Quantization, Absorbing Regions and Practical Message Passing Decoders

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Joint work with:
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- **Question: How does the quantization choice affect the performance?**

Classification of errors

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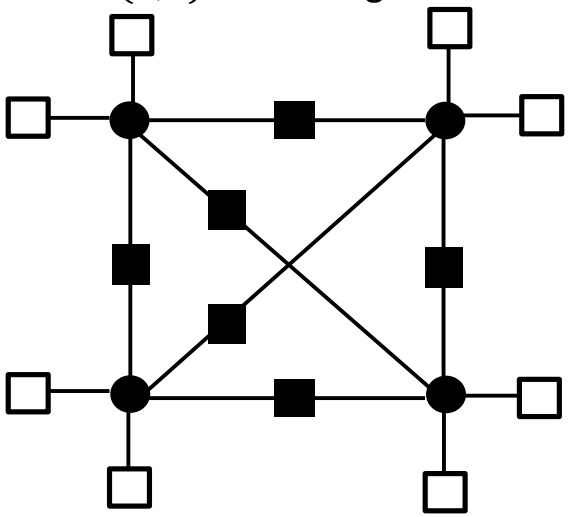
  - (c,d) absorbing set errors
  - Non-absorbing set errors
  - Non-converging errors

A (c,d) absorbing set includes c number of variable nodes and d number of unsatisfied checks. In addition, each variable node is connected to more satisfied checks than unsatisfied checks.

(4,8) Absorbing set



L. Dolecek, *et al.*, "Analysis of absorbing sets and fully absorbing sets of array-based LDPC codes," *IEEE Trans. Info. Theory*, 2010.

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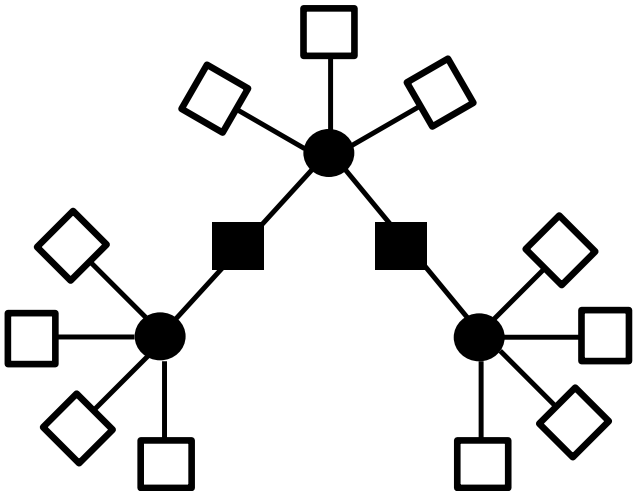
Example of non-absorbing error



Classification of errors

- Noise in the channel $\rightarrow$ error in decoding:
  - (c,d) absorbing set errors
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  - Non-converging errors
    The decoder oscillates between different objects and does not converge to a certain object.

Description of the simulations

- Extensive amount of simulations on the UCLA's Hoffman2 Cluster
- Simulations for different codes with:
  - Blocklengths in the range of 841 to 4489
  - Variable node degrees 4 and 5
- Overall, over **six months** of simulations

FER vs. SNR analysis

B. Amiri and L. Dolecek, "Experimental study on the impact of quantization on the performance of array-based LDPC codes", available at http://www.loris.ee.ucla.edu.

$$Qa.b$$

Integer part Fractional part

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Three different scenarios:

1. Vary b for constant a
   - For $a \geq 4$, b does not impact the performance
   - For $a \in \{2, 3\}$, smaller b performed better than larger b

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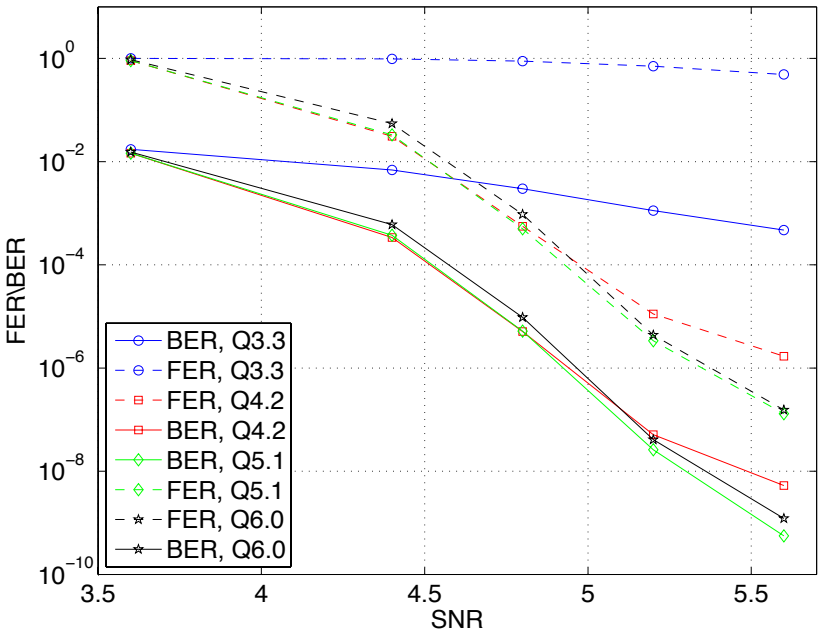
- Comparable performance for sufficiently large a
  - Example:



FER/BER vs. SNR,
N=2209,
variable node degree=4,
rate=0.9162,
a+b=6,
200 iterations

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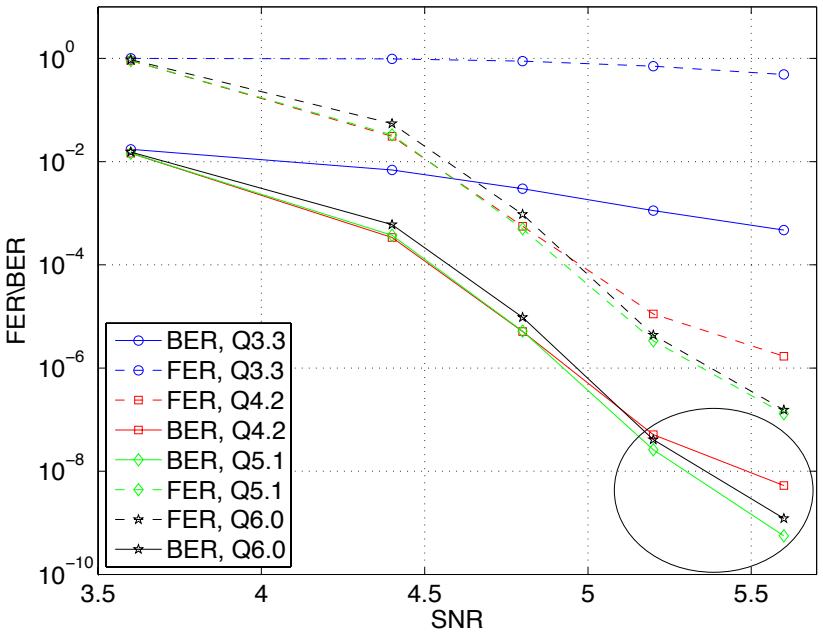

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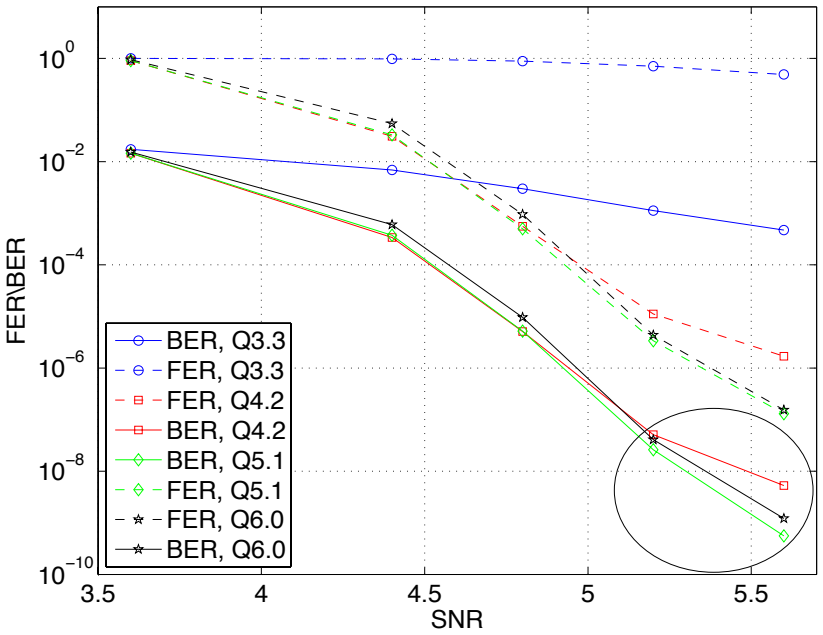

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Question: Same error profile?

Array-based codes

A class of LDPC codes with the following parity-check matrix form:

$$H_{p,\gamma} = \begin{bmatrix} I & I & I & \dots & I \\ I & \sigma & \sigma^2 & \dots & \sigma^{(p-1)} \\ I & \sigma^2 & \sigma^4 & \dots & \sigma^2(p-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & \sigma^{(\gamma-1)} & \sigma^{2(\gamma-1)} & \dots & \sigma^{(p-1)(\gamma-1)} \end{bmatrix},$$

where σ is a $p \times p$ circulant matrix.

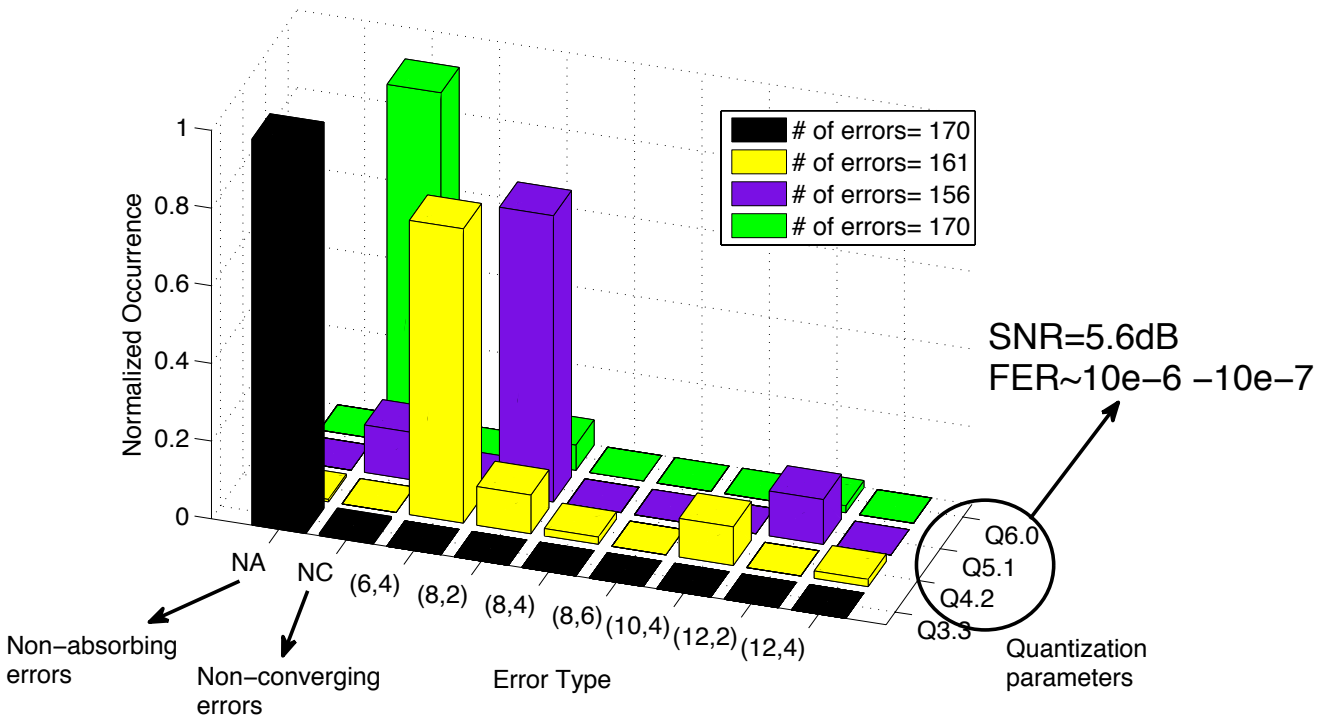
- code length $N = P^2$
- variable node degree $= \gamma$
- code rate $R = 1 - (\frac{\gamma p - \gamma + 1}{p^2})$

Error profile

- Error profile specifies the frequency of each error type
- It is well-known that error profile depends on choice of the code
- Does error profile depend on the quantization choice of the decoder?
- If so, this may help us improve the design of the decoder

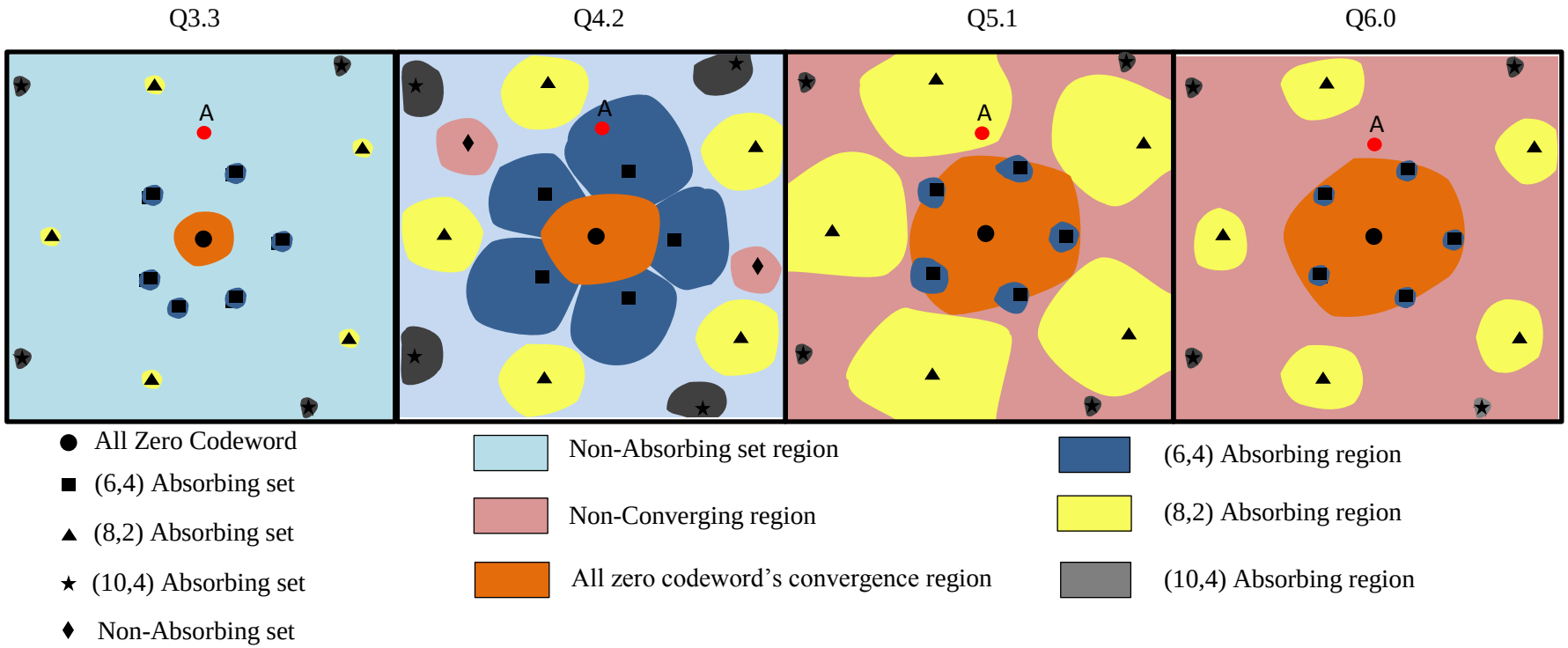
N= 2209, Column-weight = 4, R= 0.9162

- The error profile is substantially different for different choices of parameter *a*



Absorbing regions

- The set of all noise realizations which the decoder $\mathfrak{D}$ decodes as a (c, d) absorbing set is called a (c, d) **absorbing region** of $\mathfrak{D}$



Series of decoders

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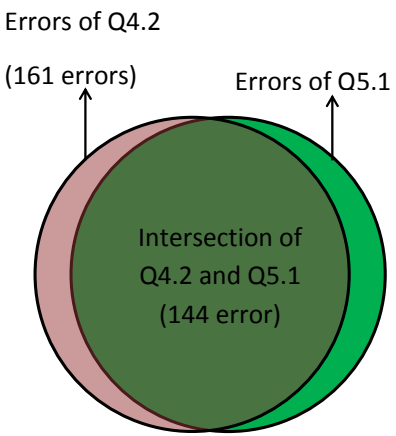
  - Q4.2 and Q5.1 $\rightarrow$ large overlap
  - Q4.2 and Q6.0 $\rightarrow$ mostly disjoint



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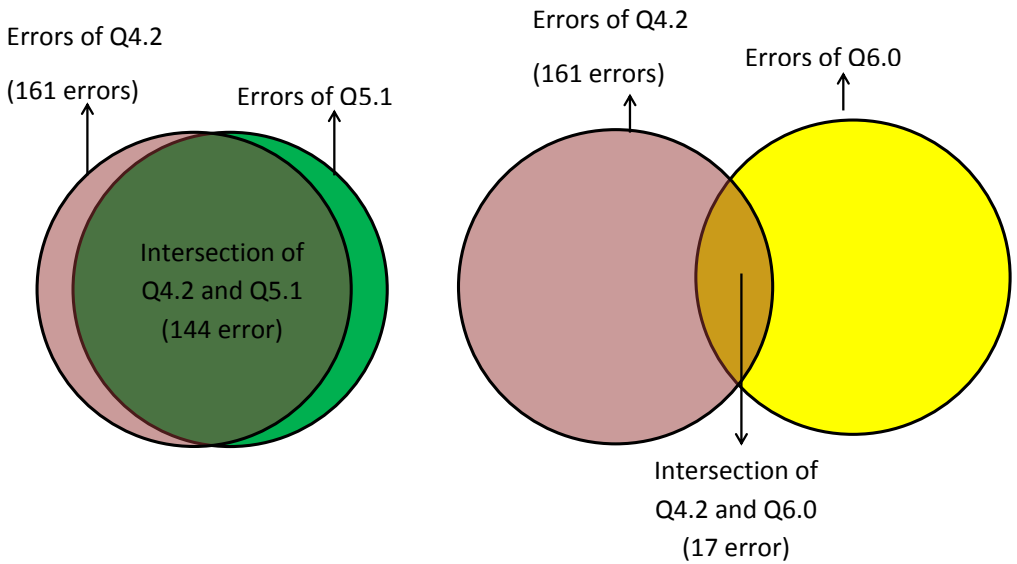

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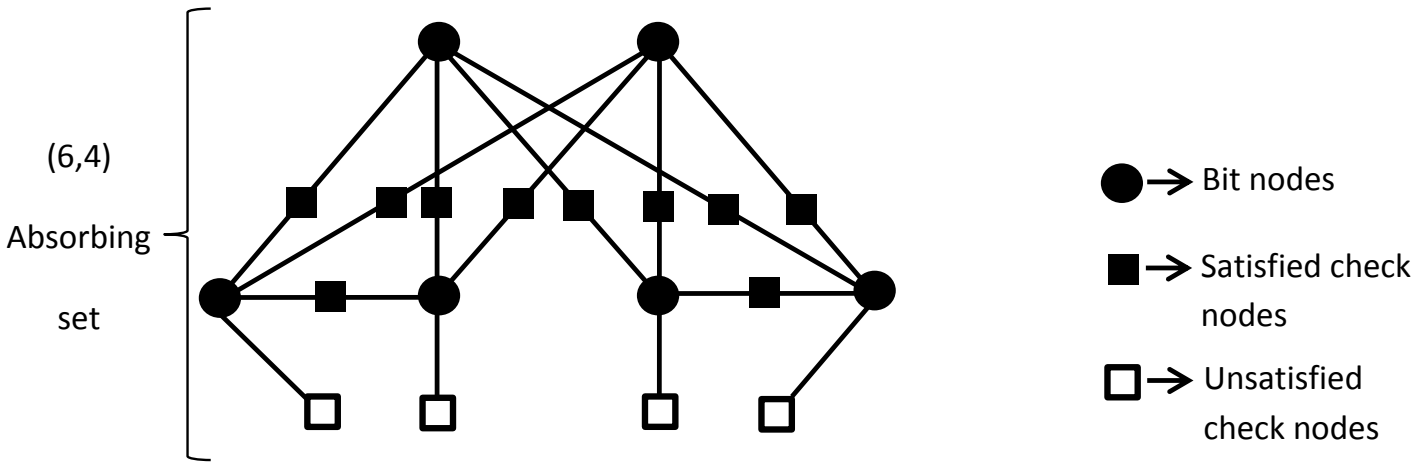
- Topological overlap
  - $(6,4)$ absorbing set $\rightarrow$ dominant error under Q4.2
  - $(8,2)$ absorbing set $\rightarrow$ dominant error under Q5.1



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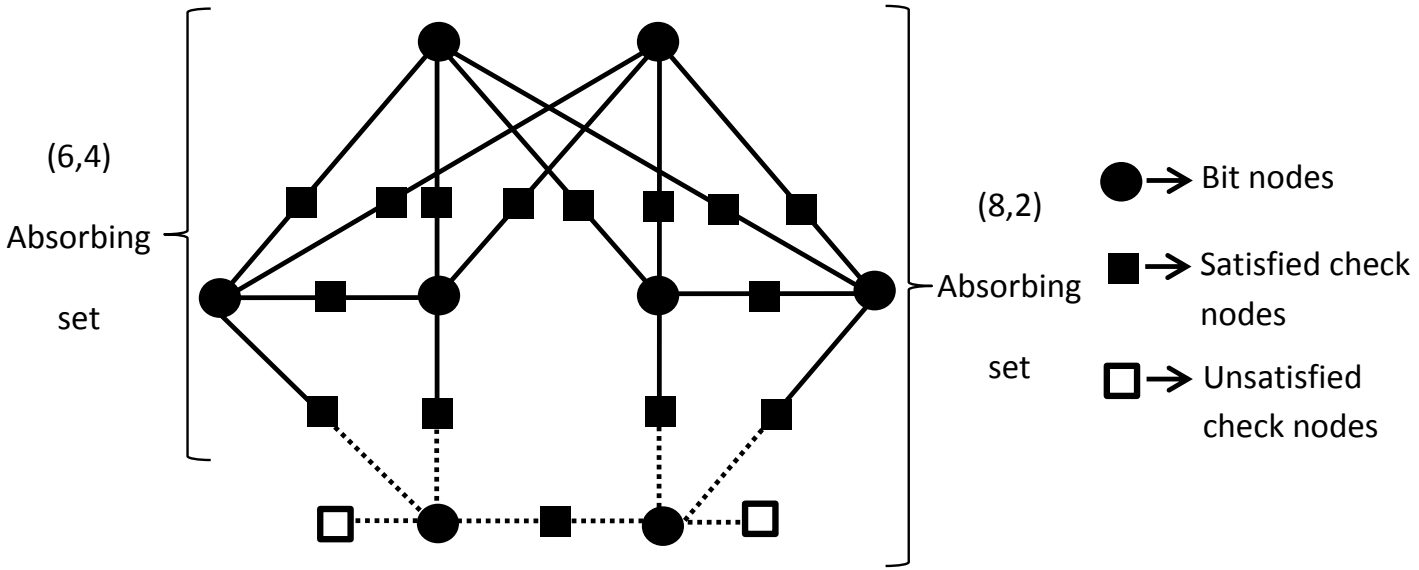

- Increasing the number of iterations improves the performance of Q6.0 since it helps most of the NC errors to converge to a codeword

Series of decoders

- We propose a decoder consisting of a series of decoders with partially disjoint errors
  - All of the received sequences from channel are decoded by decoder 1
  - Decoder 2 only decodes those sequences that caused a decoding error under decoder 1

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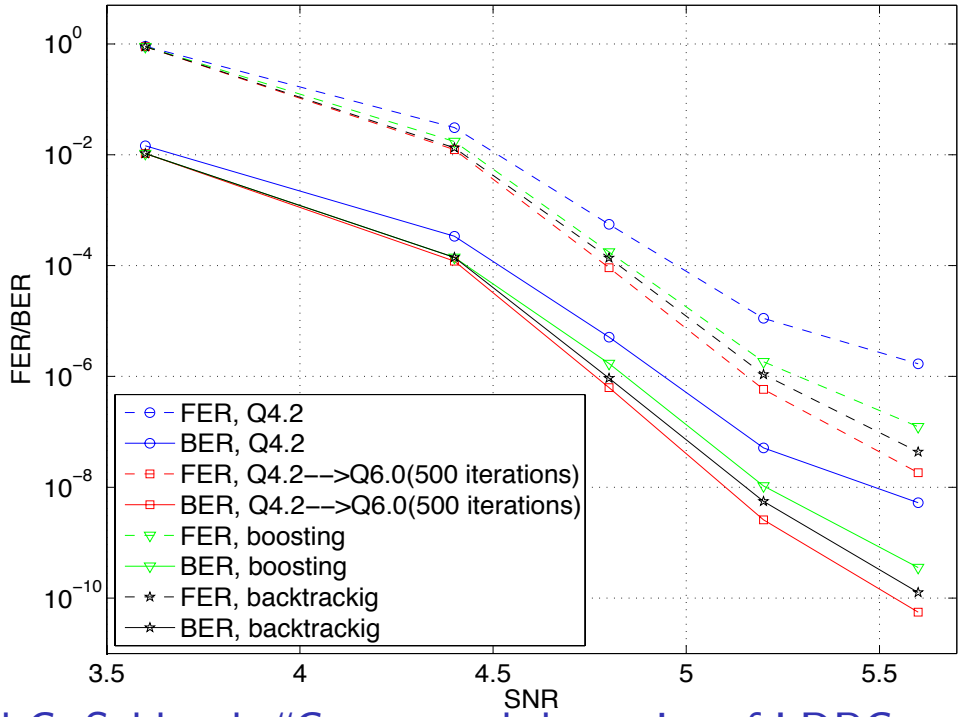
Performance comparison

Using $N = 2209$, variable degree= 4 array-based code:



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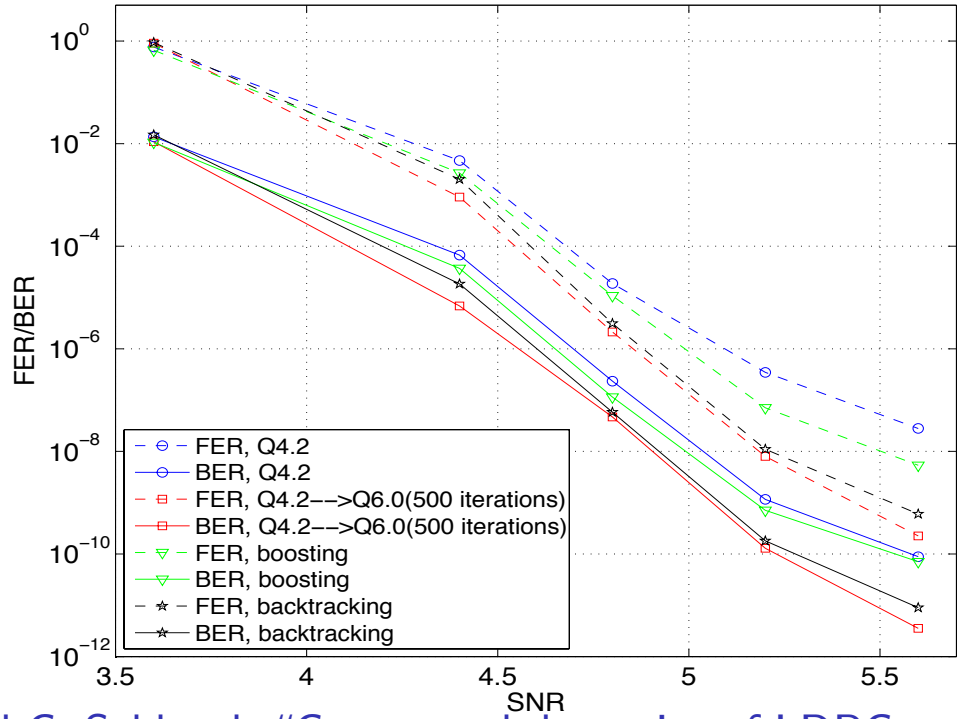
Performance comparison

Using $N = 2209$, variable degree= 5 array-based code:



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- We observed a dependency of the absorbing regions on the quantization choice
- We devised a new decoder as a series of decoders with disjoint error profiles which performs well compared to recently other proposed decoders

Thank you!