Lookup Tables

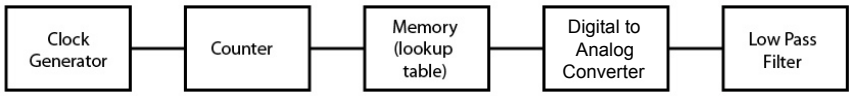
1

Example of Arbitrary Waveform Generator

- Clock Generator Output Increments a Counter
- Counter Output used as Memory (lookup table) Address
- Memory Stores One Period (or portion of period if symmetry is present) of a Waveform
- Output of Memory is Digitized Waveform Value
- This Value is Input to DAC
- DAC Output Filtered to Remove High Frequency Effects

2

Example of Arbitrary Waveform Generator



- Microcontroller can Comprise Several of These Blocks
- Lookup Table Data Allows Any Waveform to be Generated
- Could also Use an ARM with Pre- (or Post-) Indexed Addressing and let the Clock Interrupt the Processor

3

Computing the Table Values

- Consider a Sine Wave Generator
- Could Use a C Program to Calculate the Values of a Sine Wave from 0 to 90 Degrees
  - Take Advantage of Symmetry of Sine Function
  - Take Advantage of C math.h Library
- These Values Stored in the Data Section of an ARM Program
- Since Floating Point is not Supported, use Qm.n Notation – a Fixed Point Representation
  - Called 'Q notation'
  - In this example, we use Q.31 ('.' is missing on p. 137 of book)
  - Number of Integral bits often Optional so no '.' used – Wordsize Assumed to be Known
- Example C Code on page 136-7 of Textbook

4

Q*m*.*n* Notation

- Q Notation Refers to Fixed Point Number with m Integral Bits and n Fractional Bits
- Used for Hardware with No Floating Point and has Constant Resolution – Unlike Floating Point
- Resolution – Smallest Incremental Value Between Two Subsequent Values, $f(x+e)-f(x)$
  - e is Smallest Possible Value
  - e is the "Resolution"
- Integral Part is in 2's Complement Form (signed form)
- m is Number of Integral Bits NOT Including the Sign Bit
- Qm.n Requires $m+n+1$ Bits

5

Q*m*.*n* Notation (cont)

- Qm.n has a Range of $[-2^m, 2^m-2^{-n}]$
- Resolution is 2^{-n}

EXAMPLE: Q14.1

Requires $14+1+1=16$ Bits

Resolution is $2^{-1}=0.5$

Maximum Value is: $2^{14}-0.5=16K-0.5$

Minimum Value is: $-2^{14}=-16K$

All Values are $[-2^{14}, 2^{14}-2^{-1}]$

$= [-16384.0, +16383.5]$

$=$ [0x8000, 0x8001, 0x8001, …, 0xFFFF, 0x0000, 0x0001,…, 0xFFFD, 0xFFFE, 0xFFFF]

6

Q*m.n* Notation (cont)

- Q*m.n* has a Range of $[-2^m, 2^m-2^{-n}]$
- Resolution is 2^{-n}

EXAMPLE: Q6.1
Requires $6+1+1=8$ Bits
Resolution is $2^{-1}=0.5$
Maximum Value is: $2^6-0.5=64-0.5$
Minimum Value is: $-2^6=-64$
All Values are $[-2^6, 2^6-2^{-1}]$
$= [-64.0, +63.5]$
$=$ [0x80, 0x81, 0x81, …, 0xFF, 0x00, 0x01,…, 0xFD, 0xFE, 0xFF]

7

Q*m.n* Notation Conversion

- From Floating Point to Q*m.n*
  1) Multiply Floating Point Number by 2^n
  2) Round to the Nearest Integer

- From Q*m.n* to Floating Point
  1) Convert to Floating Point as if Q-value were Integer
  2) Multiply Converted Floating Point Number by 2^{-n}

8

Q31 For Sine Wave

- Q31 Allows for Accurate Representation of sin(0) through sin(89) Degrees
- What is the minimum Q31 Representation?

Q31 Range is [0x80000000,0x7FFFFFFF]

0x80000000 is Minimum Value, in Binary:

1000 0000 0000 0000 0000 0000 0000 0000

0111 1111 1111 1111 1111 1111 1111 1111

+1

1000 0000 0000 0000 0000 0000 0000 0000

Red Bits are the 31 Fractional Bits

This Bit String Represents 0.0 Decimal (as does 0x000000000)

9

Q31 For Sine Wave

- Q31 Allows for Accurate Representation of sin(0) through sin(89) Degrees
- What is the minimum non-zero Q31 Representation?

Q.31 Range is [0x80000000,0x7FFFFFFF]

0x80000001 is Minimum Value+e, in Binary:

1000 0000 0000 0000 0000 0000 0000 0001

0111 1111 1111 1111 1111 1111 1111 1110

+1

1111 1111 1111 1111 1111 1111 1111 1111

Red Bits are the 31 Fractional Bits

This Bit String Represents -SUM($2^{-1}+2^{-2}+\ldots 2^{-31}$) Decimal

10

Q31 For Sine Wave

- What is the Q31 for zero?

0.0 = 0x00000000

- What is the Q31 Representation for sin(90)?

sin(90°)=1, so must represent #1 in Q31 Format, but this is impossible in Q31 (fractions only), must use closest value

Q31 Range is [0x80000000,0x7FFFFFFF]

0x7FFFFFFF is Maximum Value, in Binary:

0111 1111 1111 1111 1111 1111 1111 1111

$=SUM(2^{-1}+2^{-2}+\ldots 2^{-31})=(15/16+15/16^2+15/16^3+\ldots+15/16^8)$

$=1-2^{-32}$ (EASY WAY TO COMPUTE)

=0.9375+0.05859375+0.00366211+0.00022888

+0.00001431+0.00000089+0.00000005+0.000000003

=0.999999999767169 MUST USE THIS FOR sin(90)=1

11

Computing the Table Values

```
#include <stdio.h>
#include <string.h>
#include <math.h>

main()
{
  int i;
  int index=0;
  signed int j[92];
  float sin_val;
  FILE *fp;

  if ((fp = fopen("sindata.twt","w"))==NULL)
    {
    printf("File could not be opened for writing\n");
    exit(1);
    }
```

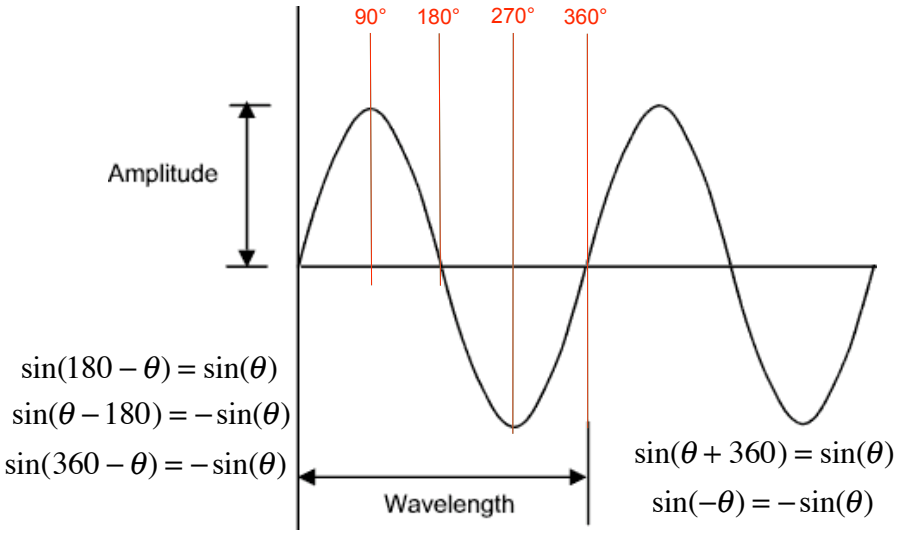
12

Computing the Table Values

```
for (i=0; i<=90; i++)  {          /* index i is in units of degrees */
    /* convert to radians */
    sin_val = sin(M_PI*i/180.0);    /* M_PI is pi constant - math.h */
    /* convert to Q31 notation */
    j[i] = sin_val * (2147483648); /* Q31 conversion factor - 2^31 */
    }
for (i=1; i<=23; i++)  {       /* Generate a line of 4 sin values */
    fprintf(fp, "DCD     ");
    fprintf(fp,"0x%x,",j[index]);
    fprintf(fp,"0x%x,",j[index+1]);
    fprintf(fp,"0x%x,",j[index+2]);
    fprintf(fp,"0x%x,",j[index+3]);
    fprintf(fp,"\n");
    index += 4;
    }
fclose(fp);
}
```

13

Sine Wave – Only need 0 through 90 deg



14

Using Symmetry with Sine Lookup Table

- First Part of Program Converts to Angle Between 0 and 90
  - Compares to 90 then 180 then 270
  - If Angle =< 90, lookup sine vlaue
  - If Angle =< 180, Angle ← 180-Angle
  - If Angle =< 270, Angle ← Angle-180
  - If Angle =< 360, Angle ← 360-Angle
- Assumes Angle NOT Greater than 360
- Lookup Value
  - Negate Value if Original Angle > 180

$$\sin(\theta - 180) = -\sin(\theta) \qquad \sin(360 - \theta) = -\sin(\theta)$$

15

ARM Program with Sine Lookup Table

```
        AREA SINETABLE, CODE
        ENTRY
;  Registers used:
;       r0 <- return value in Q.31 notation
;       r1 <- sin argumant (in degrees from 0 to 360)
;       r2 <- temp
;       r4 <- starting address of sine table
;       r7 <- copy of argument
main
   mov  r7, r1          ;make copy of argument
   ldr  r2, =270        ;const. won't fit with rotation scheme
   adr  r4, sin_data    ;load address of sin data table
   ;check if angle is less than or equal to 90
   cmp  r1, #90         ;set flags based on 90 degrees
   ble  retvalue        ;if N=1, value is LT 90, go to table
   ;check if angle is less than or equal to 180
   cmp  r1, #180        ;set flags based on 180 degrees
   rsble r1, r1, #180   ;if N=1, subtract angle from 180
```

16

ARM Program with Sine Lookup Table

```
   ble  retvalue       ;if N=1, value is LT 180, go to table
   ;check if angle is less than or equal to 270
   cmp  r1, r2         ;set flags based on 270 degrees
   suble r1, r1, #180  ;if N=1, subtract 180 from angle
   ble  retvalue       ;if N=1, value is LT 270, go to table
   ;angle must be GT 270 and LT 360
   rsb  r1, r1, #360   ;subtract angle from 360
retvalue
   ldr  r0, [r4, r1, LSL #2]   ;lookup Q.31 sine value
   ;must now determine whether or not to negate value
   cmp  r7, #180       ;set flags to determine need to negate
   rsbgt r0, r0, #0    ;negate value if N=0
done    b       done
```

17

ARM Program with Sine Lookup Table

```
   ALIGN
sin_data
   DCD  0x00000000, 0x023be164, 0x04779630, 0x06b2f1d8
   DCD  0x08edc7b0, 0x02b7eb50, 0x0d613050, 0x0f996a30
   DCD  0x11d06ca0, 0x14060b80, 0x163a1a80, 0x186c6de0
                        .
                        .
                        .
   DCD  0x7fec0a00, 0x7ffb0280, 0x7fffffff

END
```

- Complete Table on Page 138
- This Part of File Generated by the C Program

18

ARM Program with Sine Lookup Table

```
ldr r0, [r4, r1, LSL #2]       ;get sin value from table
```

- Actual Table Lookup Instruction
- `r4` Contains Starting Address of Table
- `r1` Contains Computed Degrees < 90
- Pre-Indexed Addressing
- `LSL #2` Multiplies by 4 Since Byte Addressable
- Angle is Multiplied by 4 Since 4 sine Values per Word (`DCD`)

19

FUTURE PROJECT – DIGITAL RECORDER

- Add ADC & Audio Amp Chip to Evaluator7T Board
- Interface ADC to Samsung Processor
- 2 Programs – RECORD PLAYBACK
- RECORD
  - Digitizes Speech From Microphone – store in memory
- PLAYBACK
  - Read Recorded Speech From Memory
  - Send to ADC
  - Output ADC to Audio Amplifier&Speaker

20

Jump Tables – Another Lookup Table

- Jump Tables Contain *Addresses* Instead of *Data*
- Allows a Microcontroller to Execute One of Several Subroutines Based on an Input Value; A CASE Statement
- Replaces a Series of Comparisons and Branches
- Microcontroller Receives/Computes a Value, Then Based on Value, Accesses the Jump Table
- Jump Table then "Points" to the Appropriate Subroutine to Execute

21

Jump Table Example

- Three Input Values:
  - First Value (0 or 1) Used to Determine Whether to ADD or SUBTRACT
  - Second and Third Values are Operands to be Added or Subtracted
- Uses Equate Directive (`EQU`) to Set the Number of Jump Targets
- Calls High-Level Function that Processes the Control Argument and Uses Jump Table to Call Appropriate Lower-Level Function
- Lower Level Function Executes and Returns to Main Program

22

Jump Table Example

```
        AREA Jump, CODE, READONLY      ;Name this block of code
        CODE32                         ;Following code is ARM code
num     EQU  2                         ;Number of entries in jump table
        ENTRY                          ;Mark first instr to execute
start                                  ;First instruction to call
        mov     r0, #0                 ;Set up the three parameters
        mov     r1, #3
        mov     r2, #2
        bl      arithfunc              ;Call the function
stop    b       stop                   ;Processing complete

arithfunc                              ;Label function
        cmp     r0, #num               ;function code is unsigned int.
        movhs   pc, lr                 ;if code >=num (C=1) return
        adr     r3, JumpTable          ;load address of jump table
        ldr     pc, [r3,r0,LSL #2]     ;Jump to appropriate routine
```

23

Jump Table Example

```
Jumptable
        DCD     DoAdd
        DCD     DoSub
DoAdd   add     r0, r1, r2             ;Operation 0
        mov     pc, lr                 ;Return
DoSub   sub     r0, r1, r2             ;Operation 1
        mov     pc, lr                 ;Return
        END                            ;Mark the end of file
```

- Starting Address of Jump Table in Register `r3`
- `r0` Contains 0 or 1 Condition
- Multiplied by 4 (`LSL #2`) to Account for Word Allignment
  - `DCD Do???` Are Word Aligned Addresses in Memory

24

Searching Lists/Tables

- Classical Problem Involving a Table of “Keys” and “Information”
- A “Key” is Input and a Table is Accessed to
  1) Determine if the Key and Information is Present
  2) Possibly Return the Data Associated with the Key
- Straightforward Approach is to Perform a Linear Search, Could Require Worst Case of Searching all N Keys
- If Table is Pre-processed, Other Method Can be Used Allowing "almost" Direct Access

25

Table Pre-Processing

- No Pre-Processing
  – Sequential Search, O(N)
- Storing Using a Hash Function
  – Near Constant-time Access, O(1)
- Storing in Sorted Key Order
  – Binary Search, O[lg(N)]
- We Will Consider the Binary Search Method in More Detail

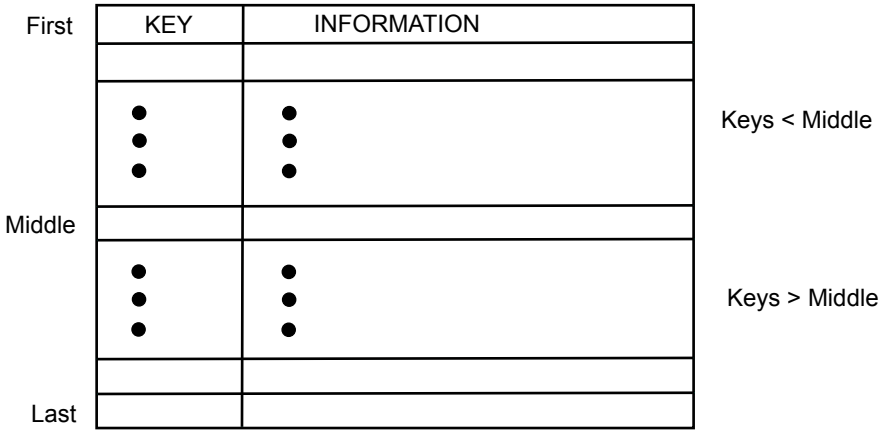
26

Binary Search Table

- Assume Keys in Ascending Numerical Order
- First Compare Input Key to Middle Key
- Can Immediately Rule Out Top or Bottom Half Based on Input Key GT or LT Middle Key
- Next Set Area to Search Equal to Top or Bottom Half of Table and Repeat
- First Discard Half, then One-Fourth, One-Eighth, etc.

27

Binary Search Table



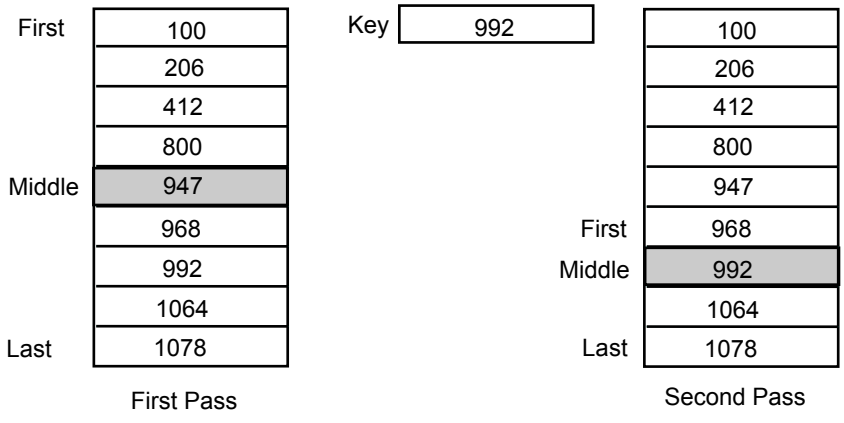
28

C Program Description

```
first = 0;
last = num_entries - 1;
index = 0;
while ((index == 0) & (first <= last ))  {
       middle = (first + last)/2;
       if (key == table[middle]) index = middle;
       else if (key < table [middle]) last = middle - 1;
       else first = middle + 1;
        }
```

29

Example – Two Passes in Search



30

Table Data Example

- Assume Table Start Address is `0x4000`
- Each Entry is 16 Bytes
  - First Word (4 Bytes) is Key
  - Remaining 12 Bytes are Character Data

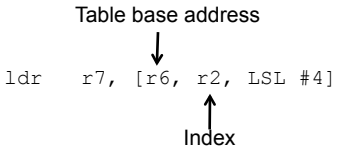
```
address = table_addr + i*size_of_entry
```

- Example Second Entry Address is

```
0x4000+2*16 = 0x4020
```

31

Example Table Structure



	4 Byte Key	12 Bytes of Data
0x4000	0x00000034	Vacuums
0x4010	0x00000234	Clothes
0x4020	0x00003403	Candy
0x4030	0x0010382C	Telephones

32

Binary Search Implementation

- Use `ldr` Instruction with Pre-indexed Addressing
- Table Base Address is Offset with a Scaled Index
- Scaling Specified as Constant `ESIZE`
- Assumes Table Size is Power of Two
  - If not Power of Two, Use a Two-Level Table
  - Entry is Key and Address Pointing to Data in Memory
- Load Table Entries with Single Pre-indexed Instruction

```
ldr r7, [r6, r2, LSL #ESIZE]
```

- Using `ESIZE` Equate Allows for Easily Changing Table Size

33

Example Code

```
NUM     EQU  14                      ;Number of entries in table
ESIZE   EQU  4                       ;log(Size) of Entry (in bytes)
        AREA   BIN-SRCH, CODE
        ENTRY                        ;Mark first instr to execute
; Registers used:
; r0 - first entry
; r1 - last entry
; r2 - middle entry
; r3 - index
; r4 - size of the entries (log 2)
; r5 - the key (what you're searching for)
; r6 - address of the list
; r7 - temp
        ldr     r5, =0x200     ;look for key-data 0x200 (PINEAPPLE)
        adr     r6, table_start        ;load address of the table
        mov     r0, #0                 ;first = 0
        mov     r1, #NUM-1             ;last - number of entries
                                       ;in list-1
```

34

Example Code (cont)

```
loop    cmp     r0, r1          ;compare first and last (first-last)
        movgt   r2, #0          ;first>last, no key found, middle=0
        bgt     done
        ;caluclate arithmetic middle index
        add     r2, r0, r1      ;first + last
        mov     r2, r2, ASR #1 ;first <- first + last/2
        ;get key of middle entry
        ldr     r7, [r6, r2, LSL #ESIZE]      ;load the entry
        cmp     r5, r7          ;compare key to middle value loaded
        ;conditionally pick to update either 'last' or 'first'
        addgt   r0, r2, #1      ;first = middle + 1
        sublt   r1, r2, #1      ;last = middle - 1
        bne     loop            ;if first NE last, continue search

done    mov     r3, r2          ;move middle to 'index' - entry found
stop    b       stop
```

35

Example Code (cont)

```
table_start
   DCD  0x004
   DCB  "PEPPERONI    "
   DCD  0x005
   DCB  "ANCHOVIES    "
   DCD  0x010
   DCB  "OLIVES       "
   DCD  0x012
   DCB  "GREEN PEPPER"
   DCD  0x018
   DCB  "BLACK OLIVES"
   DCD  0x022
   DCB  "CHEESE       "
   DCD  0x024
   DCB  "EXTRA SAUCE "
   DCD  0x026
   DCB  "CHICKEN      "
```

36

Example Code (cont)

```
DCD  0x030
DCB  "CANADIAN BAC"
DCD  0x035
DCB  "GREEN OLIVES"
DCD  0x038
DCB  "MUSHROOMS   "
DCD  0x100
DCB  "TOMATOES    "
DCD  0x200
DCB  "PINEAPPLE   "
DCD  0x300
DCB  "PINE NUTS   "
END
```

37