Estimating Production Functions with Heterogeneous Firms

Amit Gandhi, Salvador Navarro, and David Rivers

University of Wisconsin-Madison

June 21, 2008

Brief Background

Brief Background

- Production function estimation as a tool for productivity analysis

Brief Background

- Production function estimation as a tool for productivity analysis
- e.g., Estimate a production function from the input/output behavior of a sample of firms. Use residuals as productivity measures for applied work.

Brief Background

- Production function estimation as a tool for productivity analysis
- e.g., Estimate a production function from the input/output behavior of a sample of firms. Use residuals as productivity measures for applied work.

The Problem

- $y_{jt} = \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \epsilon_{jt}$

The Problem

- $y_{jt} = \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \epsilon_{jt}$
- ω_{jt} is endogenous

The Problem

- $y_{jt} = \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \epsilon_{jt}$
- ω_{jt} is endogenous
- Fixed effects impose $\omega_{jt} = \omega_j$
- IV requires good instruments

The Problem

- $y_{jt} = \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \epsilon_{jt}$
- ω_{jt} is endogenous
- Fixed effects impose $\omega_{jt} = \omega_j$
- IV requires good instruments
- The semiparametric approach initiated by Olley&Pakes (1996) and pursued further by Levinsohn&Petrin (2003) - use a model of firm behavior.

The Problem

- $y_{jt} = \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \epsilon_{jt}$
- ω_{jt} is endogenous
- Fixed effects impose $\omega_{jt} = \omega_j$
- IV requires good instruments
- The semiparametric approach initiated by Olley&Pakes (1996) and pursued further by Levinsohn&Petrin (2003) - use a model of firm behavior.
- We present a **"new"** approach that allows for richer patterns of firm heterogeneity than OP and LP.

The Scalar Unobservability Assumption

- (ω_{jt}, k_{jt}) is firm j's time t state.

The Scalar Unobservability Assumption

- (ω_{jt}, k_{jt}) is firm j's time t state.
- Firm j makes a time t static input decision $m_{jt} = f_t(\omega_{jt}, k_{jt})$ or dynamic input decision $i_{jt} = g_t(\omega_{jt}, k_{jt})$.

The Scalar Unobservability Assumption

- (ω_{jt}, k_{jt}) is firm j's time t state.
- Firm j makes a time t static input decision $m_{jt} = f_t(\omega_{jt}, k_{jt})$ or dynamic input decision $i_{jt} = g_t(\omega_{jt}, k_{jt})$.
- Take inverse of input demand function

$$\omega_{jt} = \phi_t(k_{jt}, m_{jt})$$

The Scalar Unobservability Assumption

- (ω_{jt}, k_{jt}) is firm j's time t state.
- Firm j makes a time t static input decision $m_{jt} = f_t(\omega_{jt}, k_{jt})$ or dynamic input decision $i_{jt} = g_t(\omega_{jt}, k_{jt})$.
- Take inverse of input demand function

$$\omega_{jt} = \phi_t(k_{jt}, m_{jt})$$

.

- Control for endogeneity nonparametrically

$$y_{jt} = \alpha l_{jt} + \Phi_t(m_{jt}, k_{jt}) + \epsilon_{jt}$$

Evolution of TFP

- First order Markov assumption

$$\omega_{jt} = g(\omega_{jt-1}) + \eta_{jt}$$

Evolution of TFP

- First order Markov assumption

$$\omega_{jt} = g(\omega_{jt-1}) + \eta_{jt}$$

- Time to build assumption

$$K_{jt} = d(K_{jt-1}, i_{jt-1})$$

Evolution of TFP

- First order Markov assumption

$$\omega_{jt} = g(\omega_{jt-1}) + \eta_{jt}$$

- Time to build assumption

$$K_{jt} = d(K_{jt-1}, i_{jt-1})$$

- These imply that

$$\eta_{jt} \perp k_{jt}$$

Evolution of TFP

- First order Markov assumption

$$\omega_{jt} = g(\omega_{jt-1}) + \eta_{jt}$$

- Time to build assumption

$$K_{jt} = d(K_{jt-1}, i_{jt-1})$$

- These imply that

$$\eta_{jt} \perp k_{jt}$$

- Use this moment to estimate β

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.

- However from the firm's point of view, ω_{jt} and ϵ_{jt} enter asymmetrically into the profit maximization problem.

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.
- However from the firm's point of view, ω_{jt} and ϵ_{jt} enter asymmetrically into the profit maximization problem.
- Use FOC **for a static input** & Production Function to solve for $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.
- However from the firm's point of view, ω_{jt} and ϵ_{jt} enter asymmetrically into the profit maximization problem.
- Use FOC **for a static input** & Production Function to solve for $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$
- Use same moment conditions as OP and LP to estimate model

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.
- However from the firm's point of view, ω_{jt} and ϵ_{jt} enter asymmetrically into the profit maximization problem.
- Use FOC **for a static input** & Production Function to solve for $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$
- Use same moment conditions as OP and LP to estimate model
- As opposed to OP and LP, we can allow for price heterogeneity (but still price taking)

Introducing Unobserved Heterogeneity (what we do)

- Basic problem is that ω_{jt} and ϵ_{jt} enter symmetrically into the production function, i.e., $(\omega_{jt} + \epsilon_{jt})$.
- However from the firm's point of view, ω_{jt} and ϵ_{jt} enter asymmetrically into the profit maximization problem.
- Use FOC **for a static input** & Production Function to solve for $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$
- Use same moment conditions as OP and LP to estimate model
- As opposed to OP and LP, we can allow for price heterogeneity (but still price taking)
- If we add a CES demand system (as in De Loecker) we can easilly (no assumption on existence of biyection) allow for market power in product market.

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$, assume L_{jt} is a static input (but any static input will do)

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$, assume L_{jt} is a static input (but any static input will do)

FOC for labot implies

$$\alpha = \frac{W_{jt} L_{jt}}{P_{jt} Q_{jt}}$$

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$, assume L_{jt} is a static input (but any static input will do)

FOC for labot implies

$$\alpha = \frac{W_{jt} L_{jt}}{P_{jt} Q_{jt}}$$

In terms of observables

$$\begin{aligned}
\ln\left(\frac{P_{jt} Y_{jt}}{W_{jt} L_{jt}}\right) &= -\ln(\alpha) + \varepsilon_{jt} \\
y_{jt} &= \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \varepsilon_{jt}
\end{aligned}$$

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$, assume L_{jt} is a static input (but any static input will do)
FOC for labot implies

$$\alpha = \frac{W_{jt} L_{jt}}{P_{jt} Q_{jt}}$$

In terms of observables

$$\begin{aligned}
\ln\left(\frac{P_{jt} Y_{jt}}{W_{jt} L_{jt}}\right) &= -\ln(\alpha) + \varepsilon_{jt} \\
y_{jt} &= \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \varepsilon_{jt}
\end{aligned}$$

Key: we leave all endogenous variables on the left hand side!
(remember measurement error lesson?)

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$, assume L_{jt} is a static input (but any static input will do)
FOC for labot implies

$$\alpha = \frac{W_{jt} L_{jt}}{P_{jt} Q_{jt}}$$

In terms of observables

$$\begin{aligned} \ln\left(\frac{P_{jt} Y_{jt}}{W_{jt} L_{jt}}\right) &= -\ln(\alpha) + \varepsilon_{jt} \\ y_{jt} &= \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \varepsilon_{jt} \end{aligned}$$

Key: we leave all endogenous variables on the left hand side!
(remember measurement error lesson?)
More generally

$$\begin{pmatrix} s_{jt} \\ y_{jt} \end{pmatrix} = \Upsilon(\omega_{jt}, \varepsilon_{jt})$$

Application to Chilean Data

Plant level Chilean manufacturing panel data from 1979-1996
Same data set used by Levinsohn and Petrin

Application to Chilean Data

Plant level Chilean manufacturing panel data from 1979-1996
Same data set used by Levinsohn and Petrin

Table: Industry 311

Method	Labor	95% CI	Capital	95% CI
OLS	.953	.932,.947	.400	.389,.411
LP	.647	.595,.700	.399	.292,.505
GNR	.414	.402,.425	.362	.274,.391

Robustness Check : The CES

Same trick works for many production functions

Robustness Check : The CES

Same trick works for many production functions

$$
\begin{aligned}
Y_{jt} &= e^{\omega_{jt}} e^{\epsilon_{jt}} \left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho}\right)^{\frac{r}{\rho}} \\
\ln\left(\frac{P_{jt} Y_{jt}}{w_{jt} L_{jt}}\right) &= -\ln(\alpha r) - \rho \ln L_{jt} + \ln\left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho}\right) + \epsilon_{jt}
\end{aligned}
$$

Table: Industry 311

	Estimate	SE
ρ	-.51	.06
α	.14	.06
β	.86	.19
r	.69	.06

Average Labor Elasticity = .45 (SE=.02)
Average Capital Elasticity = .24 (SE=.02)

The Diewert Production Function

$$Y_{jt} = e^{\omega_{jt}} e^{\epsilon_{jt}} \left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho} + \gamma L_{jt}^{0.5\rho} K_{jt}^{0.5\rho}\right)^{\frac{r}{\rho}}$$

$$\ln \frac{P_{jt} Y_{jt}}{w_{jt} L_{jt}} = -\ln(r) + \ln\left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho} + \gamma L_{jt}^{0.5\rho} K_{jt}^{0.5\rho}\right) - \ln\left(\alpha L_{jt}^{\rho} + 0.5\gamma L_{jt}^{0.5\rho} K_{jt}^{0.5\rho}\right) + \epsilon_{jt}$$

Table: Industry 311

	Estimate	SE
ρ	-1.11	.26
α	.01	.02
β	.85	.12
γ	.14	.10
r	.70	.05

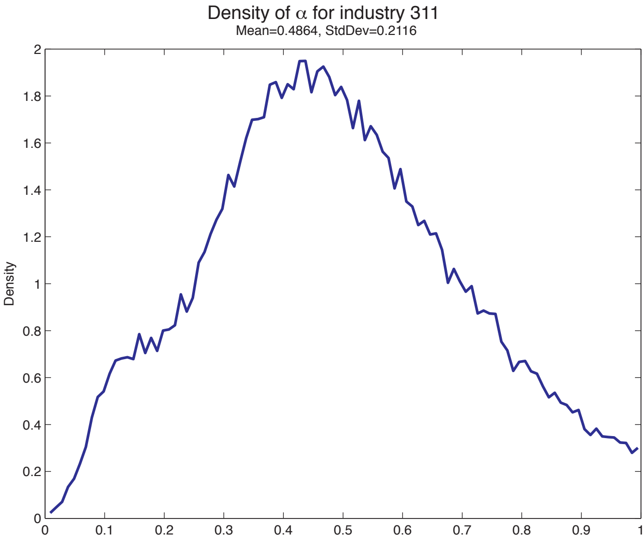
Average Labor Elasticity = .45 (SE=.02)
Average Capital Elasticity = .25 (SE=.01)

More Heterogeneity

- We show how to nonparametrically identify and estimate the distribution of α_i in a Cobb-Douglas setting with panel data

More Heterogeneity

- We show how to nonparametrically identify and estimate the distribution of α_i in a Cobb-Douglas setting with panel data



Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?

Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?
- Doesn't need to be labor, we have estimates using electricity as the static input so allowing for dynamic labor WITHOUT ad-hoc timig assumptions as in (ACF)

Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?
- Doesn't need to be labor, we have estimates using electricity as the static input so allowing for dynamic labor WITHOUT ad-hoc timig assumptions as in (ACF)
- We can easily allow for more general assumptions on TFP evolution - i.e., higher order Markov assumptions or controlled Markov process assumptions

Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?
- Doesn't need to be labor, we have estimates using electricity as the static input so allowing for dynamic labor WITHOUT ad-hoc timig assumptions as in (ACF)
- We can easily allow for more general assumptions on TFP evolution - i.e., higher order Markov assumptions or controlled Markov process assumptions
- Multiple dimensions of heterogeneity appear to be in data. Does it matter?

Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?
- Doesn't need to be labor, we have estimates using electricity as the static input so allowing for dynamic labor WITHOUT ad-hoc timig assumptions as in (ACF)
- We can easily allow for more general assumptions on TFP evolution - i.e., higher order Markov assumptions or controlled Markov process assumptions
- Multiple dimensions of heterogeneity appear to be in data. Does it matter?
- Currently extending it to allow for general patterns of heterogeneity using results from finite mixtures literature

Plus

- Can control for exit under same assumptions as OP in a simpler trivial way. Question: can we support a general exit problem?
- Doesn't need to be labor, we have estimates using electricity as the static input so allowing for dynamic labor WITHOUT ad-hoc timig assumptions as in (ACF)
- We can easily allow for more general assumptions on TFP evolution - i.e., higher order Markov assumptions or controlled Markov process assumptions
- Multiple dimensions of heterogeneity appear to be in data. Does it matter?
- Currently extending it to allow for general patterns of heterogeneity using results from finite mixtures literature
- Key application: do exports improve productivity or is it only selection (Melitz). We control for the dynamic selection process into selection (i.e. Dynamic Treatment Effects as in