ECE442 Communications
Lecture 3. Modulation and Demodulation

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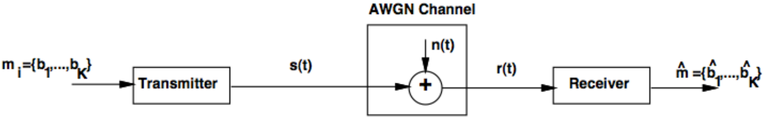
Purpose of Modulation and De-modulation

1. Modulation: mapping coded bits into base band signal.
2. Demodulation: recover the coded bits from received base band signal.
3. Requirement:
    - High spectral efficiency (use as small bandwidth as possible)
    - High power efficiency (use as small power as possible)
    - High reliability (demodulation error being as small as possible)
4. Analysis tool: theory of detection.

Some History

- In the mid-1870s, a form of amplitude modulation, initially called "undulatory currents", was the first method to successfully produce quality audio over telephone lines. In early 1900s, Reginald Fessenden demonstrated AM in audio.
- Frequency modulation was proposed by Edwin Armstrong in his paper "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation" in 1935.

Signal and System Model

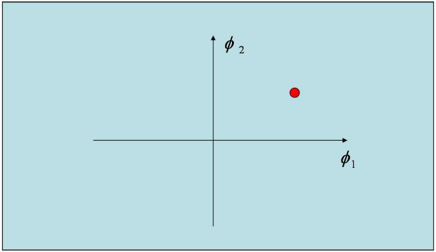


- A signal can usually be written as

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t),$$

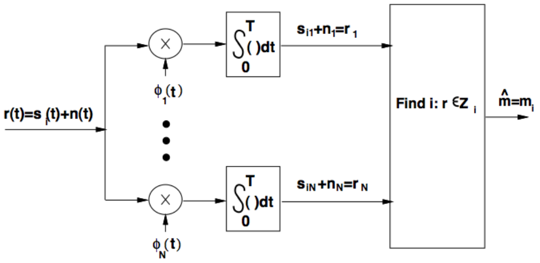
where $\phi_j(t)$ is the j-th basis function.

Signal Space and Representation



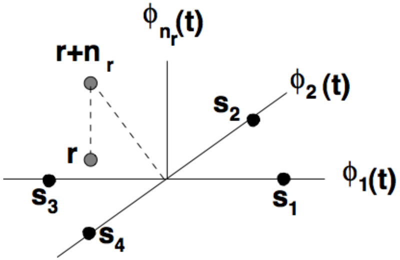
- For linear modulation (amplitude or phase modulation), the signal space is a two-dimensional vector space. The base vectors are $\phi_1(t) = \cos(2\pi f_c t)$ and $\phi_2(t) = \sin(2\pi f_c t)$.
- The representation of signal in the vector space is called *constellation*.

Receiver Structure



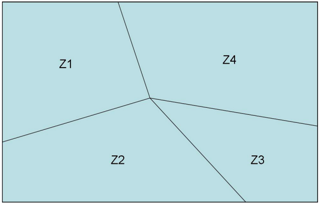
- We can use integrator to obtain $r_j = \int_0^T r(t)\phi_j(t)dt$ and the vector $r = (r_1, ..., r_N)$ is called the sufficient statistics.

Signal Projection



- The noise should not help the detection since its projection onto the signal space is zero.

Decision Regions



- Decision rules are represented by decision regions (decision i if the received signal falls r in region Z_i).
- Error probability (suppose M equal probably messages)

$$P_e = \frac{1}{M} \sum_{m=1}^{M} P(r \text{ not in } Z_m | \text{message m}).$$

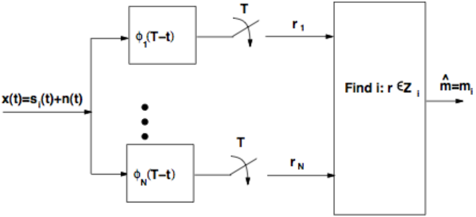
Maximum Likelihood Receiver

- Given a received signal r, define the likelihood of the j-th possible signal as

$$L(s_j) = p(r|s_j).$$

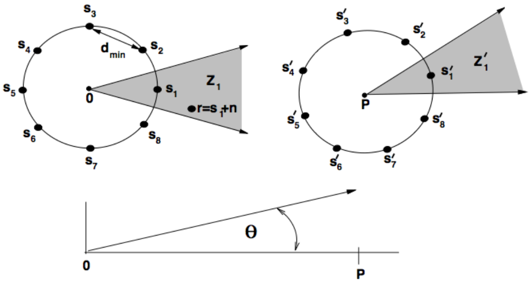
- When the noise is Gaussian, the log-likelihood is given by $l(s_j) = -\frac{1}{N_0}\|r - s_i\|^2$.
- The maximum likelihood receiver outputs the message corresponding to the signal maximizes the likelihood (log-likelihood).

Matched Filter



- We call a filter with impulse response $\psi(t) = \phi(T - t)$ the matched filter of signal $\phi(t)$.
- The maximum likelihood receiver can also be implemented using the matched filter.

Error Probability



- The error probability is given by

$$P_e = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i - s_i} p(n) dn.$$

Union Bound

- The error probability is upper bounded by

$$P_e = \sum_{i=1}^{M} p(m_i) P_e(m_i \text{ sent}) \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1, k \neq i}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}} \right).$$

- The approximation of the error can be obtained by

$$P_e \approx M d_{dim} Q\left(\frac{d_{min}}{2N_0} \right).$$

Passband Modulation Principles

- The modulated signal can be written as

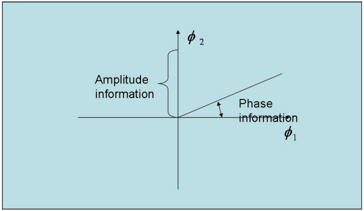
$$s(t) = a(t)\cos[2\pi(f_c + f(t))t + \theta(t) + \phi_0].$$

- In the form of in-phase and quadrature components, we have

$$s(t) = s_I(t)cos(2\pi f_c t) - s_Q(t)sin(2\pi f_c t),$$

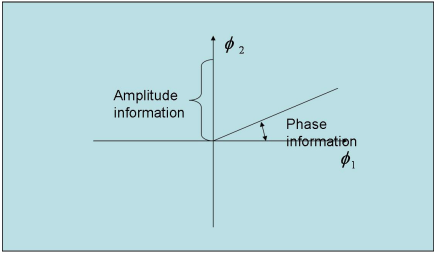
which can also be written as the complex baseband representation: $s(t) = R\left[u(t)e^{2\pi f_c t}\right]$.

Amplitude and Phase Modulation



- Information can be carried in (M is the size of constellation)
  - Amplitude only: MAPM (PAM: pulse amplitude modulation);
  - Phase only: MPSK (PSK: phase shift keying)
  - Both amplitude and phase: MQAM (QAM: quadrature amplitude modulation)
- MPAM, MPSK and MQAM are all linear modulations.

M-PAM



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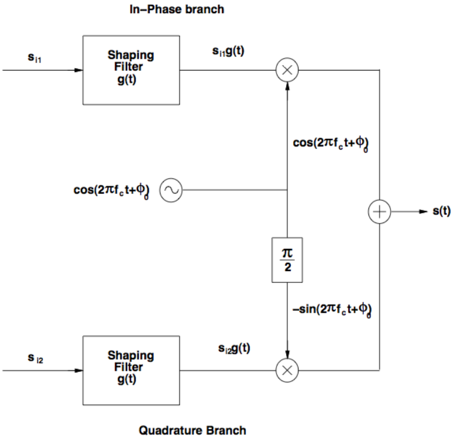
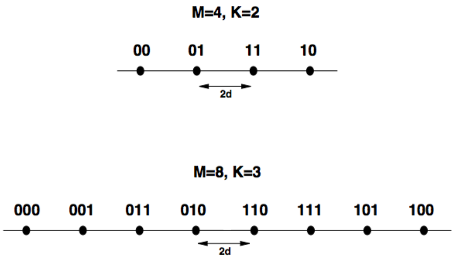
Structure of M-QAM Transmitter



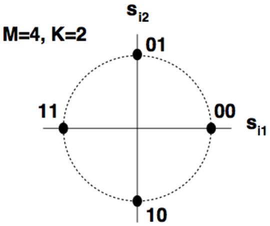
Homework 3

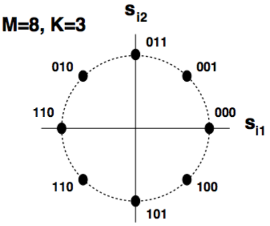
- Problem 1. Suppose that the pulse shape $g(t) = \sqrt{2/T_s}$ (T_s is the symbol period). The minimum distance in the constellation is d. Find the average energy of 8PAM and 16QAM.
- Problem 2. Consider a channel with Doppler spread $B_d = 50Hz$. Then, what time separation is required in samples of the received signal such that the samples are approximately independent?
- Problem 3. Consider 16QAM. Find an upper bound and an approximation of the demodulation error rate is $\frac{d_{min}}{\sqrt{2N_0}} = 10dB$.
- Deadline: Sept. 23rd, 2013.

Gray Encoding



The constellation mapping is usually done by Gray encoding, where the messages associated with signal amplitudes that are adjacent to each other differ by one bit.

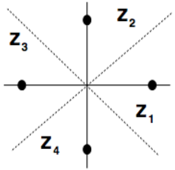


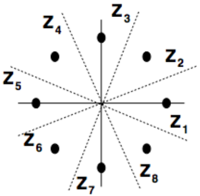


- In MPSK, the information is encoded in the phase:

$$s_i(t) = Ag(t)cos\left[2\pi f_c t + \frac{2\pi(i-1)}{M}\right].$$

Demodulation of MPSK



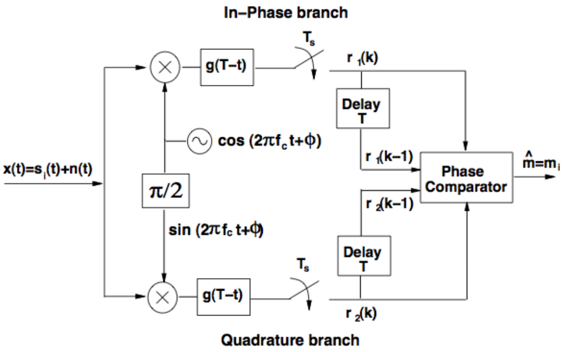


- The decision regions of MPSK are shown above.

Differential Modulation

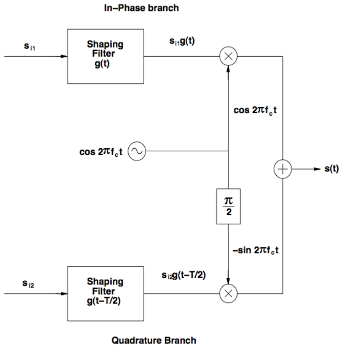
- MPSK and MQAM require coherent demodulation, namely the original phase needs to be known (by using pilot symbol).
- When channel changes very fast, it is difficult to estimate the original phase (unless you use many pilot symbols, but this causes too much overhead). Then, we need differential modulation.
- Differential modulation conveys information in the changes. DPSK: if bit 0, keep the same phase; if bit 1, change the phase by π. Example: 00100110 $\rightarrow$ $00\pi\pi\pi0\pi$.

Demodulation of DPSK



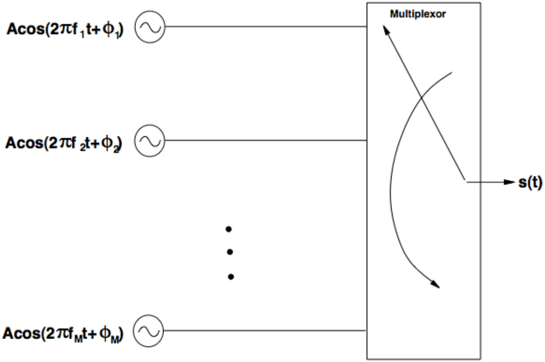
- Differential modulation is less sensitive to a random drift in the carrier phase.
- If the channel has a nonzero Doppler frequency, the signal phase can decorrelate between symbol times.

Frequency Modulation

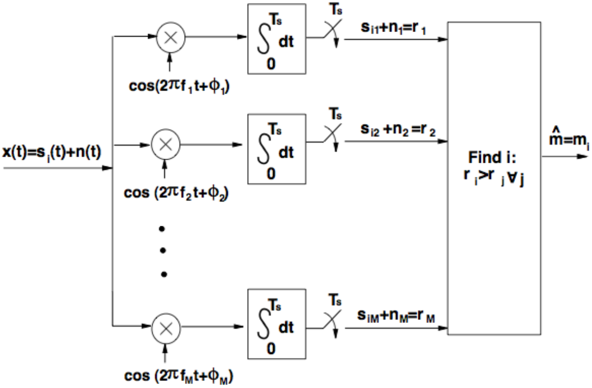


- The frequency modulation signal is given by $s_i(t) = A\cos(2\pi f_i t + \phi_i)$. The frequency spacing should be $0.5/T_s$.

Modulation of FSK



Demodulation of FSK



MSK

- MSK is a binary FSK where $\phi_1 = \phi_2$ and $2\Delta f_c = 1/2T_s$.
- Why called MSK? $2\Delta f_c = 1/2T_s$ is the requirement of minimum frequency separation for keeping orthogonality. (exercise)
- MSK was originally used by Data Transmission Co. in 1972, for spectral efficient communications.

Continuous-Phase FSK (CPFSK)

- In order to eliminate the phase continuity, we frequency modulate a single frequency carrier with a modulating waveform:

$$s(t) = A\cos\left(2\pi f_c t + 2\pi\beta\int_{-\infty}^{t} u(\tau)d\tau\right),$$

- $u(\tau)$ is a MPAM modulated signal (What if $g(t) = \delta(t)$?)

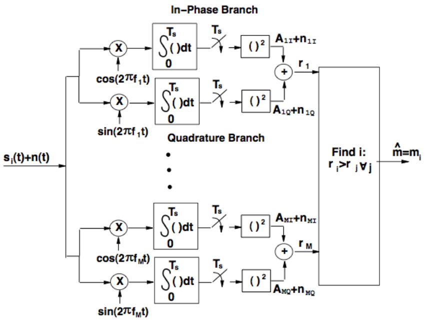
$$u(\tau) = \sum_n A_n g(t - nT_s).$$

- CPFSK is a special case of continuous phase modulation (CPM):

$$s(t) = A\cos\left(2\pi f_c t + 2\pi\sum_{k=-\infty}^{n} I_k q(t - kT)\right).$$

- Both CPFSK and CPM have memory.

Noncoherent Detection of FSK



- For each carrier frequency f_j, $j = 1, ..., M$, the received signal is multiplied by a noncoherent in-phase and quadrature carrier at that frequency, integrated over a symbol time, sampled and then squared.

Pulse Shaping

- For amplitude and phase modulation the bandwidth of the baseband and passband modulated signal is a function of the bandwidth of the pulse shape $g(t)$.
- If $g(t)$ is a rectangular pulse, then the envelope of the signal is constant. However, it has very high spectral sidelobes.
- Pulse shaping is a method to reduce the slidelobe energy relative to a rectangular pulse.

Nyquist Criterion

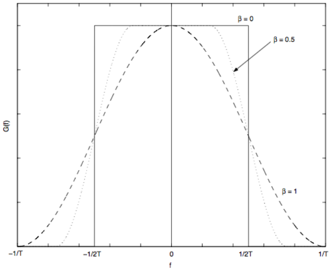
- In order to avoid the inter-symbol-interference (ISI), the pulse should satisfy

$$p(kT_s) = \begin{cases} p_0 = p(0), & k = 0 \\ 0 & k \neq 0 \end{cases}.$$

- In the frequency domain this translates to

$$\sum_{l=-\infty}^{\infty} P\left(f + \frac{l}{T_s}\right) = p_0 T_s.$$

Useful Pulse Shapes



The following pulse shapes all satisfy the Nyquist criterion:

- Rectangular pulses
- Cosine pulses
- Raised cosine pulses

Pulse Shaping for CPFSK

- The most common pulse shape used in FSK is the Gaussian pulse shape:
$$g(t) \propto \exp(-\pi^2 t^2/\alpha^2).$$
- Larger α results in a higher spectral efficiency.
- When the Gaussian pulse shape is applied to MSK, it is called GMSK, which has a high power efficiency.

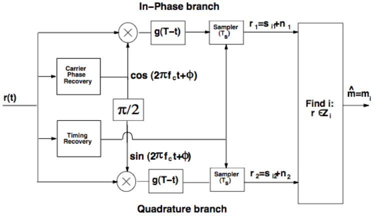
Homework 4

- Problem 1. Derive the Nyquist criterion of inter-symbol interference in the frequency domain.
- Problem 2. Verify (either analytically or numerical) that the cosine pulses satisfy the Nyquist criterion.
- Problem 3. Show that the minimum frequency separation for FSK such that the $\cos(2\pi f_j t)$ and $\cos(2\pi f_i t)$ are orthogonal is $\Delta f = \min_{i \neq j} |f_i - f_j| = 0.5/T_s$.

Symbol Synchronization and Carrier Phase Recovery

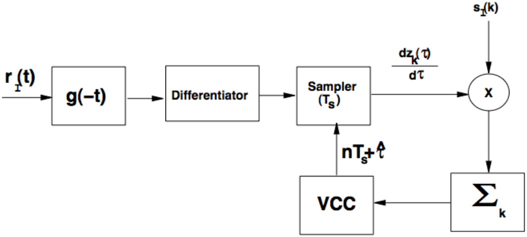
- One of the most challenging tasks of a digital demodulator is to acquire accurate symbol timing and carrier phase information.
- Timing information is needed to delineate the received signal associated with a given symbol.
- The carrier phase information is needed in all coherent demodulators for both amplitude/phase and frequency modulation.

Receiver Structure



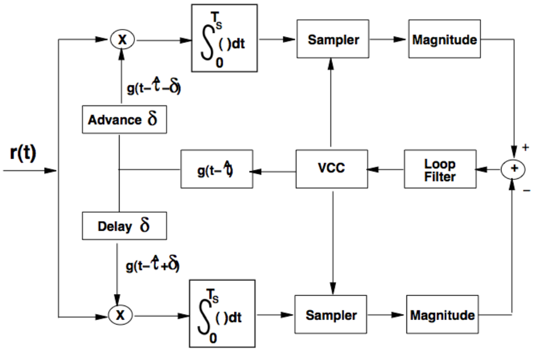
- We denote by θ the unknown parameter vector (ϕ, τ), where ϕ is the unknown phase and τ is the unknown timing.
- We can estimate them using likelihood function.

Phase Recovery



- The optimal solution for the timing is given by the equation $\sum_k s_I(k) \frac{\partial}{\partial r} z_k(\tau)$=0, which can be realized by the above structure.

Early-Late Gate Syncrhonizer



- A non decision-directed timing estimation can be realized by the early-late gate synchronizer.