PHOTOMETRIC REDSHIFTS 50 YEARS AFTER

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Tamás Budavári / The Johns Hopkins University

Binning Photons

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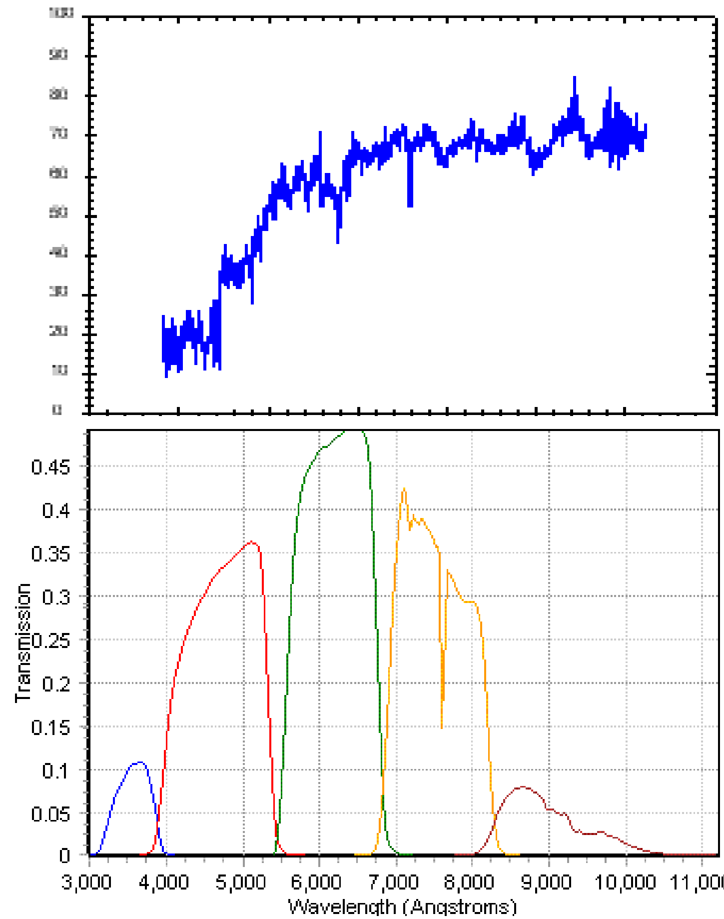
- Space (and time)
  - Position measurements
- Wavelength
  - Spectroscopy: narrow bins
  - Photometry: broad bands
- For example, SDSS
  - Spectro: $r < 17.7$ 1.6M sources
  - Photo: $r < 21+$ 360M sources

  In a fraction of the time…



Multicolor Challenges

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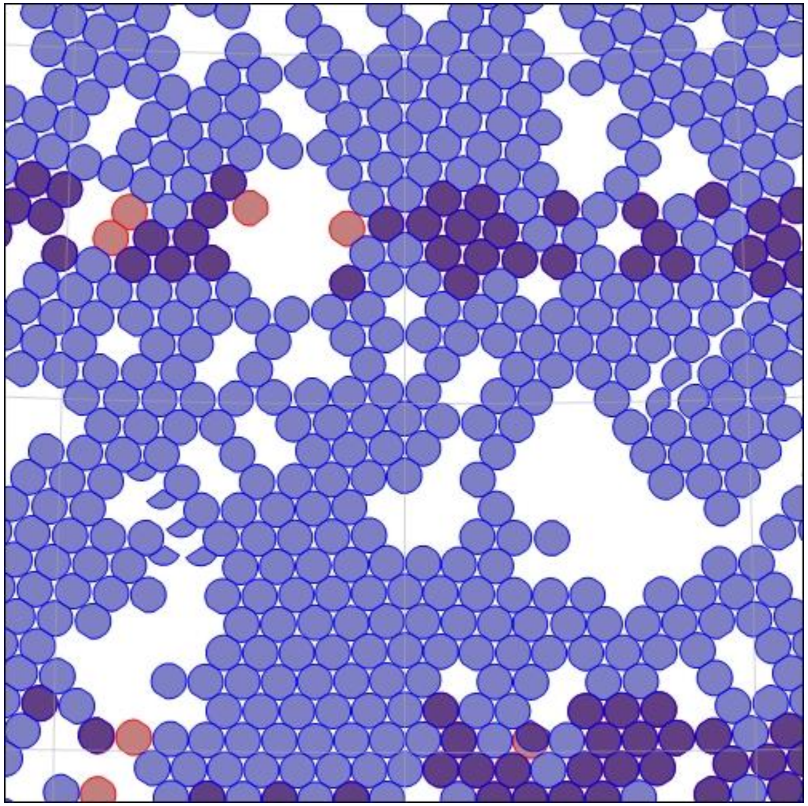
- Cross-identification of sources
  - To assemble multicolor catalogs
- Constraining physical properties
  - To interpret the data

!

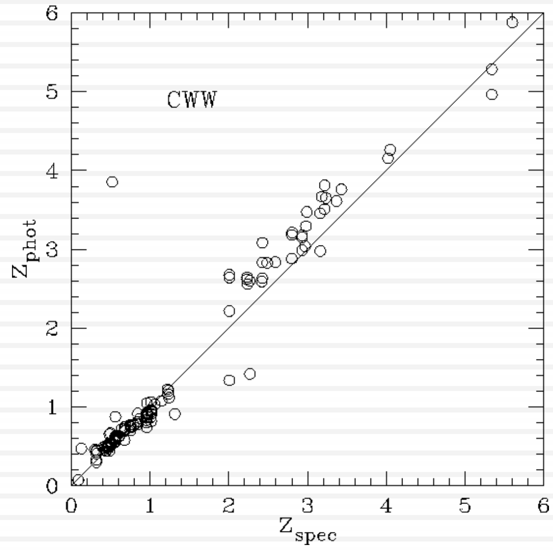
Dropouts?

- Measured fluxes
  - Extract from images
- Un-detected
  - Limit from coverage



Photometric Redshifts

- Empirical methods
- Template fitting



Historical Overview

- Baum (1962)
  - Compared average SEDs (9 bands) of ellipticals in clusters
- Koo (1985)
  - Color-shape diagrams of and Bruzual-Charlot iso-*z* tracks (-U+J+F-N)
- Loh & Spillar (1986)
  - Template fitting with 6 non-standard filters and 34 known galaxies
- Connolly et al. (1995)
  - Redshift assumed to be a linear or quadratic function of magnitudes
- Steidel et al. (1996)
  - U dropout due to Ly-alpha blanketing (< 912Å) to select $z > 2.25$

Empirical Methods

- Regression $z = F(m)$
  - Polynomial fitting
  - Piecewise linear fits
  - Nearest neighbors
  - Neural Nets
  - Support Vector Machines

- Errors?

Template Fitting

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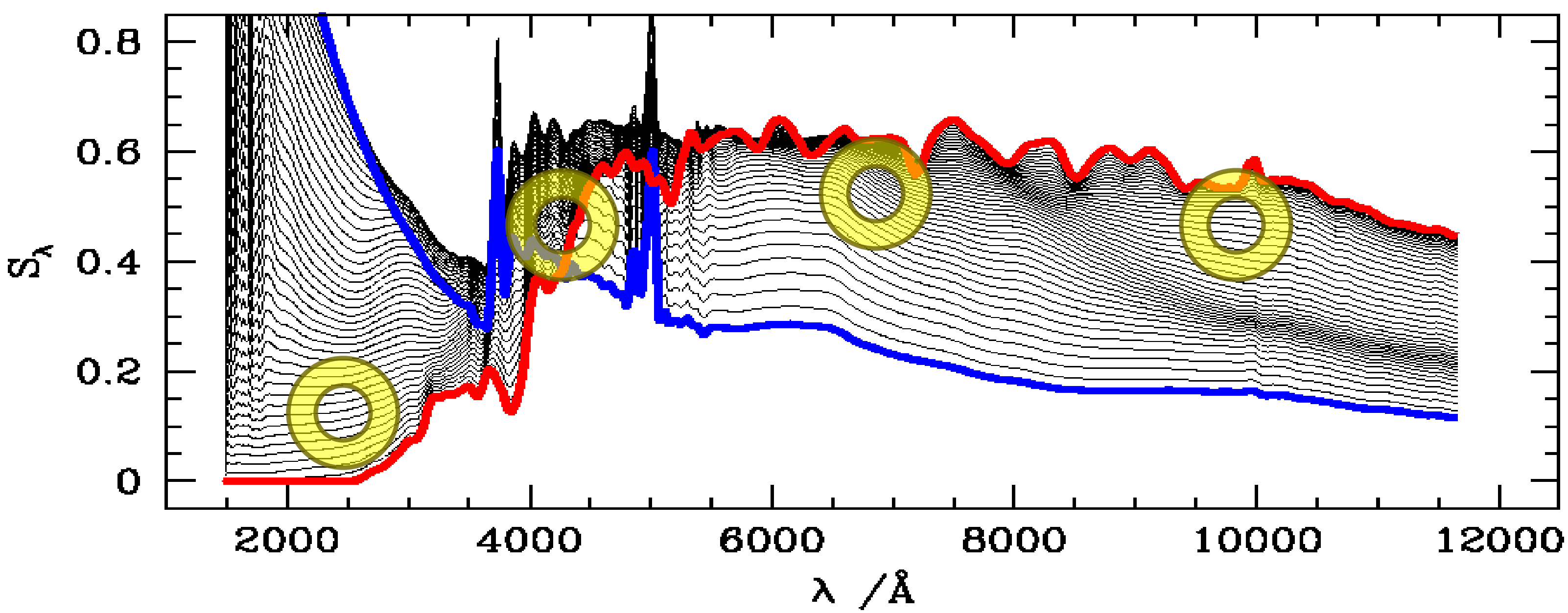
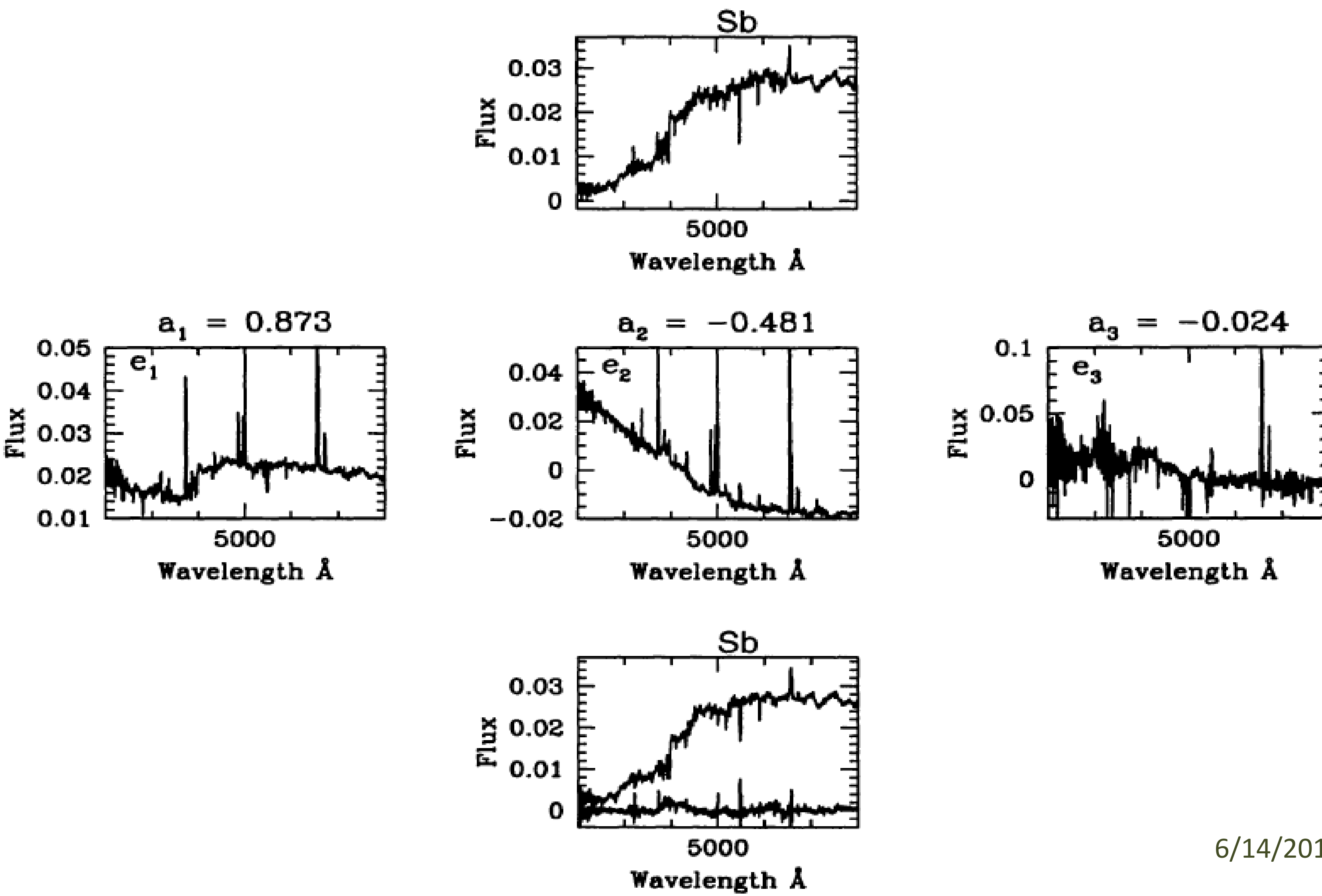
- Compare reference spectra to measurements
  - At different redshifts
  - MLE with Gaussian



Linear-combination of Eigenspectra

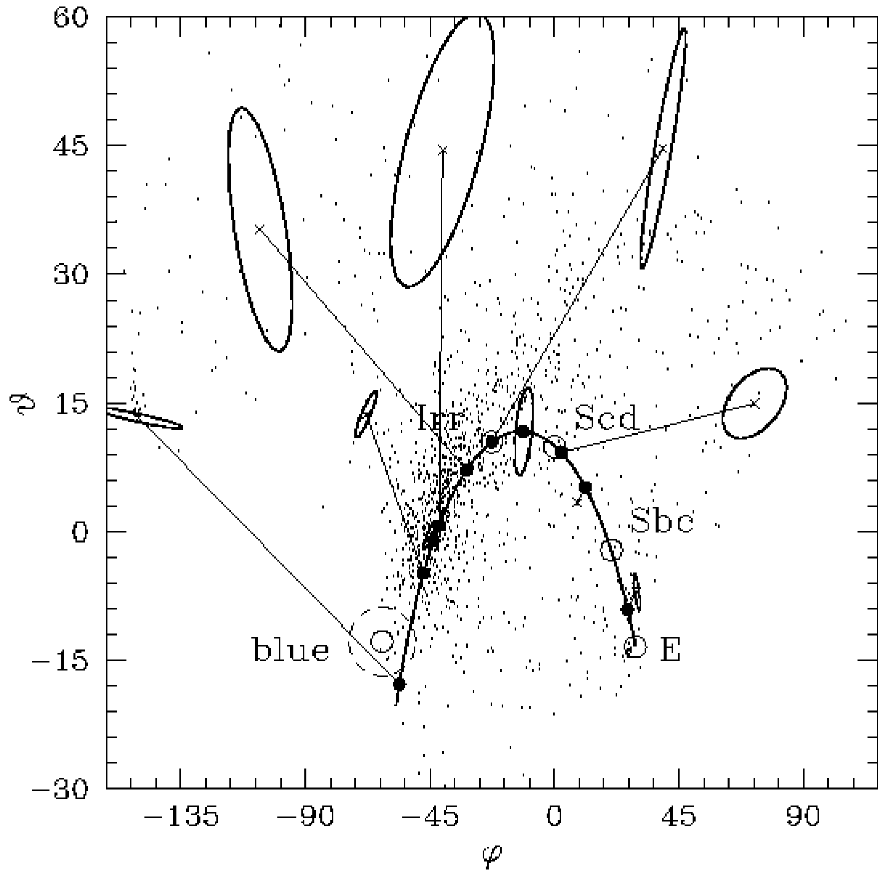


Interpolation

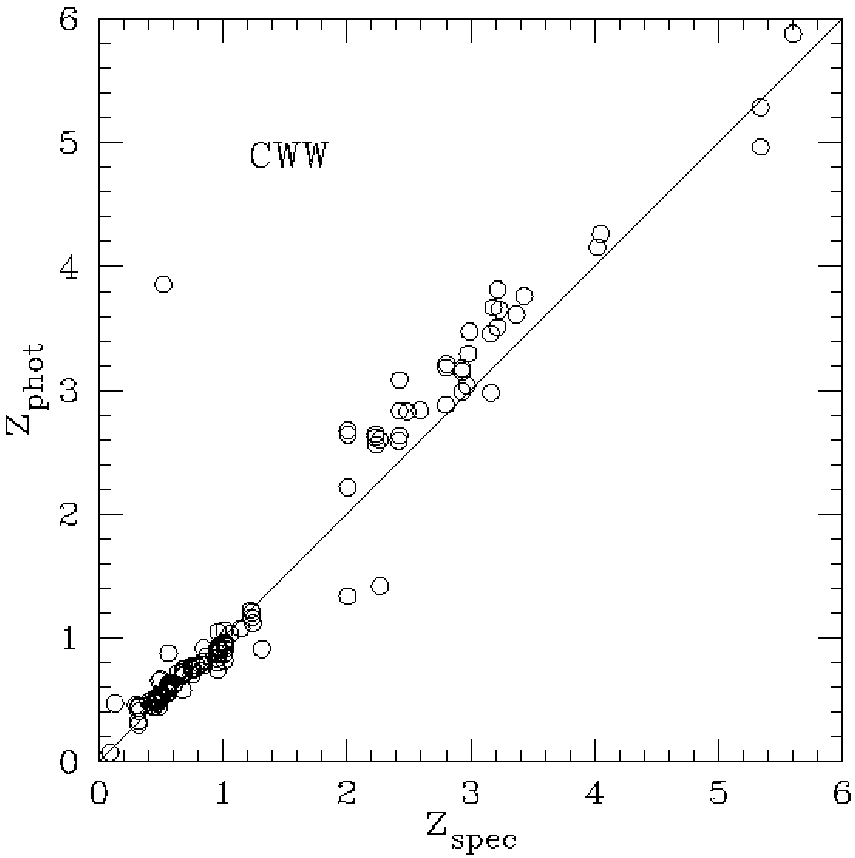
- Two mixing angles encode the spectral type when using 3 eigenspectra

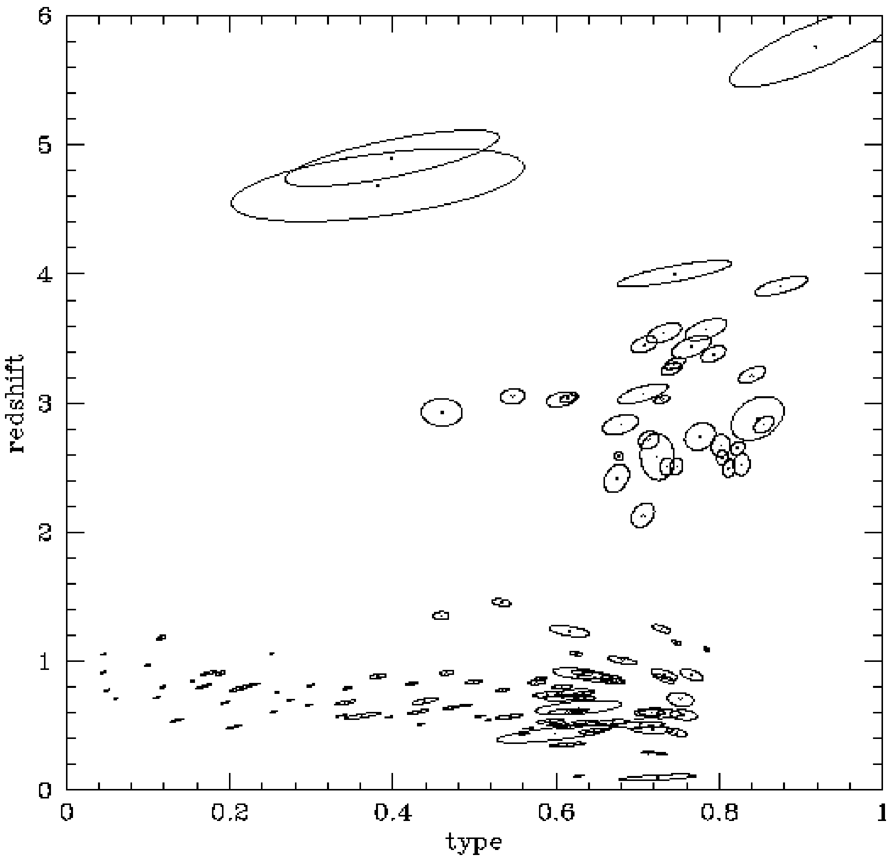
  1D type parameter connects the CWW templates

Ell	0
Sbc	0.13
Scd	0.35
Irr	0.56
blue	1



Consistent Redshift and Type





Traditional Methods

Empirical Methods

Pure fit for z over the
color hyper-plane

+ fast redshift prediction

- fitting functions and
  training sets needed
  for every survey

- unreliable extrapolation

Template Fitting

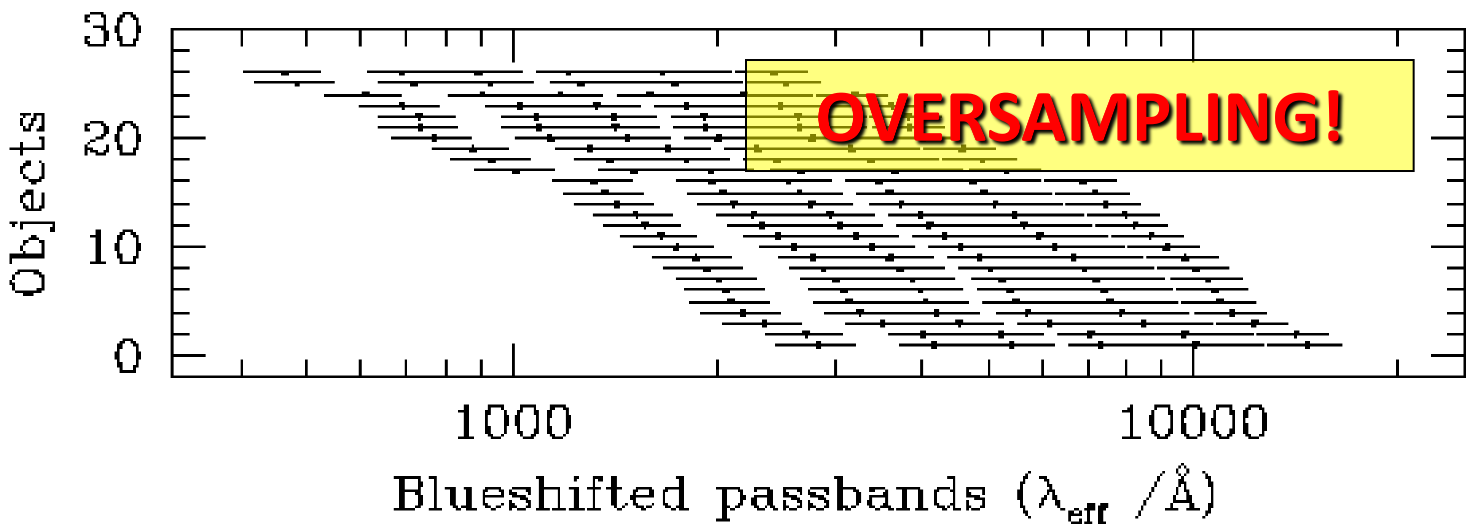
Comparison of known
SEDs to photometry

+ physics is in templates,
  no training set required

+ more outcome: spectral
  type, reddening, etc.

- templates used as come

Templates from Photometry?



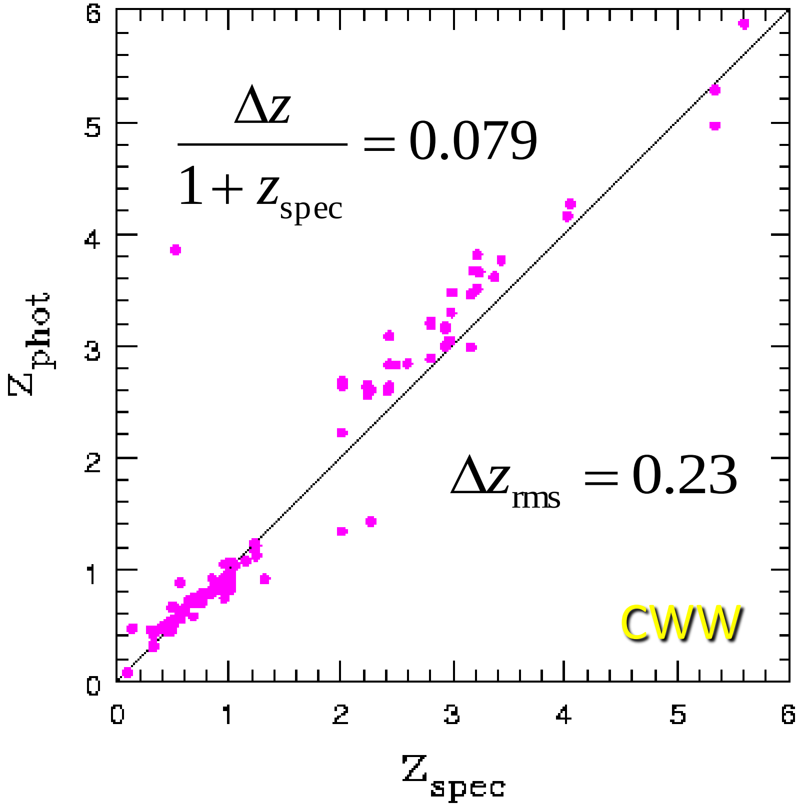
Simple Repair

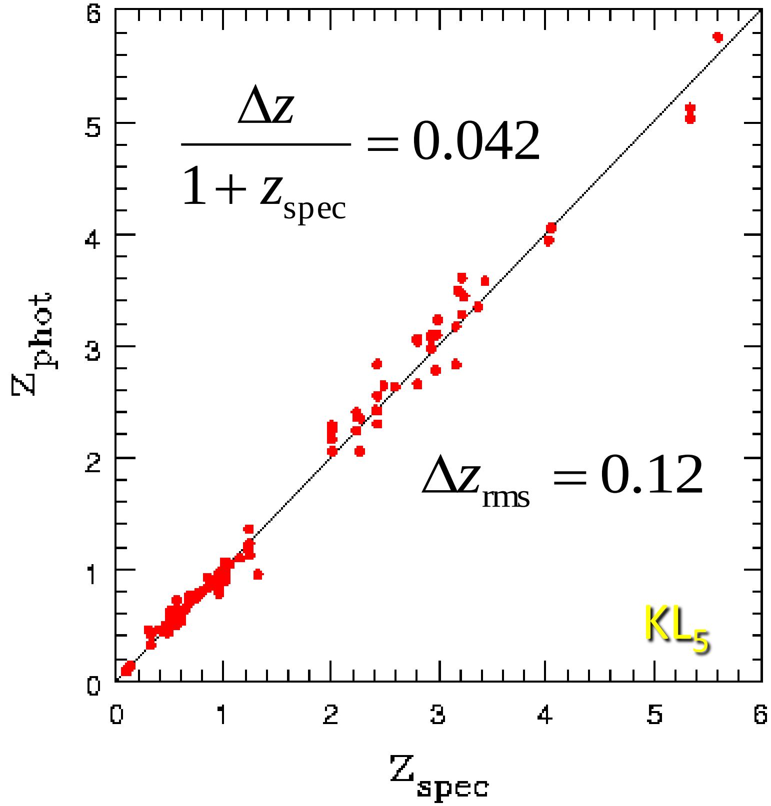
For each galaxy:

1. Compute type from known redshift
2. Derive estimate spectrum
3. Correct spectrum to photometry*

Build new basis from corrected spectra

HDF/NICMOS in 1999



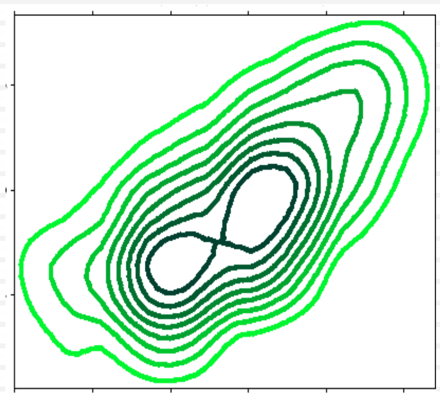


50 Years of Pragmatism

- Traditional ways
  - Empirical methods: assume a fitting function
  - Template fitting: compare to model spectra
- Very different techniques
  - For no fundamental reason
- Many open questions

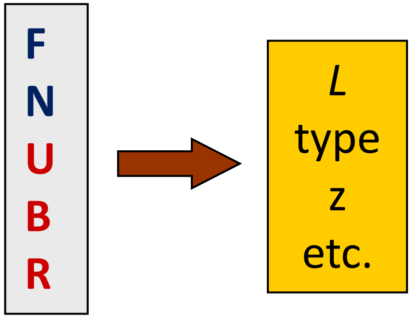
A Unified Framework

Starting from scratch…

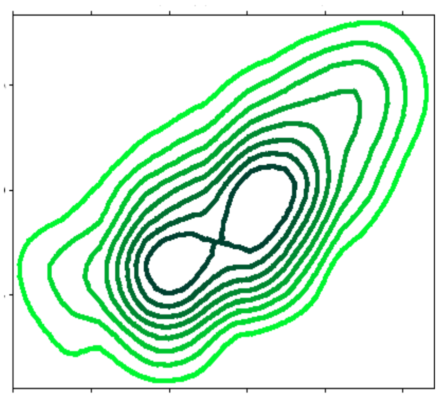


Photometric Inversion

- The general inversion problem
  - Constrain various properties consistently
  - Propagate their uncertainties and correlations



- Estimates are secondary
  - Provide probability density functions instead
  - Scientific analyses to use the full PDFs



- Make best use of observations
  - Next generation surveys are photometric only

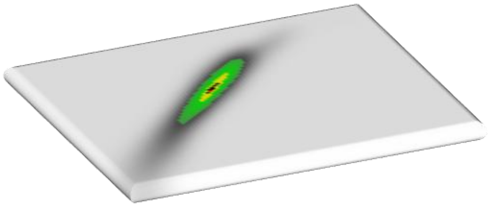
A Unified Framework

- Training and Query sets with different observables

$$
\begin{aligned}
T:&\quad \{\boldsymbol{x}_t, \boldsymbol{\xi}_t\}_{t\in T}\\
Q:&\quad \{\boldsymbol{y}_q\}_{q\in Q}\\
M:&\quad \boldsymbol{\theta}
\end{aligned}
$$

- Model yields observables for given parameter
  - Prediction via $p(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{\theta},M)$ and has prior $p(\boldsymbol{\theta}|M)$
  - Also folds in the photometric accuracy

- We are after $p(\boldsymbol{\xi}|\boldsymbol{y}_q,M)$

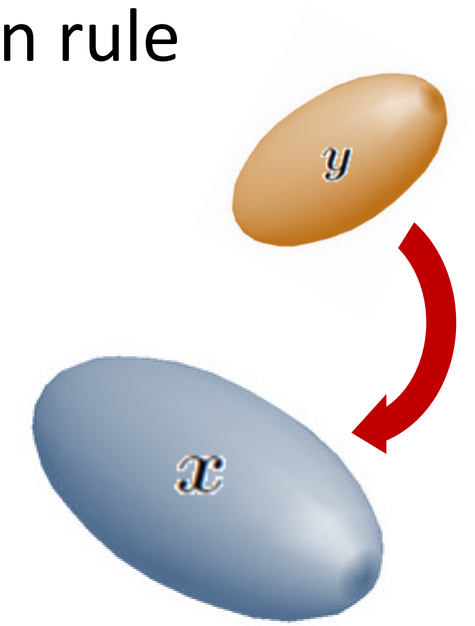


Mapping Observables

- The model provides the transformation rule

$$p(\boldsymbol{x}|\boldsymbol{y}_q, M) = \int d\boldsymbol{\theta}\; p(\boldsymbol{x}|\boldsymbol{\theta}, M)\, p(\boldsymbol{\theta}|\boldsymbol{y}_q, M)$$

with $$p(\boldsymbol{\theta}|\boldsymbol{y}_q, M) = \frac{p(\boldsymbol{\theta}|M)\, p(\boldsymbol{y}_q|\boldsymbol{\theta}, M)}{p(\boldsymbol{y}_q|M)}$$



- Think empirical conversion formulas but better
  - For example, from *UJFN* to *ugriz* with errors

Empirical Relation

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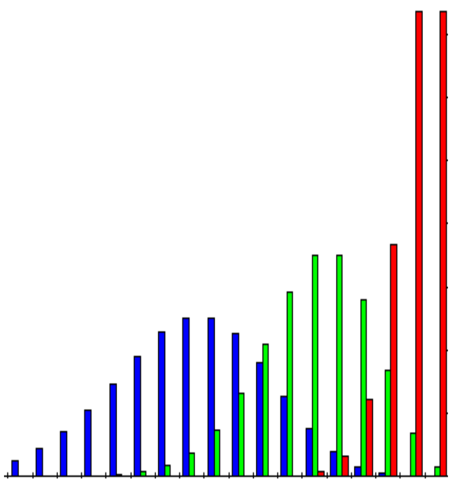
- Usually just assume a function $\xi = \hat{\xi}(x)$
  - Wrong! We know there are degeneracies…
- There is a more general relation $p(\xi|x)$
  - Usual restriction is $p(\xi|x) = \delta(|\xi - \hat{\xi}(x)|)$
  - Correct estimation

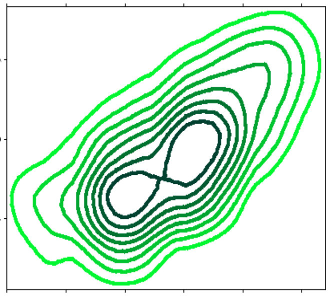
$$\boxed{p(\xi|x) = \frac{p(\xi, x)}{p(x)}}$$



- Straightforward to do if one can estimate densities
  - Also better direct methods exist

Properties of Interest

- The final constraint is



$$p(\boldsymbol{\xi}|\boldsymbol{y}_q, M) = \int d\boldsymbol{x}\; p(\boldsymbol{\xi}|\boldsymbol{x})\, p(\boldsymbol{x}|\boldsymbol{y}_q, M)$$

- If the result is unimodal (no guarantee)
  - Estimate by mean, etc.

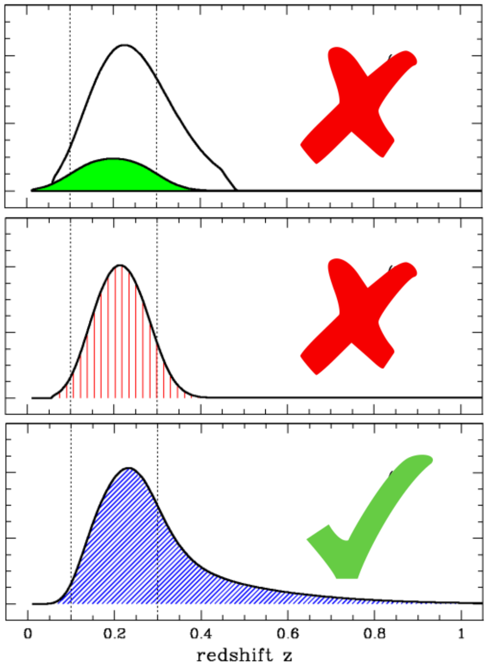
$$\bar{\boldsymbol{\xi}}(\boldsymbol{y}_q) = \int d\boldsymbol{\xi}\, \boldsymbol{\xi}\, p(\boldsymbol{\xi}|\boldsymbol{y}_q, M)$$

Properties of Samples

- Distribution of a sample *Q*
  - Directly calculated as the mean

$$p(\boldsymbol{\xi}|Q, M) = \left\langle p(\boldsymbol{\xi}|\boldsymbol{y}_q, M)\right\rangle_{q\in Q}$$

- Works for sub-samples
- No need for de-convolution →



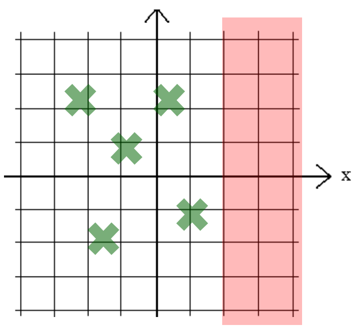
Selection Effects

- Measure of reliability is the prob of *q* making the cuts

$$P(W|\boldsymbol{y}_q, M) = \int d\boldsymbol{x}\; P(W|\boldsymbol{x})\, p(\boldsymbol{x}|\boldsymbol{y}_q, M)$$

- Include observables used in the selection criteria

$$p(\boldsymbol{\xi}|\boldsymbol{x}, T) = \frac{p(\boldsymbol{\xi}|\boldsymbol{x})\, P(T|\boldsymbol{x}, \boldsymbol{\xi})}{P(T|\boldsymbol{x})}$$



- For instance
  - Cannot use just colors, if there was a magnitude cut
  - Cannot use just fluxes, if cut on morphology

Empirical Uncertainty

- Parameters yield observables $\bar{\boldsymbol{x}}(\boldsymbol{\theta})$ and $\bar{\boldsymbol{y}}(\boldsymbol{\theta})$
  - Say your templates and redshifts sent through the filters

- Photometric accuracy
  - Normal distribution with full covariance matrix

$$p(\boldsymbol{x}_t, \boldsymbol{y}_q | \boldsymbol{\theta}, M) = N_x\left(\boldsymbol{x}_t | \bar{\boldsymbol{x}}(\boldsymbol{\theta}), \mathbf{C}_t\right) \times N_y\left(\boldsymbol{y}_q | \bar{\boldsymbol{y}}(\boldsymbol{\theta}), \mathbf{C}_q\right)$$

- Otherwise approximate with nearest

Numerical Integration

- Measure of reliability from MCMC
  - Average over parameter chain from $p(\boldsymbol{\theta}|\boldsymbol{y}_q, M)$

$$p(\boldsymbol{x}|\boldsymbol{y}_q, M) = \left\langle N_x\left(\boldsymbol{x}|\bar{\boldsymbol{x}}(\boldsymbol{\theta}_i),\mathbf{C}_t\right)\right\rangle$$

- If within boundaries, ignore the effect
  - Training points naturally drawn from their distribution

$$p(\boldsymbol{\xi}_r|\boldsymbol{y}_q, T, M) \propto \sum_{t \in T} p(\boldsymbol{\xi}_r|\boldsymbol{x}_t, T)\frac{p(\boldsymbol{x}_t|\boldsymbol{y}_q, M)}{p(\boldsymbol{x}_t|T)}$$

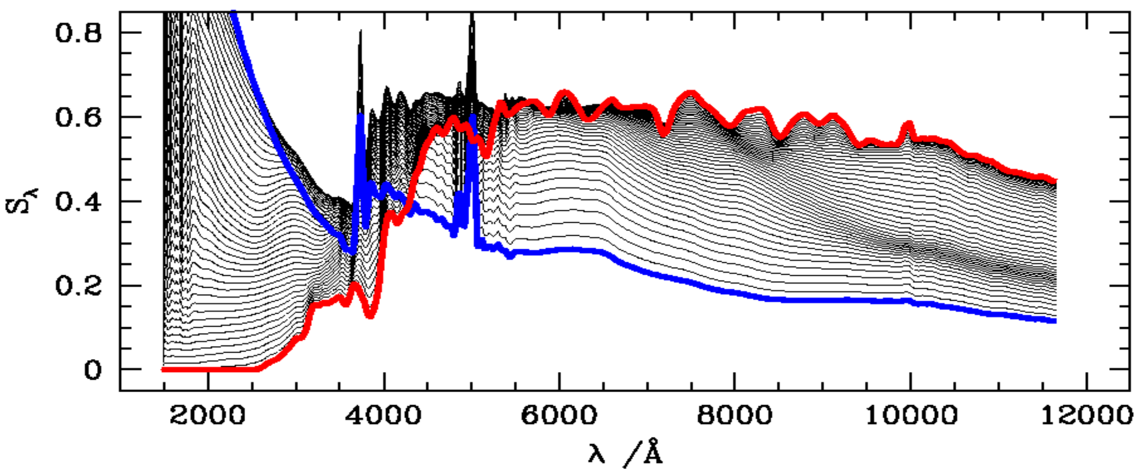
Traditional Methods

As corner cases of the Unified Framework

Template Fitting

- Artificial training set $\{x_t, \xi_t\} = \{\bar{x}(\theta_t), \bar{\xi}(\theta_t)\}$
  - From a grid of model points
  - No errors $p(x|\theta, M) = \delta(|x - \bar{x}(\theta)|)$
- Analytic result

$$p(\xi|y_q, M) \propto \sum_{t \in T} \delta(|\xi - \xi_t|)\, p(\theta_t|M)\, N(y_q|\bar{y}(\theta_t), C_q)$$

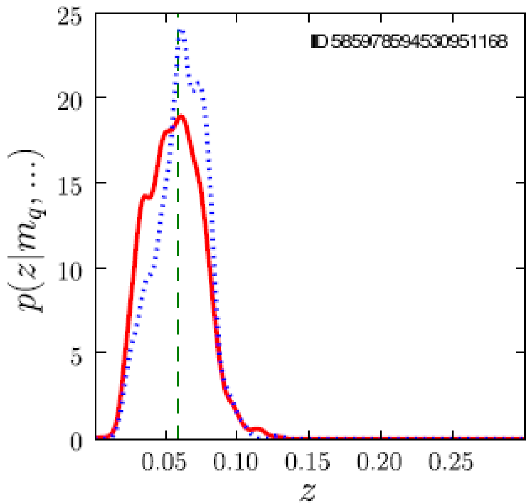


Improved Empirics

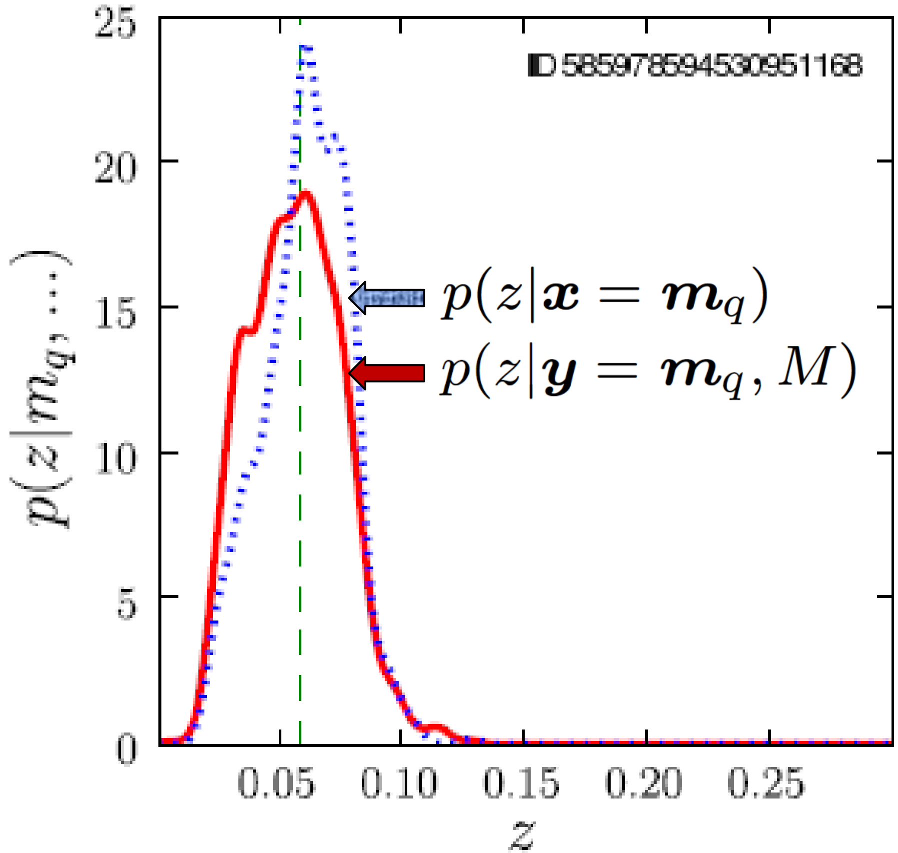
- Minimalist model
  - Normal distributions, same quantities: $\bar{x}(\theta) = \theta$ and $\bar{y}(\theta) = \theta$
  - With simple prior, the mapping is analytic , e.g., for flat

$$p(\boldsymbol{x}_t|\boldsymbol{y}_q, M) = \int d\boldsymbol{\theta}\ N(\boldsymbol{x}_t|\boldsymbol{\theta}, \mathbf{C}_t)\, N(\boldsymbol{\theta}|\boldsymbol{y}_q, \mathbf{C}_q)$$

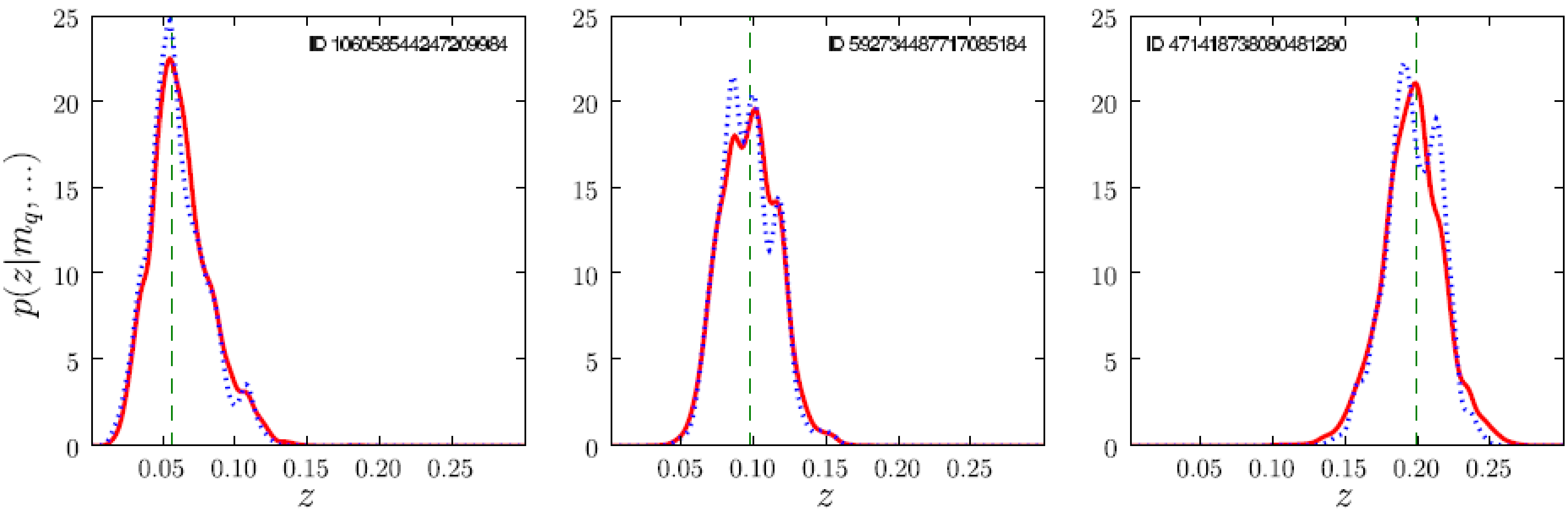
- Empirical relation
  - Fitting function as before or rather
  - General relation from densities
- Numerical summation $\rightarrow$



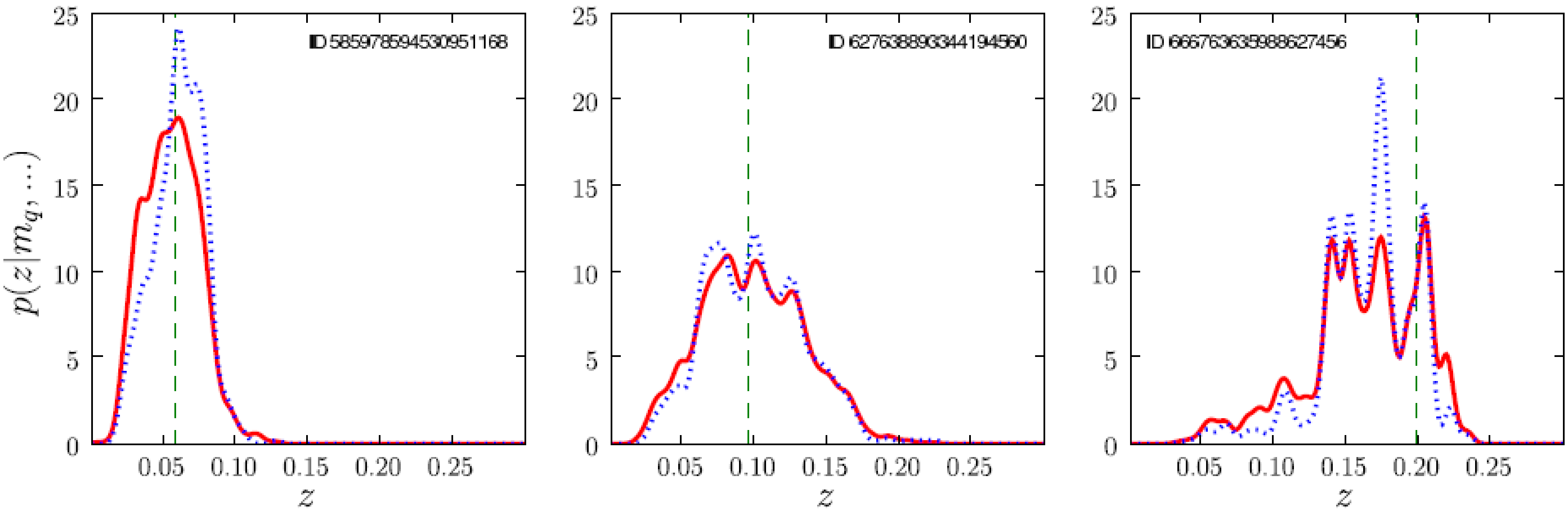
It works!


ID 585978594530951168
$p(z|\boldsymbol{x} = \boldsymbol{m}_q)$
$p(z|\boldsymbol{y} = \boldsymbol{m}_q, M)$
$p(z|m_q, \dots)$
z
0
5
10
15
20
25
0.05
0.10
0.15
0.20
0.25

Red Galaxies



Blue Galaxies



Advanced Methods

- Mapping observables via models
  - Any complete basis on wavelength range
  - Physics in the prior
- Relation of properties
  - Conditional densities
- Empirical but with templates
  - Unified framework at its best

$$
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Q:&\quad \{\boldsymbol{y}_q\}_{q\in Q}\\
M:&\quad \boldsymbol{\theta}\\
&\quad p(\boldsymbol{\theta}|M)\\
&\quad p(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{\theta},M)
\end{aligned}
$$

Concerning Priors

- Measured densities in the Query set

$$p(\boldsymbol{y}|Q)$$

- Consistent models should match that

$$p(\boldsymbol{y}|M) = \int d\boldsymbol{\theta}\, p(\boldsymbol{y}|\boldsymbol{\theta}, M)\, p(\boldsymbol{\theta}|M)$$

- Deconvolution yields empirical prior

$$p(\boldsymbol{y}|M) = p(\boldsymbol{y}|Q)$$

Summary

- Solve the photometric inversion problem
  - Real constraints from first principles
  - Works with incomplete data
- Put traditional methods in context
  - Explain their formal relations
  - Points out their strengths and weaknesses
- Suggests new directions
  - Next generation photo-z methods
  - Design better training sets

Room for Improvements

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- Understand photometric uncertainties
  - No error estimates are reliable today
- Better conditional density estimation
  - In presence of sharp boundaries
- Realistic models with good priors
  - That match the data