c}_\lambda}}$$

Higher root rate = better method.

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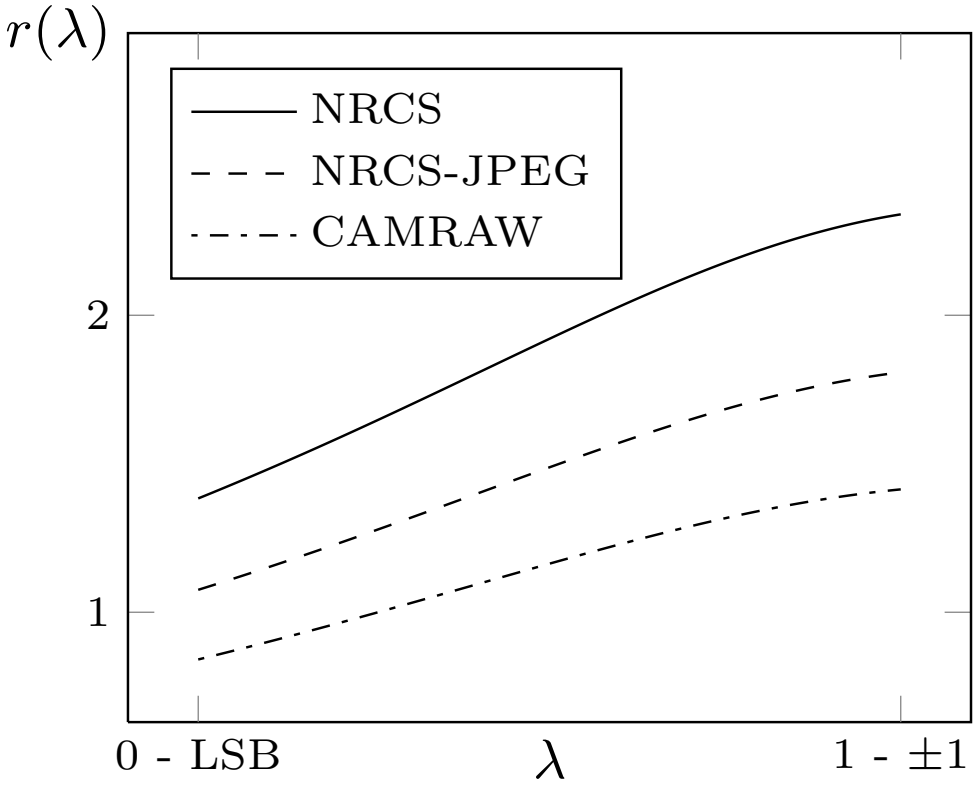
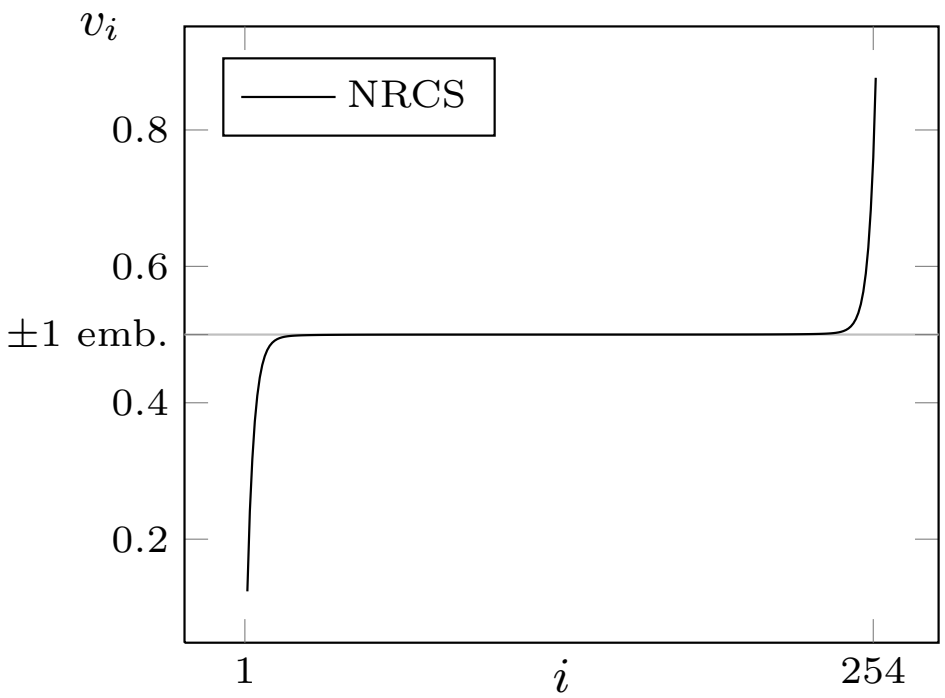


Pevný, Bass, Fridrich: Steganalysis by Subtractive Pixel Adjacency Matrix, submitted to ACM MM&SEC 2009, Princeton, NJ

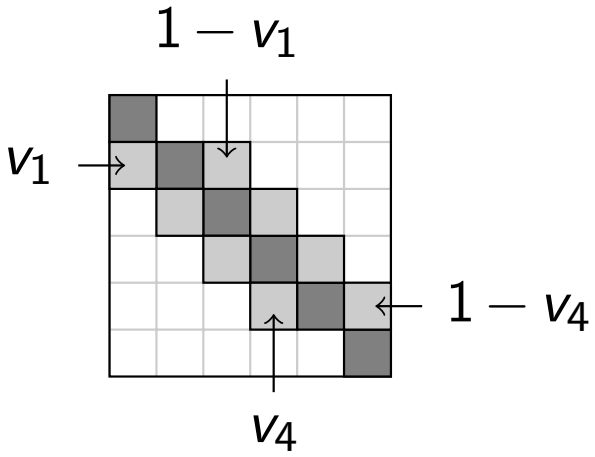
Higher root rate = better method.

± 1 is Asymptotically Optimal

What embedding method minimizes Fisher information rate and modifies cover by at most 1?



Class of embedding operations:



± 1 embedding is asymptotically optimal as number of grayscales $N \to \infty$.

Conclusion and Future Directions

Steganographic capacity of ε-SECURE stegosystems $\approx r\sqrt{n}$.

We coin a new term for constant r - the Root Rate.

Root rate

- is determined by Fisher information rate.
- can be expressed in a closed-form amenable to optimization (under mentioned assumptions).
- was used to compare spatial domain stegosystems.

Future work: use this framework for JPEG images.

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Thank you!

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