Quantization of the Diagonal Resistance: Density Gradients and the Empirical Resistance Rule in a 2D System

W. Pan, J. S. Xia, H. L. Stormer, D. C. Tsui, C. L. Vicente, E. D. Adams, N. S. Sullivan, L. N. Pfeiffer, K.W. Baldwin and K.W. West

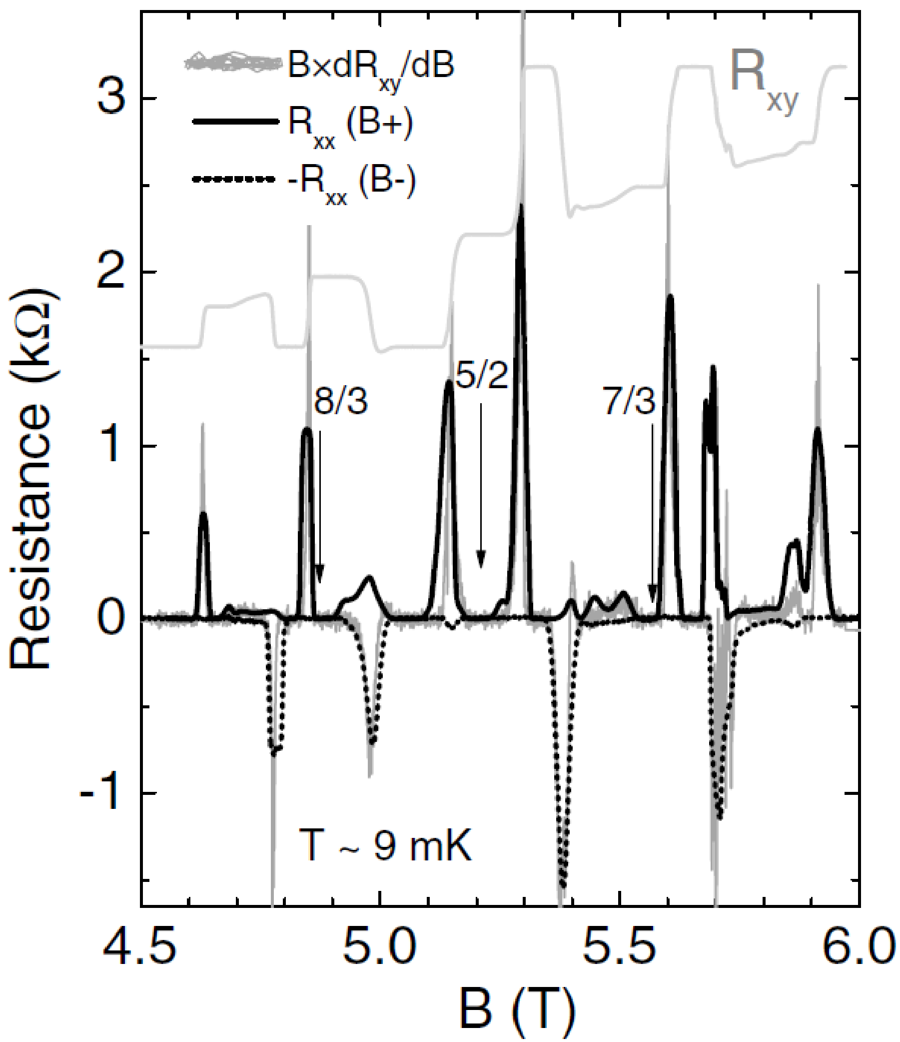
Experiment

- 30 nm GaAs/AlGaAs QW symmetrical δ-doping (@100 nm).
- Rotated wafer with In-contacts
- n=3x10^{11} cm^{-2} ; μ=31x10^{6} cm^{2}/Vs
- Dil. fridge + demagnetization stage
- $T_{fridge?}$ =6 mK

What to expect

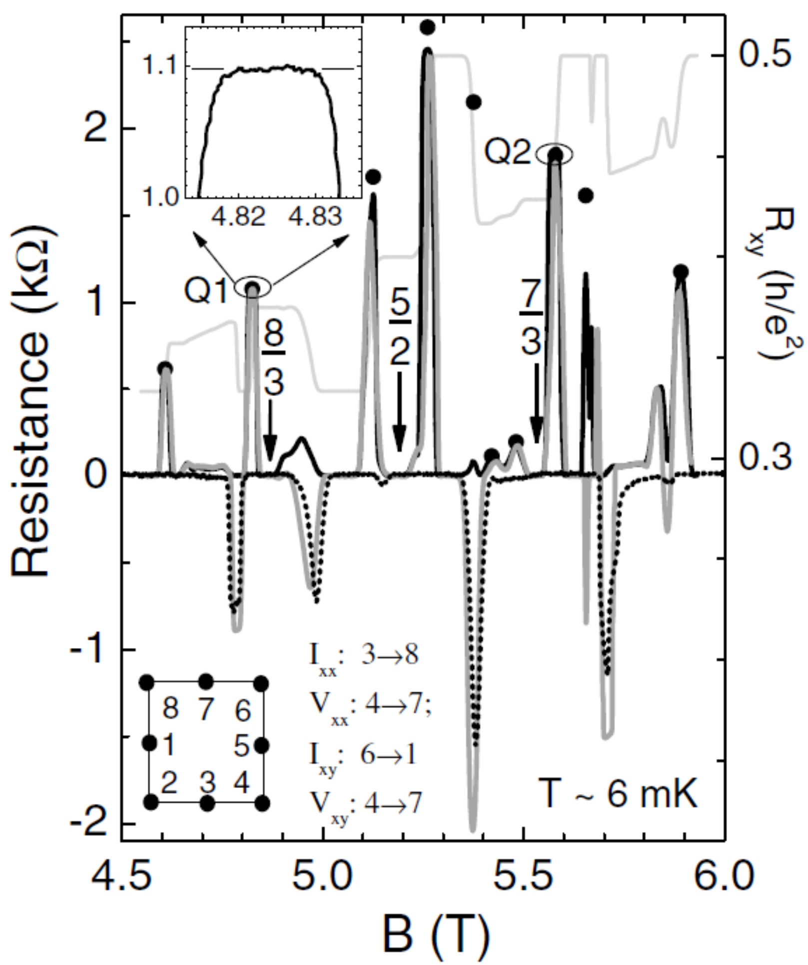
- Quantization of the diagonal resistance, R_{xx}, at the edges of several quantum Hall states.
- Quantized R_{xx} values close to the difference between the two adjacent Hall plateaus in the off-diagonal resistance, R_{xy}.
- Peaks in R_{xx} occur at different positions in positive and negative magnetic fields.
- R_{xx} features can be explained quantitatively by a density gradient.
- R_{xx} is determined by R_{xy} and unrelated to the diagonal resistivity ρ_{xx}.
- Unexpected light on the empirical resistivity rule for 2D systems.

9 mK



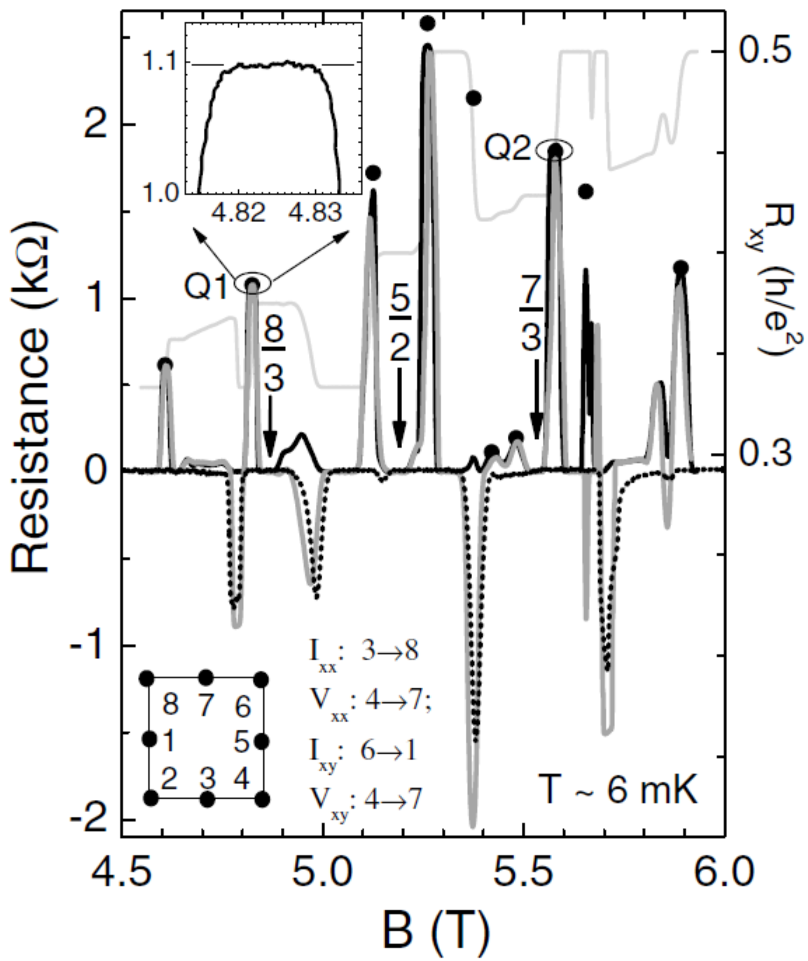
- R_{xx} and R_{xy} @ 9 mK
- FQHE states apparent in 5/2, 7/3, 8/3
- 4 RIQHE states
- Sharp peaks matching the rising flanks of R_{xy}.
- For B- the peaks are situated with the dropping flanks of R_{xy}
- Testing the resistivity rule :
  0.003 x $R_{diff}(=B \times dR_{xy}/dB)$
  → positive parts are in R_{xx}B+ and negative parts in R_{xx}B-

6 mK



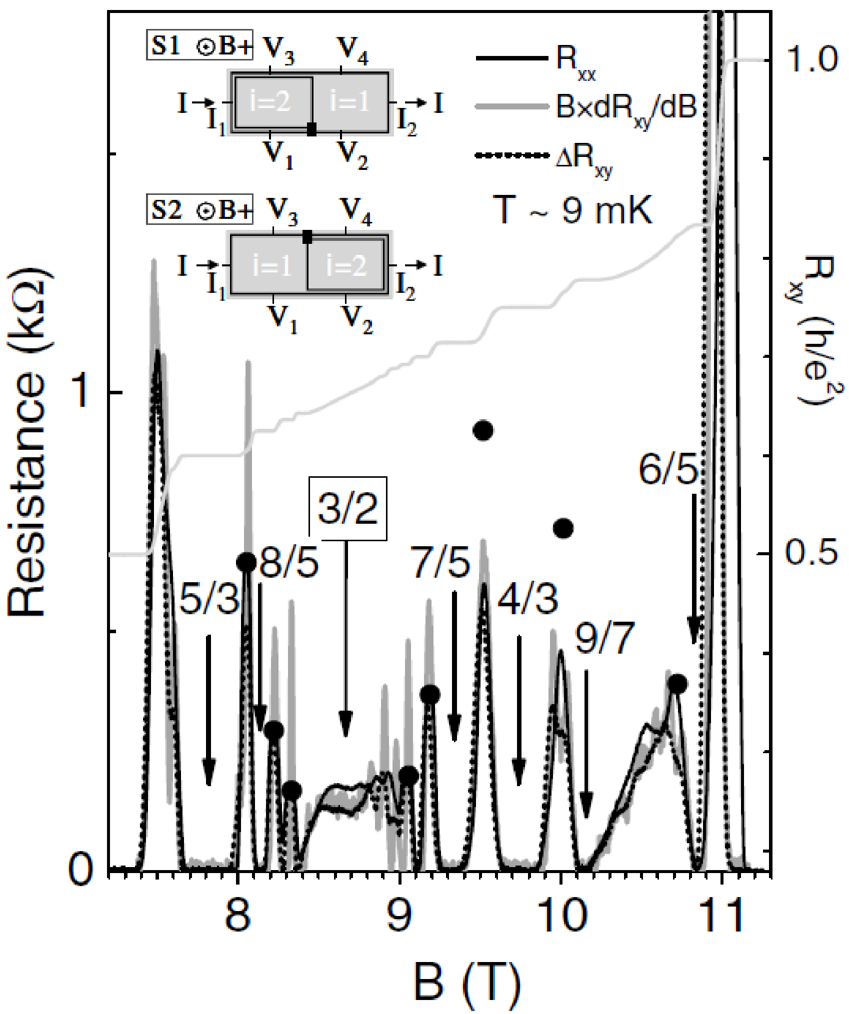
- T=6 mK. Main features similar to 9 mK.
- BUT: when the Hall resistance varies abruptly, R_{xx} assumes a flat top. (upper inset near 8/3)
- similar quantization observed at 5.58 T close to 7/3
- R_{xy} shows no anomaly, but varies monotonically between plateaus
- R_{xx} plateaus appear only for T <9 mK, width increases with decreasing T

6 mK



- R_{xx} plateau values close to the difference between adjacent quantum Hall plateaus in R_{xy}.
- Solid dots represent the value of ΔR_{xy} calculated from the adjacent R_{xy} pairs.
- This holds for all well developed peaks, even for those between adjacent FQHE states (e.g. between 2+2/5 and 2+3/8 or 2+3/8 and 2+1/3).

Testing the boundaries



general test

- R_{xx}~ΔR_{xy} also holds for the well-developed peaks in the lowest Landau level.
- R_{xx} (black) and R_{xy} (light gray) traces for $2>\nu>1$.
- The solid dots represent ΔR_{xy}.
- Their position, in many cases, matches closely the height of the strong peaks in R_{xx},
- But no quantization of R_{xx} in this filling factor range

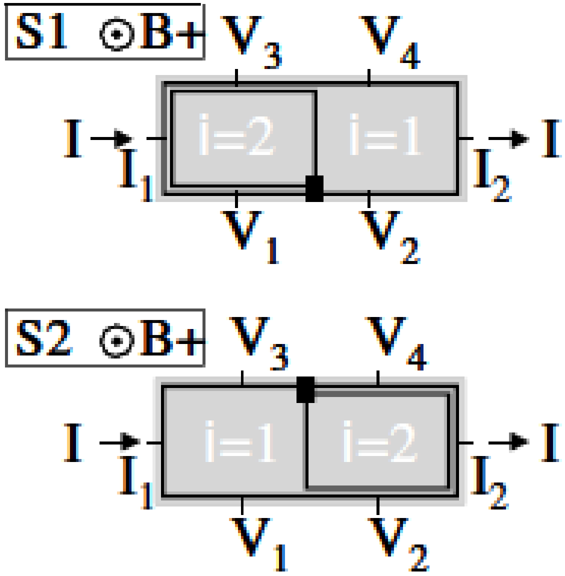
Modeling

- Plateaus in R_{xx} previously observed in macroscopically inhomogeneous 2D systems, e.g. Si MOSFETs with spatially varying electron densities.

- R_{xx} quantization and strong peaks in R_{xx} in one B-field direction

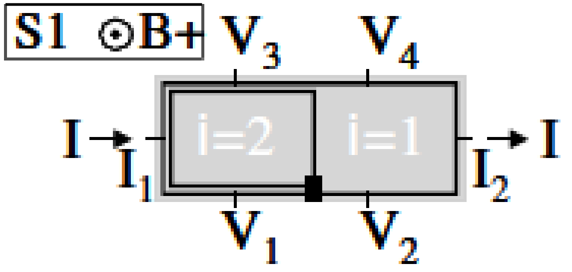
- practically vanishing R_{xx} in the opposite field direction.

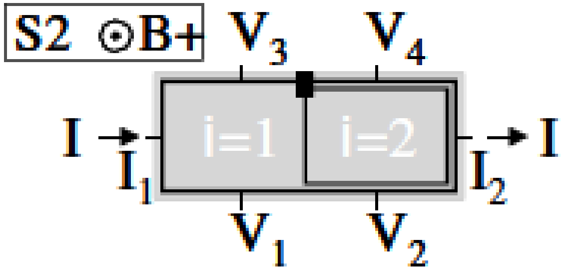
- In all cases, the observations are best explained in the edge channel picture.



Modeling

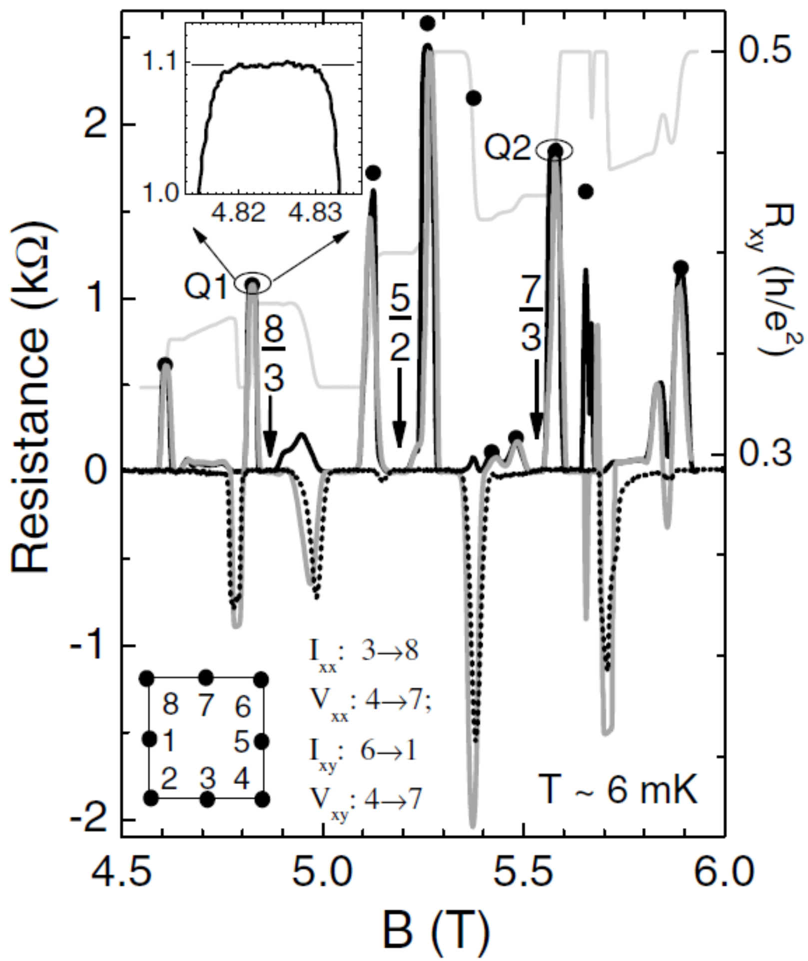
- At the boundary between two regions of different filling factors ν_1 and ν_2 ($\nu_1 > \nu_2$), a total of ν_1-ν_2 edge channels are being reflected.
- This leads to a voltage drop between these two regions equivalent to R_{xx}=$(h/e2)$x$(1/\nu_2 - 1/\nu_1)$.
- a very small, unintentional density gradient exists in the 2D specimens.





- The width ΔB~0.01 T of the Q1 translates directly into a difference of 0.5% in density between the requisite voltage probes
- Having a distance of about 5 mm, this represents an 1%/cm density gradient.

Putting the model to use

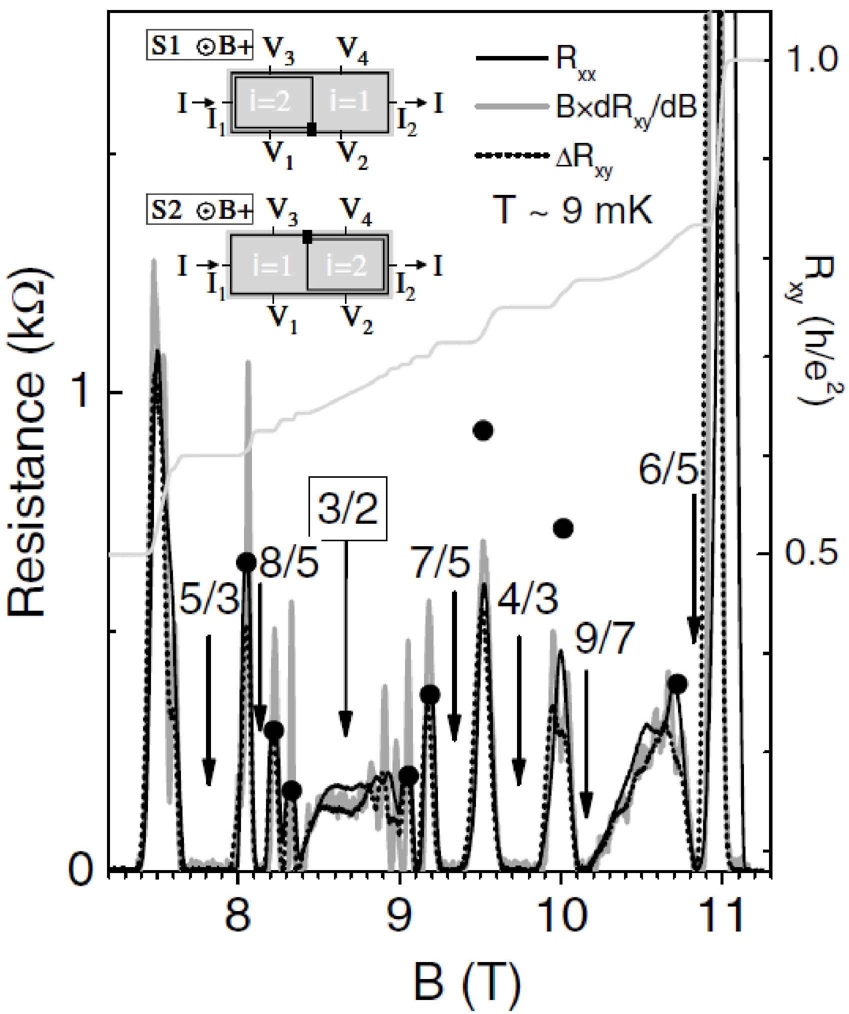


- assume a density n_1 at voltage probe 4 and a density n_2 at voltage probe 7

$$\Delta R_{xy} = R_{xy}(n_1)\text{-}R_{xy}(n_2) = R_{xy}(n)\text{-}R_{xy}(n+\Delta\nu)$$

- This is performed numerically, by subtracting the 6 mK R_{xy} trace from the same trace with a B-field axis compressed by $2\Delta n/(n_1+n_2)$ ~0.5%.
- The resulting data are plotted as a thick gray trace.
- Positive part of ΔR_{xy} reproduces $R_{xx}B+$ almost perfectly
- negative part of ΔR_{xy} matches the inverted $R_{xx}B$-.

2. LL



- 2. LL : R_{xy} for a density difference of 0.5% also reproduces R_{xx} between ν=1 and ν=2 *quantitatively.*

- An exception arises around B 11 T, where the R_{xy} peak exceeds R_{xx} by about a factor of 2.

Model

- model based on density inhomogeneity can produce R_{xx} almost perfectly from R_{xy}.
- density gradient can explain the origin of the entire resistivity rule $R_{xx}=c \times R_{\mathrm{diff}}$ in terms of R_{xy}.
- small density difference Δn between R_{xx} voltage probes creates at any given B a difference in Hall voltage

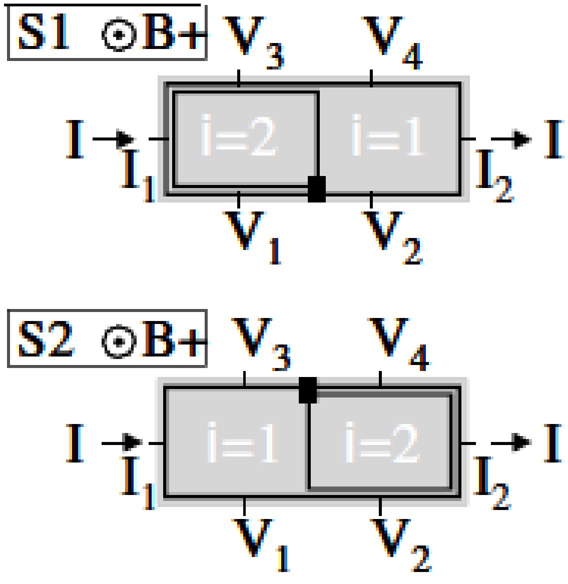
$$
\begin{aligned}
\Delta V_{xy}(n,B) &= I x [R_{xy}(n,B) - R_{xy}(n+\Delta n,B)] \\
&\approx I \times dR_{xy}(n,B)/dn \times \Delta n \\
&= I \times dR_{xy}(n,B)/dB \times B/n \times \Delta n \\
&= I \times B \times dR_{xy}(n,B)/dB \times \Delta n/n \\
& \qquad dB/B = dn/n
\end{aligned}
$$

→ $\Delta R_{xy} = \Delta V_{xy}/I = B \times dR_{xy}(n,B)/dB \times (\Delta n/n) = c \times R_{\mathrm{diff}}$

- resistivity rule can simply be derived from a density gradient.
- $c=\Delta n/n$.

B polarity dependence

- edge channel structure depends only on B direction
- The whole specimen is encircled by the *i* =1 edge state, whereas only a part of it contains a second *i* =2 edge channel.
- Edge channels represent equipotentials.
- external current → ‘‘hot spots’’ (shown as black squares), at which the potential along the edge suddenly jumps.



- The $V_1 \rightarrow V_2$ voltage drop in S1 reflects a quantized value of R_{xx}.
- The opposite side of the specimen (V_3; V_4) is in the zero-resistance state.
- If we reverse the ordering of *i* =1 and *i* =2, as in S2, the hot spot
- switches sides and occurs between V_3 and V_4 while V_1-V_2 vanishes.

Discussion

- hot spot remains on the same side of the sample on sweeping B and the density gradient is observed in R_{xx} on that side only.
- If B field is swept through anomalous regions of R_{xy}, (RIQHE states) the spatial order of quantum numbers is *reversed* (as shown in S2), the hot spot switches sides,
- voltage drop appears in V_3-V_4, and V_1-V_2 enters the zero-resistance state.
- no R_{xx} features observed between V_1 and V_2 but on the opposite side of the sample. (not tested here)
- under reversed B field,
  - V_1,V_2 contacts only show features in R_{xx} when R_{xy} experiences an anomalous, sudden *drop.*
  - sudden *rises* in R_{xy} the hot spot resides on the opposite side of the sample, → V_1-V_2 shows zero-resistance and no spike.
- All data were taken below 20 mK and the gradient model holds for the entire temperature range

Thank you for your attention