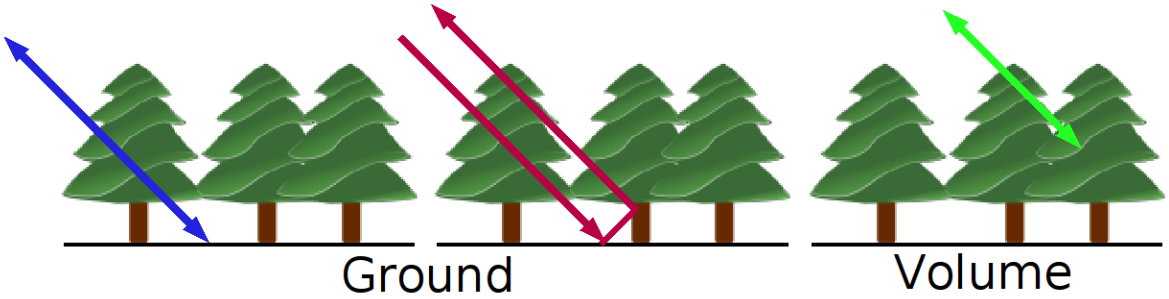
Improvement of Vegetation Parameter Retrieval from Polarimetric SAR Interferometry using a Simple Polarimetric Scattering Model

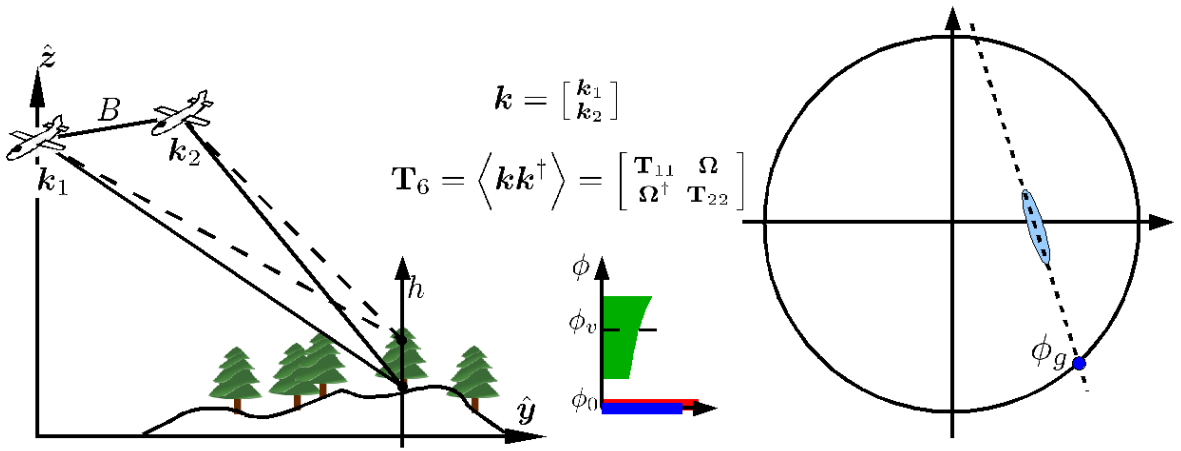
Maxim Neumann, Laurent Ferro-Famil, Andreas Reigber

Freeman–Durden Type Decomposition



Freeman, Durden, 1998, Yamaguchi *et al.*, 2005/06, Freeman, 2007

PolInSAR RVoG Model



Treuhaft and Siqueira, 2000, Papathanassiou and Cloude, 2001, Cloude and Papathanassiou, 2003

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \boldsymbol{\Omega} \\ \boldsymbol{\Omega}^\dagger & \mathbf{T} \end{bmatrix}$$

$$\mathbf{T} = f_g \mathbf{T}_g + f_v \mathbf{T}_v$$

$$\boldsymbol{\Omega} = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v$$

M. Neumann, L. Ferro-Famil, A. Reigber: IGARSS'2008

Polarimetric Ground Component

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^\dagger & \mathbf{T} \end{bmatrix} \rightarrow \begin{cases} \mathbf{T} & = f_g \mathbf{T}_g + f_v \mathbf{T}_v \\ \mathbf{\Omega} & = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v \end{cases}$$

- Ground = surface + double–bounce
- No restrictions

$$\mathbf{T}_g = \mathbf{R}_{T(\psi_g)} \begin{bmatrix} 1 & \beta & 0 \\ \beta^\dagger & \beta_{22} & 0 \\ 0 & 0 & \beta_{33} \end{bmatrix} \mathbf{R}^{\mathsf{T}}_{T(\psi_g)}$$

Polarimetric Volume Component

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^\dagger & \mathbf{T} \end{bmatrix} \rightarrow \begin{cases} \mathbf{T} & = f_g \mathbf{T}_g + f_v \mathbf{T}_v \\ \mathbf{\Omega} & = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v \end{cases}$$

- Effective particle shape
  - particle scattering anisotropy: $\delta \in \mathbb{C}$
- Polarization orientation angles distribution
  - orientation randomness: $\tau \in [0, 1]$
  - mean orientation: $\widetilde{\psi}$

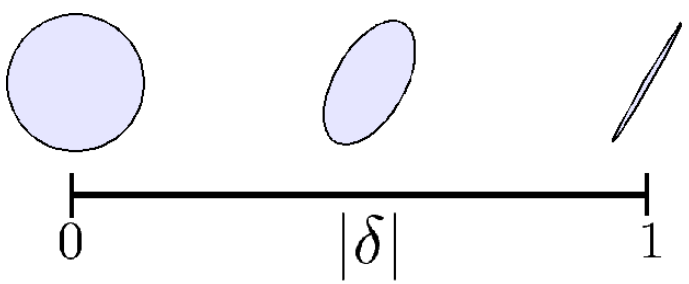
Particle Scattering Anisotropy

$$\langle \mathbf{F} \rangle = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \frac{a+b}{2} \begin{bmatrix} 1+\delta^* & 0 \\ 0 & 1-\delta^* \end{bmatrix}$$

Particle scattering anisotropy: $\delta = \left(\frac{a-b}{a+b}\right)^*$

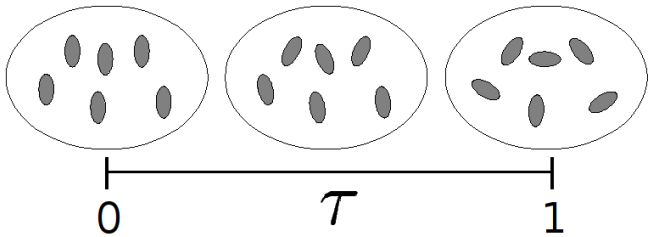
Effective particle shapes:

- $|\delta| \to 0 \Longrightarrow$ sphere, disk
- $|\delta| \to 1 \Longrightarrow$ dipole



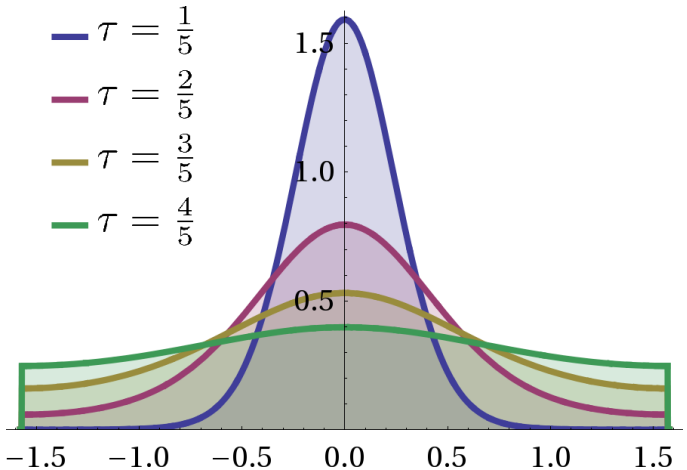
Orientation Distribution

Degree of orientation randomness: $\tau \in [0, 1]$



- $\tau \rightarrow 0 \Longrightarrow$ aligned (no randomness)
- $\tau \rightarrow 1 \Longrightarrow$ complete random orientations

Circular normal polarization orientation angles distribution:



Polarimetric Volume Component

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^\dagger & \mathbf{T} \end{bmatrix} \rightarrow \begin{cases} \mathbf{T} & = f_g \mathbf{T}_g + f_v \mathbf{T}_v \\ \mathbf{\Omega} & = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v \end{cases}$$

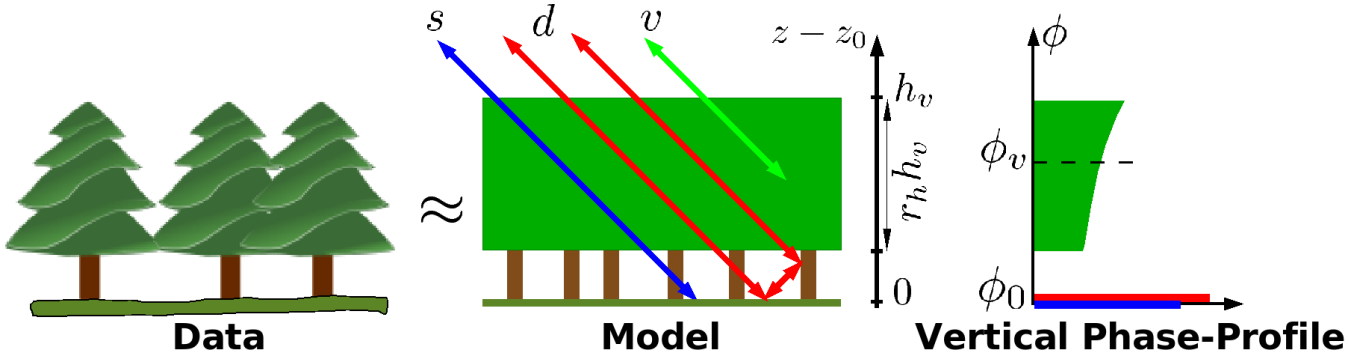
- Effective particle shape
  - particle scattering anisotropy: $\delta \in \mathbb{C}$
- Polarization orientation angles distribution
  - orientation randomness: $\tau \in [0, 1]$
  - mean orientation: $\widetilde{\psi}$

$$\mathbf{T}_v = \mathbf{R}_{T(2\widetilde{\psi})} \begin{bmatrix} 1 & g_c\delta & 0 \\ g_c\delta^* & \frac{(1+g)}{2}|\delta|^2 & 0 \\ 0 & 0 & \frac{(1-g)}{2}|\delta|^2 \end{bmatrix} \mathbf{R}^{\mathsf{T}}_{T(2\widetilde{\psi})}$$

with $g = \frac{I_2(\kappa)}{I_0(\kappa)}$, $g_c = \frac{I_1(\kappa)}{I_0(\kappa)}$, $\tau = I_0(\kappa)e^{-\kappa}$

Interferometric Components (RVoG)

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^\dagger & \mathbf{T} \end{bmatrix} \rightarrow \begin{cases} \mathbf{T} & = f_g \mathbf{T}_g + f_v \mathbf{T}_v \\ \mathbf{\Omega} & = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v \end{cases}$$



$$\gamma_g = e^{i\phi_0}\gamma_{sys}$$

$$\gamma_v = e^{i\phi_0}\gamma_{sys}\gamma_{temp}\gamma_{vol}$$

ϕ_0: ground reference phase
γ_{sys}: system decorrelation
γ_{temp}: temporal decorrelation
γ_{vol}: volume decorrelation

$$\gamma_{vol} = e^{ik_z(1-r_h)h_v} \int f_0(z) e^{ik_z z} dz,\ f_0(z) = \frac{e^{\frac{2\sigma}{\cos\theta_0} z}}{\int e^{\frac{2\sigma}{\cos\theta_0} z'} dz'}$$

K. Papathanassiou, S. Cloude, A. Reigber, W.M. Boerner, IGARSS 2000,

R. Treuhaft, P. Siqueira, Radio Science, 2000

Parameter Inversion Problem I

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau,\ h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

- Polarimetric response

$$\hat{\mathbf{T}} = f_g \mathbf{T}_g + f_v \mathbf{T}_v$$

$$\mathbf{T}_g = \begin{bmatrix} 1 & \beta & 0 \\ \beta^\dagger & \beta_{22} & 0 \\ 0 & 0 & \beta_{33} \end{bmatrix}, \qquad \mathbf{T}_v = \begin{bmatrix} 1 & g_c\delta & 0 \\ g_c\delta^* & \frac{(1+g)}{2}|\delta|^2 & 0 \\ 0 & 0 & \frac{(1-g)}{2}|\delta|^2 \end{bmatrix}$$

$\Longrightarrow$ 9 unknowns but only 5 equations :(

- Interferometric coherence diversity

$$\hat{\mathbf{\Omega}} = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v = \begin{bmatrix} A+iB & C+iD & 0 \\ E+iF & G+iH & 0 \\ 0 & 0 & K+iL \end{bmatrix}$$

+ 4 unknowns + 10 equations :)

Parameter Inversion Problem I

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau,\ h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

- Polarimetric response

$$\hat{\mathbf{T}} = f_g \mathbf{T}_g + f_v \mathbf{T}_v$$

$$\mathbf{T}_g = \begin{bmatrix} 1 & \beta & 0 \\ \beta^\dagger & \beta_{22} & 0 \\ 0 & 0 & \beta_{33} \end{bmatrix}, \qquad \mathbf{T}_v = \begin{bmatrix} 1 & g_c\delta & 0 \\ g_c\delta^* & \frac{(1+g)}{2}|\delta|^2 & 0 \\ 0 & 0 & \frac{(1-g)}{2}|\delta|^2 \end{bmatrix}$$

$\Longrightarrow$ 9 unknowns but only 5 equations :(

- Interferometric coherence diversity

$$\hat{\mathbf{\Omega}} = f_g\gamma_g \mathbf{T}_g + f_v\gamma_v \mathbf{T}_v = \begin{bmatrix} A+iB & C+iD & 0 \\ E+iF & G+iH & 0 \\ 0 & 0 & K+iL \end{bmatrix}$$

+ 4 unknowns + 10 equations :)

Parameter Inversion Problem II

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau,\ h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

- Structural parameters: ambiguous

$$\gamma_g = e^{i\phi_0}\gamma_{sys}, \quad \gamma_v = e^{i\phi_0}\gamma_{sys}\gamma_{temp}\gamma_{vol(h_v,\sigma,r_h)}$$

$\Longrightarrow$ use multiple baselines

- Temporal decorrelation: $\Longrightarrow |\gamma_v|$ corrupted
  $\Longrightarrow$ $\arg\gamma_v$ independent of temporal decorrelation

Parameter Inversion Problem II

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau,\ h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

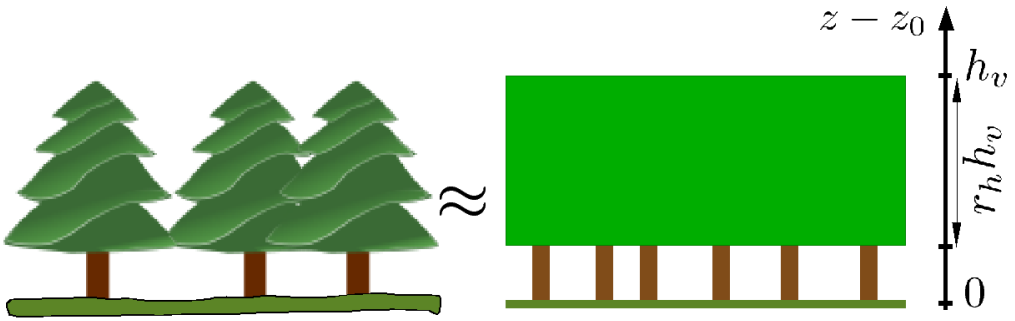
- Structural parameters: ambiguous

$$\gamma_g = e^{i\phi_0}\gamma_{sys}, \quad \gamma_v = e^{i\phi_0}\gamma_{sys}\gamma_{temp}\gamma_{vol(h_v,\sigma,r_h)}$$

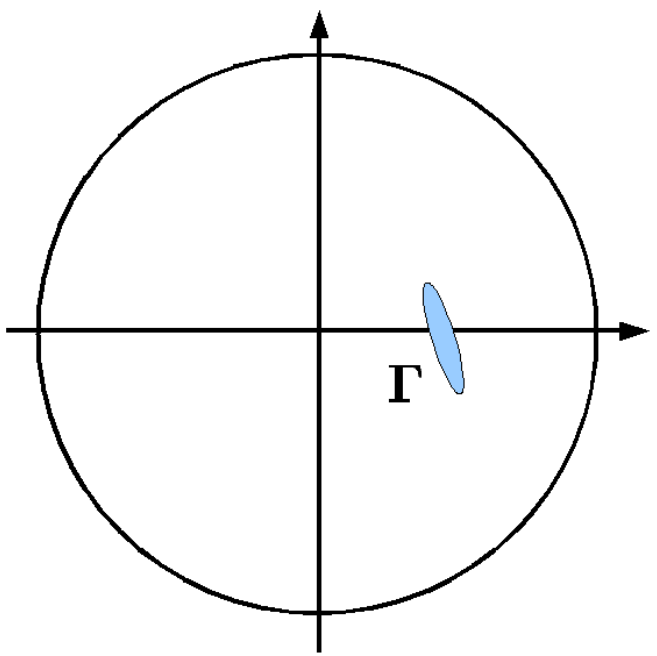
  $\Longrightarrow$ use multiple baselines

- Temporal decorrelation: $\Longrightarrow |\gamma_v|$ corrupted
  $\Longrightarrow$ $\arg\gamma_v$ independent of temporal decorrelation

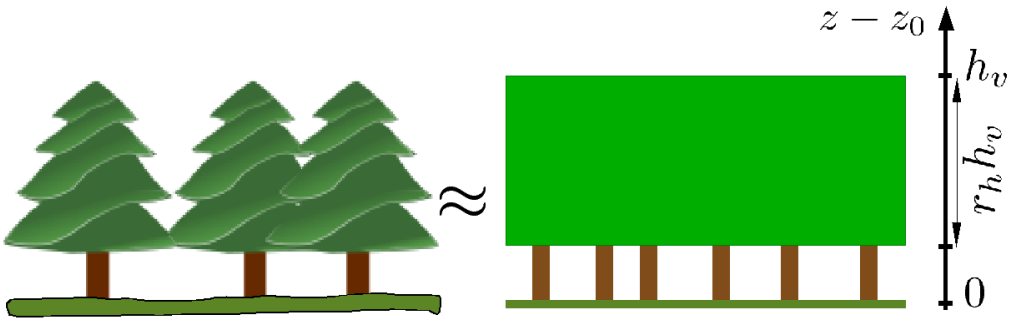
Parameter Retrieval Framework Scheme



1. Preprocessing

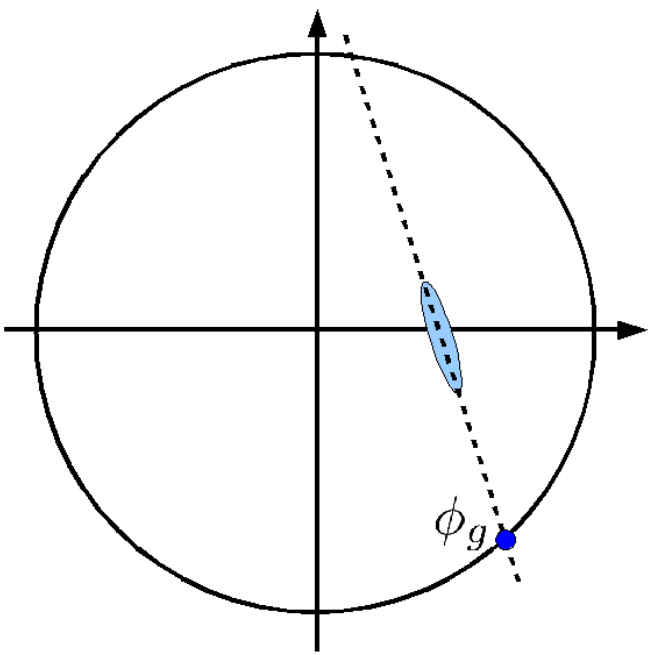


Parameter Retrieval Framework Scheme

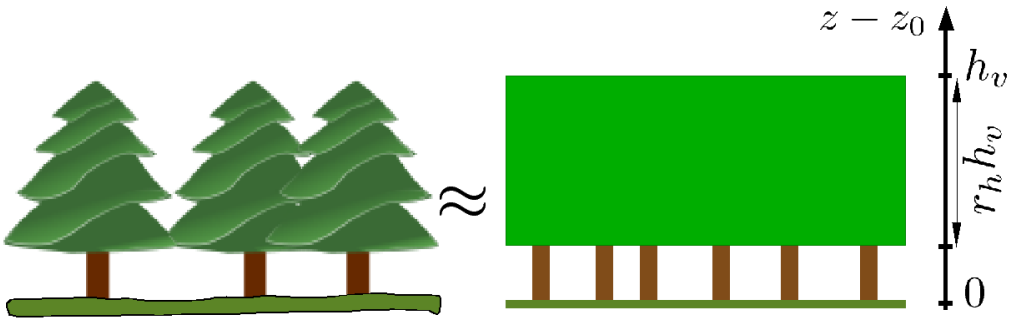


1. Preprocessing

2. Linear structure estimation
$\Longrightarrow \phi_0^i, \gamma_g^i$



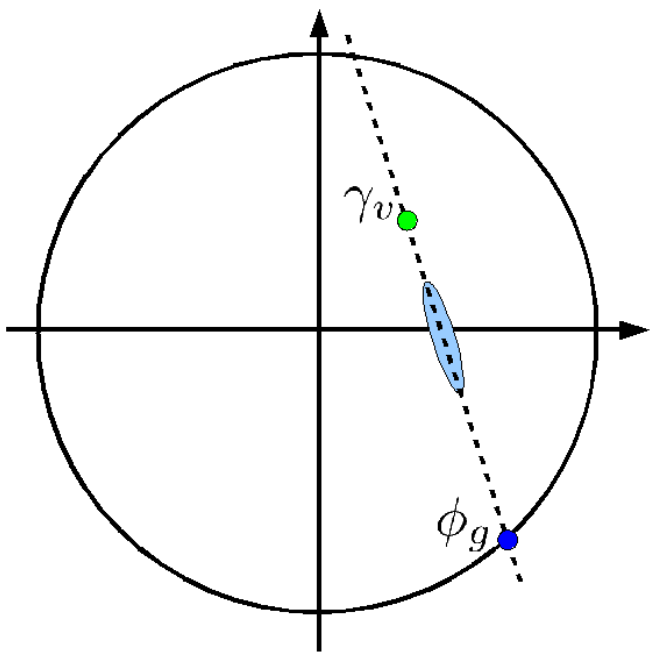
Parameter Retrieval Framework Scheme



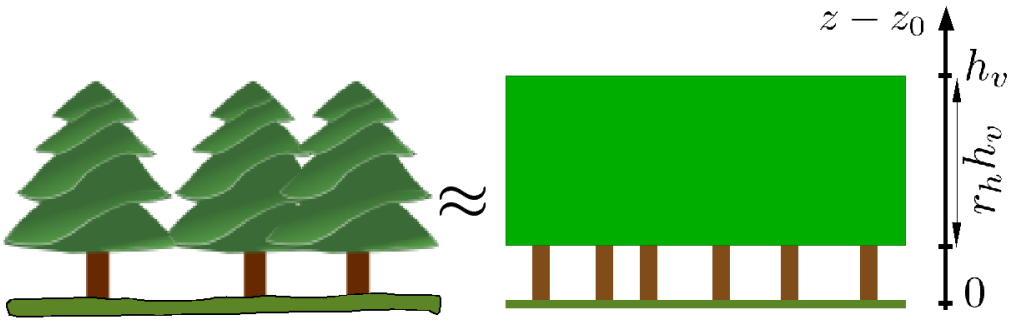
1. Preprocessing

2. Linear structure estimation
$\Longrightarrow \phi_0^i, \gamma_g^i$

3. Estimation of polarimetric and interferometric parameters $\Longrightarrow \delta, \tau, \beta, \beta_{22}, \beta_{33}, f_v, f_g, \gamma_v^i$



Parameter Retrieval Framework Scheme

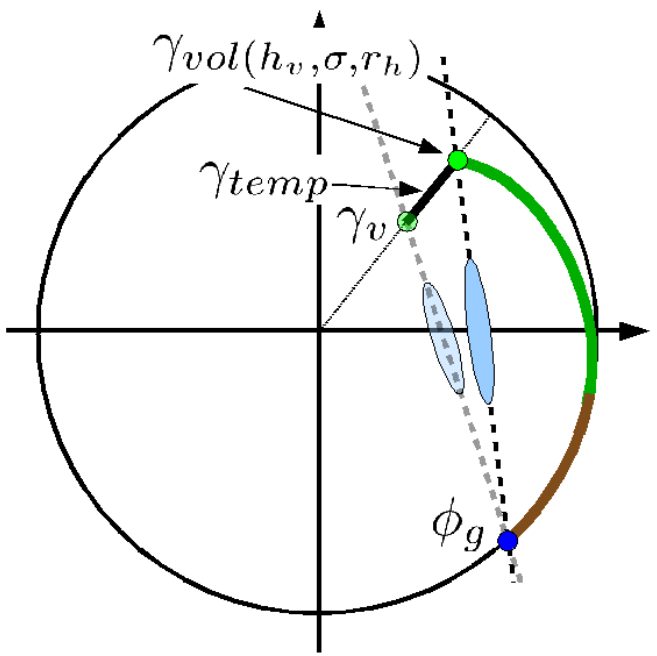


1. Preprocessing

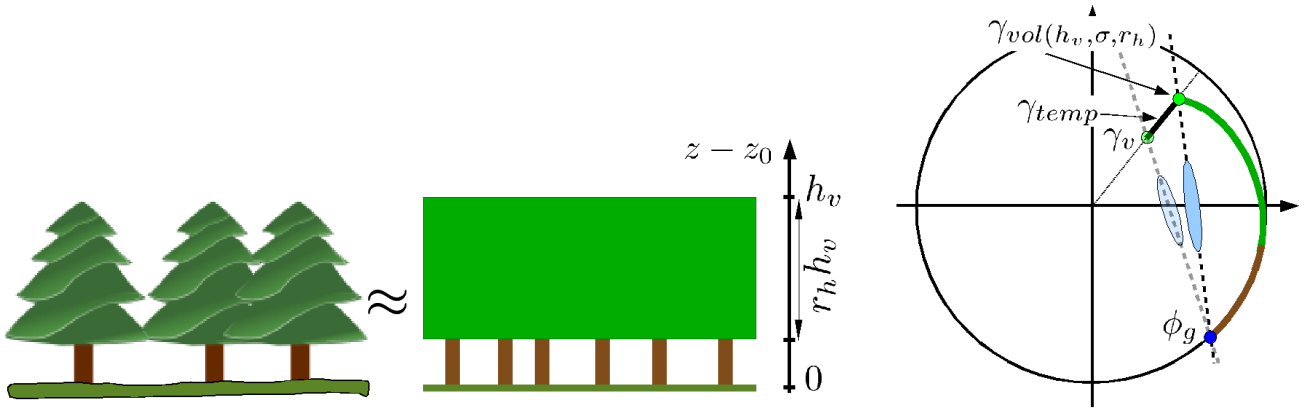
2. Linear structure estimation
$\Longrightarrow \phi_0^i, \gamma_g^i$

3. Estimation of polarimetric and interferometric parameters $\Longrightarrow \delta, \tau, \beta, \beta_{22}, \beta_{33}, f_v, f_g, \gamma_v^i$

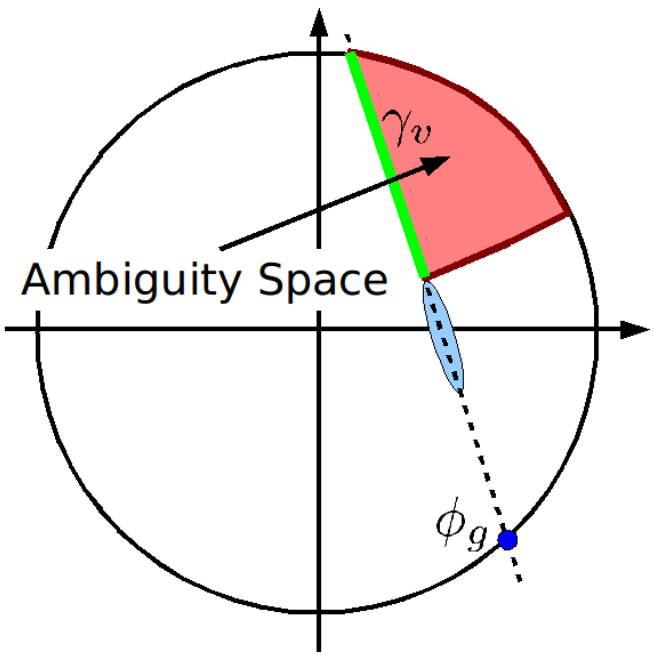
4. Estimation of structural parameters
$\Longrightarrow h_v, \sigma, r_h, \gamma_{temp}^i$



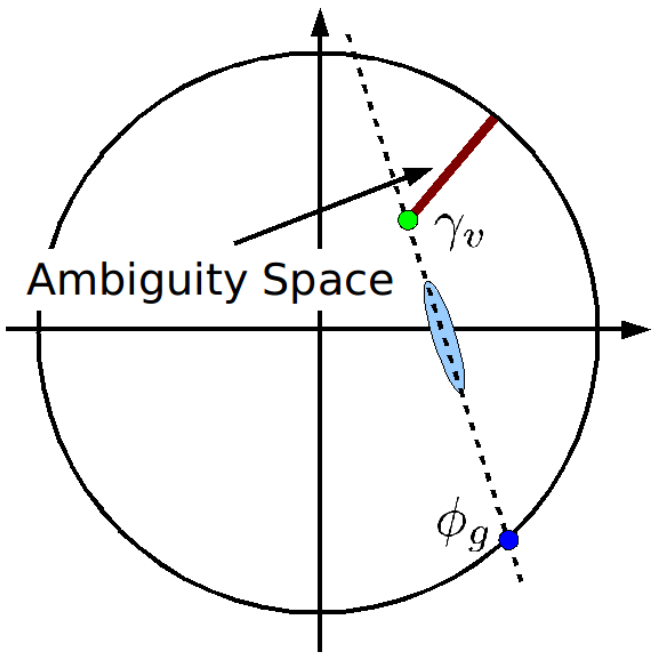
Comparison with RVoG Inversion: Ambiguity Space



RVoG Model



With scattering model



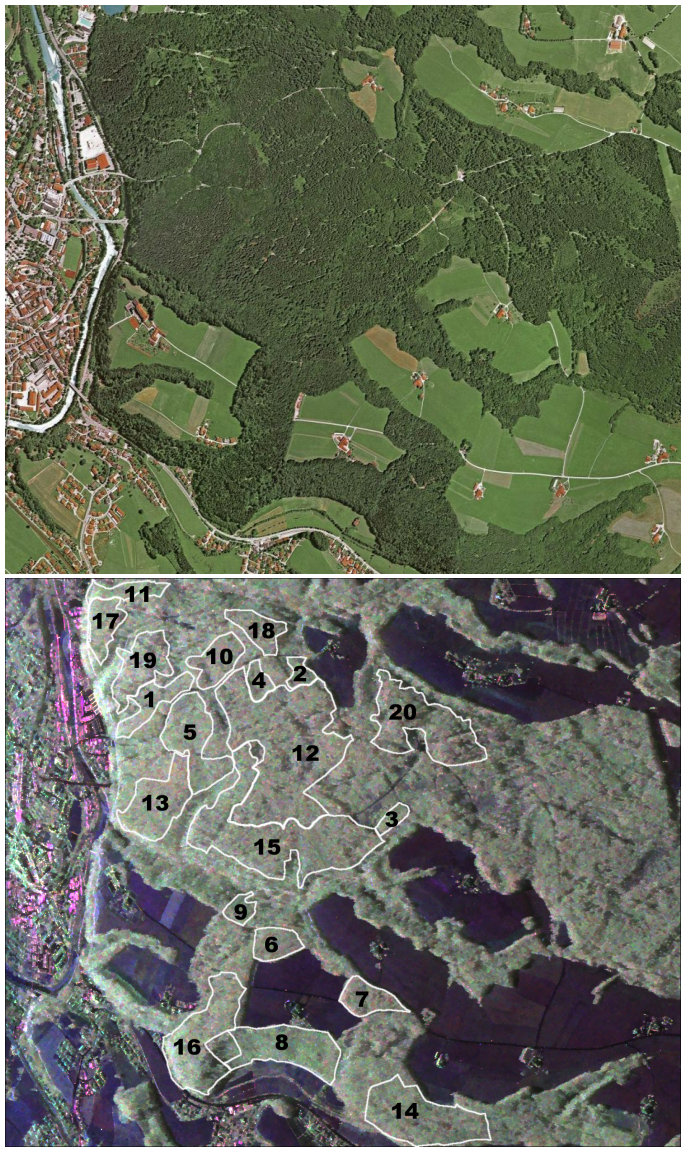
Test Site Traunstein

- Traunstein test site, 11.11.2003
- E–SAR L–band
- Incidence angles: 25°– 56°
- Resolution: 1.5m (slant range) x 95cm (azimuth)
- Data:
  - Track 1: 09:00
  - Track 2: 08:50
  - Track 3: 08:40
  - Track 4: 08:00
- Used baselines:
  - 1–2: 10min, 5m baseline
  - 1–3: 20min, 10m baseline
  - 1–4: 1h, 0m baseline

Test Site Traunstein

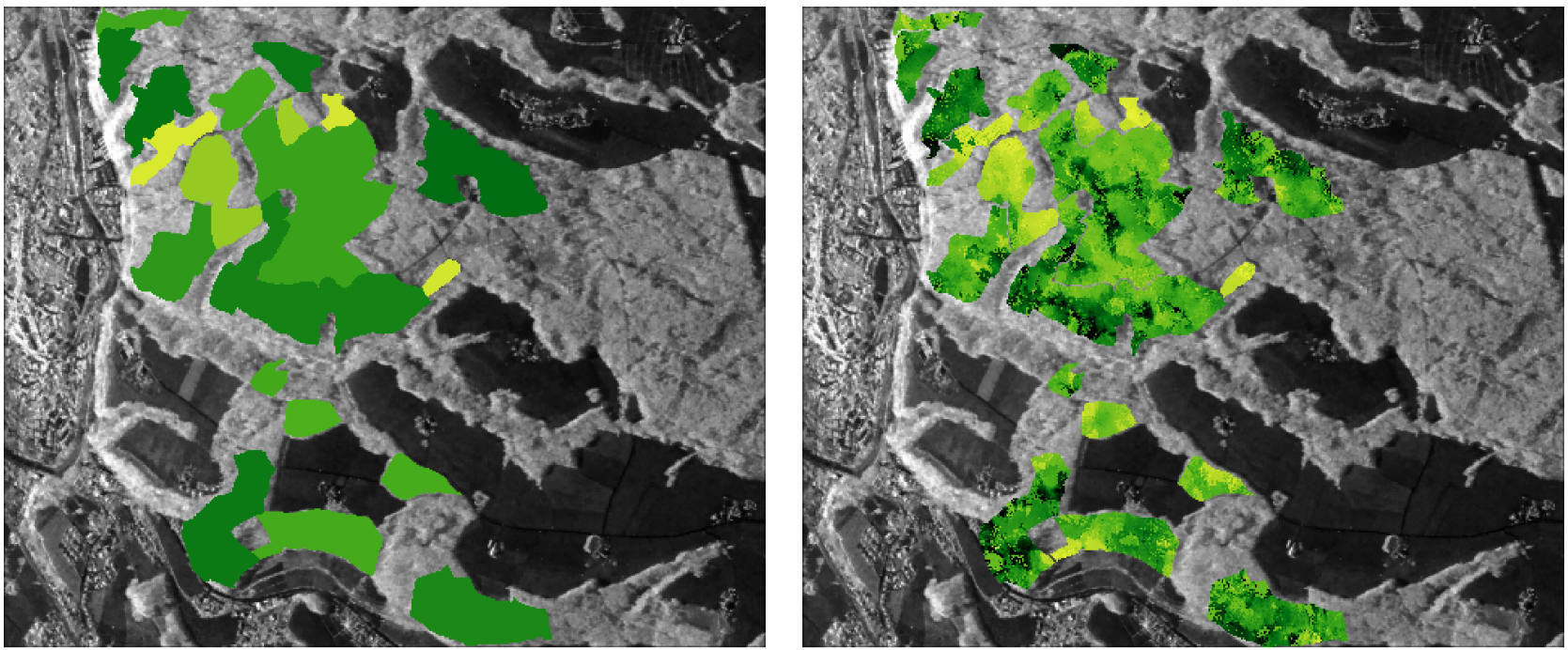
	S	T	Species	Avg. h_v	$\pm h_v$
1	G	c	Fi-Ta-Bu	12.46m	±2.3m
2	G	c	Fi	13.00m	±0m
3	G	d	Ah-Bu-Fi	13.05m	±0m
4	R	c	Fi-Bu	18.66m	±3.6m
5	G	c	Fi-Ta-Bu	19.68m	±8.6m
6	G	c	Fi-Bu-Ei	26.3m	±1.5m
7	G	c	Fi-Bu-Ei	26.93m	±2.3m
8	G	d	Bu-Ah-Es	27.20m	±3.1m
9	G	d	Bu-Fi-La	27.32m	±0m
10	M	c	Fi-Bu-La	27.43m	±2.2m
11	G	c	Fi-Bu-La	27.62m	±2.2m
12	M	c	Fi-Bu-Ta	28.43m	±1.6m
13	M	c	Fi-Bi-Bu	30.13m	±2.7m
14	M	c	Fi-Bu-Es	32.49m	±1.6m
15	R	c	Fi-Bu-Ta	33.14m	±2.7m
16	R	c	Fi-Bu-Ah	34.34m	±1.8m
17	M	c	Fi-Bu-Ei	34.59m	±2.5m
18	R	c	Fi-Ta-Ki	34.66m	±1.4m
19	R	c	Fi-Ta-Bu	35.23m	±1.2m
20	R	c	Fi-Bu	36.10m	±1.8m

G: growth, M: mature, R: regenerating.
c: coniferous, d: deciduous.
Fi: north spruce, Ki: Scots pine, Ta: white fir, La: Eur. larch, Bu: Eur. beech, Es: ash, Ah: maple, Ei: oak, Bi: birch.



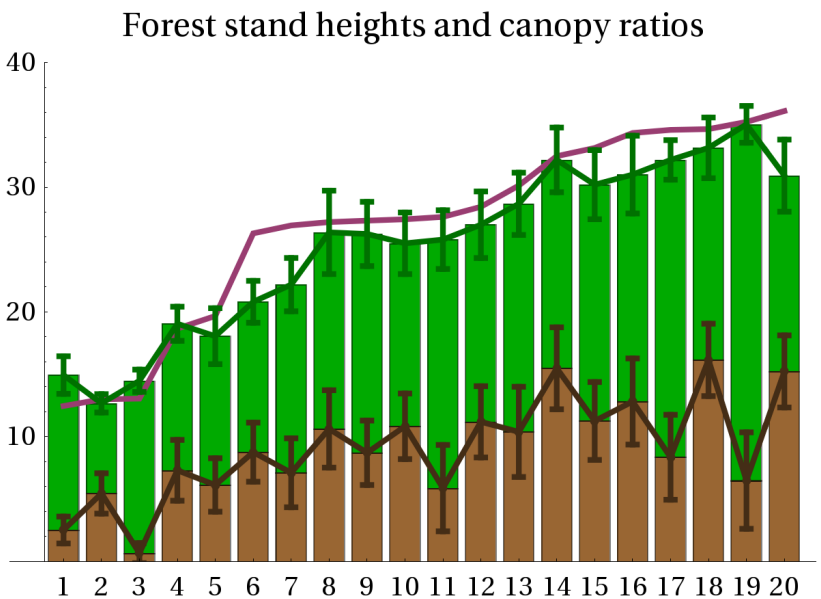
Experimental Results: Vegetation Height

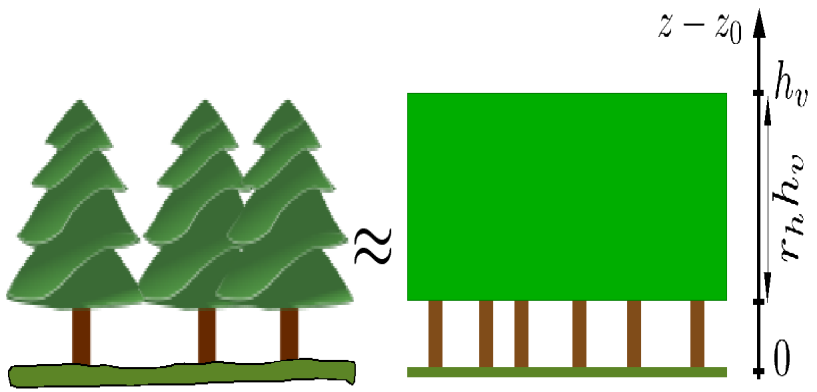
$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$



Experimental Results: Vegetation Height

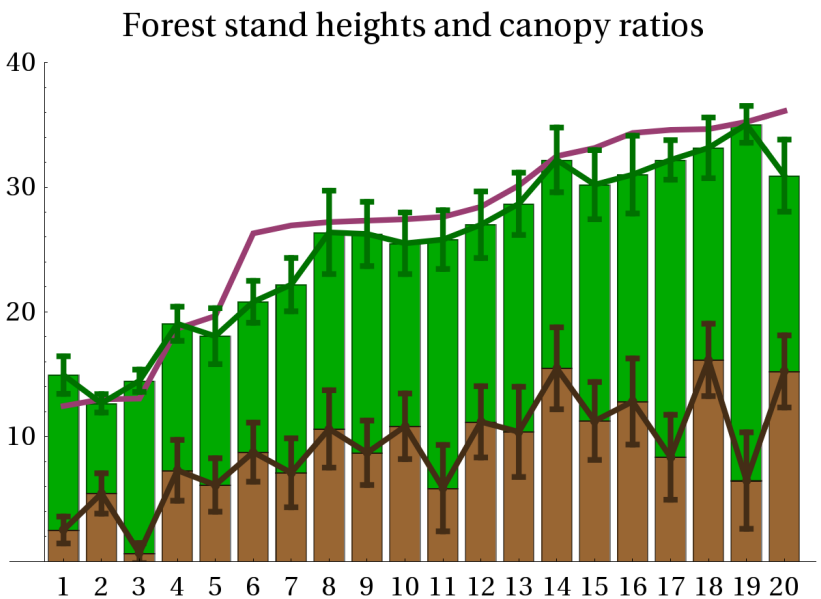
$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

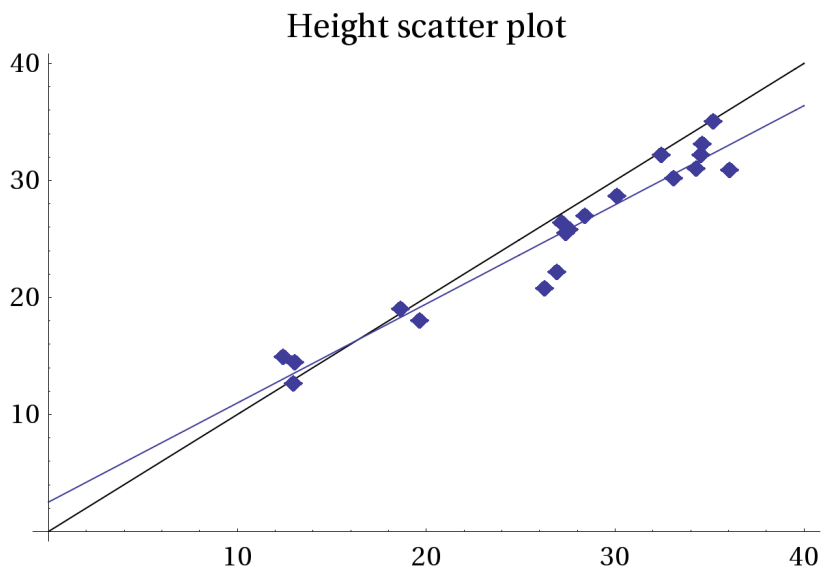




Experimental Results: Vegetation Height

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

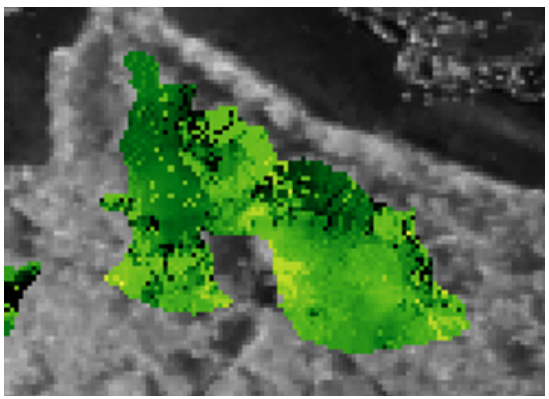




Experimental Results: Vegetation Height

Evaluation Stand 20

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

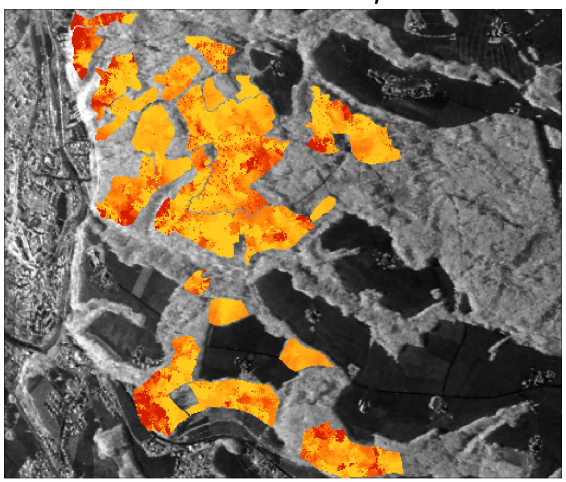


- Radar data: 2003. Ground truth: 1998!
- Forest inventory: height = 36.10m $\pm$ 1.8m
- Dominated by spruce and beech tree species.
- Quite heterogeneous: roads, clear cuts, ...

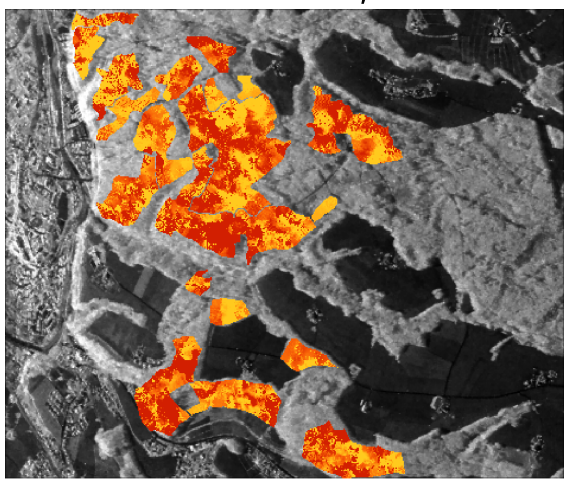
Experimental Results: Temporal Decorrelation

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

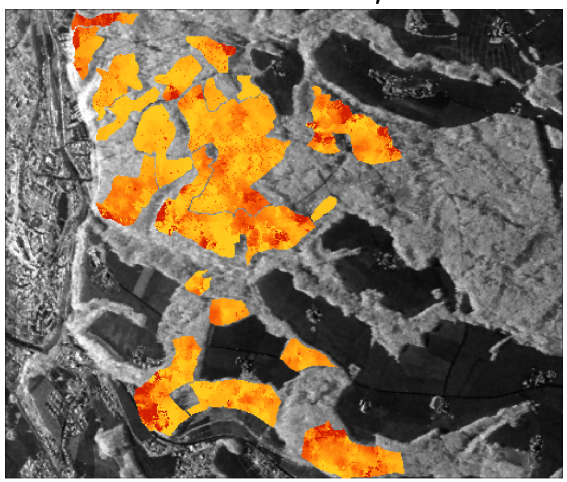
1–2: 10min, 5m

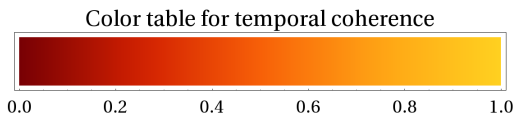


1–3: 20min, 10m



1–4: 60min, 0m

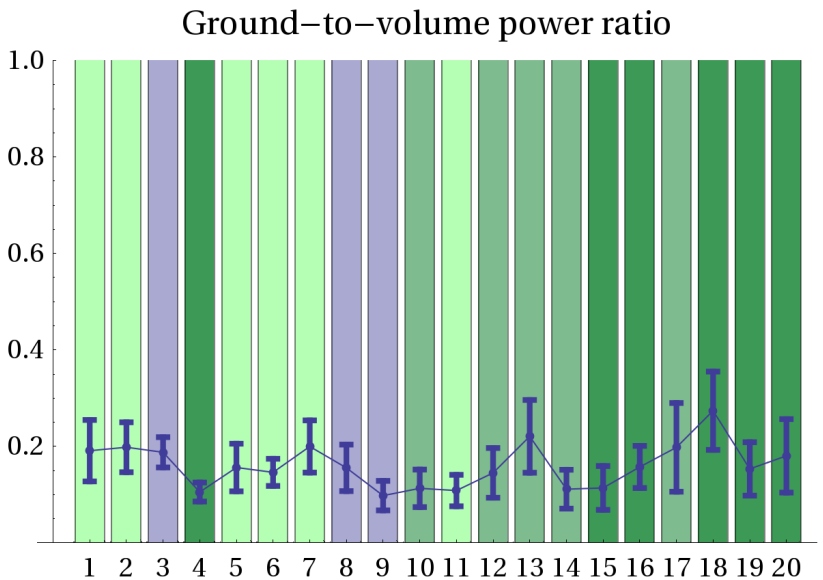




- Short temporal scale: temporal decorrelation mostly caused by wind in the canopy.
- γ_{temp}: baseline dependent?

Experimental Results: Ground–To–Volume Power Ratio

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

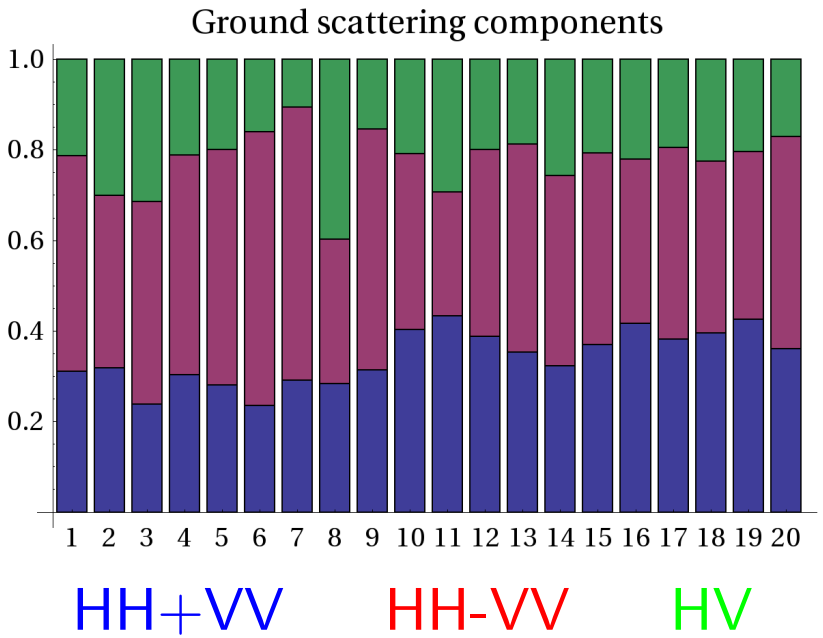


- Ground–to–volume ratio: $\frac{P_g}{P_v}$
- $P_g = f_g\,\mathrm{trace}\,\mathbf{T}_g$: total ground scattering power
- $P_v = f_v\,\mathrm{trace}\,\mathbf{T}_v$: total volume scattering power

Experimental Results: Ground Scattering

Pauli–Components

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$



- All Pauli components present.
- HV: small, but present.

Tree Morphology

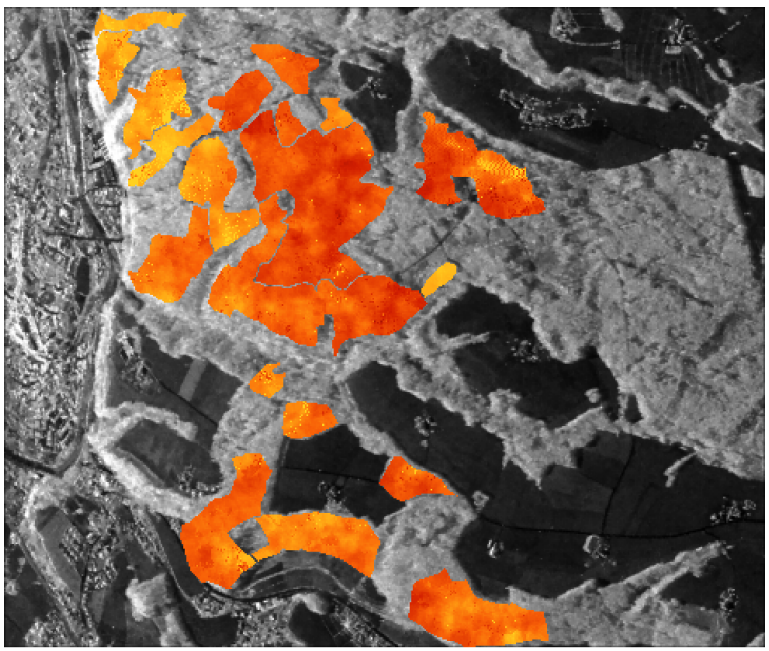
Coniferous trees

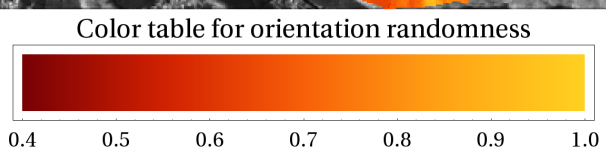
Deciduous trees

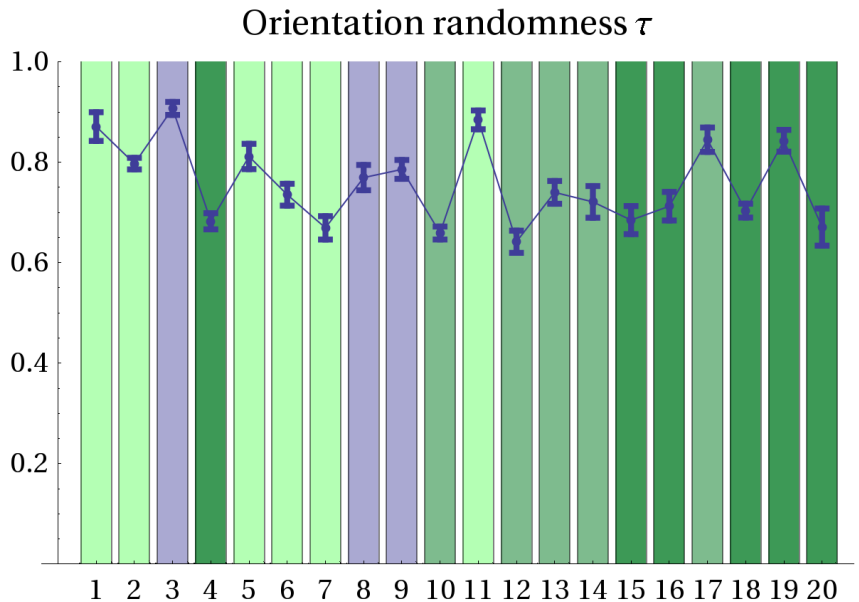
- τ: depends on incidence angle and preferred orientation.
- Coniferous trees: more horizontal.
- Deciduous trees: more random and more vertical.

Experimental Results: Degree of Orientation Randomness

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$





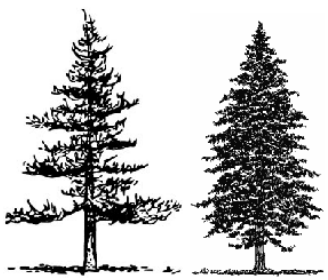


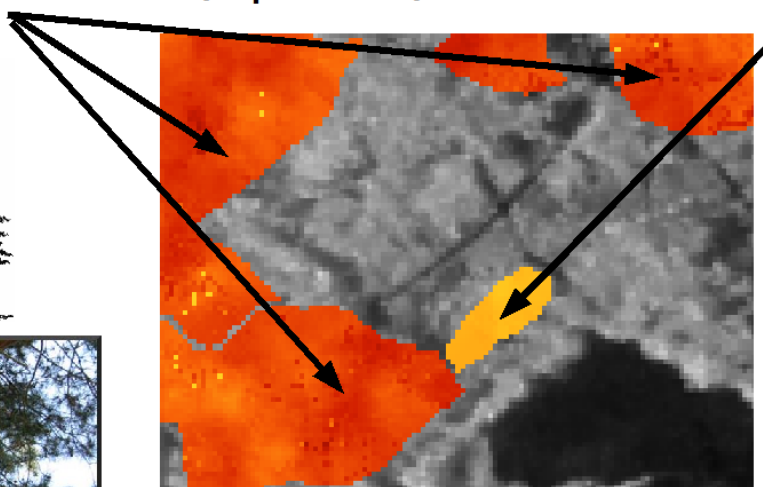
- Low SDEV.
- Dependent on incidence angle.
- Discriminates between different forest stands.

Experimental Results: Degree of Orientation Randomness

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

Coniferous Trees (Spruce)

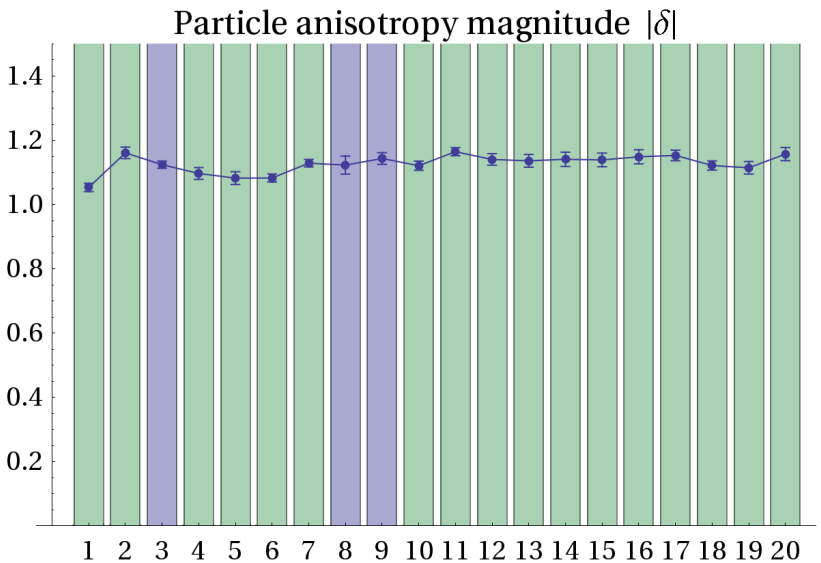


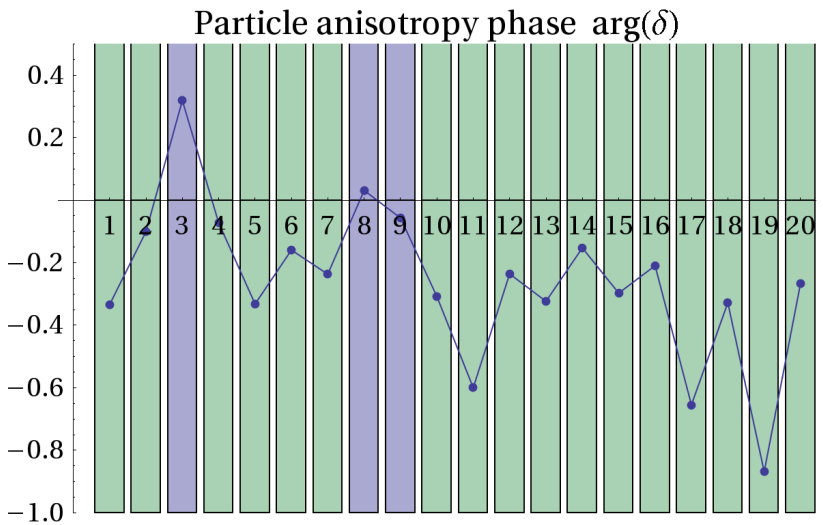


Deciduous Trees (Maple Trees)

Experimental Results: Particle Anisotropy

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$

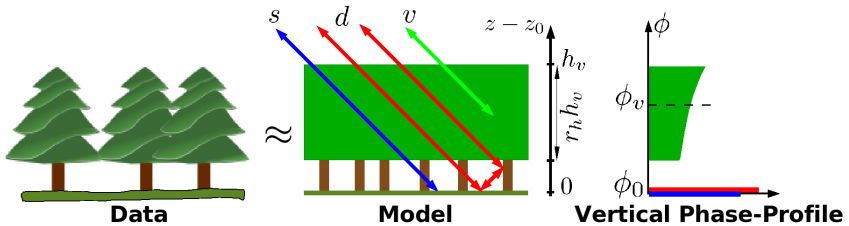




- Anisotropy magnitude $> 1 \Longrightarrow$ possibly multiple scattering.
- Phase as an indicator for forest species type?

Summary

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^\dagger & \mathbf{T} \end{bmatrix} \rightarrow \begin{cases} \mathbf{T} & = f_g \mathbf{T}_g + f_v \mathbf{T}_v \\ \mathbf{\Omega} & = f_g \gamma_g \mathbf{T}_g + f_v \gamma_v \mathbf{T}_v \end{cases}$$



- Ground: merge of surface and double–bounce scattering
- Volume: scattering anisotropy, orientation randomness
- Vertical structure: height, extinction, canopy–fill–factor

- Enhancement of structural parameter retrieval
- Estimation of vegetation characteristics
- Estimation of the ground component under vegetation
- Temporal decorrelation estimation and compensation

Perspectives

- Vegetation parameter retrieval and classification: forests, agriculture.
- Other volumetric media: ice, snow, dry soil/sand, ...
- Ground characteristics under vegetation.
- Combination of multiple data sources (multi–baseline, multi–altitude, multi–angular, multi–frequency) in a single parameter retrieval framework.
- Multi–temporal monitoring of ecosystem dynamics (carbon dynamics in forestry)

Thank You!
Questions?

PolInSAR: Number of Observables

- General quad–pol case: $\mathbf{T}_8 \in \mathbb{C}^{8\times 8} \Longrightarrow$ 64 observables.
- Reciprocity: $S_{hv} = S_{vh} \Longrightarrow \mathbf{T}_6 \in \mathbb{C}^{6\times 6} \Longrightarrow$ 36 observables.
- Polarimetric stationarity: $\mathbf{T}_{11} = \mathbf{T}_{22} \Longrightarrow$ 27 observables.
- Reflection symmetry: $\langle S_{aa} S_{ab}^{*} \rangle = 0 \Longrightarrow$ 15 observables.

$$\mathbf{T}_8 = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{\Omega} \\ \mathbf{\Omega}^{\dagger} & \mathbf{T}_{22} \end{bmatrix}$$

$$\mathbf{T}_{11} = \begin{bmatrix} A_1 & E_1+iF_1 & G_1+iH_1 & I_1+iJ_1 \\ E_1-iF_1 & B_1 & K_1+iL_1 & M_1+iN_1 \\ G_1-iH_1 & K_1-iL_1 & C_1 & O_1+iP_1 \\ I_1-iJ_1 & M_1-iN_1 & O_1-iP_1 & D_1 \end{bmatrix}$$

$$\mathbf{T}_{22} = \begin{bmatrix} A_2 & E_2+iF_2 & G_2+iH_2 & I_2+iJ_2 \\ E_2-iF_2 & B_2 & K_2+iL_2 & M_2+iN_2 \\ G_2-iH_2 & K_2-iL_2 & C_2 & O_2+iP_2 \\ I_2-iJ_2 & M_2-iN_2 & O_2-iP_2 & D_2 \end{bmatrix}$$

$$\mathbf{\Omega} = \begin{bmatrix} Q+iR & S+iT & U+iV & W+iX \\ Y+iZ & a+ib & c+id & e+if \\ g+ih & k+il & m+in & o+ip \\ q+ir & s+it & u+iv & w+ix \end{bmatrix}$$

PolInSAR: Number of Observables

- General quad–pol case: $\mathbf{T}_8 \in \mathbb{C}^{8\times 8} \Longrightarrow$ 64 observables.
- Reciprocity: $S_{hv} = S_{vh} \Longrightarrow \mathbf{T}_6 \in \mathbb{C}^{6\times 6} \Longrightarrow$ 36 observables.
- Polarimetric stationarity: $\mathbf{T}_{11} = \mathbf{T}_{22} \Longrightarrow$ 27 observables.
- Reflection symmetry: $\langle S_{aa} S_{ab}^{*} \rangle = 0 \Longrightarrow$ 15 observables.

$$\mathbf{T}_6 = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{\Omega} \\ \mathbf{\Omega}^{\dagger} & \mathbf{T}_{22} \end{bmatrix}$$

$$\mathbf{T}_{11} = \begin{bmatrix} A_1 & E_1+iF_1 & G_1+iH_1 \\ E_1-iF_1 & B_1 & K_1+iL_1 \\ G_1-iH_1 & K_1-iL_1 & C_1 \end{bmatrix}$$

$$\mathbf{T}_{22} = \begin{bmatrix} A_2 & E_2+iF_2 & G_2+iH_2 \\ E_2-iF_2 & B_2 & K_2+iL_2 \\ G_2-iH_2 & K_2-iL_2 & C_2 \end{bmatrix}$$

$$\mathbf{\Omega} = \begin{bmatrix} Q+iR & S+iT & U+iV \\ Y+iZ & a+ib & c+id \\ g+ih & k+il & m+in \end{bmatrix}$$

PolInSAR: Number of Observables

- General quad–pol case: $\mathbf{T}_8 \in \mathbb{C}^{8\times 8} \Longrightarrow$ 64 observables.
- Reciprocity: $S_{hv} = S_{vh} \Longrightarrow \mathbf{T}_6 \in \mathbb{C}^{6\times 6} \Longrightarrow$ 36 observables.
- Polarimetric stationarity: $\mathbf{T}_{11} = \mathbf{T}_{22} \Longrightarrow$ 27 observables.
- Reflection symmetry: $\langle S_{aa} S_{ab}^{*} \rangle = 0 \Longrightarrow$ 15 observables.

$$\mathbf{T}_6^{PS} = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^{\dagger} & \mathbf{T} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} A & E+iF & G+iH \\ E-iF & B & K+iL \\ G-iH & K-iL & C \end{bmatrix}$$

$$\mathbf{\Omega} = \begin{bmatrix} Q+iR & S+iT & U+iV \\ Y+iZ & a+ib & c+id \\ g+ih & k+il & m+in \end{bmatrix}$$

PolInSAR: Number of Observables

- General quad–pol case: $\mathbf{T}_8 \in \mathbb{C}^{8\times 8} \Longrightarrow$ 64 observables.
- Reciprocity: $S_{hv} = S_{vh} \Longrightarrow \mathbf{T}_6 \in \mathbb{C}^{6\times 6} \Longrightarrow$ 36 observables.
- Polarimetric stationarity: $\mathbf{T}_{11} = \mathbf{T}_{22} \Longrightarrow$ 27 observables.
- Reflection symmetry: $\langle S_{aa} S_{ab}^{*} \rangle = 0 \Longrightarrow$ 15 observables.

$$\mathbf{T}_6^{PS,refl} = \begin{bmatrix} \mathbf{T} & \mathbf{\Omega} \\ \mathbf{\Omega}^{\dagger} & \mathbf{T} \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} A & E+iF & 0 \\ E-iF & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

$$\mathbf{\Omega} = \begin{bmatrix} Q+iR & S+iT & 0 \\ Y+iZ & a+ib & 0 \\ 0 & 0 & m+in \end{bmatrix}$$

Experimental Results: Number of Looks L

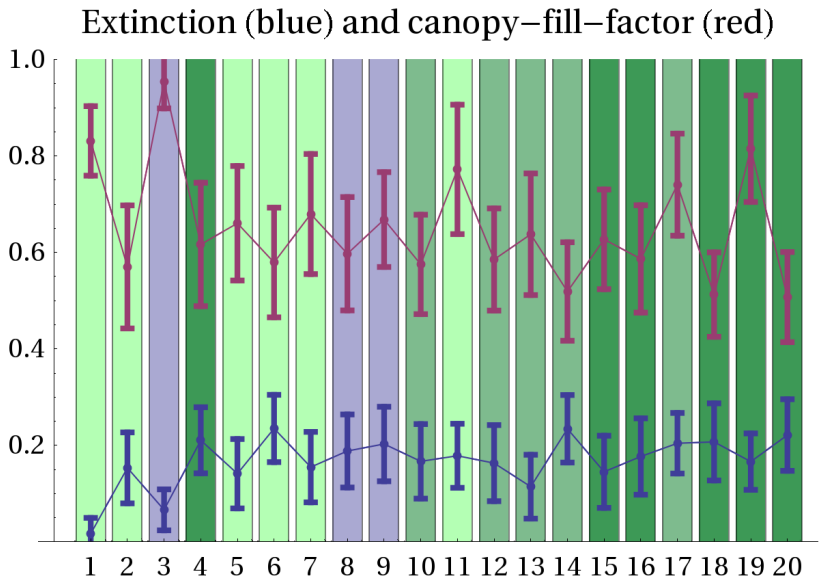
Height estimation statistics for different numbers of looks:

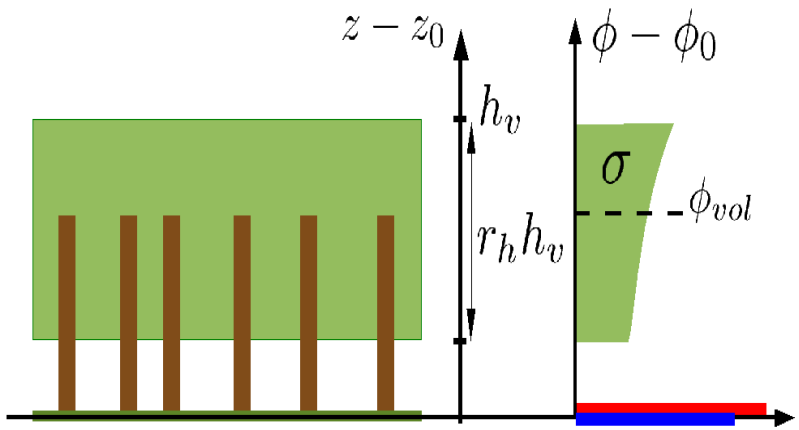
L	Bias	Abs. error	RMSE	SDEV
450 looks	-2.13	4.74	5.87	5.10
900 looks	-1.75	4.47	5.61	4.88
1350 looks	-1.60	4.34	5.43	4.77
1800 looks	-1.51	3.97	4.97	4.33

- Accuracy increasing with higher number of looks.
- Over some evaluation stands always good results, independent of L.

Experimental Results: Extinction and Canopy–Fill–Factor

$$\mathbf{m} = \{f_g, \mathrm{Re}\,\beta, \mathrm{Im}\,\beta, \beta_{22}, \beta_{33}, f_v, \mathrm{Re}\,\delta, \mathrm{Im}\,\delta, \tau, h_v, r_h, \sigma, \gamma_{sys}, \gamma_{temp}, \phi_0\}$$





- Both are very volatile, probably caused by ambiguities.