

# Chapter 8 Hypothesis Testing

Chapter problem: Does the MicroSort method of gender selection increase the likelihood that a baby will be girl?

- MicroSort: a gender-selection method developed by Genetics & IVF.
  - MicroSort XSORT is to increase the likelihood of a baby girl
  - MicroSort YSORT is to increase the likelihood of a baby boy
- Result:
  - 726 used XSORT, 668 couples did have a baby girl, a success rate of 92.0%
- **Question:** can we actually support the claim that XSORT is effective in increasing the probability of a girl?

# 8.1 Overview

Definition – In statistics, a **hypothesis** is a claim or statement about a property of a population.

A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.

Examples: Genetics, Business, Medicine, Aircraft Safety, Quality Control.

Rare Event Rule for Inferential Statistics:

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

# 8.1 Overview

e.g.1 Gender Selection. A product named “Gender Choice”

- The Ad claims “increase your chances of having a girl up to 80%”
- Assumption: Gender Choice had no effect
- If 100 couples using Gender Choice have 100 babies
  - a. Consisting of 52 girls
  - b. Consisting of 97 girls
- What should we conclude about the assumption?

Sol.

- a. There is no sufficient evidence that Gender Choice is effective (common sense) (52 out of 100 is not significant)
- b. Explained it in two ways. One explanation is that extremely rare event happened. The other is that the assumption is not true, i.e. Gender Choice is effective. (97 out of 100 is significant)

# 8.2 Basics of Hypothesis Testing

## Key Concept

- Null hypothesis
- Alternative hypothesis
- Test statistics
- Critical region
- Significance level
- Critical value
- P-value
- Type-I error
- Type-II error

# 8.2 Basics of Hypothesis Testing – Part I

## Objective for Part I

- Given a claim, identify the null hypothesis and alternative hypothesis, express them in symbolic form
- Given a claim and sample data, calculate the value of the test statistics
- Given a significance level, identify the Critical value(s).
- Given a value of the test statistic, identify the P-value
- State the conclusion of a hypothesis test in simple, non-technical terms.

# 8.2 Basics of Hypothesis Testing – Part I

Need to understand this example (show concept and process)

e.g.2 Chapter problem (continue). Key points:

- Claim: for couples using XSORT, the proportion of girls is  $p > 0.5$
- Working assumption: The proportion of girls is  $p = 0.5$  (with no effect from XSORT)
- The sample resulted in 13 girls among 14 births, so the sample proportion is  $\hat{p} = 13/14 = 0.929$ .
- **Assuming** that  $p = 0.5$ ,  $P(13 \text{ girls or more in } 14 \text{ births}) = 0.0016$ .
- Two possible explanations for the results of 13 girls in 14 births:
  - either the random chance event (with prob. 0.0016) has occurred,
  - or the proportion of girls born to couple using Gender Choice is greater than 0.5.
- Because the random chance is so small, we reject the random chance and conclude that there is sufficient evidence to support a claim that XSORT is effective.

# 8.2 Basics of Hypothesis Testing – Part I

## Working with the Stated Claim: Null and Alternative Hypothesis

- The **Null hypothesis** (denoted by  $H_0$ ) is that the parameter (e.g. proportion, mean, and SD) equal to some claim value. E.g.

$$H_0: p = 0.5, \quad H_0: \mu = 98.6, \quad H_0: \sigma = 15,$$

- The **Alternative hypothesis** (denoted by  $H_1$ ) is that the parameter has a value that differs from the one in null hypothesis.

Proportions	$H_1: p > 0.5,$	$H_1: p < 0.5,$	$H_1: p \neq 0.5,$
means:	$H_1: \mu > 98.6,$	$H_1: \mu < 98.6,$	$H_1: \mu \neq 98.6,$
SD's:	$H_1: \sigma > 15,$	$H_1: \sigma < 15,$	$H_1: \sigma \neq 15,$

- Always use **equal symbol** in null hypothesis  $H_0$ .

# 8.2 Basics of Hypothesis Testing – Part I

Working with the Stated Claim: Null and Alternative Hypothesis

Procedure of Identifying the null and alternative hypothesis

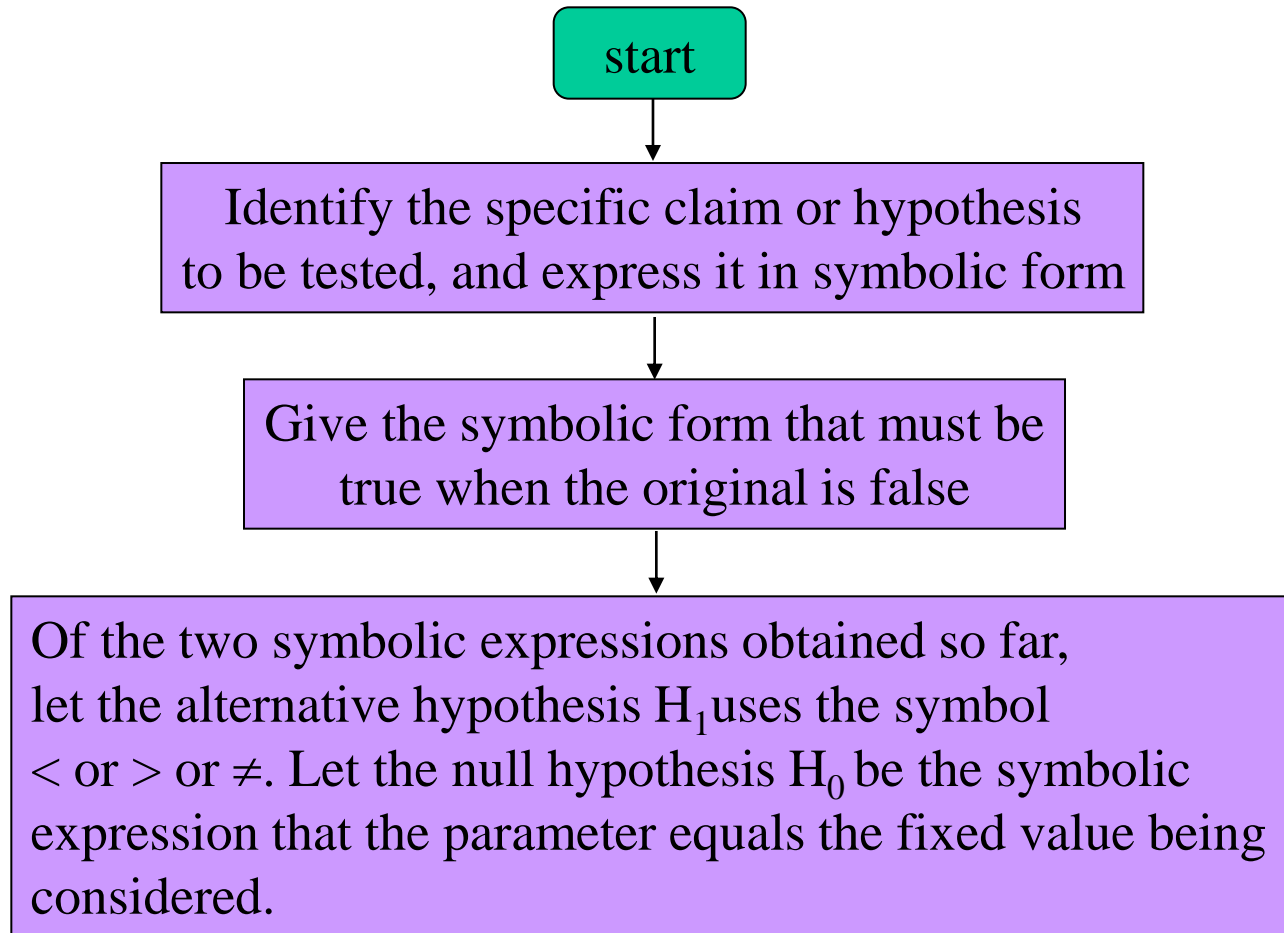


Figure 8.2  
Identifying  
 $H_0$  and  $H_1$



## 8.2 Basics of Hypothesis Testing – Part I

Working with the Stated Claim: Null and Alternative Hypothesis

e.g.3 Identifying the Null and Alternative hypotheses. Claim: the mean weight of airline passengers (including carry on baggage) is at most 195 pounds.

Sol. Following Figure 8-2.

Step 1.  $\mu \leq 195$ . (translate “mean weight ... is at most 195 lbs”)

Step 2. If  $\mu \leq 195$  is false, then  $\mu > 195$

Step 3. Therefore we determine  $H_0$  and  $H_1$ ,  $H_1: \mu > 195$ , and  $H_0: \mu = 195$

# 8.2 Basics of Hypothesis Testing – Part I

## Converting Sample Data to a Test Statistic

Test statistic for proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for mean

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{or} \quad t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Test statistic for SD

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

# 8.2 Basics of Hypothesis Testing – Part I

## Converting Sample Data to a Test Statistic

e.g.4 Finding the Test Statistic. Claim: XSORT increase the likelihood of having a baby girl. Preliminary results from a test of the XSORT involved 14 couples who gave birth to 13 girls and 1 boy. Use the given claim and the preliminary results to calculate the value of the test statistic. Use the format given, so that a normal distribution is used to approximate a binomial distribution.

Sol.  $H_0: p = 0.5,$

$H_1: p > 0.5,$

test statistic: 
$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{14}}} = 3.21$$

**Interpretation:** z score of 3.21 is unusual. It fall within the range of values considered to be significant because they are so far above 0.5 that they are not likely to occur by chance. (see graph in next page)

## 8.2 Basics of Hypothesis Testing – Part I

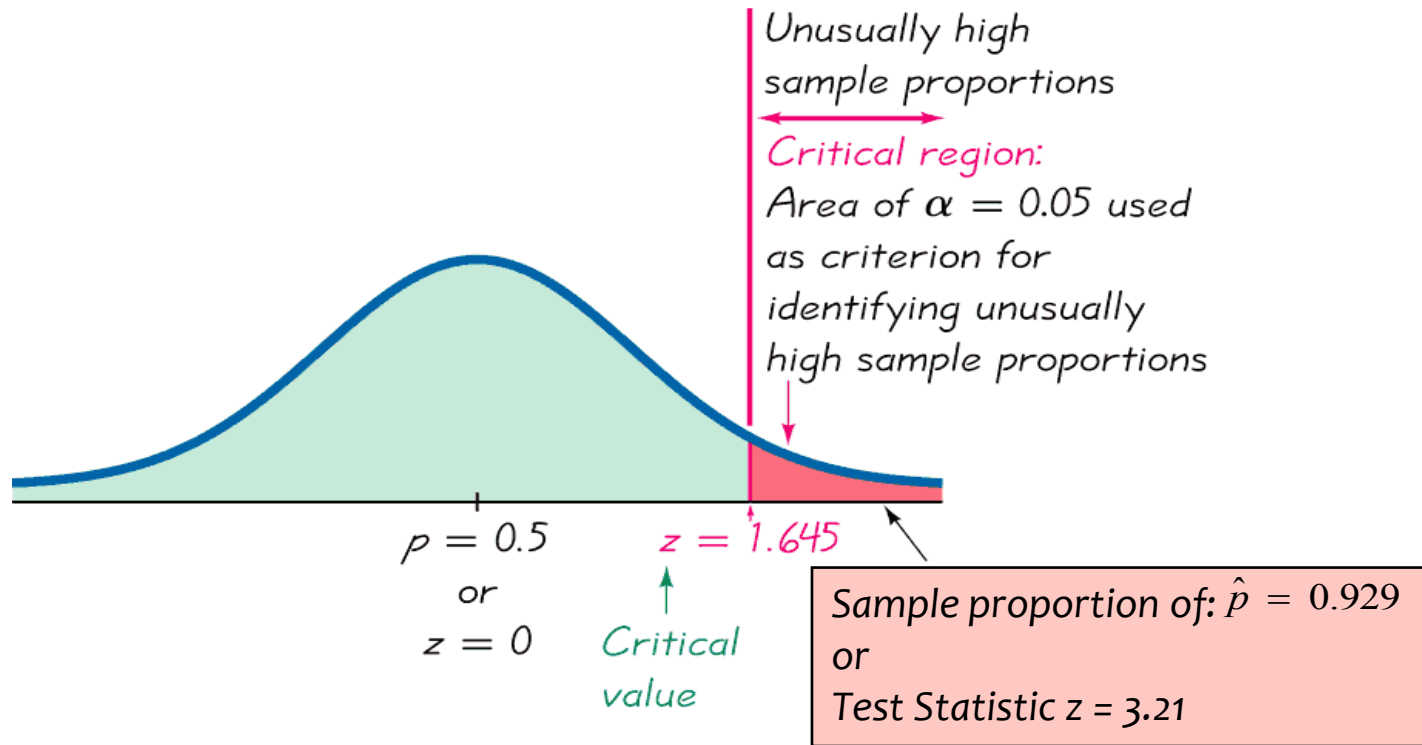


Figure 8-3 Critical Region, Critical Value, Test Statistic

My comment: this is a **right tail** test.

## 8.2 Basics of Hypothesis Testing – Part I

### Tools for Assessing the Test Statistic: Critical Region, Significant Level, Critical Value, and P-Value

- The critical region (or rejection region) is the set of all values of the test statistics that cause us to reject the null hypothesis. For example, Figure 8-3
- The significant level (denoted by  $\alpha$ , the same  $\alpha$  introduced in 7.2) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true. **If the test statistic falls into the critical region, we reject the null hypothesis.**
- The critical value is any value that separates the critical region from the values of the test statistic that do not lead to rejection of the null hypothesis.

## 8.2 Basics of Hypothesis Testing – Part I

e.g.5 and e.g.6 Finding Critical Values. Using the significance level of  $\alpha = 0.05$ , find the critical  $z$  values for each of the following alternative hypothesis. (Assuming normal distribution can be used to approximate the binomial distribution):

- a.  $H_1: p \neq 0.5$
- b.  $H_1: p < 0.5$
- c.  $H_1: p > 0.5$

Sol. Draw Figure 8–4 on the blackboard.

- By examining the **alternative hypothesis**, we can determine whether a test is right-tailed ( $H_1: >$ ), left-tailed ( $H_1: <$ ) or two-tailed. (Draw Figure 8-6).
- The tail corresponds to the critical region containing the values that would conflict significantly with the null hypothesis.

## 8.2 Basics of Hypothesis Testing – Part I

The P-Value (or probability value) is the probability of getting a value of the test statistic that is at least as extreme as the one representing the sample data, assume that the null hypothesis is true. (next page, Figure 8 – 5) P-value can be found after finding the area in Figure 8-5.

# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions

The standard procedure of hypothesis testing requires that we always test the null hypothesis first. So the initial conclusion will be one of the following:

- Reject the null hypothesis
- Fail to reject the null hypothesis

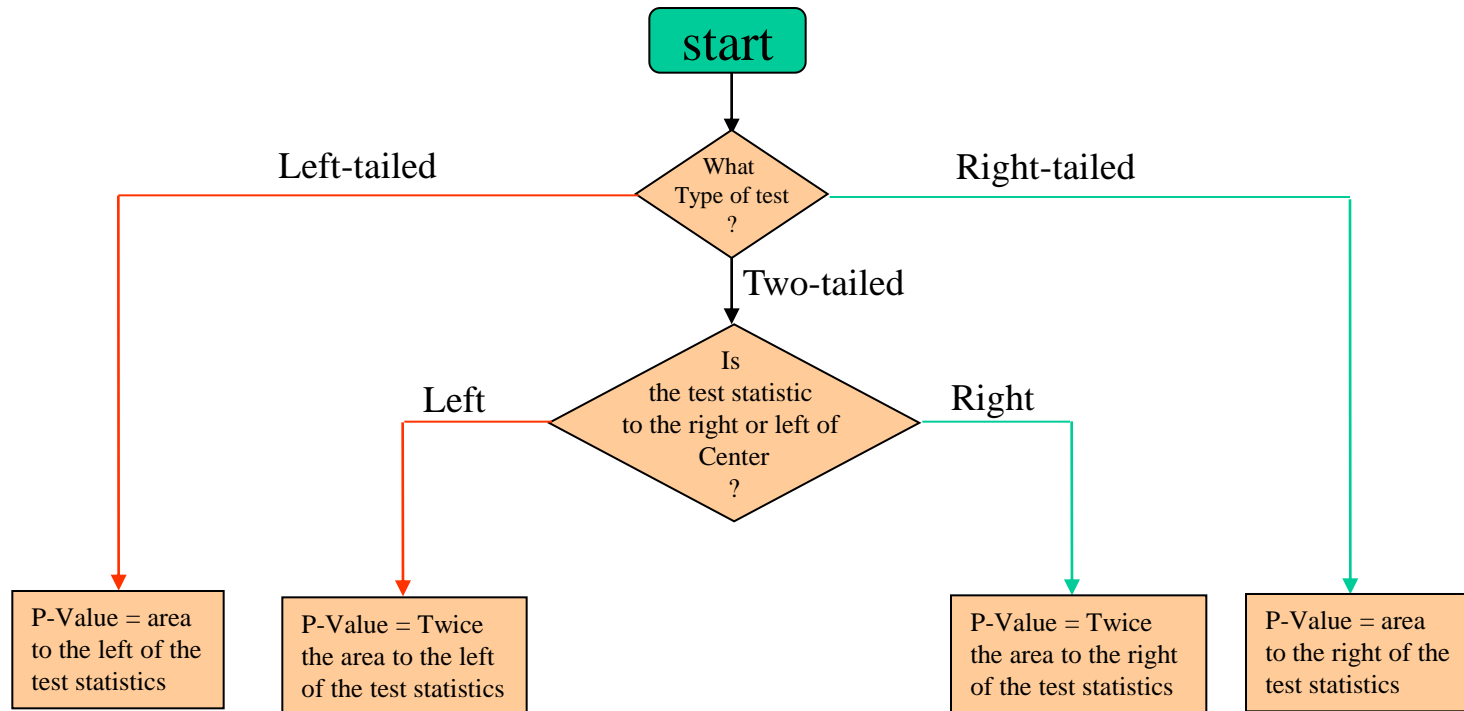
A memory tool useful for interpreting the P-values:

- If the P is low, the null must go.
- If the P is high, the null will fly.



# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions



Draw 4 pictures in class accordingly.

Figure 8 – 5 Procedure for Finding P-values

# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions

### Decision Criterion:

- **Traditional method**: reject  $H_0$  if test statistic falls within the critical region. Fail to reject  $H_0$  if test statistic does not fall within the critical region
- **P-Value method**: reject  $H_0$  if  $P\text{-value} \leq \alpha$  (where  $\alpha$  is the significance level, such as 0.05). Fail to reject  $H_0$  if  $P\text{-value} > \alpha$ .
- **Another option**: instead of using the significance level, simply identify P-value and leave the decision to the reader.
- **CI method**: because a CI estimate of a population parameter contains the likely values of that parameter, reject a claim that the population parameter has a value that is not included in CI.

# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions

e.g.7 Finding P-Values for a Critical Region in the Right Tail.

Claim: XSORT increase the likelihood of having a baby girl.

Preliminary results from a test of the XSORT involved 14 couples who gave birth to 13 girls and 1 boy. Find P-Value, and interpret the P-value

Sol. Consider  $H_1 : p > 0.5$ .  $z = 3.21$  (e.g.4), area to the right of  $z = 3.21$  is 0.0007. So P-value = 0.0007.

**Interpretation:**  $P = 0.0007$ , it shows that there is very small chance of getting the sample result that led to a test statistic  $z = 3.21$ . It is very unlikely that we will get 13 (or more) girls in 14 births by chance. This suggest that XSORT is effective for gender selection.

# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions

e.g.8 Finding P-Values for a Critical Region in two tails. Claim: with the XSORT, the likelihood of having a baby girl is different from  $p = 0.5$ . Preliminary results from a test of the XSORT involved 14 couples who gave birth to 13 girls and 1 boy. Find P-Value, and interpret the P-value. (same problem like e.g.7,  $H_1$  changed)

Sol. Consider  $H_1 : p \neq 0.5$ .  $z = 3.21$  (e.g.4), area to the right of  $z = 3.21$  is 0.0007. So P-value =  $2(0.0007) = 0.0014$ .

**Interpretation:**  $P = 0.0014$ , it shows that there is very small chance of getting the sample result that led to a test statistic  $z = 3.21$ . It is very unlikely that we will get 13 (or more) girls in 14 births by chance. This suggest that with XSORT, the likelihood of having a baby girl is different from 0.5.

# 8.2 Basics of Hypothesis Testing – Part I

## Decisions and Conclusions

### Wording the Final Conclusion.

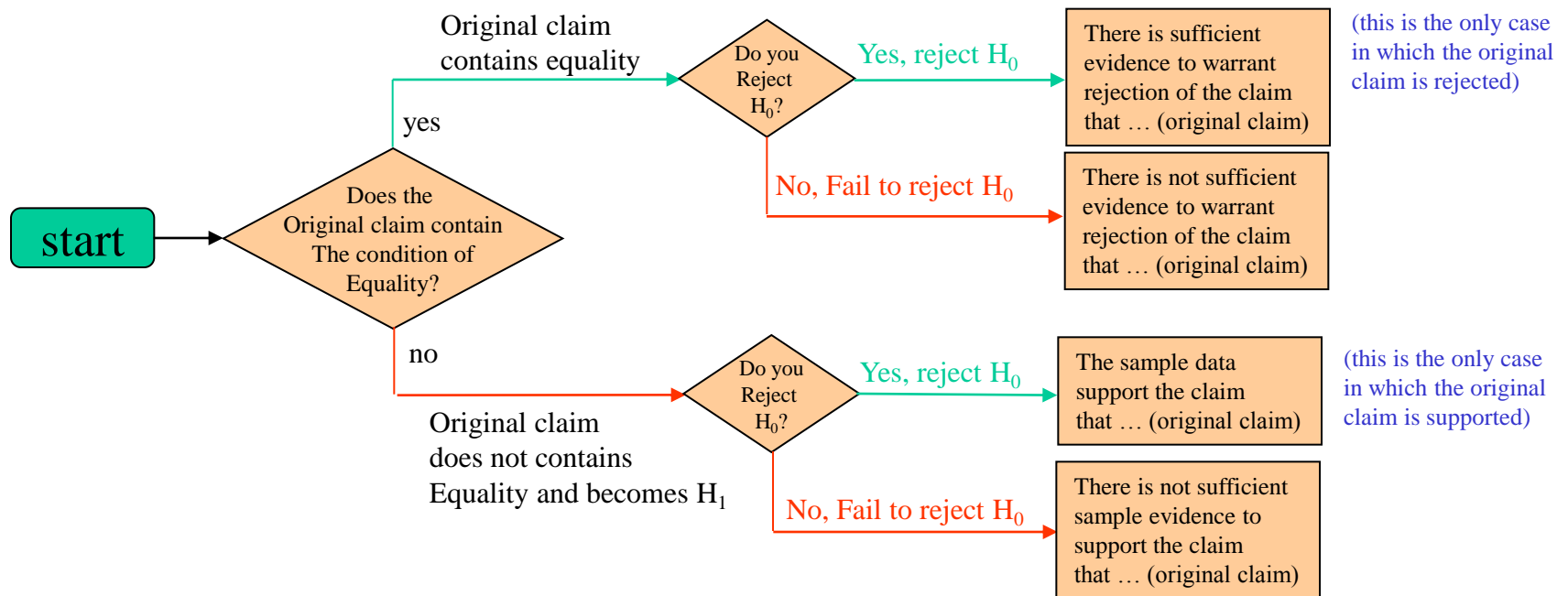


Figure 8 – 7 Wording of Final Conclusion

# 8.2 Basics of Hypothesis Testing – Part I

## Wording the Final Conclusion

e.g.9 Stating the Final Conclusion.

- XSORT. 13 Girls in 14 birth after using XSORT.
- $H_1: p > 0.5$ ,  $H_0: p = 0.5$
- P-Value = 0.0007
- **reject** the null hypothesis of  $p = 0.5$ .

State the conclusion in simple non-technical terms.

Ans: by Figure 8 – 7, the conclusion is as simple as follows: “The sample data support the claim that the XSORT method of gender selection increases the likelihood of a baby girl”

# 8.2 Basics of Hypothesis Testing – Part I

## Errors in Hypothesis Tests

### Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	Type I error (rejecting a true null hypothesis) $\alpha$	Correct Decision
	We fail to reject the null hypothesis	Correct Decision	Type II error (Failing to reject a false null hypothesis) $\beta$

Table 8 – 1  
Type I and  
Type II errors

# 8.2 Basics of Hypothesis Testing – Part I

## Errors in Hypothesis Tests

### Notation

$\alpha$  (alpha) = probability of a type I error (the probability of rejecting  $H_0$  when it is true)

$\beta$  (beta) = probability of a type II error (the probability of failing to reject  $H_0$  when it is false)

e.g.10 XSORT.  $H_1 : p > 0.5$ . Claim: XSORT increase the likelihood of a baby girl.

- a. Type I error is made when XSORT has no effect, we conclude that XSORT is effective.
- b. Type II error is made when XSORT is effective, we conclude that XSORT has no effect.



# 8.2 Basics of Hypothesis Testing – Part I

## Comments

- Don't use “Accept” for “Fail to Reject”
- Don't use Multiple Negatives

## Comprehensive Hypothesis Test

- Details of P-value method, traditional method, and CI method
- Next three pages (examples in next section)

# 8.2 Basics of Hypothesis Testing – Part I

## P-Value Method

Step 1. Identify the specific claims and put it in symbolic form

Step 2. Give the symbolic form that must be true when the original claim is false

Step 3. Obtain  $H_1$  and  $H_0$

Step 4. Select the significance level  $\alpha$  based on the seriousness of a type I error.  
make  $\alpha$  small if consequences of rejecting a true  $H_0$  are severe. (for example 0.05 and 0.01)

Step 5. Identify the statistic (such as normal, t, chi-square).

Step 6. Find the test statistic and find the P-value (see Figure 8 – 6, page 16).  
Draw a graph and show the test statistic and P-value

Step 7. Reject  $H_0$  if the P-value is less than or equal to the significance level  $\alpha$ .  
Fail to reject  $H_0$  if the P-value is greater than  $\alpha$ .

Step 8. Restate this previous decision in simple, non-technical terms, and address the original claim.

# 8.2 Basics of Hypothesis Testing – Part I

## Traditional Method

Step 1. Identify the specific claims and put it in symbolic form

Step 2. Give the symbolic form that must be true when the original claim is false

Step 3. Obtain  $H_1$  and  $H_0$

Step 4. Select the significance level  $\alpha$  based on the seriousness of a type I error.  
make  $\alpha$  small if consequences of rejecting a true  $H_0$  are severe. (for example 0.05 and 0.01)

Step 5. Identify the statistic (such as normal, t, chi-square).

**Step 6.** Find the test statistic, the critical values and the critical region. Draw a graph

**Step 7.** Reject  $H_0$  if test statistic is in the critical region. Fail to reject  $H_0$  if the test statistic is not in the critical region.

Step 8. Restate this previous decision in simple, non-technical terms, and address the original claim.

# 8.2 Basics of Hypothesis Testing – Part I

## Confidence Method

- Construct a CI with a confidence level selected as in Table 8–2
- Because a CI estimate of a population parameter contains the likely values of that parameter, **reject a claim** that the population parameter has a value that is not included in the CI.
- For one-tail hypothesis test with significance level  $\alpha$ , construct a CI with CL of  $1 - 2\alpha$ . See Table 8-2

		Two-Tailed Test	One-Tailed Test
Significance	0.01	99%	98%
Levels for	0.05	95%	90%
Hypothesis	0.10	90%	80%
Test			

Table 8-2 CL for CI

## 8.3 Testing a Claim About a Proportion

- Introduction
- The P-Value Method
- Traditional Method
- CI Method

# 8.3 Testing a Claim About a Proportion

- Introduction

The following are examples of the types of claims we will be able to test:

- **Genetics.** The Genetics & IVF Institute claims that its XSORT method allows couples to increase the probability of having a baby girl, so that the proportion of girls with this method is **greater than 0.5**.
- **Medicine.** Pregnant women can correctly guess the sex of their babies so that they are correct **more than 50%** of the time.
- **Entertainment.** Among the television sets in use during a recent Super Bowl game, **64%** were turned to the Super Bowl.

## 8.3 Testing a Claim About a Proportion

- Introduction

### Requirements for Testing Claims About a Population Proportion $p$

1. Simple random sample
2. Conditions for a binomial distribution are satisfied
3. The condition  $np \geq 5$  and  $nq \geq 5$  are both satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with  $\mu = np$  and  $\sigma = \sqrt{npq}$ . See 6-6).

# 8.3 Testing a Claim About a Proportion

- Introduction

## Notation

$n$  = sample size or number of trials

$$\hat{p} = \frac{x}{n} \text{ (sample proportion)}$$

$p$  = population proportion (used in the null hypothesis)

$$q = 1 - p$$

Test Statistic for Testing a Claim About a Proportion

$$\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

**P-value:** use the standard normal distribution (Table A-2) and Figure 8-5

**Critical Values:** Use the standard normal distribution (Table A-2)



## 8.3 Testing a Claim About a Proportion

e.g.1 Testing the effectiveness of the XSORT. Among 726 babies born to couples using the XSORT, 668 of the babies were girls and the others are boys. Use these results with 0.05 significance level to test the claim that among babies born to couples using the XSORT method, the proportion of girls is greater than 0.5 that is expected with no treatment.

Summary of the claim and the sample data:

Claim:  $p > 0.5$

Sample data:  $n = 726$  and  $\hat{p} = \frac{668}{726} = 0.920$

3 requirements: simple random sample; fixed # of independent trials (726), two outcomes,  $p = 0.5$ ,  $q = 0.5$ ;  $np = nq = 726(0.5) = 363 \geq 5$

# 8.3 Testing a Claim About a Proportion

## The P-Value Method

Step 1.  $p > 0.5$  (does not include equality)

Step 2. Opposite:  $p \leq 0.5$

Step 3.  $H_0: P = 0.5$

$H_1: P > 0.5$

Step 4. Select  $\alpha = 0.05$

Step 5. Sampling distribution  $\hat{p}$  is approximated by a normal distribution.

Step 6. Test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.92 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{726}}} = 22.63$$

P-Value = area to the **right** of  $z = 22.63$ , Figure 8-5. By A-2, for  $z = 3.5$  and higher, P-Value = 0.0001

Step 7. Because  $0.0001 < 0.05$ , we reject the Null hypothesis

Step 8. We conclude that there is sufficient sample evidence to support the claim that among babies born to couples using the XSORT, the proportion of girls is greater than 0.5. It does appears that the XSORT **is effective**.

# 8.3 Testing a Claim About a Proportion

## The Traditional Method

Step 6. Test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.92 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{726}}} = 22.63$$

Now find the critical value (instead of P-value). For  $\alpha = 0.05$ , right-tailed test, using table A-2 (or **TI-84**), we find the critical value = 1.645.

Step 7. the test statistics falls within the critical region, reject the Null hypothesis

Step 8. We conclude that there is sufficient evidence to support the claim that among babies born to couples using the XSORT, the proportion of girls is greater than 0.5. It does appears that the XSORT **is effective**.

# 8.3 Testing a Claim About a Proportion

## The CI Method

The claim of  $p > 0.5$  can be tested with a 0.05 significance level by constructing a 90% CI. For one-tailed hypothesis tests, with significance level  $\alpha$  require a confidence interval with a CL of  $1 - 2\alpha$ . (see Table 8 – 2).

If we want a significance level of  $\alpha = 0.05$  in right-tail test, we use 90% confidence level with the method of section 7.2 to get this result,

$$\hat{p} = 0.92, \hat{q} = 0.08, \text{ for 90\% CL, } z_{\frac{\alpha}{2}} = 1.645, n = 726$$

Therefore, margin of error

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \sqrt{\frac{(0.92)(0.08)}{726}} \approx 0.0166$$

Therefore, the CI is  $(\hat{p} - E, \hat{p} + E) = (0.92 - 0.0166, 0.92 + 0.0166) = (0.9304, 0.9366)$ .

Because we have 90% confidence that the true value is contained in  $(0.9304, 0.9366)$ , we have sufficient evidence to support the claim that

$p > 0.5$ . (same conclusion as the P-method and the traditional method) <sup>36</sup>

## 8.3 Testing a Claim About a Proportion

### Caution

- P-value method and traditional method always yield the same result
- CI method is different, may yields different result as those by P-value method or traditional method (exercise 29).
- Good strategy:
  - CI for **estimating** proportion.
  - P-value and traditional method for **hypothesis testing**.

## 8.3 Testing a Claim About a Proportion

e.g.2 **Finding the number of success  $x$ .** A study addressed the issue of whether pregnant women can correctly guess the sex of their baby. Among 104 recruited subjects, 55% guessed the sex of the baby. How many of 104 women made correct guess?

Ans.

$$0.55 \times 104 = 57.2 \approx 57$$

## 8.3 Testing a Claim About a Proportion

e.g.3 **Can a pregnant women predict the sex of her baby?** See e.g.2.  
Among 104 recruited subjects, 57 guessed the sex of the baby.  
Use the sample data to test the claim that the success rate of such guesses is no different from 50% success rate expected with random guesses. Use  $\alpha = 0.05$  significance level?

Solution. The P-Value Method

Step 1.  $p = 0.50$

Step 2. Opposite:  $p \neq 0.50$

Step 3.  $H_0: p = 0.50$   
 $H_1: p \neq 0.50$

Step 4.  $\alpha = 0.05$

Step 5. Sampling distribution  $\hat{p}$  is approximated by a normal distribution.

## 8.3 Testing a Claim About a Proportion

Step 6. Test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{57}{104} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{104}}} = 0.98$$

P-Value = **twice the area** to the right of  $z$  ( $= 0.98$ ). By A-2, for  $z = 0.98$  the area to the right is  $1 - 0.8365 = 0.1635$ . Therefore, P-Value  $= 2(0.1635) = 0.327$  (**also TI-84**)

Step 7. Because P-value  $> 0.05$ , we fail to reject the Null hypothesis

**Interpretation:** There is not sufficient evidence to warrant rejection of the claim that women who guess the sex of their babies have a success rate of 50%.



## 8.3 Testing a Claim About a Proportion

### The Traditional Method

Step 6. Test statistic

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{57}{104} - 0.5}{\sqrt{\frac{(0.5)(0.5)}{104}}} = 0.98$$

for two-tail test, the critical values are  $z = -1.96$  and  $z = 1.96$   
(also TI-84)

Step 7. The test statistic = 0.98 would not fall in the critical region.  
So we fail to reject the null hypothesis.

**Interpretation:** There is not sufficient evidence to warrant rejection of the claim that women who guess the sex of their babies have a success rate of 50%.

# 8.3 Testing a Claim About a Proportion

## Confidence Method

If we want a significance level of  $\alpha = 0.05$  in two-tail test, we use 95% confidence level with the method of section 7.2 to get this result,  $\hat{p} = 57/104 = 0.548$ ,  $\hat{q} = 0.452$ , for 95% CL,  $z_{\frac{\alpha}{2}} = 1.96$ ,  $n = 104$

Therefore, margin of error

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \sqrt{\frac{(0.548)(0.452)}{104}} \approx 0.095653$$

Therefore, the CI is  $(\hat{p} - E, \hat{p} + E) = (0.548 - 0.096, 0.548 + 0.096) = (0.452, 0.643)$ .

Because we have 95% confidence that the true value is contained in (0.452, 0.643), which includes 0.5, we do **NOT** have sufficient evidence to reject the claim that  $p = 50\%$  rate. Three methods leads to the **same conclusion**.

# 8.3 Testing a Claim About a Proportion

Part 2: Exact method using binomial distribution for testing claims about  $p$  (population proportion).

To test hypotheses using the exact binomial distribution, use the binomial probability distribution with P-value method. Find the P-values as follows:

- Left-tailed test: p-value =  $P(\text{getting } x \text{ or fewer success in } n \text{ trials})$
- Right-tailed test: p-value =  $P(\text{getting } x \text{ or more success in } n \text{ trials})$
- Two-tailed test:
  - If  $\hat{p} > p$ , p-value =  $2 \times P(\text{getting } x \text{ or more success in } n \text{ trials})$
  - If  $\hat{p} < p$ , p-value =  $2 \times P(\text{getting } x \text{ or fewer success in } n \text{ trials})$

e.g.4 Re-do e.g.3 (Using Statdisk)  $\hat{p} = 57/104 = 0.548 > 0.5$ , so

P-value =  $2 \times 0.188792 = 0.377584 > 0.05$ . Conclusion: not unusual, so we fail to reject the null hypothesis.

## 8.3 Testing a Claim About a Proportion

Rationale for the Test Statistic. When using a normal distribution to approximate the binomial distribution, use  $\mu = np$ ,  $\sigma = \sqrt{npq}$

$$z = \frac{x - \mu}{\sigma} = \frac{x - np}{\sqrt{npq}}$$

# 8.3 Testing a Claim About a Proportion

## Using Technology

- STATDISK
  - Analysis → Hypothesis testing → Proportion-One Sample
- TI-83/84 PLUS
  - yes

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

- Introduction
- The P-Value Method
- Traditional Method
- CI Method

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

### Requirements

1. Simple random sample
2.  $\sigma$  is known
3. Either the population is normally distributed or  $n > 30$

Test Statistic for Testing a Claim about a Mean (with  $\sigma$  known)

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

**P-Value:** use the standard normal distribution (Table A-2) and refer to Figure 8 – 5

**Critical value:** Use the standard normal distribution (Table A-2, or TI-84)

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

e.g.1 **Overloading boats**. People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data set 1 in Appendix B, we obtain sample statistics:  $n = 40$ ,  $\bar{x} = 172.55$  lb. Research from several other sources suggests that the standard deviation of men's weight  $\sigma = 26$  lb. Test the claim that  $\mu$  greater than 166.3 lb, which is the weight in the National Transportation and Safety board's recommendation M-04-04. Use  $\alpha = 0.05$ , and use the P-value method.

**Sol. Requirement.** Simple random sample.  $\sigma$  is known. Sample size  $n = 40$  (greater than 30). **Requirements are satisfied.**



## 8.4 Testing a Claim About a Mean: $\sigma$ Known

### Sol. **P-Value Method.**

Step 1.  $\mu > 166.3$  lb

Step 2. Alternative:  $\mu \leq 166.3$  lb

Step 3.  $H_0: \mu = 166.3$  lb

$H_1: \mu > 166.3$  lb

Step 4.  $\alpha = 0.05$

Step 5. Sample statistic: sample mean,  $n = 40 > 30$ .

Step 6. Test statistic:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26}{\sqrt{40}}} = 1.52$$

using  $z = 1.52$ , we find the P-value = 0.0643 (**TI-84**)

Step 7. Because the P-value of  $0.0643 > 0.05$ , we fail to reject the null hypothesis.

**Interpretation.** There is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the NTSB recommendation.

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

### e.g.2 **Traditional Method.**

Step 6. For  $\alpha = 0.05$ . Since the alternative use  $>$ . The critical value is  $z = 1.645$ . The test statistic of  $z = 1.52$  does not fall within the critical region. Therefore we fail to reject the null hypothesis.

e.g.3 **CI method.** For a one-tailed hypothesis test with 0.05 significance level, we construct 90% CL. Now,  $n = 40$ ,  $\bar{x} = 172.55$ . Assume  $\sigma = 26$ . Now use the method in Sec 7–3.

$$z_{\frac{\alpha}{2}} = 1.645, \text{ thus margin of error } E = 1.645 \sigma / \sqrt{n}$$

$$= 1.645 (26) / \sqrt{40} = 6.7625$$

$$\text{therefore, CI} = (172.55 - 6.7625, 172.55 + 6.7625)$$

$$= (165.79, 179.31) \quad (\text{include } 166.2 \text{ lb})$$

Conclusion: can't support that mean is greater than 166.3 lb.

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

**Comments.** When testing a claim, we make an assumption (null hypothesis) of equality. We then compare the assumption and the sample results to form one of the following conclusions:

- If the sample results can easily occur when the assumption (null hypothesis) is true, we attribute the relatively **small discrepancy** between the assumption and the sample results to chance.
- If the sample results cannot easily occur when the assumption (null hypothesis) is true, we explain the relatively **large discrepancy** between the assumption and the sample results by concluding that the assumption is not true, so we reject the assumption.

## 8.4 Testing a Claim About a Mean: $\sigma$ Known

### Using Technology

- STATDISK
  - Didn't try
- EXCEL
  - Extremely tricky to use
- TI-83/84 PLUS
  - Yes

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

- Introduction
- Traditional Method
- The P-Value Method
- CI Method

# 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

## Requirements

1. Simple random sample
2.  $\sigma$  is **NOT** known
3. Either the population is normally distributed or  $n > 30$

Test Statistic for Testing a Claim about a Mean (with  $\sigma$  Not known)

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

**P-Value and Critical value:** use Table A-3 and use  $df = n - 1$  for the number of degrees of freedom (See Figure 8 – 5 for P-value Procedures.)

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

### Comments:

- Normal Requirement – not a strict requirement. No outliers (t-dist **robust**)
- Sample size – use simplified criterion,  $n > 30$ .

### Important properties of Student t Distribution (**done before**)

- Different for different sample size
- Bell-shaped like Normal Distribution, but with greater variability
- Mean = 0 just as standard Normal distribution
- SD of Student t Distribution varies sample size and is **greater than 1** (standard Normal distribution has SD = 1)
- As sample size  $n$  gets larger, Student t-distribution gets closer to standard Normal distribution

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

e.g.1 **Overloading boats**. People have died in boat accidents because an obsolete estimate of the mean weight of men was used. Using the weights of the simple random sample of men from Data set 1 in Appendix B, we obtain sample statistics:  $n = 40$ ,  $\bar{x} = 172.55$  lb,  $s = 26.33$  lb. Test the **claim** that  $\mu$  greater than 166.3 lb, which is the weight in the National Transportation and Safety board's recommendation M-04-04. Use  $\alpha = 0.05$ , and use the P-value method.

**Sol. Requirement.** Simple random sample.  $\sigma$  is not known. Sample size  $n = 40$  (greater than 30). **Requirements are satisfied.**



## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

**Sol. Traditional Method.**

Step 1.  $\mu > 166.3$  lb

Step 2. Alternative:  $\mu \leq 166.3$  lb

Step 3.  $H_0: \mu = 166.3$  lb

$H_1: \mu > 166.3$  lb

Step 4.  $\alpha = 0.05$

Step 5. Sample statistic: sample mean. Student t Distribution is satisfied, so **we use t-distribution**.

Test statistic: 
$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}} = \frac{172.55 - 166.3}{\frac{26.33}{\sqrt{40}}} = 1.501$$

The critical value of  $t = 1.685$  is found by referring to Table A-3.  $Df = 40 - 1 = 39$ . Because this test is right-tailed with  $\alpha = 0.05$ , refer to the column indicating an area of 0.05 in one tail. **(TI-84)**

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

Step 7. Because the test statistic of  $t = 1.501$  does not fall in the critical region, we fail to reject the null hypothesis.

**Interpretation.** We fail to reject the null hypothesis. There is not sufficient evidence to support a conclusion that the population mean is greater than 166.3 lb, as in the NTSB recommendation.

# 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

## Finding P-Values with the Student $t$ Distribution

- P-Value approach is more difficult in this section than in last section
- Reason: Table A-3 usually does not allow you to get exact P-Value. Thus we have following recommendations:
- Use software or a **TI-83/84 (yes)** (STATDISK provides P-Values for  $t$  tests)
- If technology is not available, use table A-3 to identify a **range** of P-Values as follows:
  - Use df to locate the relevant row of table A-3
  - Then determine where the test statistic lies relative to the  $t$  values in that row
  - Based on a comparison of the  $t$  test statistic and the  $t$  values in the row of table A-3, identify a range of values by referring to the area values given at the top of table A-3

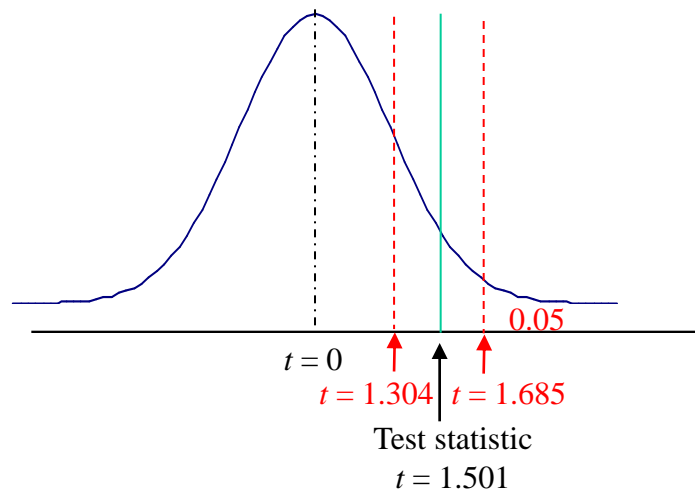
# 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

e.g.2 Finding P-Value. Use table A-3 to find a range of values for the P-Value corresponding to the test statistic of  $t = 1.501$  from the preceding example. Note  $df = 40$ , the test is right-tailed.

Sol. Go to table A-3, in the row  $df = 40$ . Refer to Table A-3, the test statistic of  $t = 1.501$  falls between the table values of 1.685 and 1.304, so the “area in one tail” (to the right of the test statistic) is between 0.05 and 0.10 (Figure below). We can see that area to the right of  $t = 1.501$  is greater than 0.05. although we can't find the exact P-value from table A-3, **we can conclude** that  $P\text{-value} > 0.05$ . Because  $P\text{-value} > 0.05$ , we fail to reject the null hypothesis. There is no sufficient evidence to support the claim that the mean weight of men is greater than 166.3 lb.

**Technology:**

**$P\text{-value} = 0.07 > 0.05$**



## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

e.g.3 Finding P-Value. Use table A-3 to find a range of values for the P-Value corresponding to the t-test with these components (based on the claim that  $\mu = 120$  lb for the sample weights of women listed in Data Set 1 in Appendix B): Test statistic  $t = 4.408$ , sample,  $n = 40$ ,  $\alpha = 0.05$ , alternate hypothesis is  $H_1: \mu \neq 120$  lb

Sol.  $df = 40 - 1 = 39$ . Test statistic  $t = 4.408$  is greater than every value in the row. So the P-value is less than 0.01.

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

### CI Method.

- Can use CI for testing a claim about  $\mu$  when  $\sigma$  is unknown
- For a 2-tailed hypothesis test with  $\alpha = 0.05$  significance level, we construct 95% CI
- For a 1-tailed hypothesis test with  $\alpha = 0.05$  significance level, we construct 90% CI (see table 8-2)

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

e.g.4 Consider e.g.1  $n = 40$ ,  $\bar{x} = 172.55$  lb,  $s = 26.33$  lb. with  $\sigma$  unknown.  $\alpha = 0.05$ . We can test the claim that  $\mu > 166.3$  lb. Construct 90% CI. It is  $165.54 < \mu < 179.56$  (see sec 7-4, **TI-84**). Because the assumed value of

- 1)  $\mu = 166.55$  is contained in the CI, we cannot reject the null hypothesis.
- 2) Based on the 40 sample values provide, we don't have sufficient evidence to support the claim that the mean weight is greater than 166.3 lb.
- 3) Based on CI, the true value is likely to any thing between 165.54 lb and 179.56 lb, including 166.3 lb.

## 8.5 Testing a Claim About a Mean: $\sigma$ Not Known

e.g.5 **Small Sample: Monitoring Lead in Air.** Listed below are measured amounts of lead (in micrograms per cubic meter, or  $\mu\text{g}/\text{m}^3$ ) in the air. The Environmental Protection Agency has established an air quality standard for lead of  $1.5 \mu\text{g}/\text{m}^3$ . The measurement below constitute an simple random sample recorded in Building 5 of the World Trade Center site on different days immediately after September 11, 2001. use a 0.05 significance level to test the **claim** that the sample is from a population with a mean greater than the EPA standard of  $1.5 \mu\text{g}/\text{m}^3$ .

5.40, 1.10, 0.42, 0.73, 0.48, 1.10

**Sol. Requirement.** Simple random sample;  $\sigma$  is not known; Sample size  $n = 6$ . Use Statdisk to generate Quartile plot. Not close to a straight line. So it is NOT from a normal distribution.

Requirements are NOT satisfied.



# 8.5 Testing a Claim About a Mean: $\sigma$ NOT Known

## Using Technology

- STATDISK
  - E.g.5
- TI-83/84 PLUS
  - Tried many times

## 8.6 Testing a Claim About a SD or Variance

- Introduction
- The Traditional Method
- The P-Value Method
- CI Method

## 8.6 Testing a Claim About a SD or Variance

### Introduction

- Testing a claim made about a population SD  $\sigma$  or population variance  $\sigma^2$
- Use chi-square distribution ( $\chi^2$  distribution)

## 8.6 Testing a Claim About a SD or Variance

### Requirements

1. Simple random sample
2. The population has a normal distribution. (This is a much stricter requirement than when testing claims about means, as in section 8 – 4 and 8 – 5.)

Test Statistic for Testing a Claim about a  $\sigma$  or  $\sigma^2$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

**P-Value Critical value:** use Table A-4 and use  $df = n - 1$  for the number of degrees of freedom (Table A – 4 is based on cumulative areas from the right.)

## 8.6 Testing a Claim About a SD or Variance

### **Important properties of Chi-Square Distribution**

- All values of  $\chi^2$  are non-negative. The distribution is not symmetric
- Different  $\chi^2$  distributions for different df
- The critical values are found in table A- 4 using  $df = n - 1$

## 8.6 Testing a Claim About a SD or Variance

### Find the critical value

- Locate the row, df
- Locate the column using significance level  $\alpha$  (assuming  $\alpha = 0.05$ )
  - Right-tailed test: because the area to the right of the critical value is 0.05, locate 0.05 at the top of table A – 4.
  - Left-tailed test: with left-tailed area of 0.05, the area to the right of the critical value is 0.95, so locate 0.95 at the top of the table A – 4
  - Two-tailed test: unlike the normal and standard distribution, the critical values in this  $\chi^2$  test will be two different positive values. Divide a significance level of 0.05 between the left and right tails, so the areas to the two critical values are 0.975 and 0.025, respectively. Locate 0.975 and 0.025 in the table A – 4

## 8.6 Testing a Claim About a SD or Variance

e.g.1. **Quality Control of Coins**. A common goal in business and industry is to improve the quality of goods or services by reducing variation. Quality control engineers want to ensure that a product has an acceptable mean, but they also want to produce items of consistent quality so that there will be few defects. If weights of coins have an specified mean but too much variation, some will have weights that are too low or too high, so that the vending machine will not work correctly. consider a simple random sample of 37 weights of post-1983 pennies listed in Data set 20 in Appendix B. They have sample statistics  $\bar{x} = 2.499$  g and  $s = 0.01648$  g. U.S. Mint specification requires that pennies be manufactured so that the mean weight 2.500 g. A hypothesis test will verify that the sample appears to come from a population with a mean of 2.500 g as required, but use 0.05 significance level to test the **claim** that the population of weight has a standard deviation less than the specification of 0.0230 g.

Sol. Requirement. 1) Simple Random Sample, yes. 2) Normal distribution, yes. STATDISK plot (histogram or quartile plot).

## 8.6 Testing a Claim About a SD or Variance

### Traditional Method.

Step 1.  $\sigma < 0.023$

Step 2. Alternative:  $\sigma \geq 0.023$

Step 3.  $H_0: \sigma = 0.023$

$H_1: \sigma < 0.023$  (original claim)

Step 4.  $\alpha = 0.05$

Step 5. Claim is about  $\sigma$ , we use  $\chi^2$  distribution

Step 6. Sample statistic: 
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(37-1)(0.01648)^2}{(0.023)^2} = 18.483$$

df = 36 (**Not in table**), left-tailed test,  $\alpha = 0.05$ . Find that critical value is between 18.493 and 26.509. (TI-84: **23.269**)

Step 7. The test statistic is in the critical region ( $18.483 < 18.493$ , left critical region), we reject the null hypothesis.



## 8.6 Testing a Claim About a SD or Variance

### **Traditional Method.**

**Interpretation.** There is sufficient evidence to support the claim that the SD of weights is less than 0.0230 g. So the manufacturing process is acceptable.

## 8.6 Testing a Claim About a SD or Variance

e.g.2 Same example.

### **P-Value Method using Table A-4**

- Cannot find the exact P-value (A-4 only include selected  $\alpha$ ).  
But **TI-84** can, P-value = 0.0069
- $P\text{-value} < 0.05$ , we reject the null hypothesis, and arrive at the same conclusion given in e.g.1

## 8.6 Testing a Claim About a SD or Variance

### CI Method

- Continue previous example. Use the method described in 7 – 5
  - Use sample data ( $n = 37$ ,  $s = 0.01648$  g) to construct this 90% CI:  
 $0.01385 \text{ g} < \sigma < 0.02050 \text{ g}$ .  
(use [EXCEL](#) to find critical values  $\chi_L^2$  and  $\chi_R^2$ . Then use the following:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

to find the confidence interval.  $\chi_L^2 \approx 21.34$ ,  $\chi_R^2 \approx 54.43$ .)

- This confidence interval is to the left of 0.0230. So we support the claim that SD is less than 0.023g. We reach the same conclusion found with the traditional method and P-method.

## 8.6 Testing a Claim About a SD or Variance

### Technology

- STATDISK – Analysis -> Hypothesis Testing -> StDev-One Sample -> provide required entries -> Evaluate
  - Will display test statistic, P-Value, Conclusion and CI
- T1-83/84 Plus (**can't find critical value using TI-84, no inverse chi square function**)
- Use EXCEL to find the critical value.