Logik für Informatiker
Formal proofs for propositional logic

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Strategies and tactics in Fitch

1. *Understand* what the sentences are saying.
2. *Decide* whether you think the conclusion follows from the premises.
3. If you think it does not follow, or are not sure, try to find a *counterexample*.
4. If you think it does follow, try to give an *informal proof*.
5. If a *formal proof* is called for, use the *informal proof to guide* you in finding one.
6. In giving consequence proofs, both formal and informal, don't forget the tactic of *working backwards*.
7. In working backwards, though, always check that your *intermediate goals are consequences* of the available information.

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Strategies in Fitch, cont'd

- Always try to *match* the situation in your proof with the *rules* in the book (see book appendix for a complete list)
- Look at the *main connective* in a *premise*, apply the corresponding *elimination rule* (forwards)
- Or: look at the *main connective* in the *conclusion*, apply the corresponding *introduction rule* (backwards)

P	Q	P → Q
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

Game rule: $P \rightarrow Q$ is replaced by $\neg P \vee Q$.

Formalisation of conditional sentences

- The following English constructions are all translated $P \rightarrow Q$: If P then Q; Q if P; P only if Q; and Provided P, Q.
- Unless P, Q and Q unless P are translated: $\neg P \rightarrow Q$.
- Q is a logical consequence of $P_1, \ldots, P_n$ if and only if the sentence $(P1 \wedge \cdots \wedge P_n) \rightarrow Q$ is a logical truth.

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Conditional Elimination ($\to$ Elim)

$$
\begin{array}{l|l}
 & P \to Q \\
 & \vdots \\
 & P \\
 & \vdots \\
\triangleright & Q
\end{array}
$$

Conditional Introduction ($\rightarrow$ Intro)

$$
\begin{array}{l|l}
 & \begin{array}{|l} P \\ \hline \vdots \\ Q \end{array} \\
\triangleright & P \rightarrow Q
\end{array}
$$

Biconditionals

P	Q	P $\leftrightarrow$ Q
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

Game rule: $P \leftrightarrow Q$ is replaced by $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

Biconditional Elimination ($\leftrightarrow$ Elim)

$$
\begin{array}{c|l}
 & P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\
 & \vdots \\
 & P \\
 & \vdots \\
\triangleright & Q
\end{array}
$$

Biconditional Introduction (↔ Intro)

$$
\begin{array}{l|l}
 & \quad \begin{array}{|l} P \\ \hline \vdots \\ Q \end{array} \\
 & \quad \begin{array}{|l} Q \\ \hline \vdots \\ P \end{array} \\
\triangleright & P \leftrightarrow Q
\end{array}
$$

Reiteration (Reit)

$$\begin{array}{l|l} & P \\ & \vdots \\ \triangleright & P \end{array}$$