

Logik für Informatiker

Formal proofs for propositional logic

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WiSe 2009/10

Strategies and tactics in Fitch

- 1 *Understand* what the sentences are saying.
- 2 *Decide* whether you think the conclusion follows from the premises.
- 3 If you think it does not follow, or are not sure, try to find a *counterexample*.
- 4 If you think it does follow, try to give an *informal proof*.
- 5 If a *formal proof* is called for, use the *informal proof to guide* you in finding one.
- 6 In giving consequence proofs, both formal and informal, don't forget the tactic of *working backwards*.
- 7 In working backwards, though, always check that your *intermediate goals are consequences* of the available information.

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Strategies in Fitch, cont'd

- Always try to *match* the situation in your proof with the *rules* in the book (see book appendix for a complete list)
- Look at the *main connective* in a *premise*, apply the corresponding *elimination rule* (forwards)
- Or: look at the *main connective* in the *conclusion*, apply the corresponding *introduction rule* (backwards)

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Game rule: $P \rightarrow Q$ is replaced by $\neg P \vee Q$.

Formalisation of conditional sentences

- The following English constructions are all translated $P \rightarrow Q$:
If P then Q ; Q if P ; P only if Q ; and Provided P , Q .
- Unless P , Q and Q unless P are translated: $\neg P \rightarrow Q$.
- Q is a logical consequence of P_1, \dots, P_n if and only if the sentence $(P_1 \wedge \dots \wedge P_n) \rightarrow Q$ is a logical truth.

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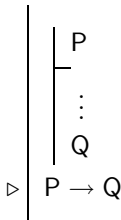
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Conditional Elimination (\rightarrow Elim)

$$\begin{array}{l} | \\ | \\ P \rightarrow Q \\ | \\ \vdots \\ | \\ P \\ | \\ \vdots \\ \triangleright | \\ | \\ Q \end{array}$$

Conditional Introduction (\rightarrow Intro)



Biconditionals

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

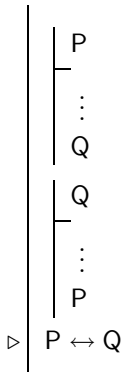
Game rule: $P \leftrightarrow Q$ is replaced by $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

Biconditional Elimination

(\leftrightarrow Elim)

		$P \leftrightarrow Q$ (or $Q \leftrightarrow P$)
		\vdots
		P
		\vdots
\triangleright		Q

Biconditional Introduction (\leftrightarrow Intro)



Reiteration (Reit)

$$\begin{array}{|l} P \\ \vdots \\ P \end{array}$$

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