

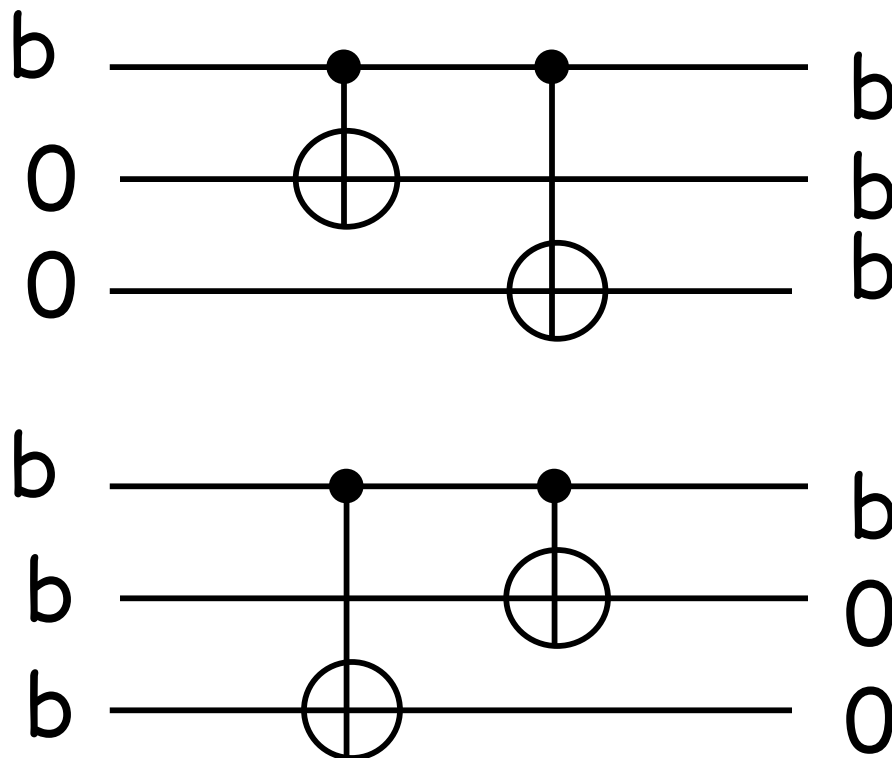
Quantum Computing Lecture 14a (notes on QEC)

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Classical Error Correcting Codes

- Suppose errors in our physical system for storing 0 and 1 cause each physical bit to be toggled independently with probability p
- We can reduce the probability of error to be in $O(p^2)$ by using a "repetition code"
- e.g. : encode a logical 0 with the state 000 and a logical 1 with the state 111

Reversible networks for encoding and decoding



Classical Error Correcting Codes

- After the errors occur, decode the logical bits by taking the majority answer of the three bits and correct the encoded bits

- So
- | | |
|-----------------------|-----------------------|
| $000 \rightarrow 000$ | $111 \rightarrow 111$ |
| $001 \rightarrow 000$ | $011 \rightarrow 111$ |
| $010 \rightarrow 000$ | $101 \rightarrow 111$ |
| $100 \rightarrow 000$ | $110 \rightarrow 111$ |

Classical Error Correcting Codes

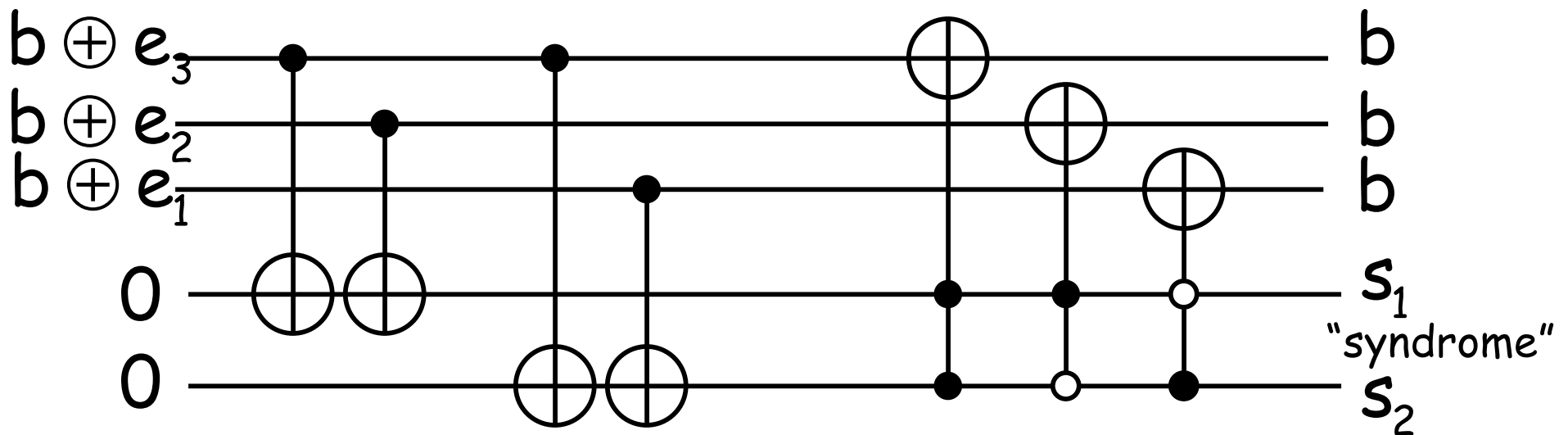
- As long as less than 2 errors occurred, we will keep the correct value of the logical bit
- The probability of 2 or more errors is

$$3p^2(1 - p) + p^3 = 3p^2 - 2p^3 \in O(p^2)$$

(which is less than p if $p < \frac{1}{2}$)

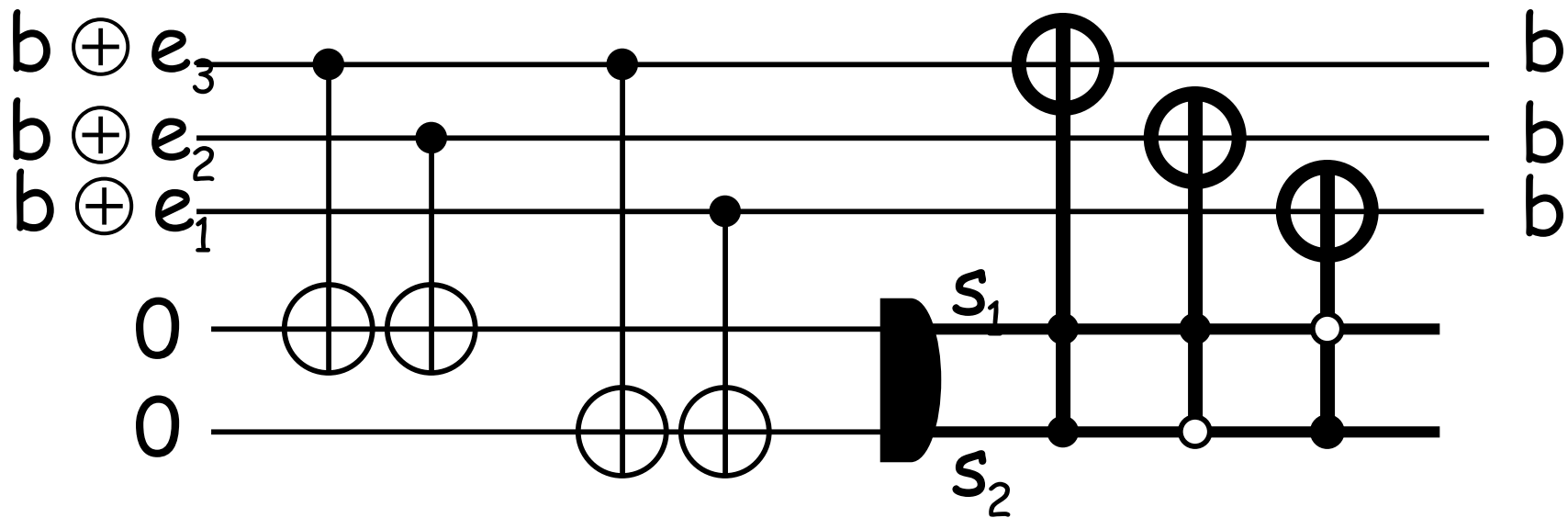
Reversible network for error correction

- Assume that $e_3 + e_2 + e_1 \leq 1$ $e_i \in \{0,1\}$

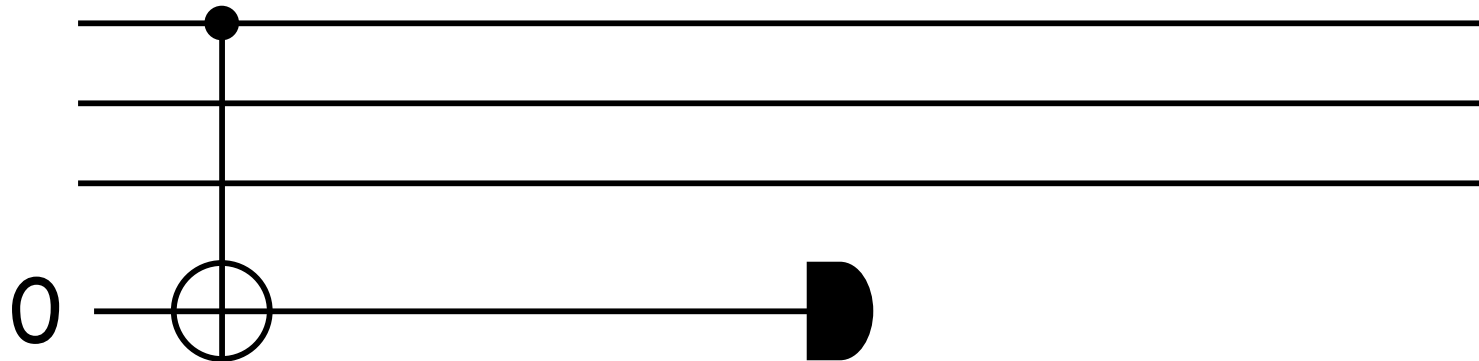


- If $s_1 s_2 = 00$ then no error occurred
- Otherwise, the error occurred in bit j where $j = 2s_1 + s_2$

Equivalently

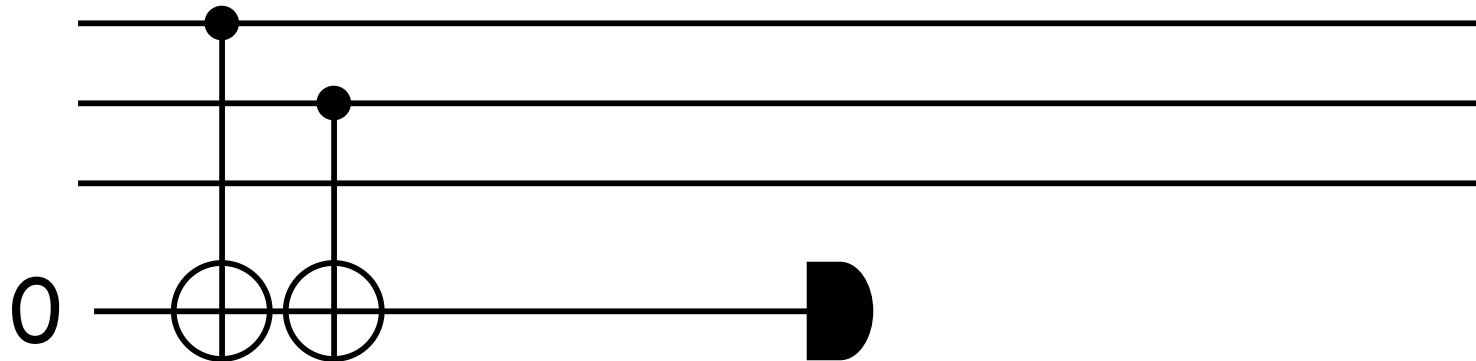


Stabilizer measurement??



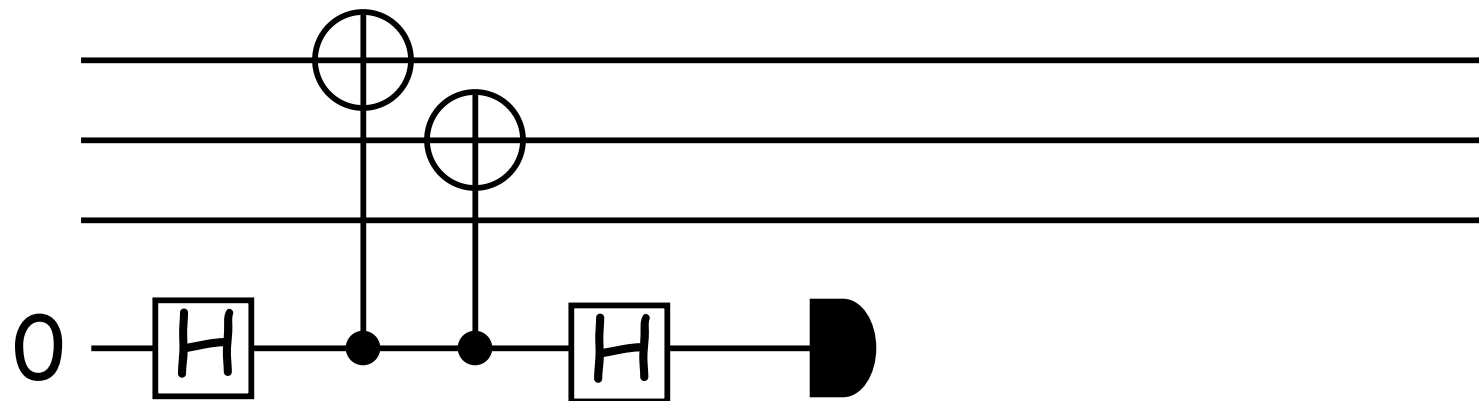
- This is implementing a Z_1 measurement (interpreting 0 as +1, and 1 as -1)

Stabilizer measurement??



- This is implementing a Z_1Z_2 measurement

Stabilizer measurement??



- This is implementing a X_1X_2 measurement

Notation clarification

- For an n -qubit system Z_j denotes

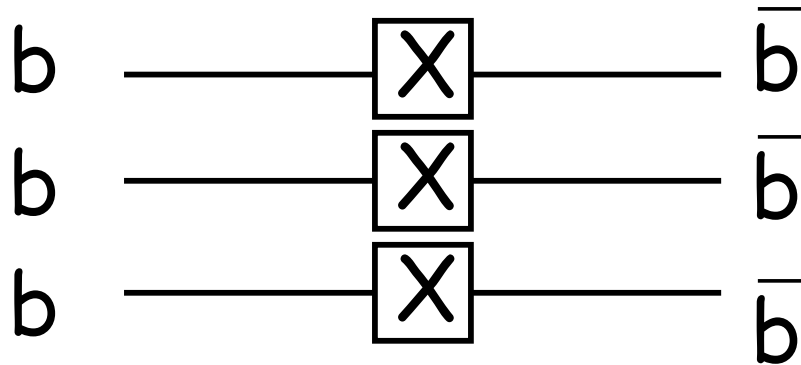
$$\underbrace{I \otimes I \otimes \dots \otimes I}_{j-1} \otimes Z \otimes \underbrace{I \otimes \dots \otimes I}_{n-j}$$

- E.g. $n=3$, then

$$Z_1 Z_2 = (Z \otimes I \otimes I)(I \otimes Z \otimes I) = (Z \otimes Z \otimes I)$$

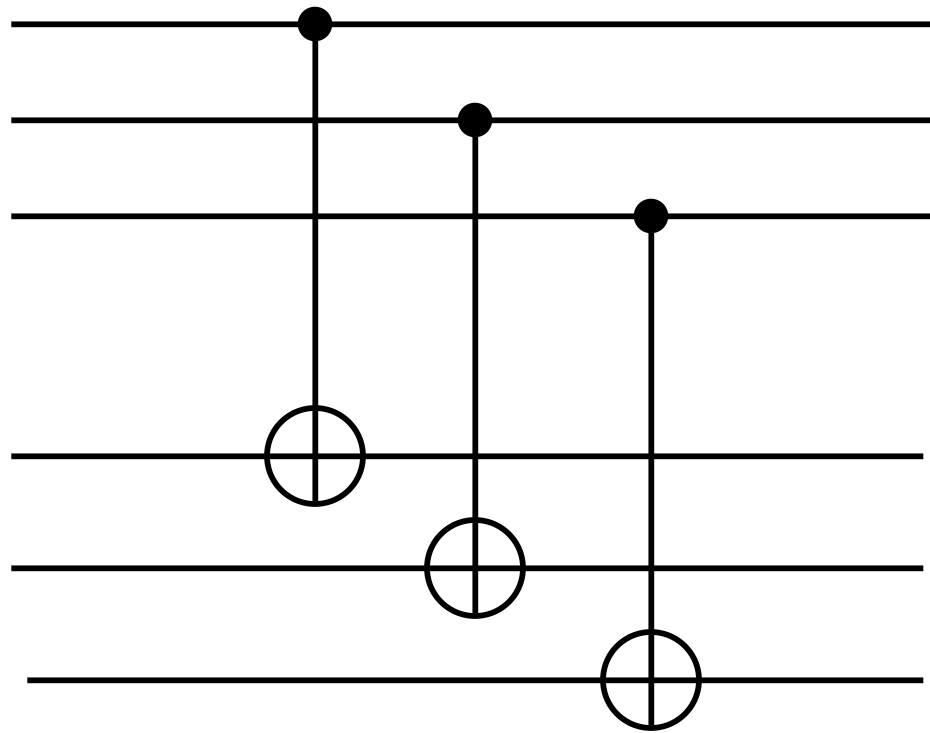
Perform operations on logical bits

- e.g. NOT gate



Perform operations on logical bits

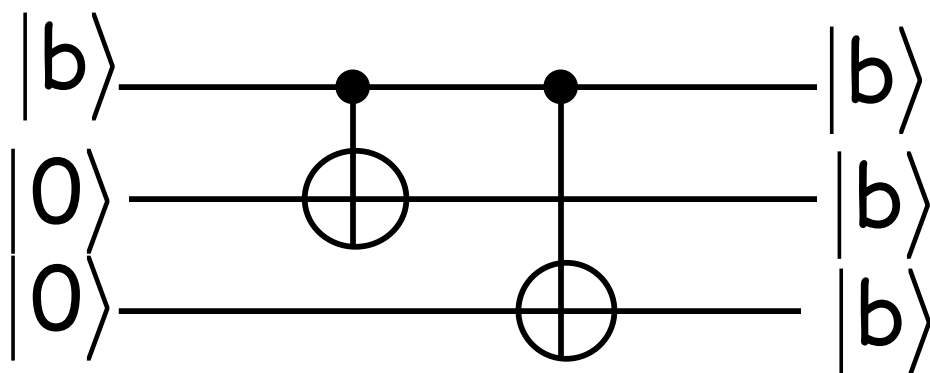
- e.g. c-NOT gate



Quantum Error Correcting Codes

- e.g. : encode a logical $|0\rangle$ with the state $|000\rangle$ and a logical $|1\rangle$ with the state $|111\rangle$

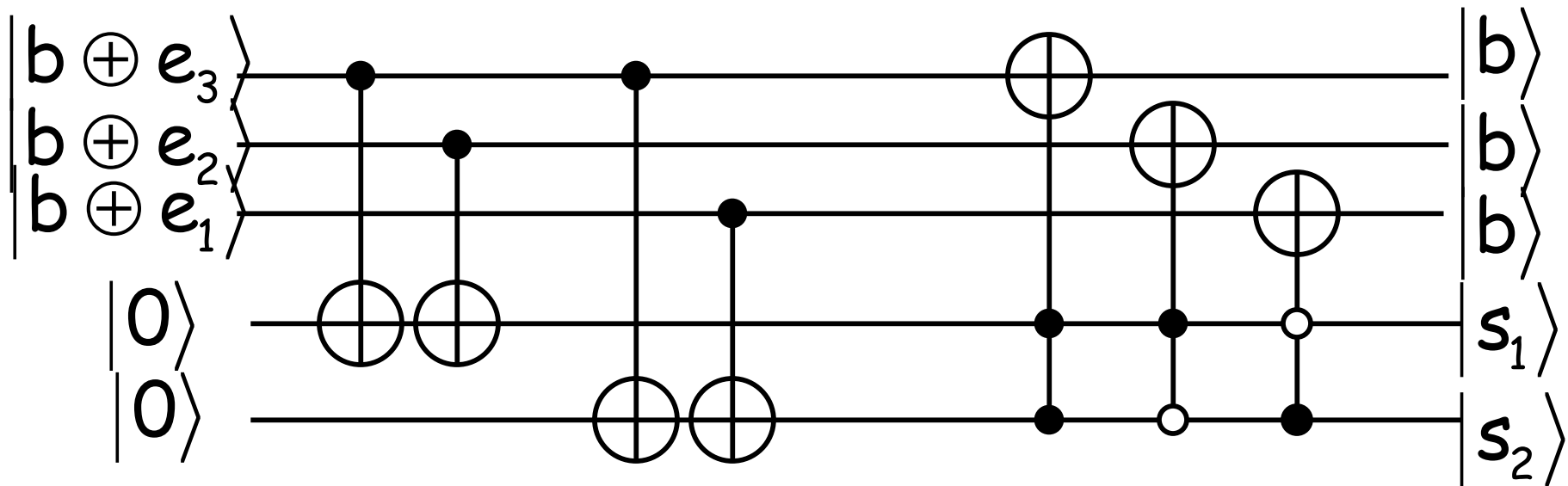
Quantum network for encoding



$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle \rightarrow \alpha|0\rangle|0\rangle|0\rangle + \beta|1\rangle|1\rangle|1\rangle$$

Quantum network for correcting errors

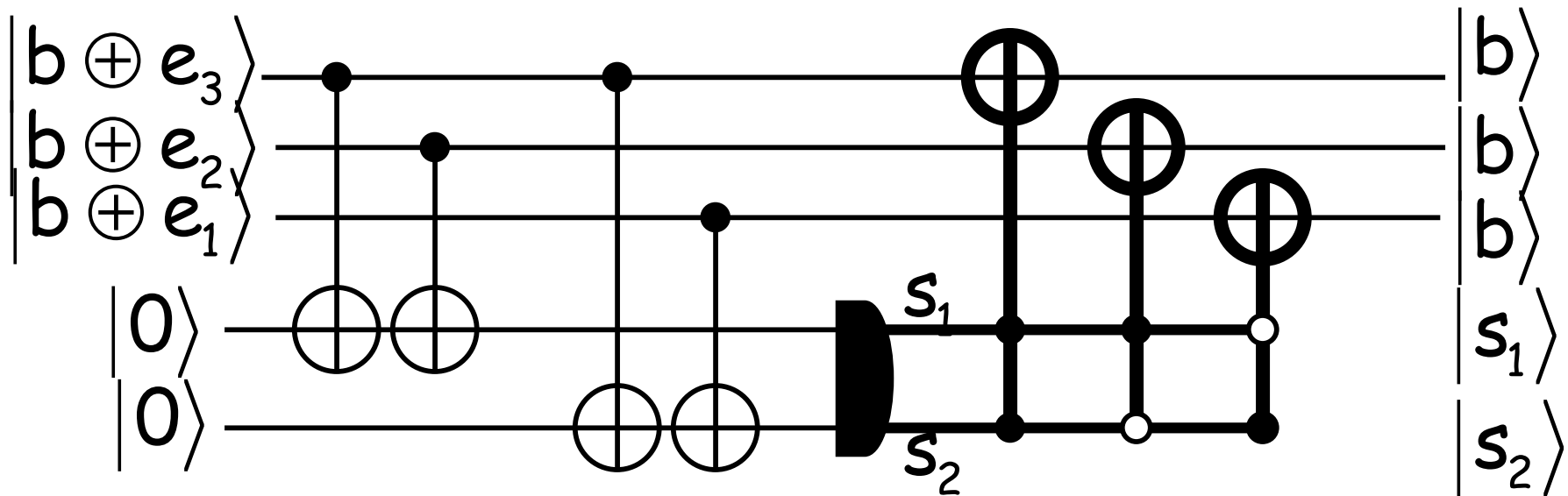
● Assume that $e_3 + e_2 + e_1 \leq 1$ $e_i \in \{0,1\}$



$$\alpha |e_3\rangle |e_2\rangle |e_1\rangle + \beta |1 \oplus e_3\rangle |1 \oplus e_2\rangle |1 \oplus e_1\rangle \rightarrow$$

$$\alpha |0\rangle |0\rangle |0\rangle + \beta |1\rangle |1\rangle |1\rangle$$

Equivalently



Perform operations on logical bits

- e.g. Hadamard gate

