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Modeling and Characterization of High-Force/High-Stroke Piezoelectric Actuator

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Acknowledgements

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Outline

- State-of-the-art in actuator characterization
- Modeling and experimental results
  - Quasi-static and dynamic modeling
  - Experimental set-up
  - Comparison of modeled and empiric data
- Design tools
  - Mechanical and electrical envelopes
  - Design guidelines
- Conclusions

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State-of-the-art in actuators characterization

Goal

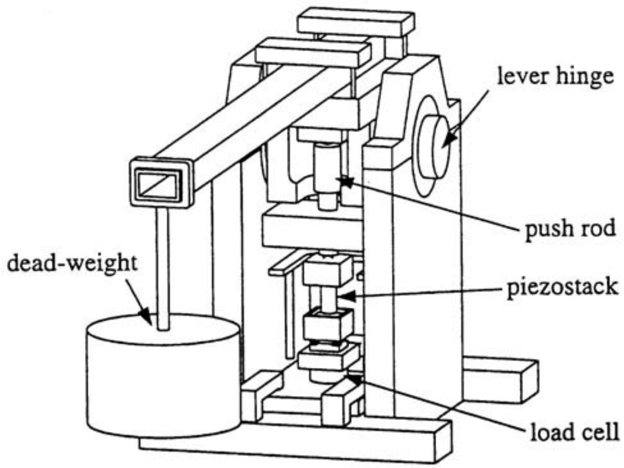
Provide characterization of actuator force, displacement, energy and power under various load conditions

Typical manufacturer data

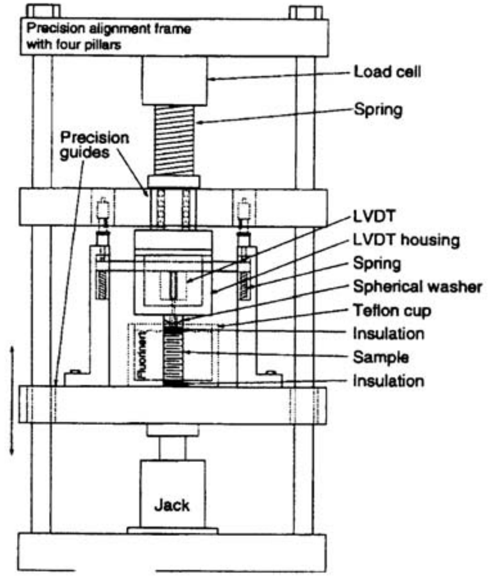
Physik Instrumente, Kinetic Ceramics, PiezoSystems Jena, Etrema Inc., etc.

Response under variable pre-stress and voltage cycle

Lee *et al*. (1999), Mitrovic *et al.* (1999), Pan *et al.* (2000), Straub *et al*. (1999), Mitrovic (2000)

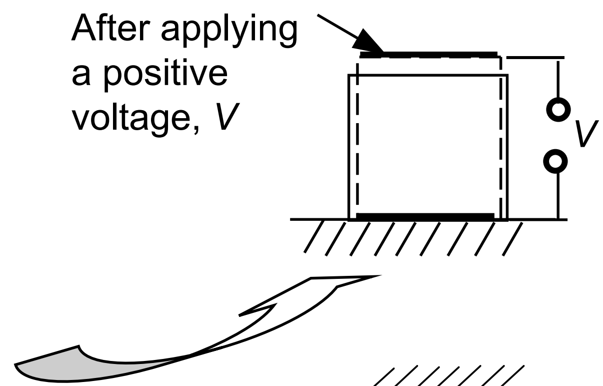


Lee *et al*. (1999),



Pan *et al.* (2000)

Quasi-static piezoelectric actuator modeling



Hypotheses:

a) linear constitutive equations

b) equilibrium condition

How it works as an actuator…

$$S_3 = s_{33}^E T_3 + d_{33} E_3 \quad \Rightarrow \quad u = L\frac{s_{33}^E}{A}F + L\frac{d_{33}}{t}V$$

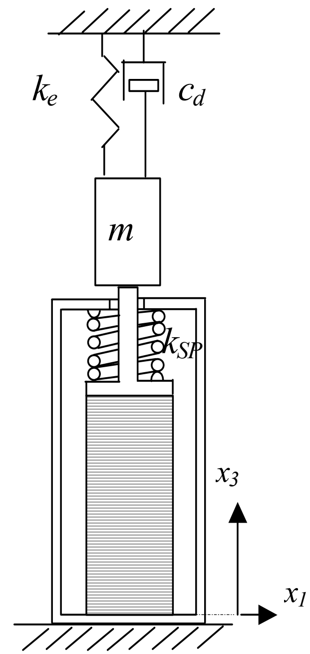
Model development:

Apply the loads in 3 steps:

a) Apply the internal pre-stress due to the internal spring, F_0

b) Apply the external force $F^{(b)}$

c) Apply the voltage V



Equations:

$$F_e^{(c)} = F_e^{(b)} - A\frac{d_{33}\cdot V}{t\cdot s_{33}^{(E)}}\frac{k_e}{k_e + k_{SP} + k_{ST}^{(E)}}$$

$$u_3^{(c)} = \frac{\left(1+\frac{k_{SP}}{k_{ST}^{(E)}}\right)F_0 + F_e^{(1)}}{\left(k_{ST}+k_{SP}\right)} + L\frac{d_{33}}{t}k_{ST}^{(E)}\frac{V}{k_e + k_{SP} + k_{ST}^{(E)}}$$

Dynamic piezoelectric actuator modeling (I)

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Hypotheses:

- linear constitutive equations
- the piezoelectric stack is modeled as a continuous medium
- the material losses are introduced through complex coefficients:

$s_{33}^{(E)*} = s_{33}^{(E)}(1\text{-}i\eta)$; $\varepsilon_{33}^{(E)*} = \varepsilon_{33}^{(E)}(1\text{-}i\delta)$; $d_{33}^{*}=d_{33}(1\text{-}i\lambda)$

Model development:

Equation: $$\frac{\partial^2 u(x_3,\tau)}{\partial \tau^2} = c^2 \frac{\partial^2 u(x_3,\tau)}{\partial x_3^2}$$

Boundary conditions: $$u(0,\tau)=0$$ $$A\cdot T_3(L,\tau)=F_{ST}(\tau)$$

With $V(\tau)=V_0+V_a e^{i\omega\tau}$, assume $u(x_3,\tau)=\left(C_1\sin(\gamma x_3)+C_2\cos(\gamma x_3)\right)e^{i\omega\tau}-C_3 x_3$

$$F(x_3=L,\tau)=\frac{A}{s_{33}^{(E)*}}\left[\left(\frac{\partial u}{\partial x_3}\right)_{x_3=L}-d_{33}^{*}\frac{V_0+V_a e^{i\omega\tau}}{t}\right]$$

Electrical modeling:

div $\boldsymbol{D}$ = ρ_{free} $\quad Q=D_3 A \quad i=\frac{dQ}{d\tau}$

Dynamic piezoelectric actuator modeling (II)

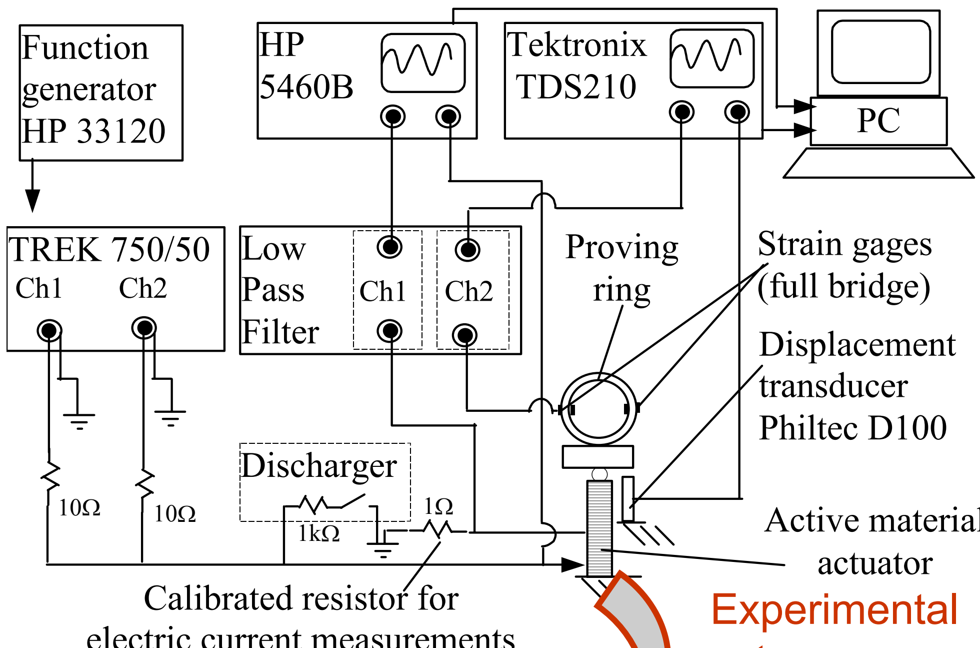
Displacement, force and electric current equations:

$$u_{ST} = \frac{d_{33}^{*} V_a \tan(\gamma L) e^{i\omega\tau}}{t\gamma \left(1 + \frac{s_{33}^{(E)*}}{A} \frac{i\omega Z_{EXT}}{\gamma} \tan(\gamma L)\right)} + \frac{F_e^{(1)}}{k_{ST}^{(E)*} + k_{sp}} + \frac{L}{t} d_{33}^{*} V_0 \frac{k_{ST}^{(E)*}}{k_{ST}^{(E)*} + k_{sp} + k_e}$$

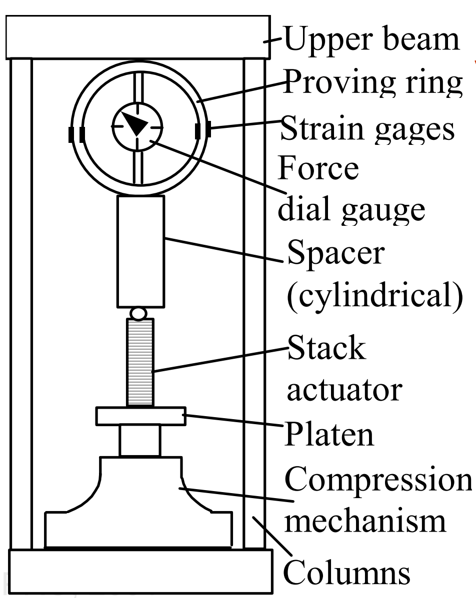
$$F_{ST}(\tau) = -F_{block} \frac{V_a}{V_0} e^{i\omega\tau} \frac{1}{\left(1 + \frac{s_{33}^{(E)*}}{A} \frac{i\omega Z_{EXT}}{\gamma} \tan(\gamma L)\right)} + F_0 + F_e^{(b)} \frac{k_{ST}^{(E)*}}{k_{ST}^{(E)*} + k_{SP}} + F_{block} \left(\frac{k_e + k_{SP}}{k_e + k_{SP} + k_{ST}^{(E)*}} + \frac{V_a}{V_0} e^{i\omega\tau} \right)$$

$$i_{ST}(\tau) = \frac{dQ}{d\tau} = \frac{L}{t} i\omega V_a e^{i\omega\tau} \left[d_{33}^{*} F_{block} \left(\frac{1}{\left(1 + \frac{s_{33}^{(E)*}}{A} \frac{i\omega Z_{EXT}}{\gamma} \tan(\gamma L)\right)} - 1 \right) + \frac{\varepsilon_{33}^{*} A}{t} \right]$$

Testing of PAHL 120/20 actuator



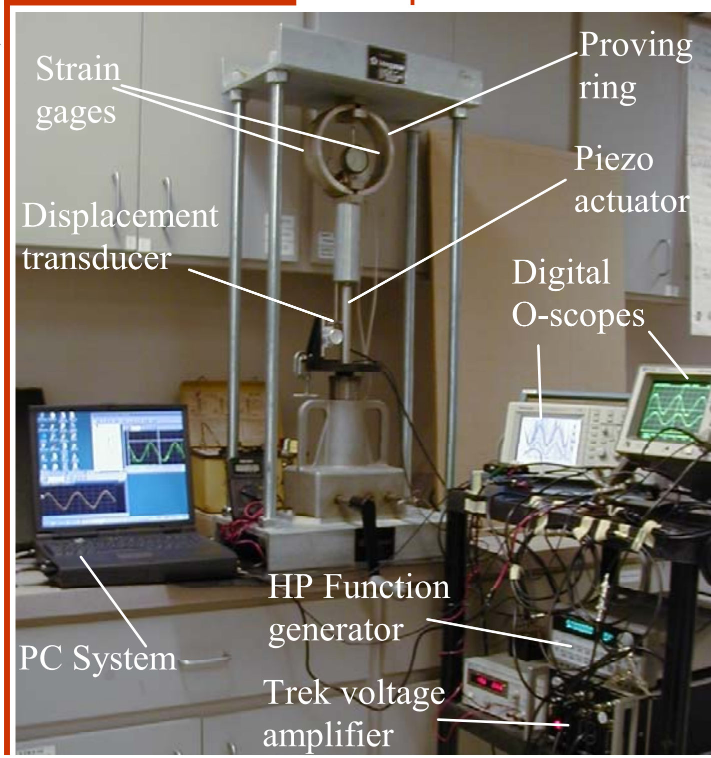
Experimental set-up schematic



Compression frame

PiezoSystems Jena PAHL 120/20 piezoelectric actuator

Experimental set-up



PAHL 120/20: Blocked force

Definition

Blocked force = the force generated by the actuator when the displacement is completely denied

Testing methods

Method 1 (quasi-static): apply first the voltage (actuator expands) and then a compressive force until the initial length is recovered

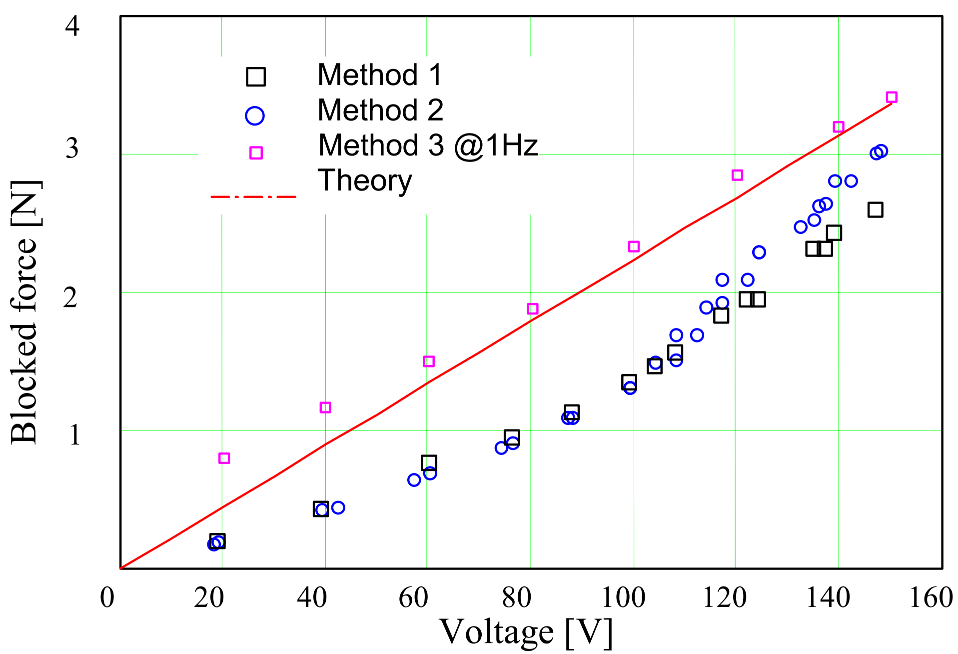
Method 2 (quasi-static): apply first a force (actuator compresses), and then a voltage until the initial length is recovered

Method 3 (dynamic): apply a biased harmonic voltage and then increase the compressive force until the maximum displacement corresponds to the undeformed actuator

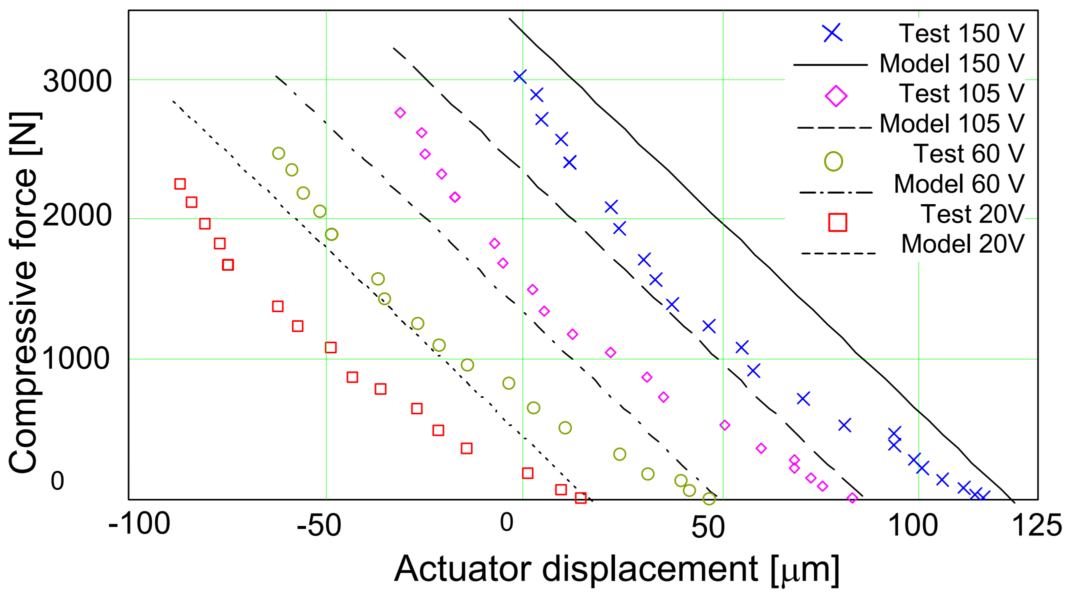
Theory

$$F_{block} = -\frac{d_{33}}{s_{33}^{E}} \frac{A}{t} V$$

Results



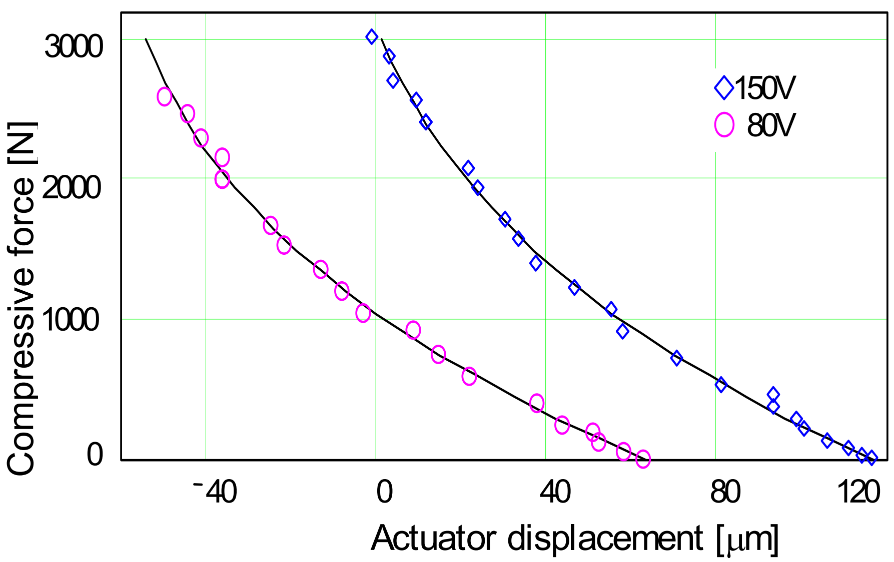
PAHL 120/20: Quasi-static test results



Linear model

Bi-cubic regression

$$u(F,V) = C_0F^3 + C_1F^2V + C_2FV^2 + C_3V^3$$
$$+ C_4F^2 + C_5FV + C_6V^2 + C_7F + C_8V$$



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PAHL 120/20: Quasi-static s_{33} and d_{33}

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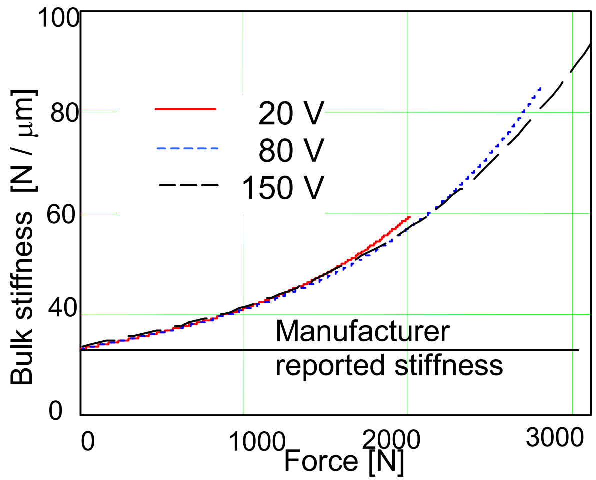
Bulk actuator stiffness:

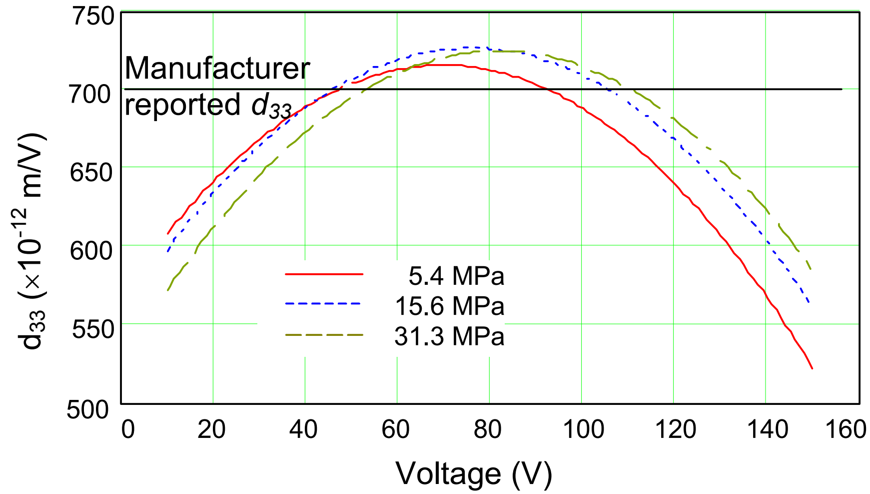
$$k_B \approx \left(\frac{\partial u}{\partial F}\right)^{-1}$$



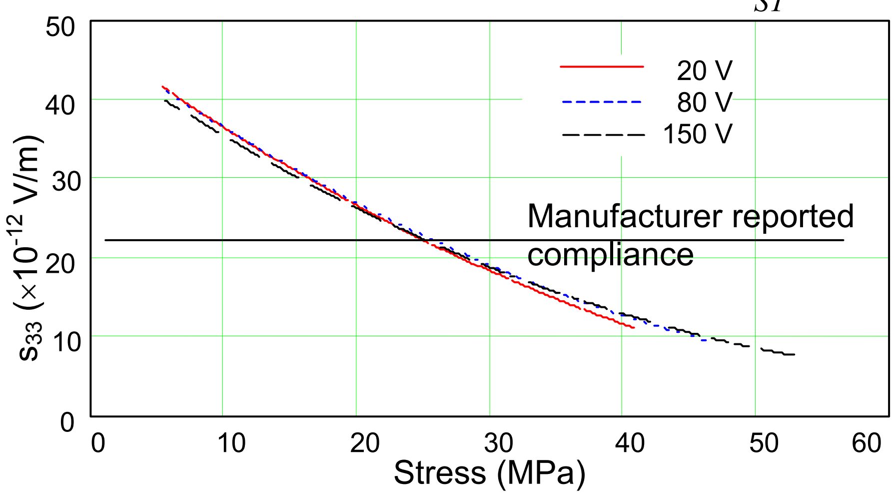
Force in the stack:

$$F_{ST} = T_{33}A = F_E + F_0 - k_{SP}\cdot u$$

Piezoelectric coefficient: $d_{33} \approx \frac{t}{L}\frac{\partial u}{\partial V}$



Compliance coefficient: $s_{33} \approx \frac{A}{L}\frac{\partial u}{\partial F_{ST}}$



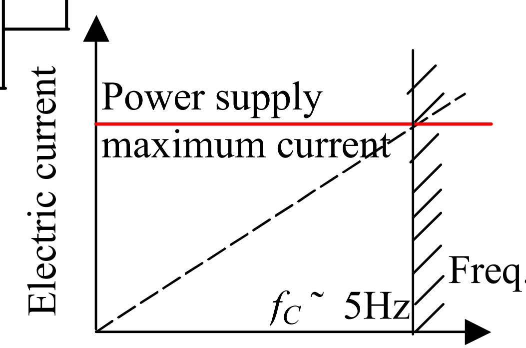
PAHL 120/20: Dynamic test results (I)

Test matrix for dynamic measurements

Frequency=5Hz
Frequency=4Hz
Frequency=2Hz
Frequency=1Hz

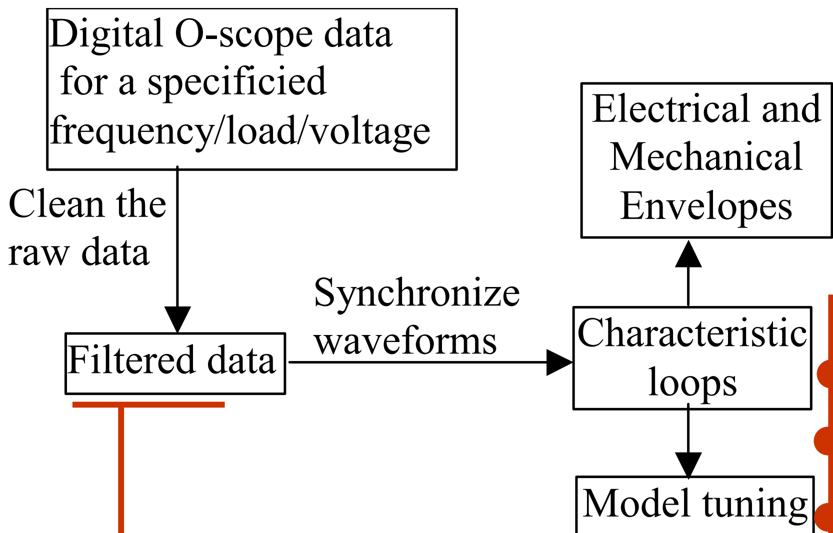
Load case / Voltage range	Load: $k_e = 6.5 \cdot 10^6$, ω_n=450Hz, ζ=0.05 Pre-stress (N)						No external load
	0	570	1150	1700	2300	2900	
0 – 20 V	✓	✓	✓	✓	✓	✓	✓
0 – 40 V	✓	✓	✓	✓	✓	✓	✓
0 – 60 V	✓	✓	✓	✓	✓	✓	✓
0 – 80 V	✓	✓	✓	✓	✓	✓	✓
0 – 100 V	✓	✓	✓	✓	✓	✓	✓
0 – 120 V	✓	✓	✓	✓	✓	✓	✓
0 – 140 V	✓	✓	✓	✓	✓	✓	✓
0 – 150 V	✓	✓	✓	✓	✓	✓	✓

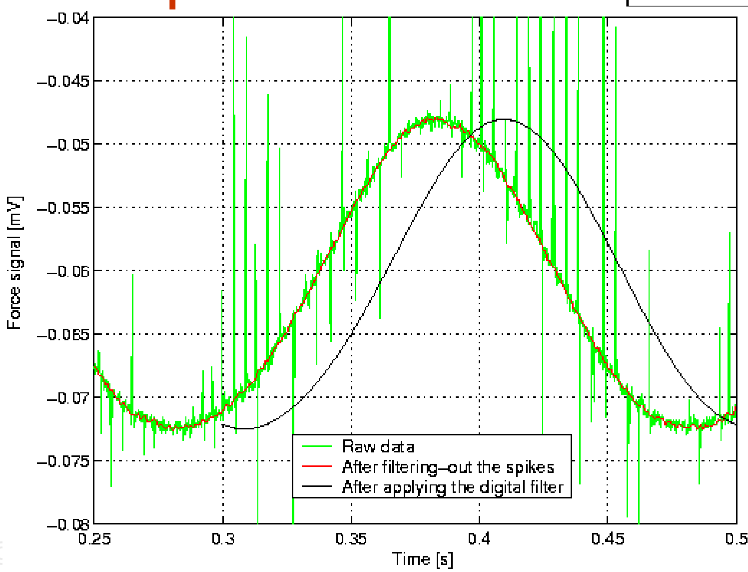
Number of processed files:
(4 frequencies)x(8 voltage cycles)x
[(6 loaded cases)x(4 signals)+(1 no load case)x(3 signals)] = **864**

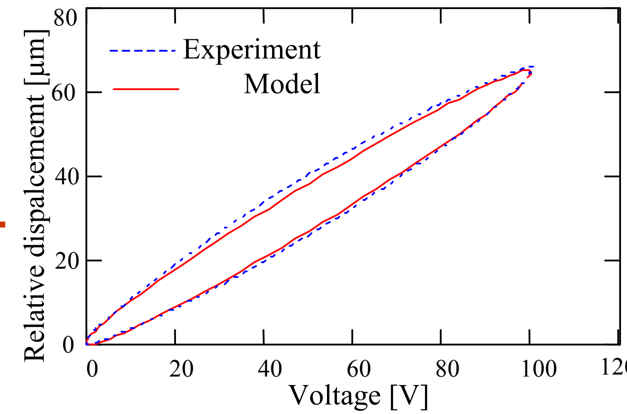


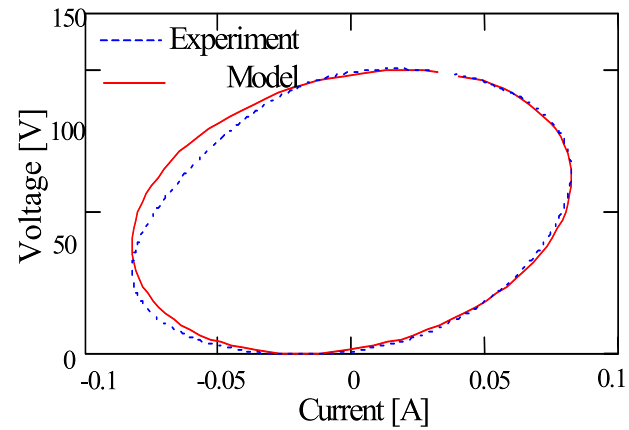
PAHL 120/20: Dynamic test results (II)

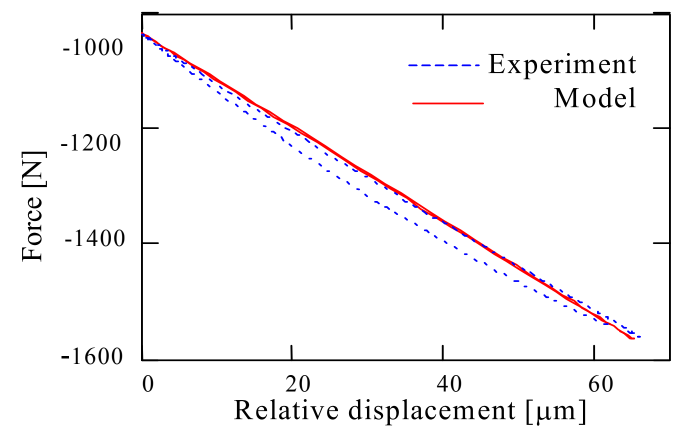
Data processing flow





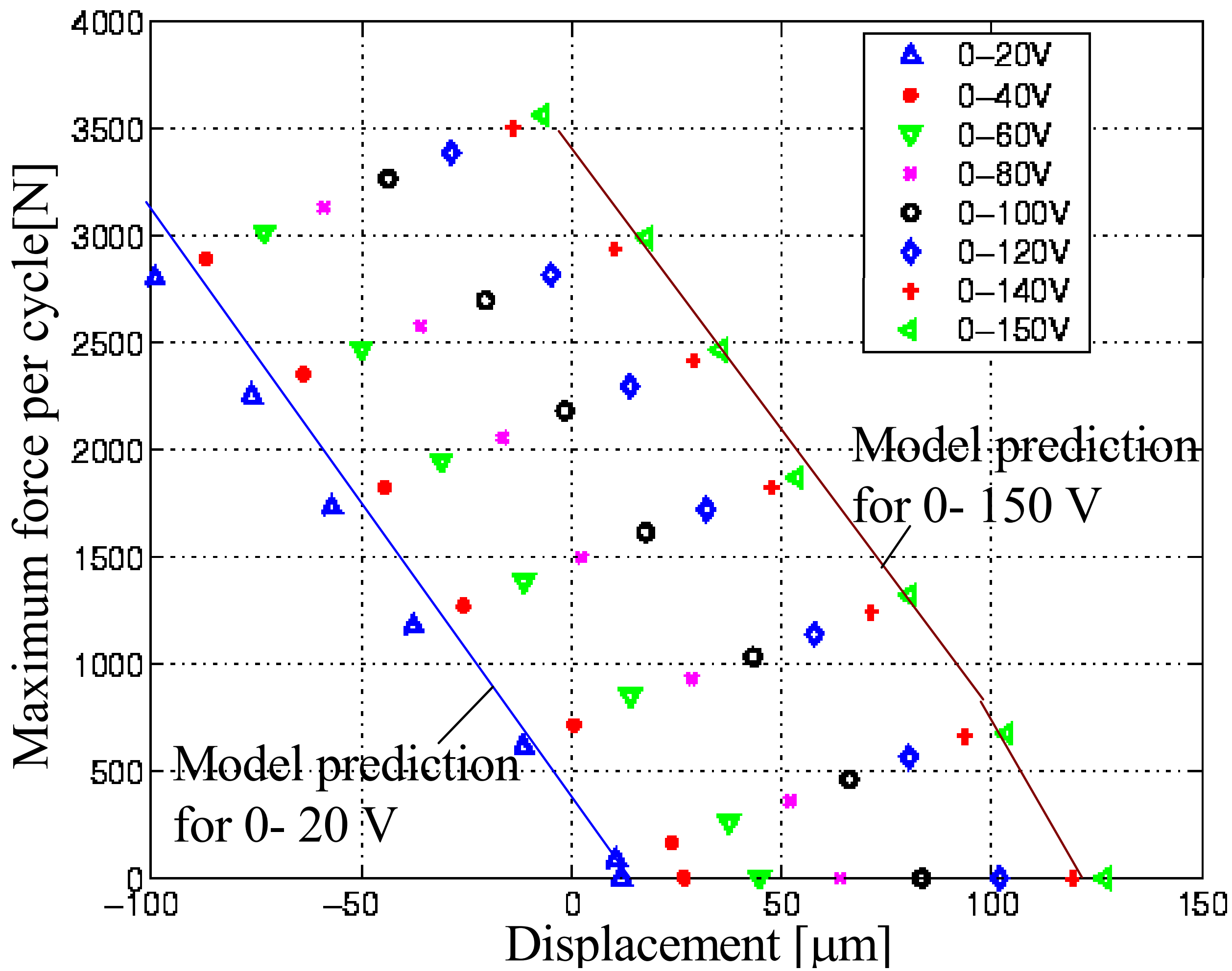






PAHL 120/20: Mechanical envelope

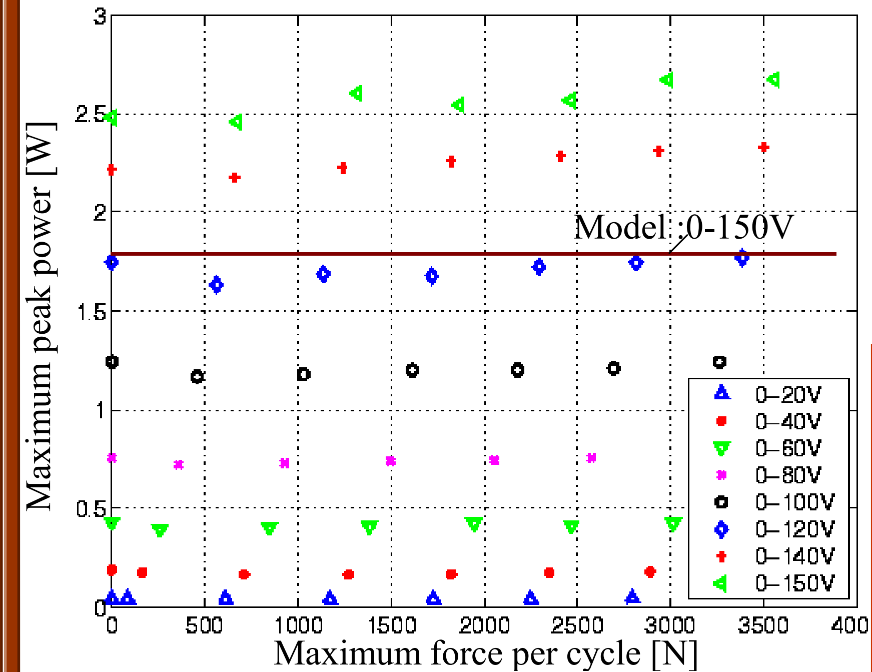
Frequency = 1Hz



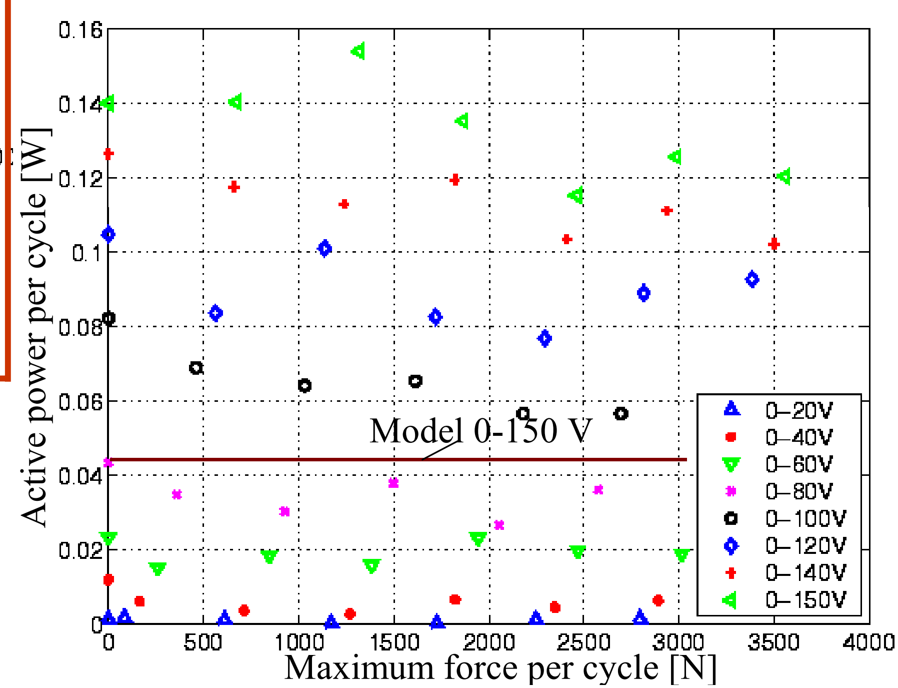
PAHL 120/20: Electrical Envelopes

Frequency = 1Hz

Peak power



Average active power per cycle



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Dynamic model improvement

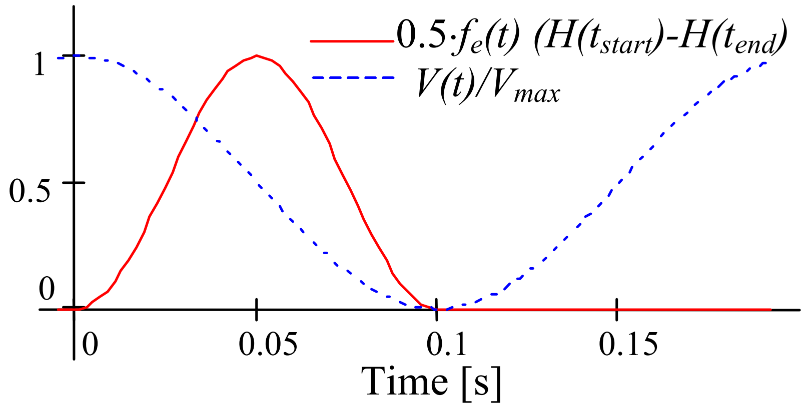
Initial model:

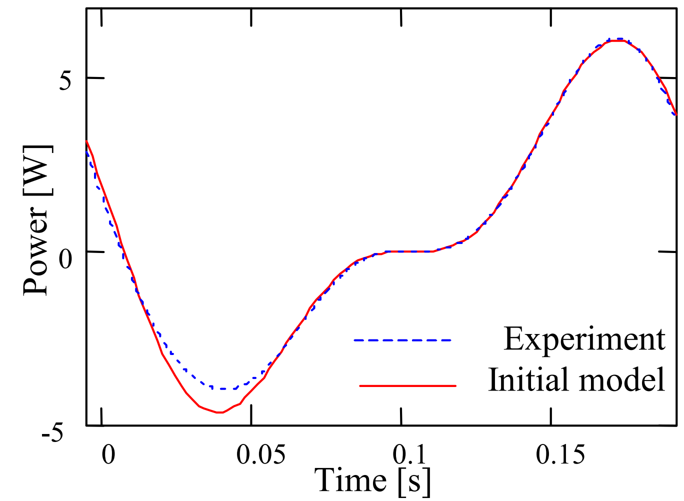
$\varepsilon_{33}^{(E)*} = \varepsilon_{33}^{(E)} (1-i\delta)$

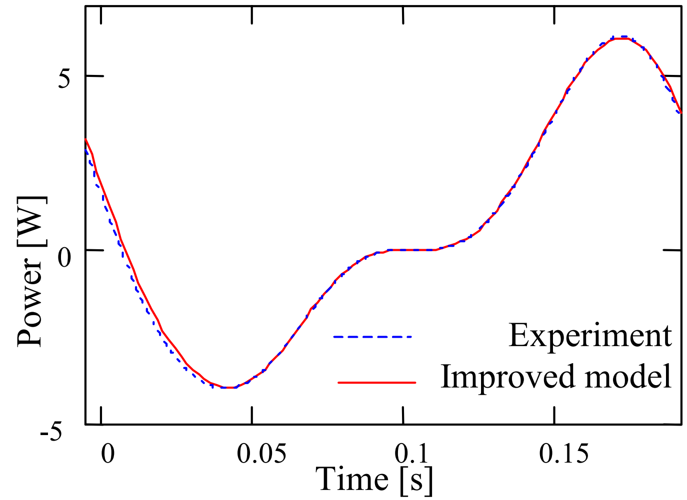
Improved model:

$$\varepsilon = \varepsilon_0 \left[1 - 0.5 \left(k_\varepsilon - 1 \right) f_e \left(t \right) \left(H \left(t_{end} \right) - H \left(t_{start} \right) \right) \right]$$

$$\delta = \delta_0 \left[1 + 0.5 \left(k_\delta - 1 \right) f_e \left(t \right) \left(H \left(t_{end} \right) - H \left(t_{start} \right) \right) \right]$$







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Actuation design with PAHL 120/20 (I)

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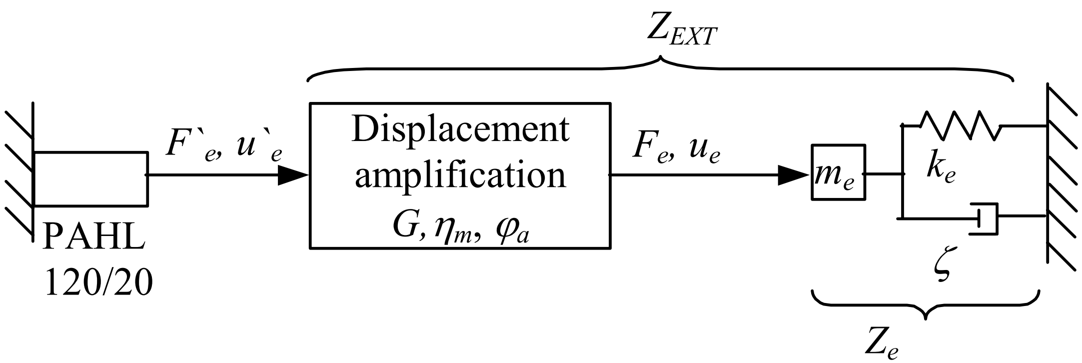
Problem formulation



$$F_e' \dot{u}_e' \eta_m = F_e \dot{u}_e \qquad u_e(\tau) = G u_e'(\tau) e^{i\varphi_a}$$

Given:

External load: m_e=10kg, ω_{ne} =150Hz; ζ=0.05

Actuation parameters: 5Hz frequency

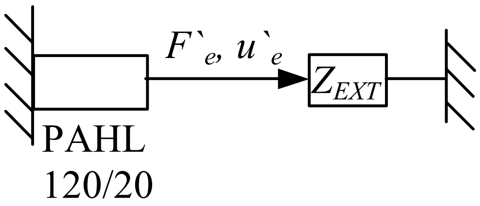
Displacement amplification parameters: G=7, η_m = 0.8, φ_a = 0^0

Required minimum displacement: 0.5 mm

Power supply ratings: 120V, maximum current i_{max} or maximum power p_{max}.

Actuation design with PAHL 120/20 (II)

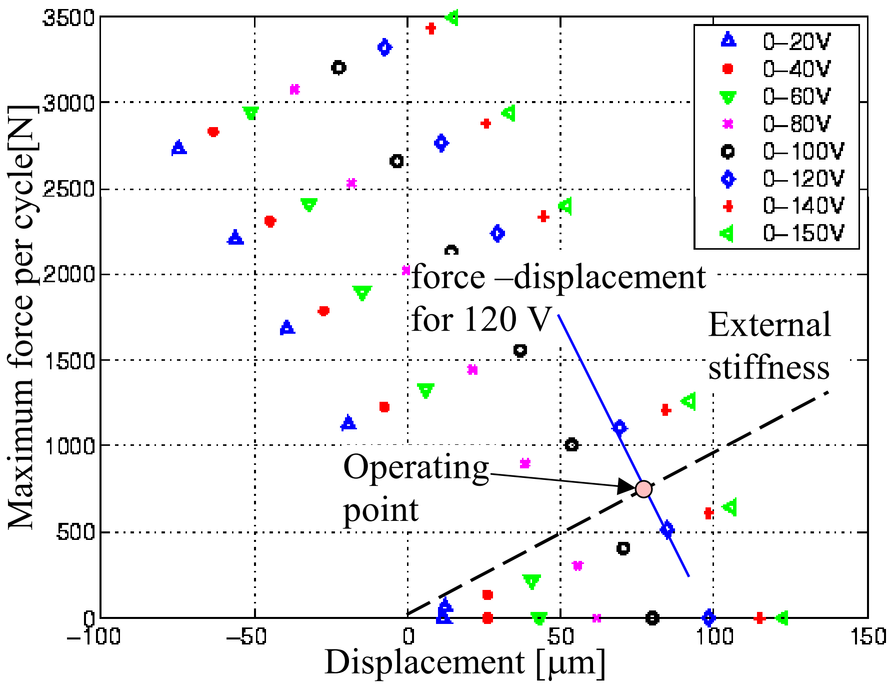
Reduce the problem to a known case:



$$Z_{EXT} = \frac{G^2}{\eta_m} Z_e e^{i\varphi_a}$$

$$k_d = i\omega Z_{EXT} = m_e\left(\omega_n^{\ 2} - \omega^2 + 2i\zeta\,\omega\,\omega_n\right)e^{i\varphi_a}\,\frac{G^2}{\eta_m}$$

Re(k_d) = 9.6·N/μm



The maximum displacement :
75 μm x G = 0.525 mm

The peak electric current:

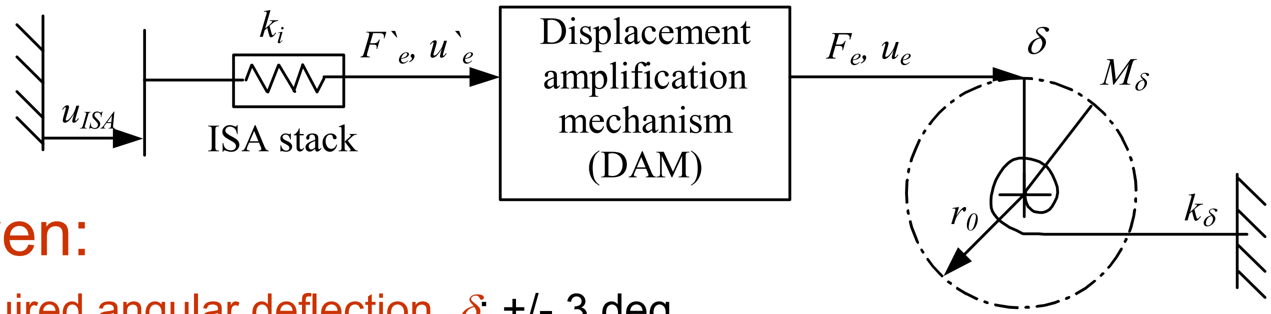
$$i_{max} \cong \omega C V_a = 2\pi f\left(\frac{L}{t}\frac{\varepsilon A}{t}\right)V_a = 0.103 \text{ A}$$

The peak power (electrical envelope)

$$p_{max} \cong 1.75 \text{ W}$$

Optimal quasi-static energy transfer (I)

Problem formulation



Given:

Required angular deflection, δ: +/- 3 deg

Aerodynamic stiffness, k_δ: 47 Nm/rad

Hinge radius, r_0: 5 mm

Actuator free induced displacement u_{ISA}: +/- 60 μm.

Definitions:

$$\eta_m = \frac{F_e \cdot u_e}{F'_e \cdot u'_e}$$: DAM work efficiency

$$r = \frac{k_i}{k_d r_0^2}$$: stiffness ratio

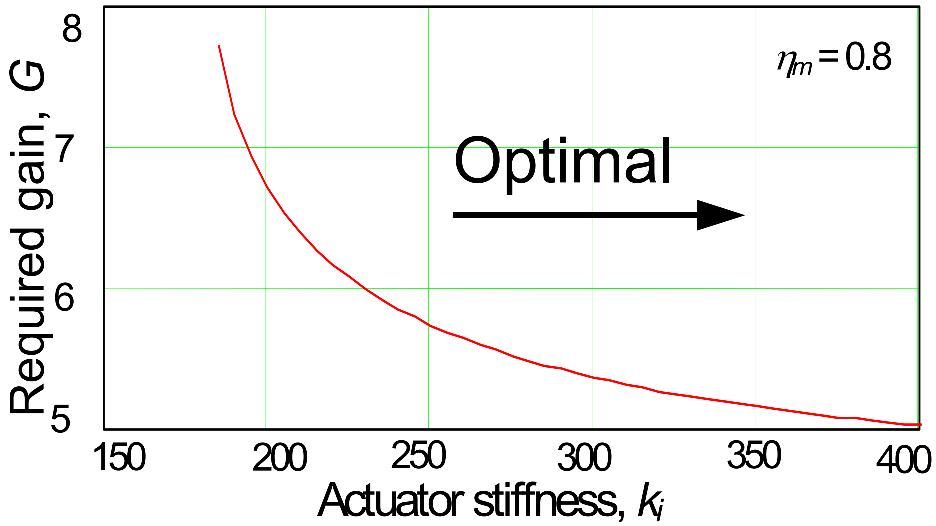
$$G = \frac{u_e}{u'_e}$$: DAM gain

$$\eta = \frac{r_0 \delta}{u_{ISA}}$$: kinematic gain

$$E_e' = \frac{E_{out}}{0.5 k_i u_{ISA}^2} = \frac{k_\delta \delta^2}{k_i u_{ISA}^2}$$: energy transfer coefficient

Optimal quasi-static energy transfer (II)

Solution



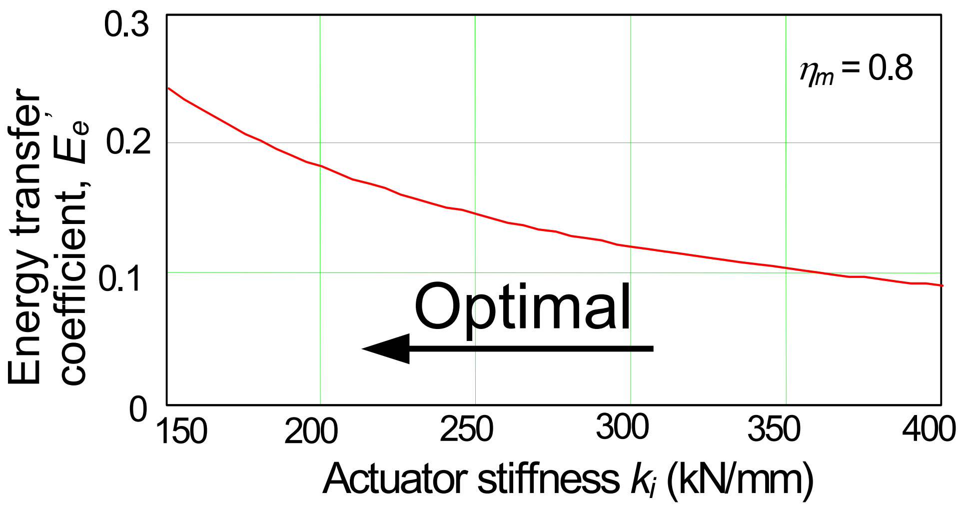
Required gain: $$G = \eta_m \frac{1 + \sqrt{1 - 4\frac{r\eta^2}{\eta_m}}}{2r\eta}$$

$$\downarrow$$

Critical actuator stiffness: $$k_i \geq k_{i\ cr} = \frac{4k_\delta}{\eta_m u_{ISA}}$$

Energy transfer coefficient:

$$E_e' = \eta^2 r$$



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Optimal quasi-static energy transfer (III)

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Optimization criteria:

Smaller required gain
- less complexity involved in the DAM design
- smaller and lighter DAM

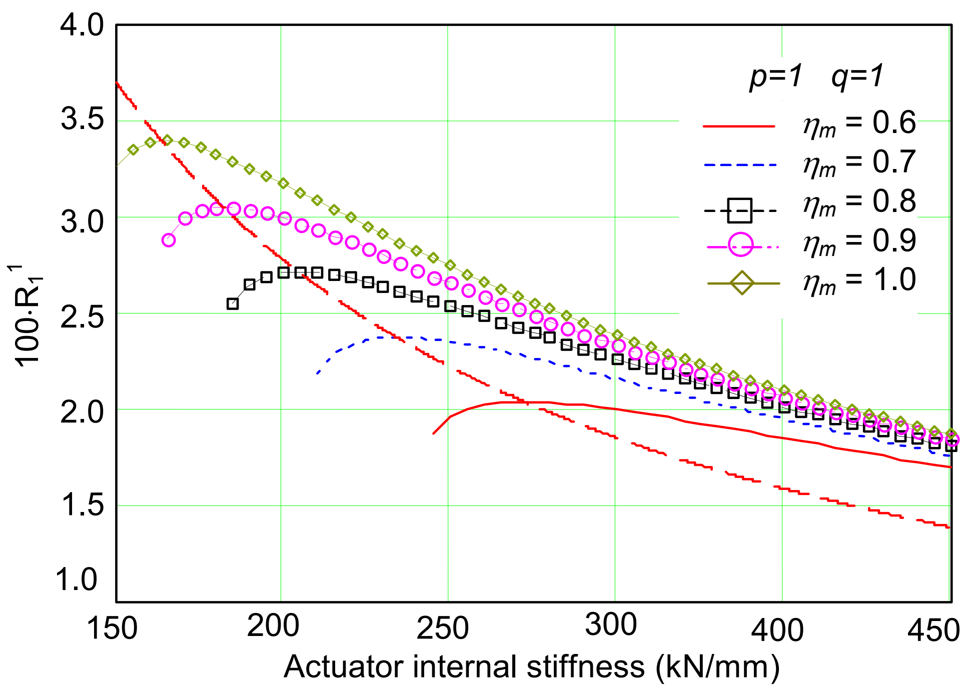
Greater energy transfer coefficient
- lighter actuator for a particular application

$$R_q^p = \frac{\left(E_e'\right)^p}{\left(G\right)^q} \qquad x = 4r\eta^2/\eta_m$$

$$R_q^p = \eta_m^p \eta^{-q} 2^{-(2p+q)} \frac{x^{p+q}}{\left(1+\sqrt{1-x}\right)^q}$$

$$\Downarrow$$

$$x_{opt} = 4\frac{p(p+q)}{(2p+q)^2}$$



Optimal quasi-static energy transfer (IV)

Actuator selection:

Option 1: PiezoSystems Jena PAHL 120/20: k_i = 30 – 80 N/μm

Option 2: Kinetic Ceramics D125120: k_i = 205 N/μm

Option 3: Physik Instrumente P247-70: k_i =400 N/μm

The critical stiffness $k_{i\ cr}$ =154 N/μm ~~Option 1~~

For equal optimal criteria weights, p=q=1, x_{opt} = 8/9

Choose η_m = 0.8

} k_{i_opt} = 200 N/μm ⇩ **Option 2**

Conclusions

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- Review of the state-of-the-art for high-force/high stroke smart material actuators characterization
- Modeling and characterization of a high-force/high-stroke piezoelectric actuator. Model improvement was addressed, based collected experimental data
- Effective design tools were proposed based on mechanical and electrical envelopes. Examples were given for the particular case of the PAHL 120/20 actuator
- Design guidelines for dynamic actuation systems incorporating piezoelectric actuators were produces