The Energy Authority

Modeling Commodity Spreads with Vector Autoregressions

Ted Kury
The Energy Authority ®
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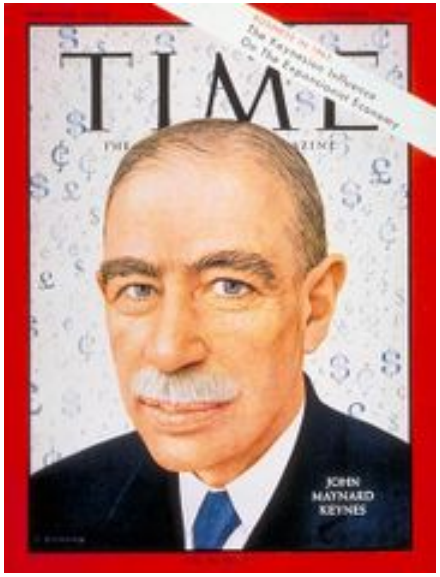
Rule #1 of Pricing Models

Pricing models can offer valuable insight into the behavior of simple or complex markets

Rule #2 of Pricing Models

"Markets tend to be rational in the long run, but markets can stay irrational longer than you can stay solvent"
J.M. Keynes (attributed)



The Problem

- Model the prices of two closely-related commodities
  - Natural gas at two different delivery points
  - Interest rates
- Prices usually tied together by some fundamental factor (e.g. transportation rates)
- Capture not only the evolution of prices, but the relationship between prices

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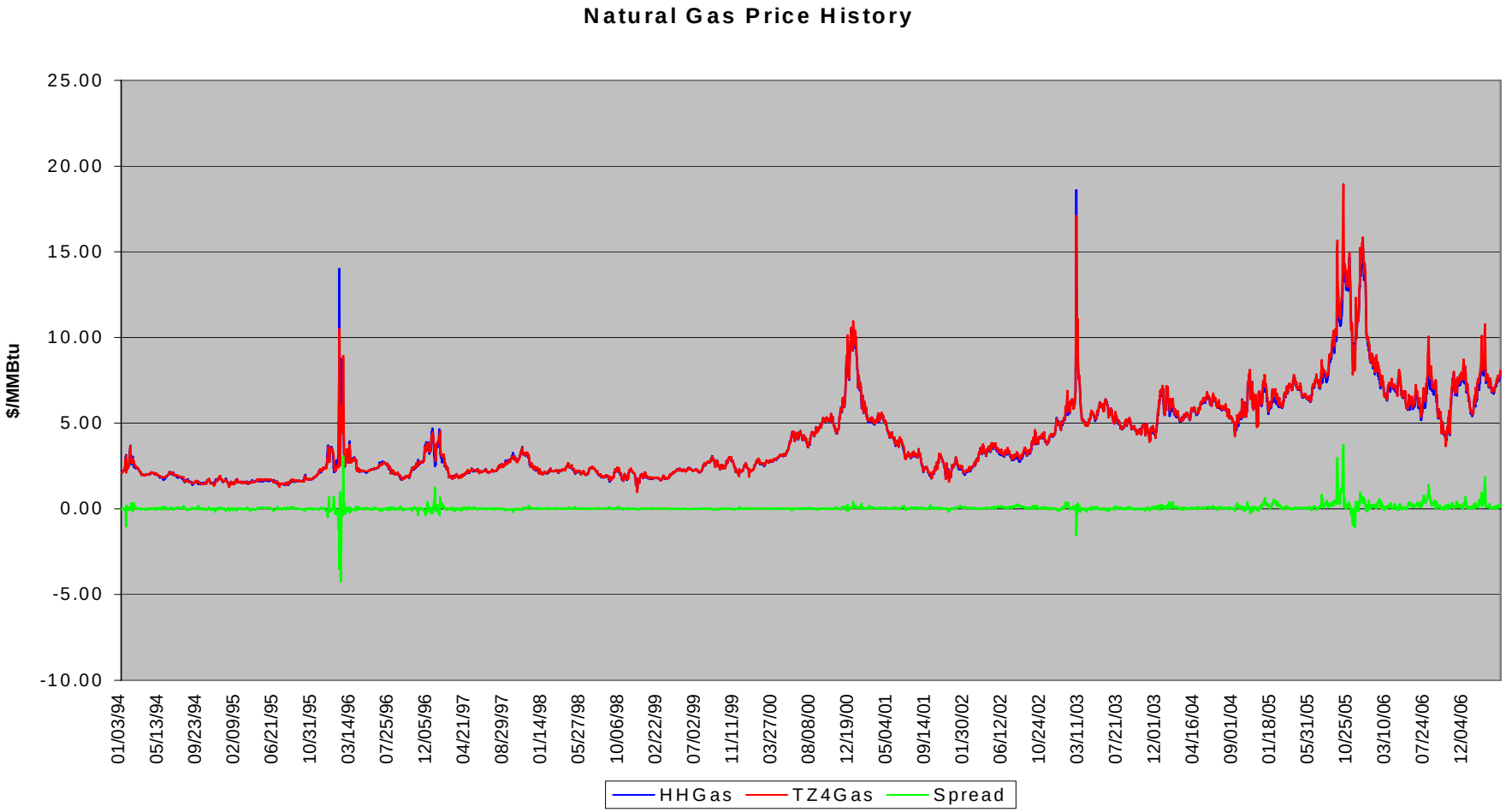
Natural Gas Time Series



Gas Price Characteristics

	Since 1994			Since 2000		
	Henry Hub	Transco Zone 4	Basis	Henry Hub	Transco Zone 4	Basis
1st %tile	1.44	1.46	-0.20	2.03	2.10	-0.12
5th %tile	1.59	1.60	-0.06	2.41	2.46	-0.04
10th %tile	1.74	1.76	-0.03	2.82	2.86	-0.01
25th %tile	2.17	2.19	0.00	3.94	3.98	0.02
75th %tile	5.61	5.66	0.07	6.66	6.81	0.12
90th %tile	7.15	7.27	0.18	7.79	8.05	0.27
95th %tile	8.01	8.22	0.30	9.64	9.92	0.40
99th %tile	12.82	13.28	0.72	13.68	14.28	0.95

Presentation Outline

- Modeling Considerations
- Traditional Energy Models
  - Modeling Prices
  - Modeling Spreads
- Vector Autoregression Framework

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Modeling Considerations

- Relative Prices
  - Spread Option
- Absolute and Relative Prices
  - Barrier Option
  - Cash Flow at Risk of Forward Purchase or Sale
  - Absolute Product (Natural Gas) Cost

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The Usual Suspects

- Closed-Form Spread Option Formulas
- Geometric Brownian Motion Price Model
- Single Factor Mean Reversion Price Model

Traditional Spread Models

- Model such as Margrabe (1978)
- Derived from Black-Scholes, so it shares its assumptions
  - Lognormal price returns
  - Independent and identically distributed shocks
  - No transaction costs

Margrabe Valuation for Spread Options

- Similar to Black and Black-Scholes

$$w(x_1, x_2, t) = x_1 N(d_1) - x_2 N(d_2)$$

where:

$$d_1 = \frac{\ln\left(x_1 / x_2\right) + \tfrac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

Geometric Brownian Motion Model (GBM)

- Use the log formulation since we're modeling a trending series

$$\ln S_t = \ln S_{t-1} + \varepsilon_t$$

where:

$$\varepsilon_t \sim N(0, \sigma^2)$$

Price Evolution Under GBM

- Let's just assume that p is the log price
- At time 1

$$p_1 = p_0 + \varepsilon_1$$

- At time 2

$$p_2 = \left[p_0 + \varepsilon_1\right] + \varepsilon_2$$

- At time t, collecting the shock terms

$$p_t = p_0 + \sum_{i=1}^{t} \varepsilon_i$$

Price Variance Under GBM

- Variance of each individual shock term

$$Var(\varepsilon_t) = \sigma^2$$

- So the variance of t terms

$$Var(p_t) = t\sigma^2$$

Modeling the Spread Between Two Prices

- Assume two prices, p and q, where

$$q = p + \eta$$

and:

$$\varepsilon_{pt} \sim N(0, \sigma_p^2)$$

$$\varepsilon_{qt} \sim N(0, \sigma_q^2)$$

$$\rho = corr(\varepsilon_p, \varepsilon_q)$$

Spread between GBM Prices

- So the basis at time t

$$q_t - p_t = p_0 + \eta_0 - p_0 + \sum_{i=1}^{t} \varepsilon_{qi} - \sum_{i=1}^{t} \varepsilon_{pi}$$

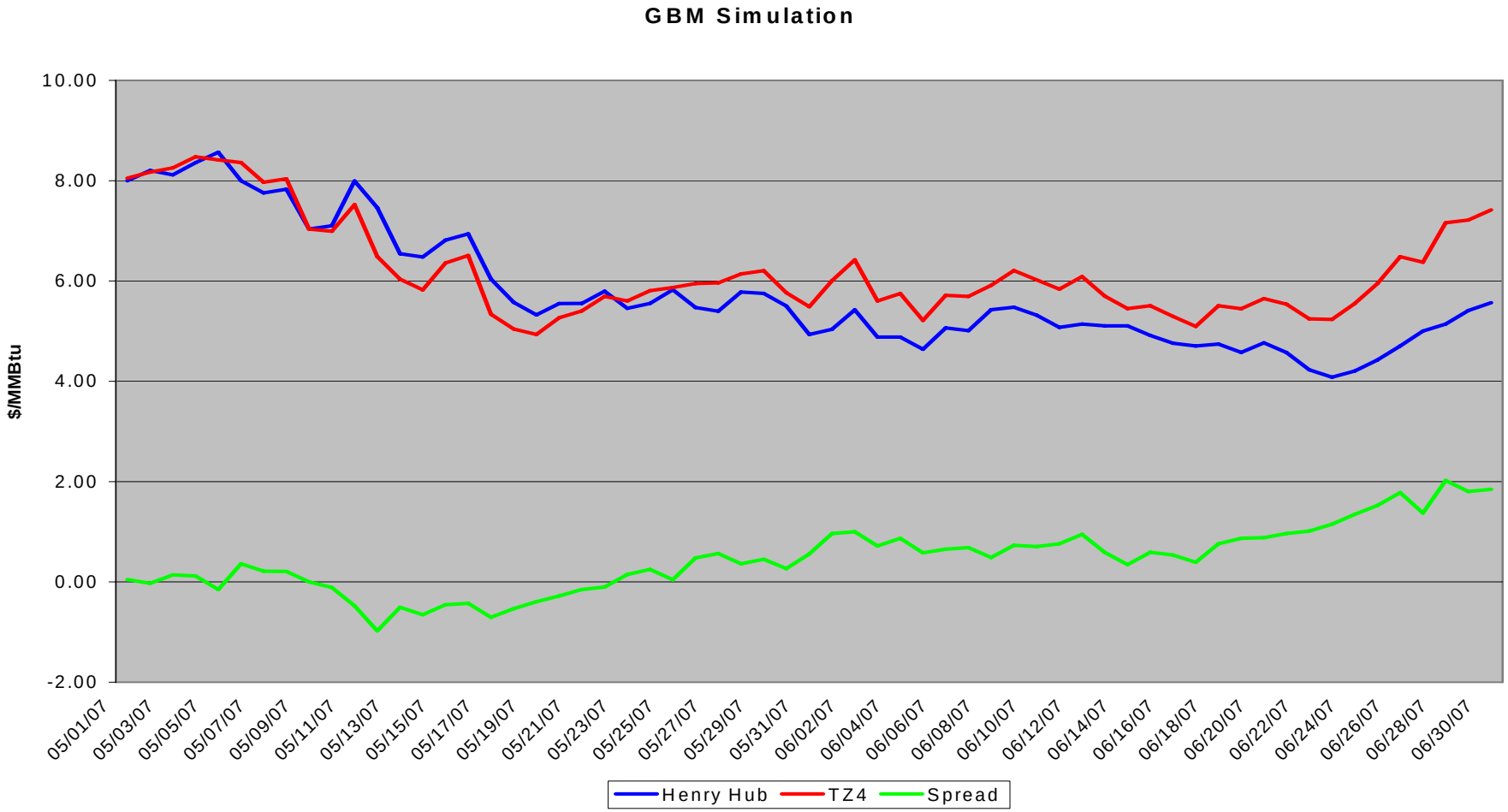
- or

$$q_t - p_t = \eta_0 + \sum_{i=1}^{t} \varepsilon_{qi} - \sum_{i=1}^{t} \varepsilon_{pi}$$

- With variance

$$Var(q_t - p_t) = t(\sigma_p^2 + \sigma_q^2 - 2\rho\sigma_p\sigma_q)$$

Sample GBM Simulation



Single Factor Mean Reverting Model (SFMR)

- Framework of Pindyck (1999) and Schwartz (1997)

$$\ln S_t = \ln S_{t-1} + \alpha(\mu - \ln S_{t-1}) + \varepsilon_t$$

where:

$$\varepsilon_t \sim N(0, \sigma^2)$$

α is mean reversion rate

μ is log of the long run equilibrium price

Price Evolution Under SFMR

- Again, assuming that p is the log price
- At time 1, rearranging terms

$$p_1 = \alpha\mu + (1-\alpha)p_0 + \varepsilon_1$$

- At time 2

$$p_2 = \alpha\mu + (1-\alpha)[\alpha\mu + (1-\alpha)p_0 + \varepsilon_1] + \varepsilon_2$$

- At time t, collecting terms

$$p_t = \alpha\mu\sum_{i=0}^{t-1}(1-\alpha)^i + (1-\alpha)^t p_0 + \sum_{i=1}^{t}\varepsilon_i(1-\alpha)^{t-i}$$

Price Variance Under SFMR

- Variance of each individual shock term

$$Var(\varepsilon_t) = \sigma^2$$

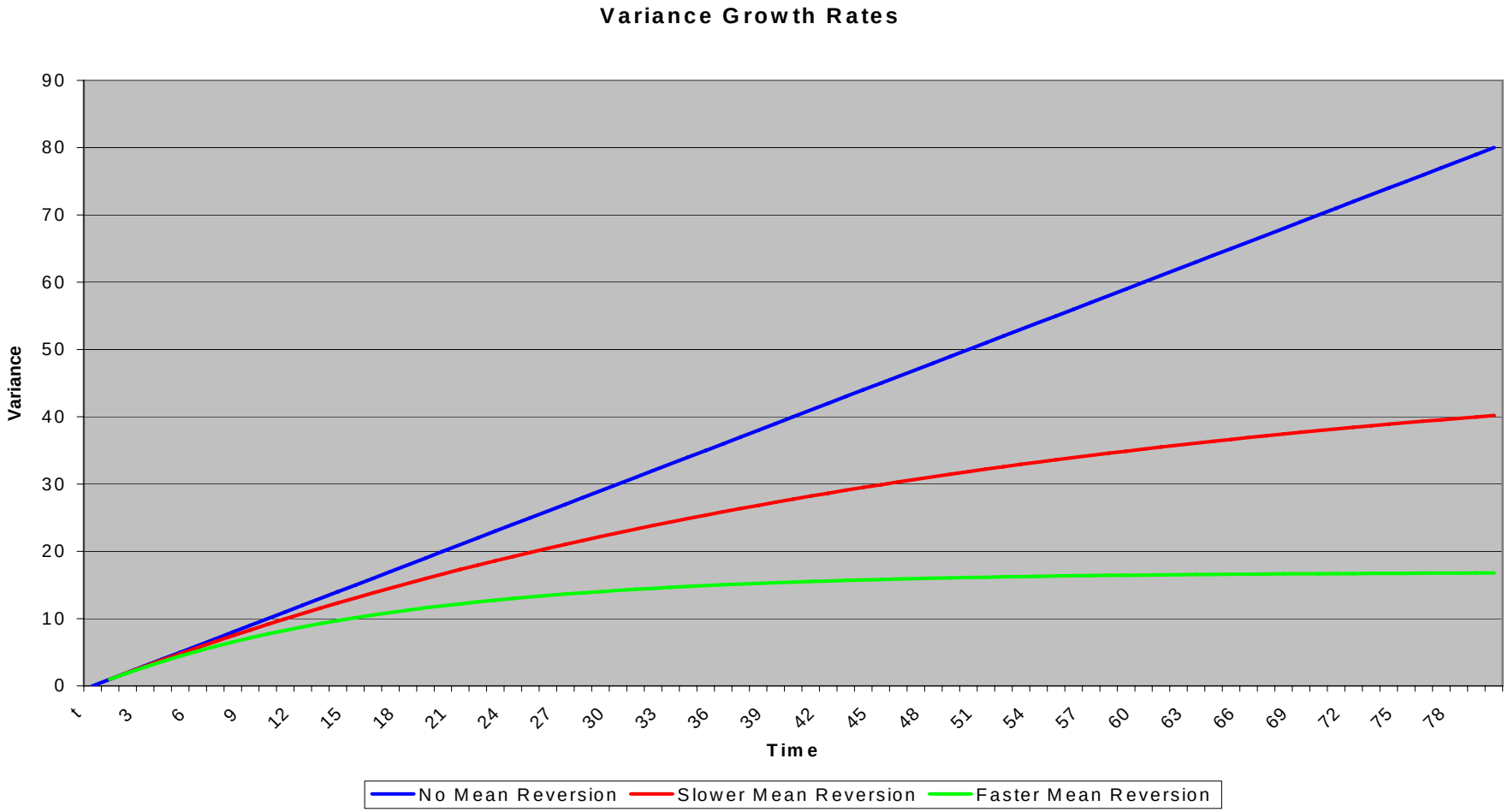
- So the variance of *t* terms

$$Var(p_t) = \sum_{i=1}^{t} \sigma^2 (1-\alpha)^{2(t-i)}$$

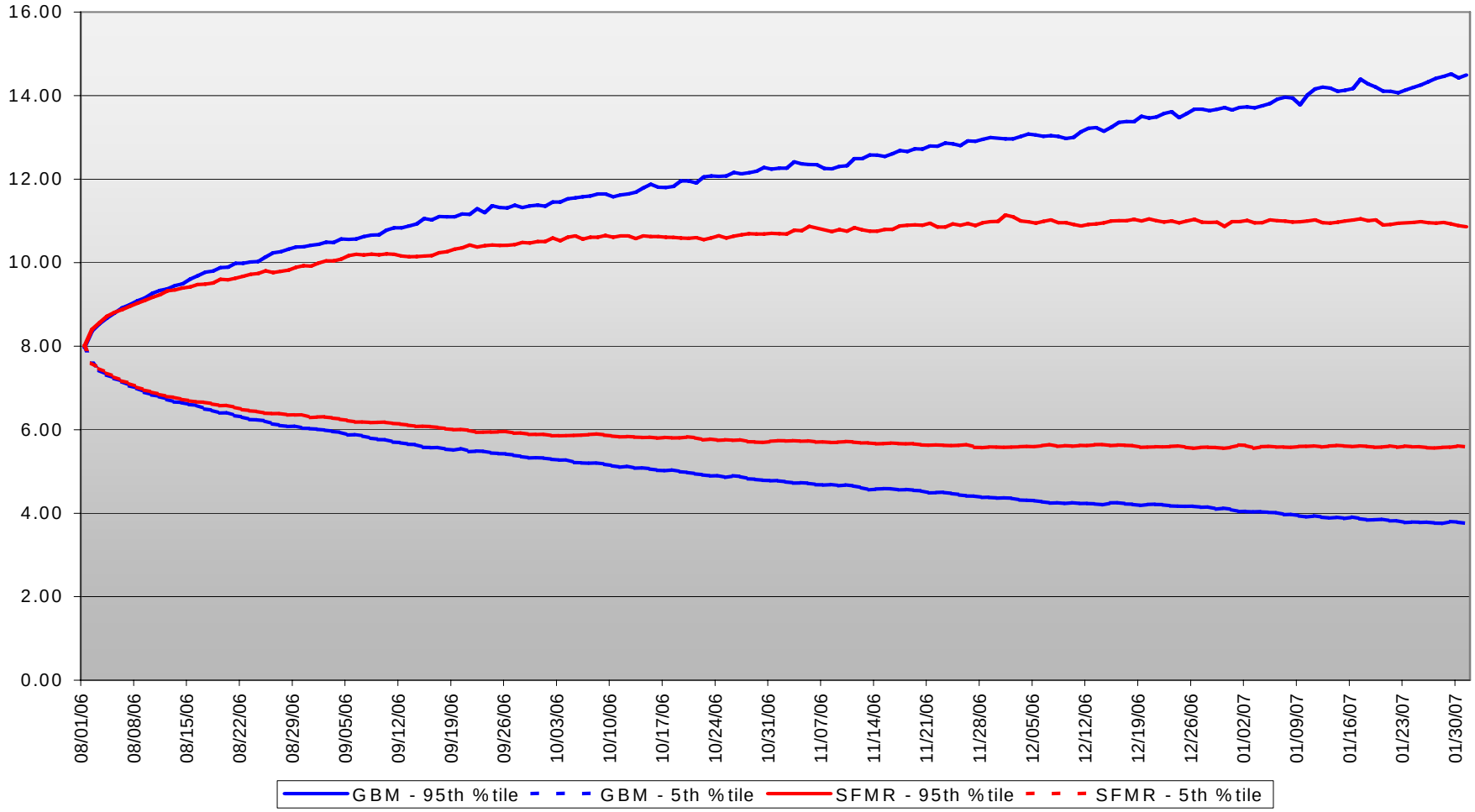
- Which reduces to

$$Var(p_t) = \frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2}\sigma^2$$

Variance Comparisons



Price Comparisons



Spread between SFMR Prices

- So the basis at time t

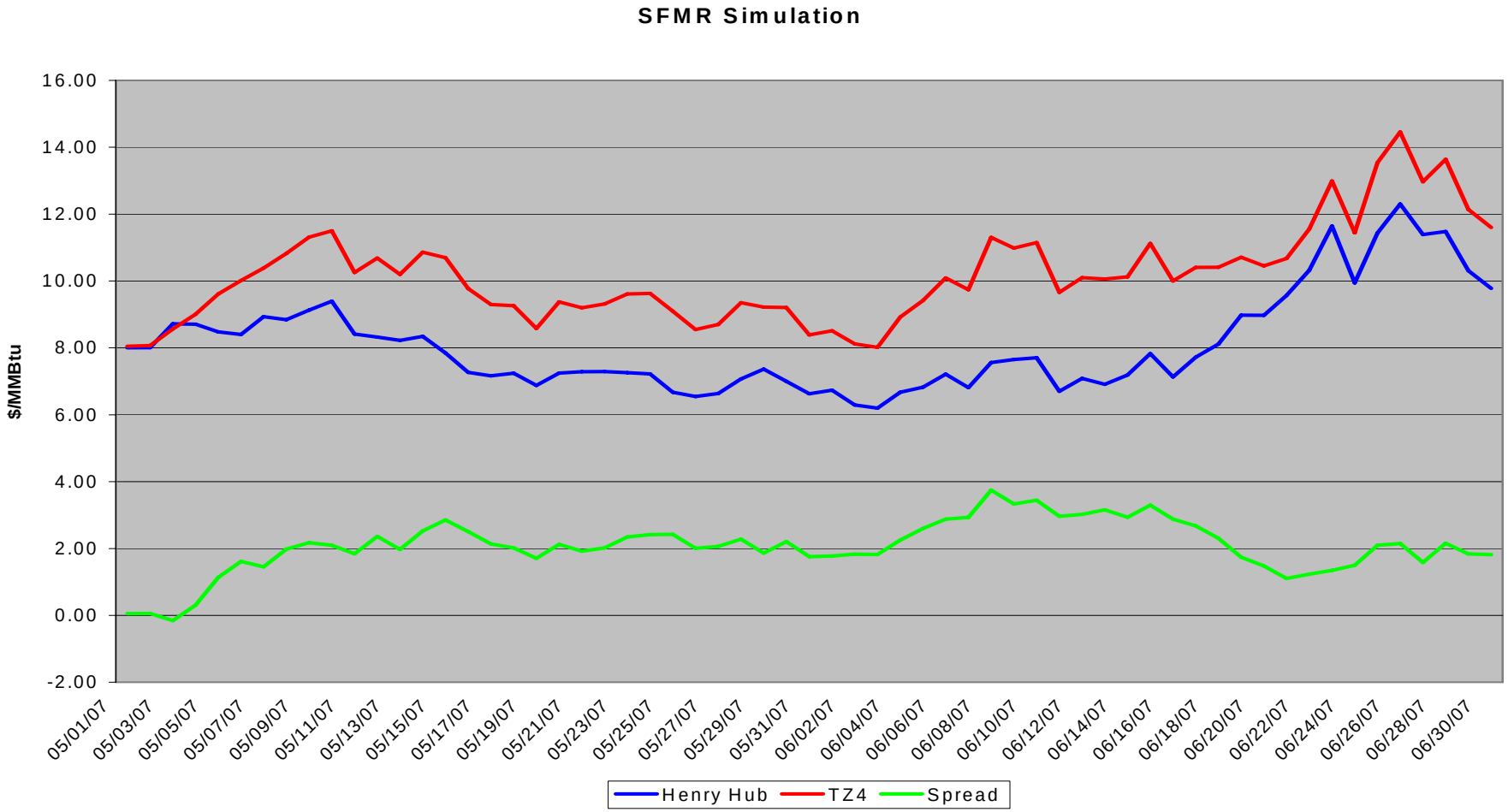
$$q_t - p_t = \left[(1-\alpha_p)^t - (1-\alpha_q)^t\right](\mu_p - p_0) + (1-\alpha_q)^t \eta_0 + \left[1-(1-\alpha_q)^t\right]\eta$$

$$+ \sum_{i=0}^{t-1} \varepsilon_{qi}\left(1-\alpha_q\right)^{t-i} - \sum_{i=0}^{t-1} \varepsilon_{pi}\left(1-\alpha_p\right)^{t-i}$$

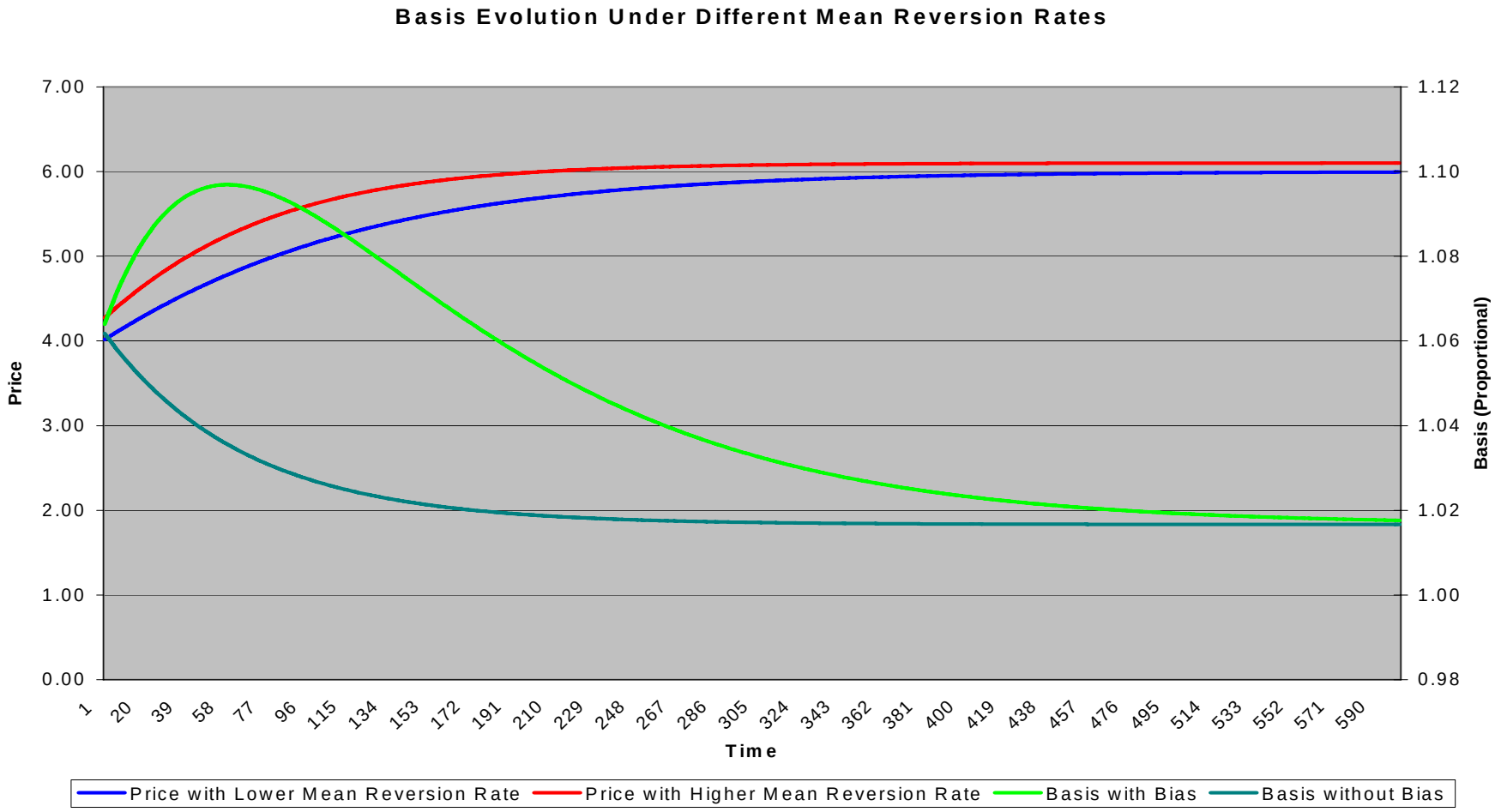
- With variance

$$Var\left(q_t - p_t\right) = \frac{1-\left(1-\alpha_p\right)^{2t}}{1-\left(1-\alpha_p\right)^2}\sigma_p^2 + \frac{1-\left(1-\alpha_q\right)^{2t}}{1-\left(1-\alpha_q\right)^2}\sigma_q^2 - 2\rho\sqrt{\frac{1-\left(1-\alpha_p\right)^{2t}}{1-\left(1-\alpha_p\right)^2}}\sigma_p\sqrt{\frac{1-\left(1-\alpha_q\right)^{2t}}{1-\left(1-\alpha_q\right)^2}}\sigma_q$$

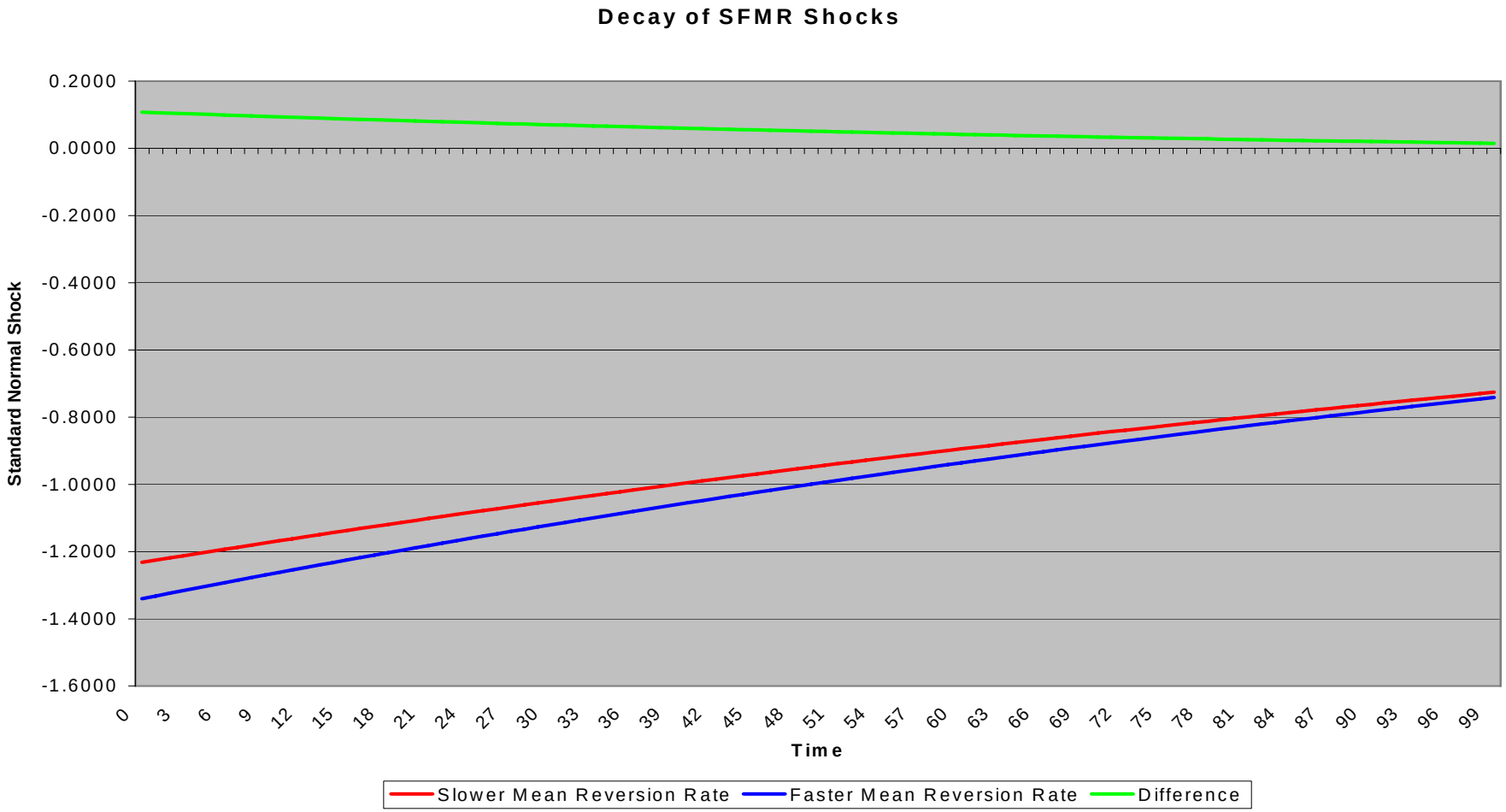
Sample SFMR Simulation



Behavior of SFMR Expected Basis



Behavior of SFMR Shocks



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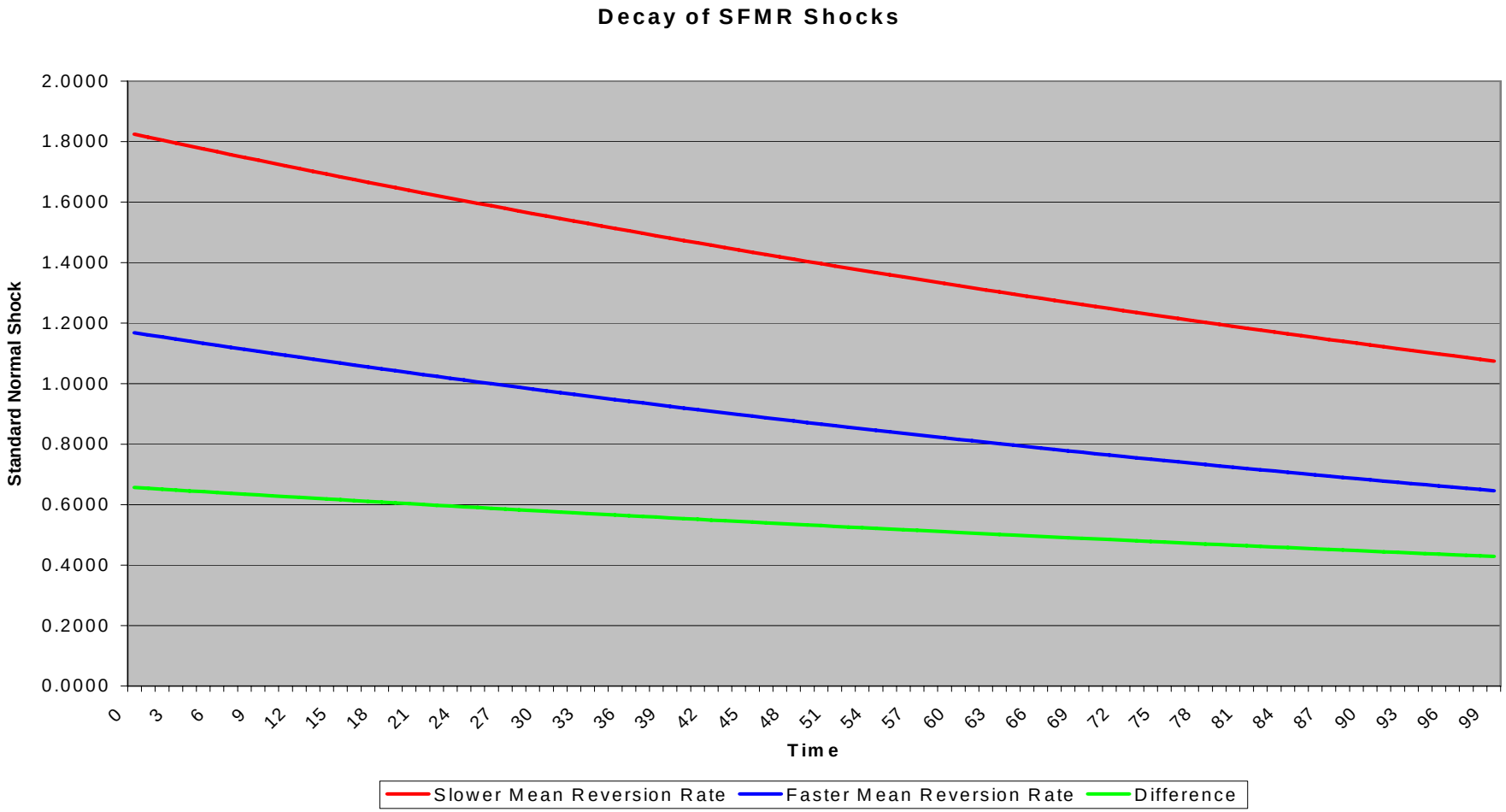

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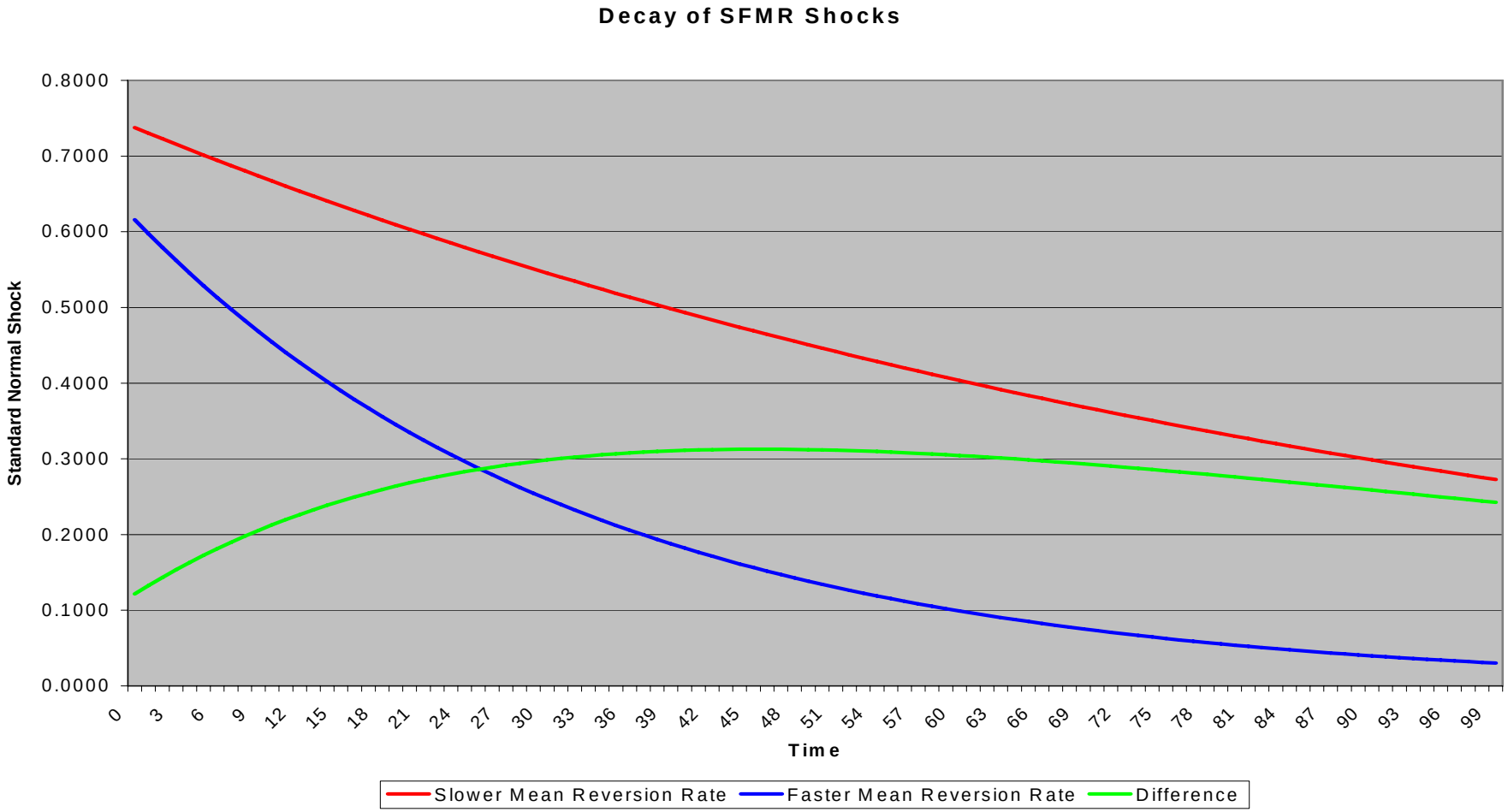

Traditional Energy Models

- Margrabe Spread Model
  - Shares assumptions, both good and bad, with Black-Scholes
- Geometric Brownian Motion
  - Fixed expected basis equal to today's basis
  - Infinite variance of spread
- Single Factor Mean Reverting
  - Variable expected basis
  - Finite variance of spread, but different mean reversion rates can lead to much different decay rates
  - Difference in shocks can increase and may diverge from what is seen in reality

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Vector Autoregressions - The Better Mousetrap

- Vector autoregression framework allows greater flexibility
  - Established methodology
  - Robust diagnostic testing
  - Multiple methodologies to handle shocks
- Future path of prices depends on
  - Historical path of all modeled prices; and
  - Future path of other prices

Vector Autoregression Model (VAR)

- Models prices of goods that are close substitutes

$$p_t = \alpha_1 + \beta_{11} p_{t-1} + \beta_{12} q_{t-1} + \varepsilon_{pt}$$

$$q_t = \alpha_2 + \beta_{21} p_{t-1} + \beta_{22} q_{t-1} + \varepsilon_{qt}$$

where:

$$\varepsilon_{pt} \sim N(0, \sigma_p^2)$$

$$\varepsilon_{qt} \sim N(0, \sigma_q^2)$$

$$\rho = corr(\varepsilon_p, \varepsilon_q)$$

Price Evolution under VAR

- Matrix representation

$$\begin{bmatrix} p_t \\ q_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} p_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{qt} \end{bmatrix}$$

- Change notation to

$$\mathbf{P_t} = \mathbf{A} + \mathbf{B}\mathbf{P_{t-1}} + \mathbf{E_t}$$

- Price at t in terms of P_0

$$\mathbf{P_t} = \mathbf{B}^t\mathbf{P_0} + \sum_{i=0}^{t-1} \mathbf{B}^i(\mathbf{A} + \mathbf{E_{t-i}})$$

The Stability of the Weighting Matrix

- Given the weighting matrix

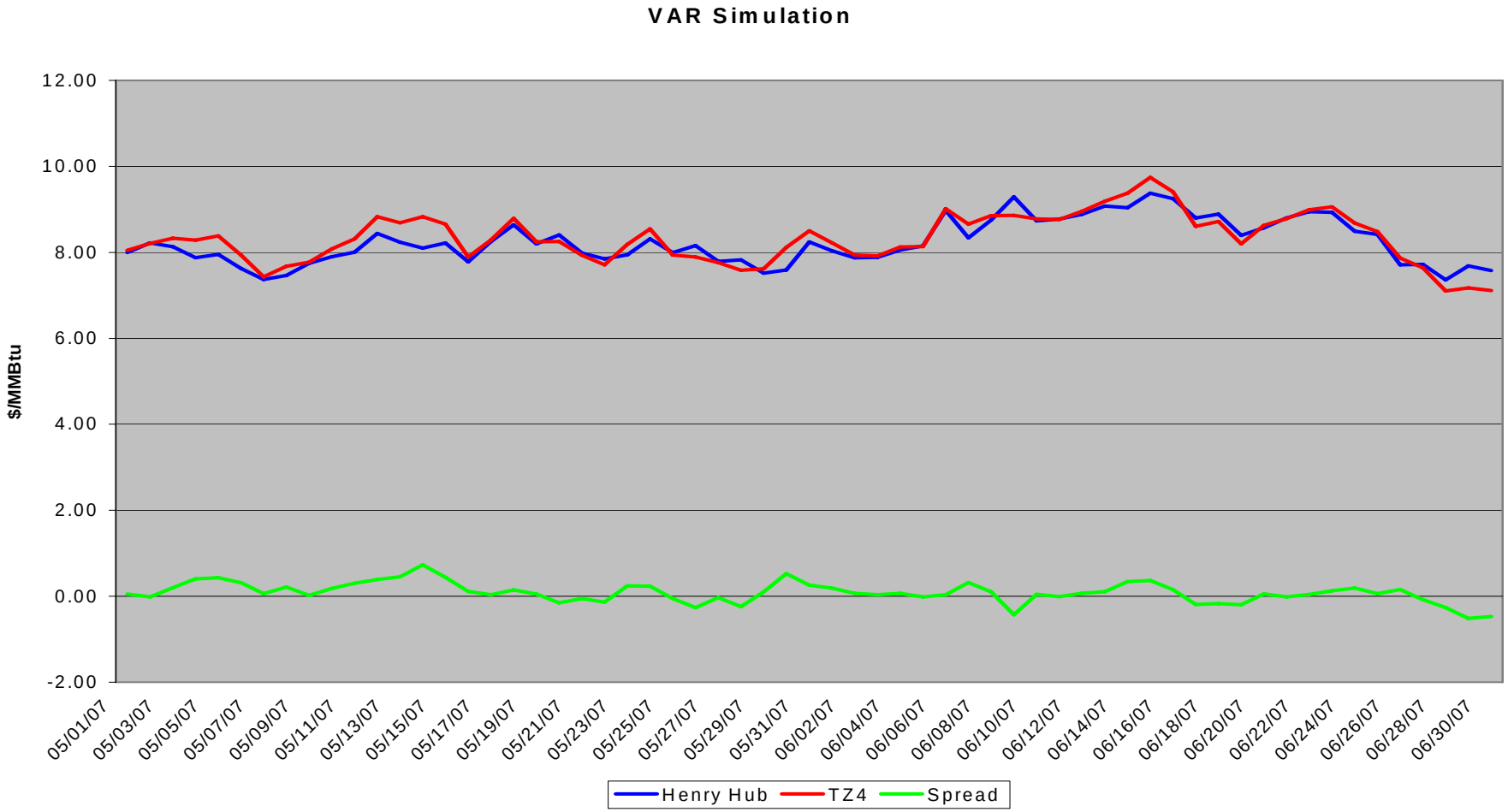
$$\mathbf{B}^1 = \begin{bmatrix} 0.726 & 0.253 \\ 0.171 & 0.822 \end{bmatrix}$$

- Subsequent powers are

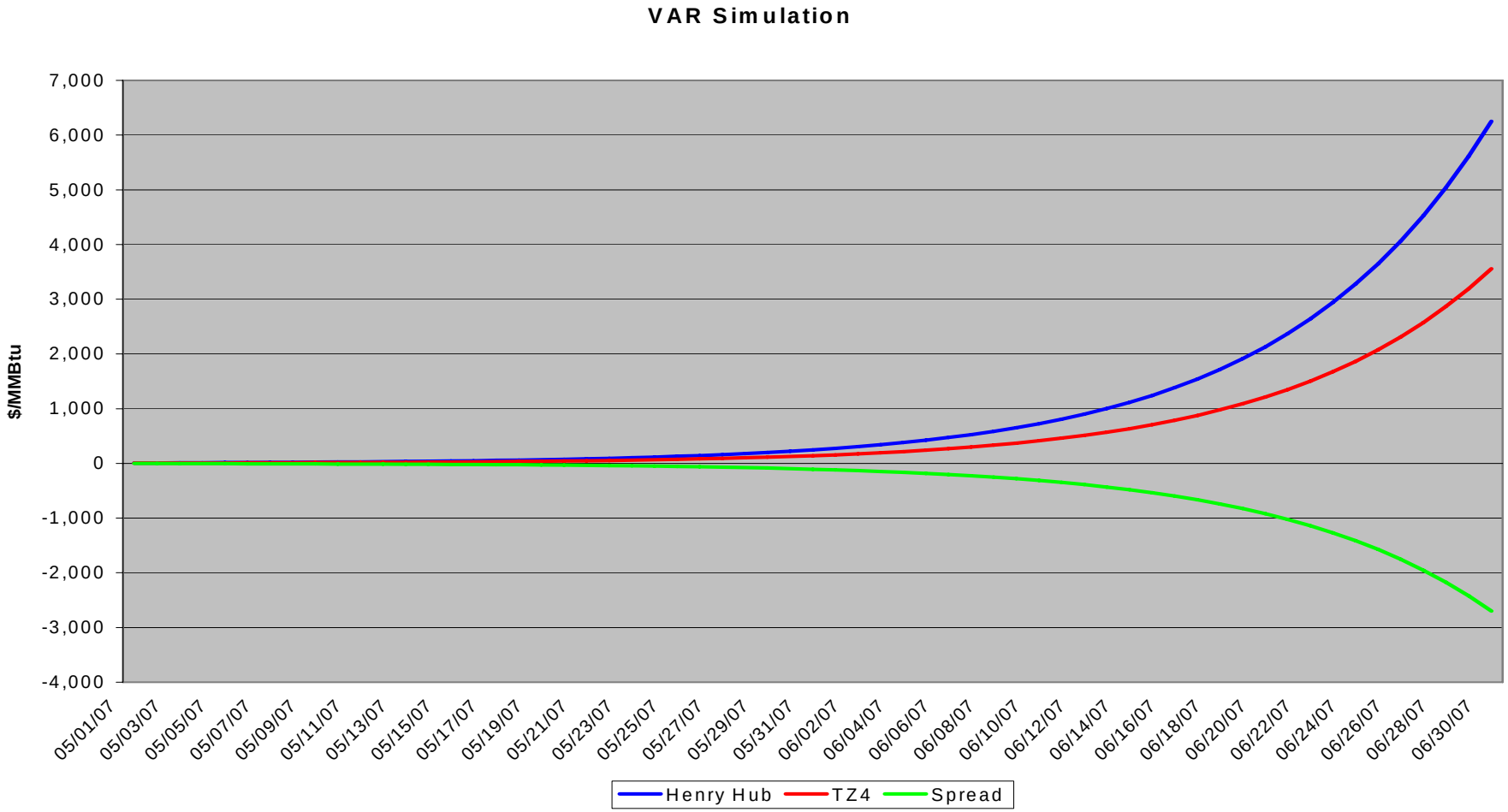
$$\mathbf{B}^2 = \begin{bmatrix} 0.571 & 0.392 \\ 0.266 & 0.719 \end{bmatrix} \qquad \mathbf{B}^3 = \begin{bmatrix} 0.482 & 0.467 \\ 0.316 & 0.659 \end{bmatrix}$$

$$\mathbf{B}^5 = \begin{bmatrix} 0.399 & 0.525 \\ 0.356 & 0.598 \end{bmatrix} \qquad \mathbf{B}^{10} = \begin{bmatrix} 0.346 & 0.524 \\ 0.355 & 0.544 \end{bmatrix}$$

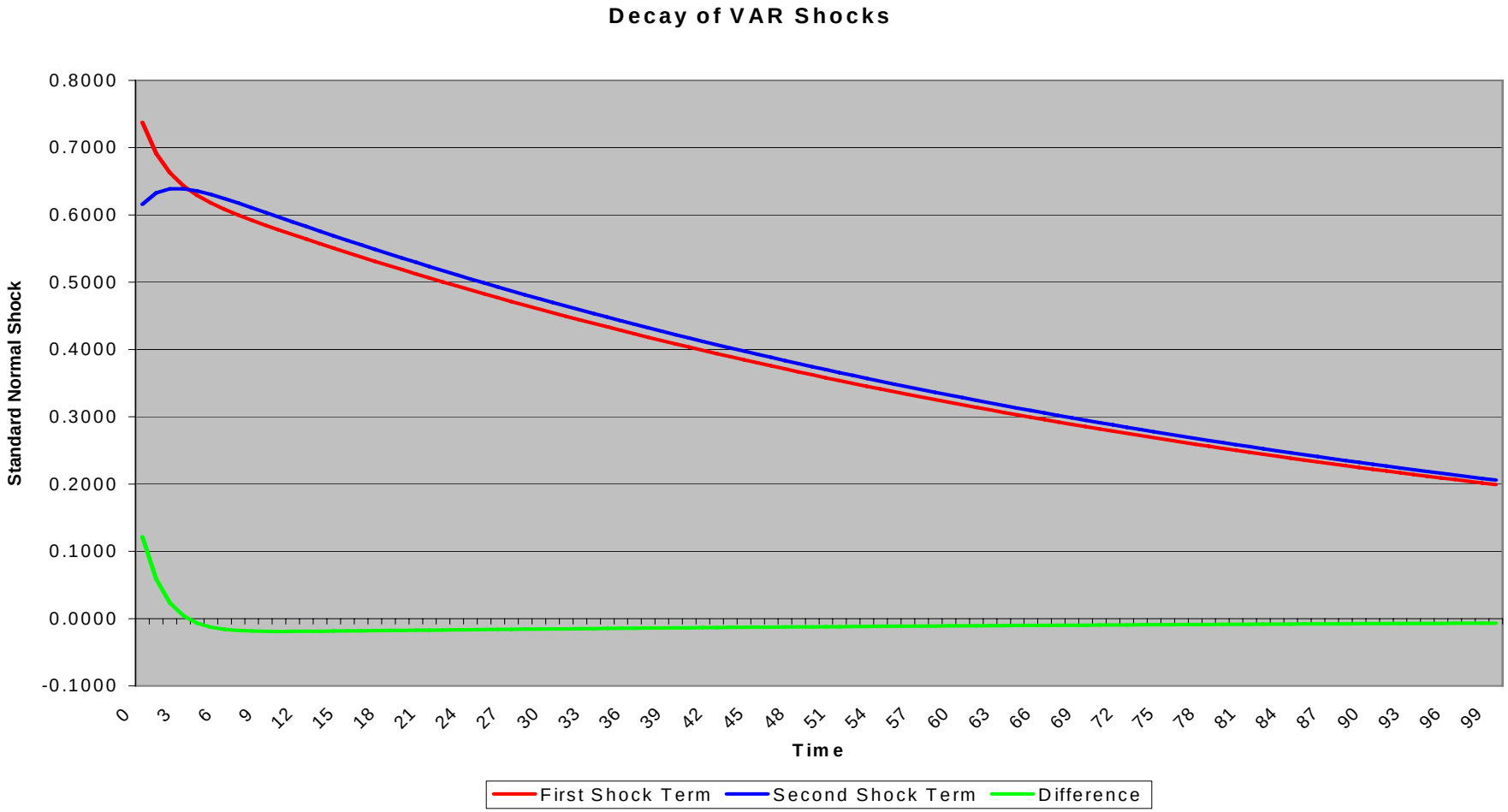
Sample VAR Simulation



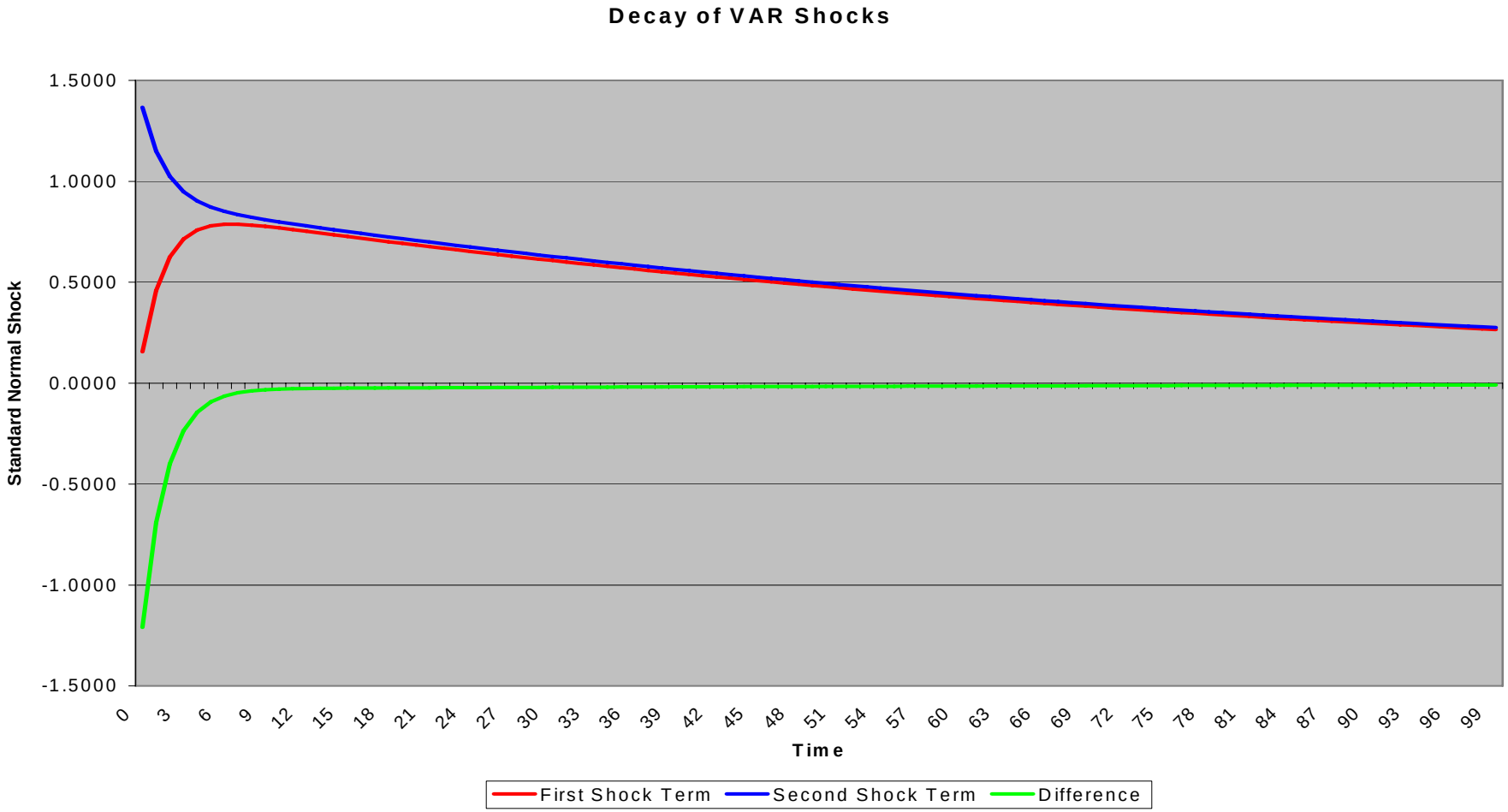
Unstable VAR Simulation



Behavior of VAR Shocks



Behavior of VAR Shocks



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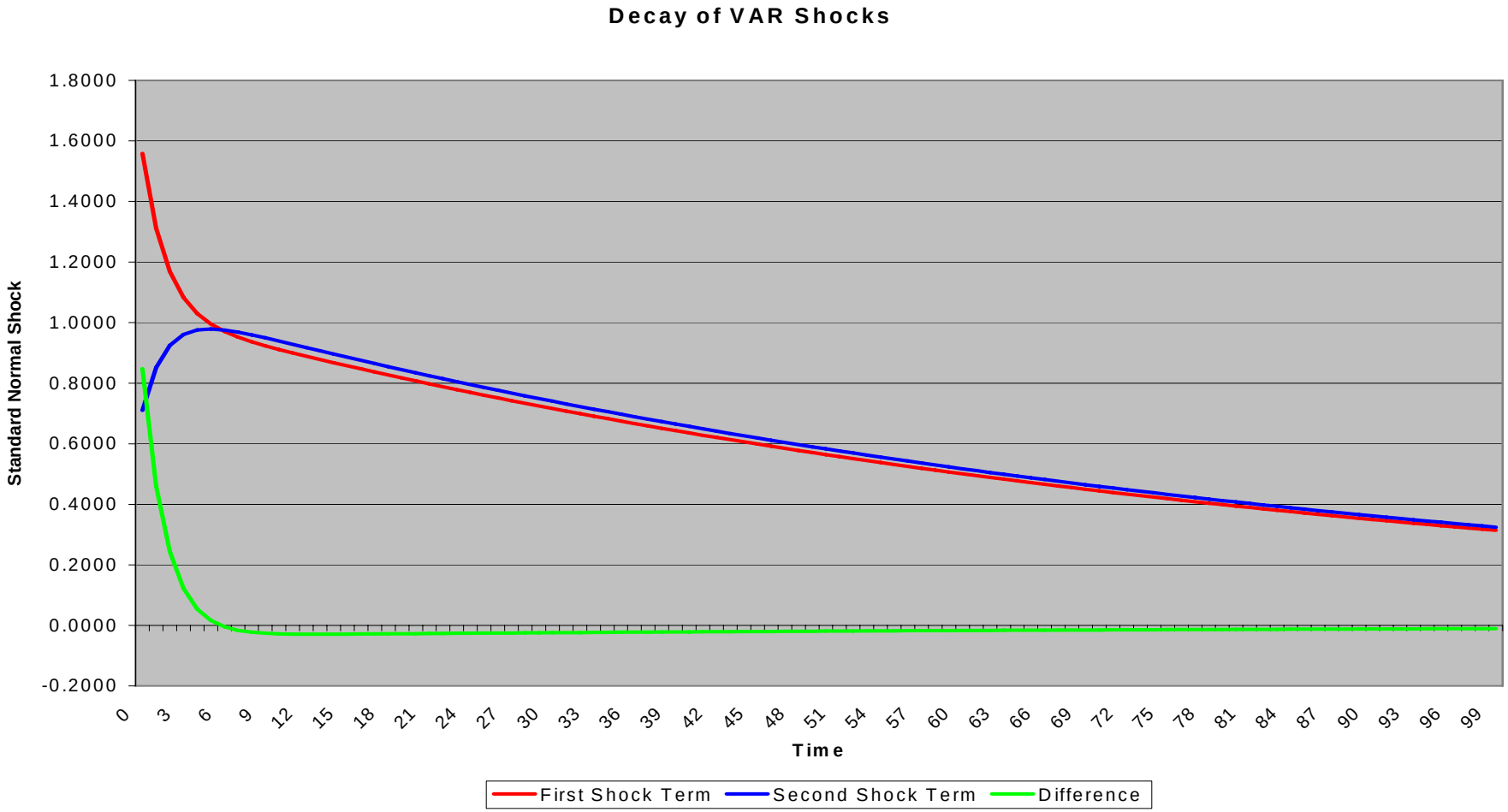


Strengths of the VAR

- Construct paths for prices without associated forward curves
- Several well established tests to determine optimal number of lags
- Two methods to correlate price shocks
  - Actual correlation and normally distributed shocks
  - Resample actual historical shocks to pick up correlation and any non-normal distributions

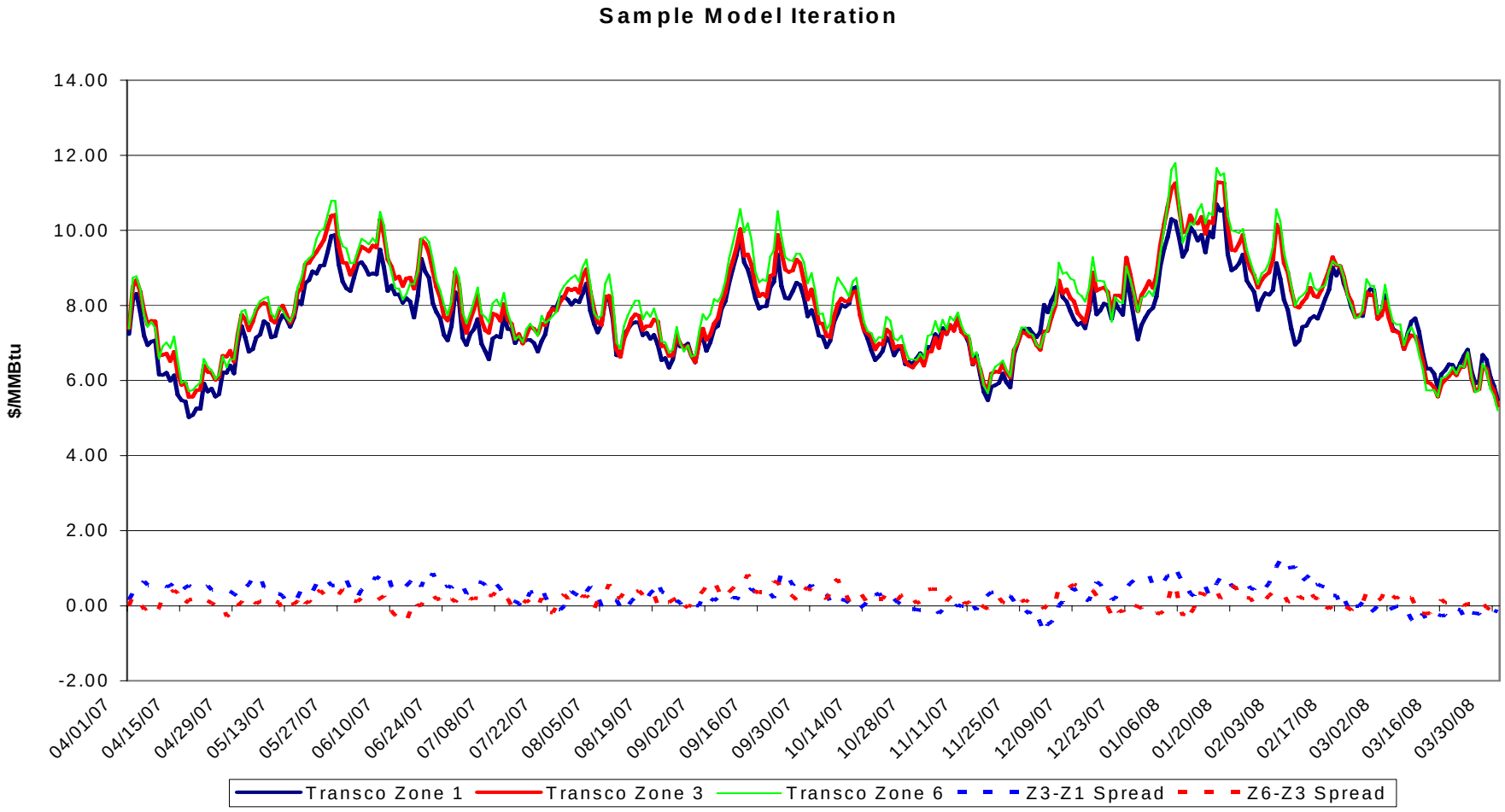
Weaknesses of the VAR

- More rigorous process to determine parameters
- More diagnostic testing of model
  - Proper functional form
  - Stable system of equations
- Number of parameters grows quickly (N^2L) and can erode your degrees of freedom, so a larger data set may be required

Pipeline Management Risk Assessment

- Resource management problem
- Value of transportation capacity
- Risk depends on the price of gas at 3 hubs
- Most conservative test shows the need for 100 lags

Pipeline Model Simulation



Summary

- Traditional models may not work well to model absolute price levels and commodity spreads
  - Infinite variance
  - Unstable mean spreads
  - Different rates of mean reversion can cause divergence over time
- Vector autoregression offers a flexible framework
  - Better captures price interactions
  - Derive future path for prices without forward curve
  - Handle non-normal and heteroscedastic shocks

Questions?

- Contact Info:

  Ted Kury

  tkury@teainc.org

  (904) 360-1444

References

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