Quantum Entanglement and the Geometry of Spacetime

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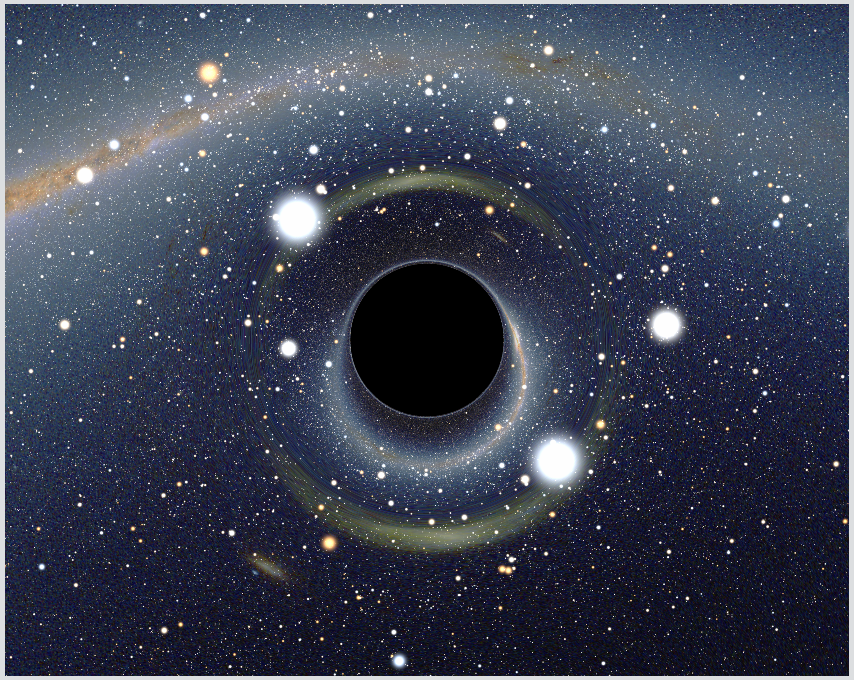
Black Hole Entropy

Bekenstein, Hawking '74:

area of event horizon

$$S = \frac{k_B c^3 a}{4 G_N \hbar} = k_B \frac{a}{4 l_P^2}$$

Planck length
(~ 10^{-33} cm)



G_N → gravity

$\hbar$ → quantum mechanics

k_B → statistical mechanics

Mysteries:

What are the “atoms” of the black hole?

Why is $S \propto a$?

Classical and Quantum Gravity

General relativity:

- Gravity is a manifestation of the curvature of spacetime
- Geometry of spacetime (metric $g_{\mu\nu}$) is dynamical

Einstein equation: $$\underbrace{G_{\mu\nu}}_{\text{curvature}} = 8\pi G_{\mathrm{N}} \underbrace{T_{\mu\nu}}_{\text{matter}} - \underbrace{\Lambda}_{\text{cosmological constant}} g_{\mu\nu}$$

Classical theory.

We know of many *quantum* theories of gravity (from string theory, . . .).
At long distances (compared to Planck length), they reduce to GR.
They have various

- numbers of dimensions
- types of matter fields
- values of Λ

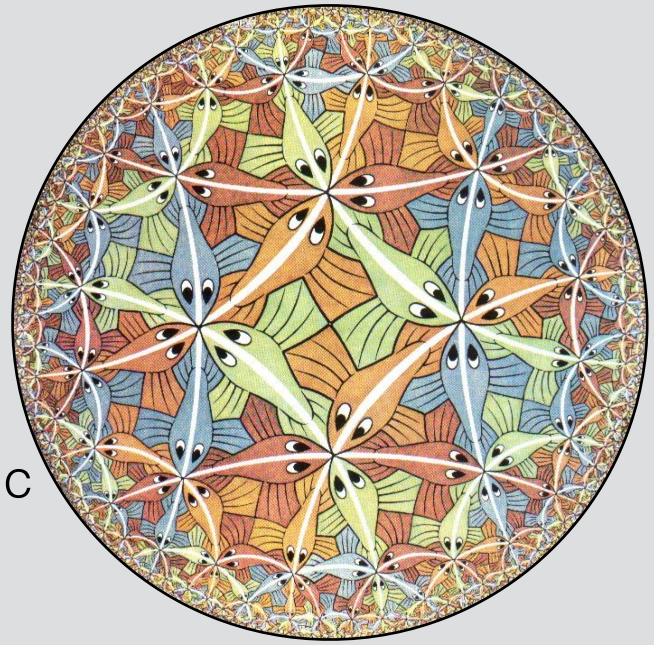
Unfortunately, we don't understand them well enough to directly answer the above questions.

Holographic Dualities

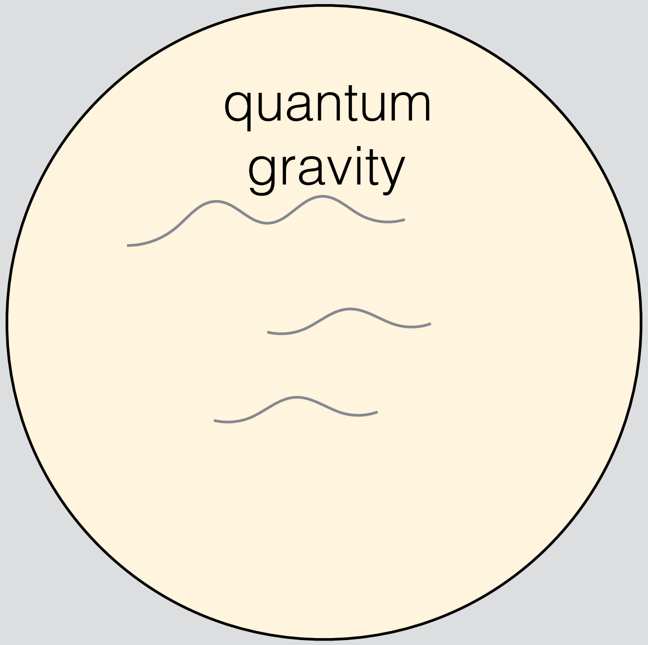
Suppose we have a quantum theory of gravity in $d+1$ dimensions ($d = 2, 3, \ldots$).

$$\Lambda = -\frac{1}{R^2} \qquad\qquad \frac{R}{l_{\mathrm{P}}} \gg 1$$

Simplest solution to Einstein equation is *anti-de Sitter (AdS) spacetime*.
No matter ($T_{\mu\nu} = 0$).
Space is hyperbolic (Lobachevsky) space.
Boundary is infinitely far away, with infinite potential wall.
Light can reach boundary (and reflect back) in finite time.



hyperbolic space



Let spacetime geometry fluctuate, fixing boundary conditions at infinity.
Closed quantum system.

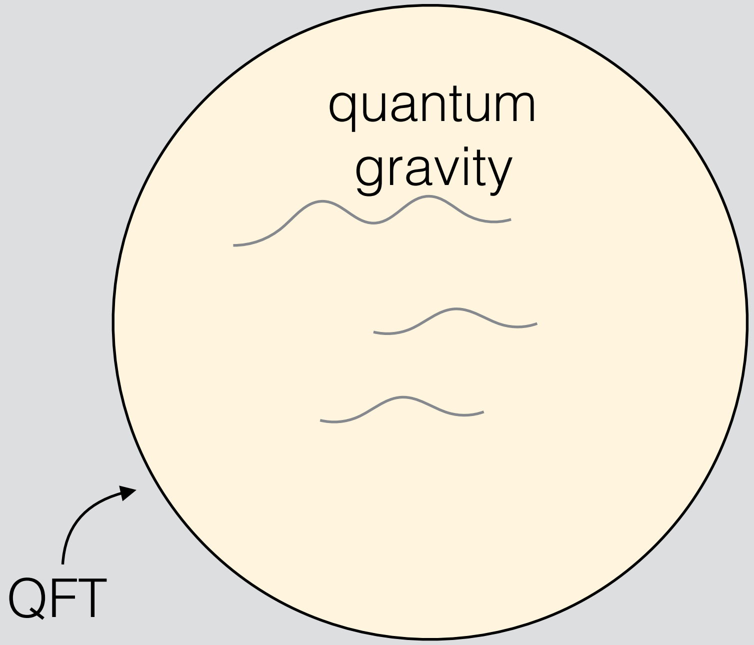
Holographic Dualities

Maldacena '97:

Quantum gravity in $d+1$ dimensions with AdS boundary conditions
= d dimensional ordinary quantum field theory (without gravity).

QFT "lives on the boundary".

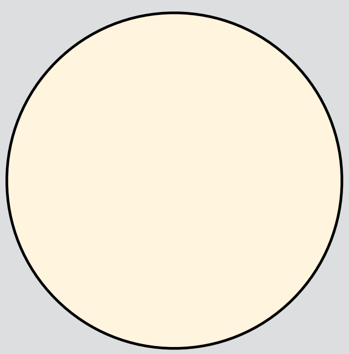
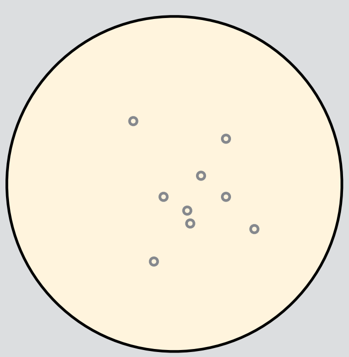
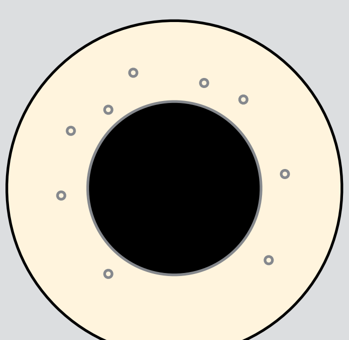
Map between the two theories is non-local.



QFT has a large number of strongly interacting fields:

$$N = \left(\frac{R}{l_{\mathrm{P}}}\right)^{d-1} = \frac{R^{d-1}}{G_{\mathrm{N}}\hbar} \gg 1$$

Holographic Dualities

	Quantum gravity		Quantum field theory
	$\dfrac{R^{d-1}}{G_{\mathrm{N}}\hbar}$		N
	$\hbar \to 0$		$N \to \infty$
	classical limit		thermodynamic limit
	general relativity		macroscopic (collective) description
	empty AdS	$S = 0$	vacuum
	thermal gas of particles in AdS	$S = \mathcal{O}(1)$	confined thermal state
	black hole	$S = \dfrac{a}{4G_{\mathrm{N}}\hbar} = \mathcal{O}(N)$	deconfined plasma

This helps to understand black hole entropy.

But mysteries remain.

Nothing special happens at a black hole horizon.

What about other surfaces? Can their areas represent entropies?

Are there entropies that are intrinsic to a system --- not thermal?

Entanglement Entropy

Classical mechanics:

definite state → certain outcome for any measurement

Quantum mechanics:

definite state → uncertain outcomes for some measurements

Example: $|\uparrow\rangle$

measurement of S_z definitely gives $+\frac{1}{2}\hbar$

measurement of S_x gives $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$ with equal probability

When only certain kinds of measurements are allowed, a definite (pure) state will *effectively* be indefinite (mixed).

Suppose a system has two parts, but we can only measure one.

Spin singlet state: $$|AB\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) \qquad S_{AB} = 0$$

A B

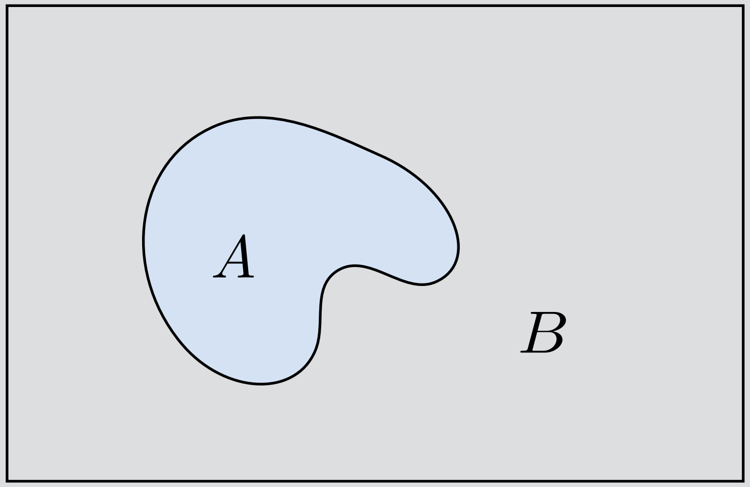
To see that this is a pure state (superposition, not mixture, of $|\uparrow\rangle|\downarrow\rangle$ and $|\downarrow\rangle|\uparrow\rangle$) requires access to both A and B.

For an observer who only sees A, effective state is mixed:

$$\rho_A = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right) \qquad S_A = k_{\mathrm{B}} \ln 2$$

In classical mechanics, if whole is in a definite state then each part is also: $S_A \leq S_{AB}$

Entanglement Entropy in QFT



In quantum field theories, spatial regions are highly entangled.

New way of thinking about QFTs.
Entanglement entropy S_A is a function of the state and the region A.

Reveals a lot about the theory:
- quantum criticality
- topological order
- renormalization-group monotones

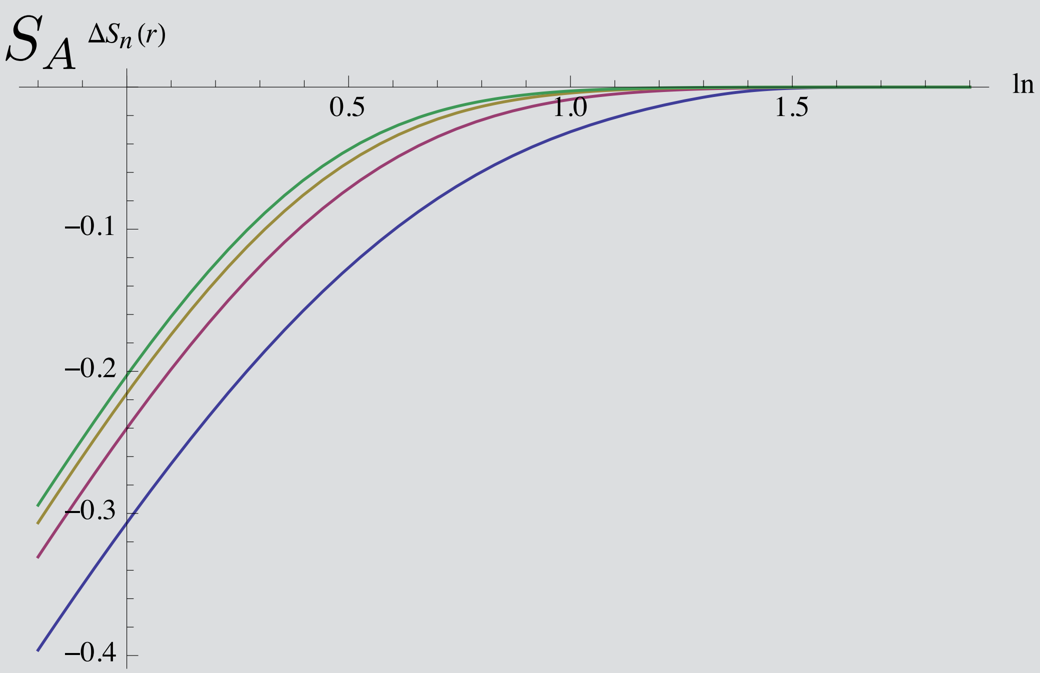
Unfortunately, difficult to compute, even in simple theories.
Standard method is *replica trick*:

1. using path integral, compute *Rényi entropies* $S_A(n) = \dfrac{1}{1-n} \ln \operatorname{tr} \rho_A^n \quad (n = 2, 3, \ldots)$

2. extrapolate to $n=1$: $\lim_{n\to 1} S_A(n) = -\operatorname{tr}(\rho_A \ln \rho_A) = S_A$

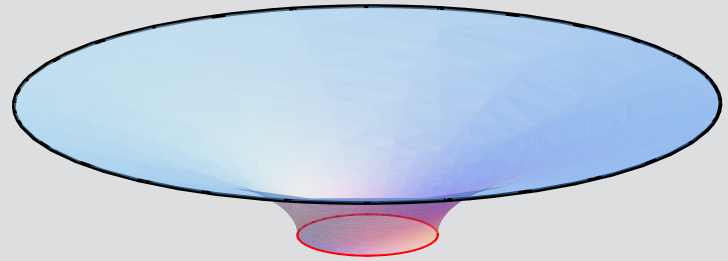
MH, Lawrence, Roberts '12:
Showed that entanglement entropy is invariant under bosonization in 1+1 dimensions.

Agón, MH, Jafferis, Kasko '13:
Calculated S_A for disk of radius r in 2+1 dimensional electromagnetism.



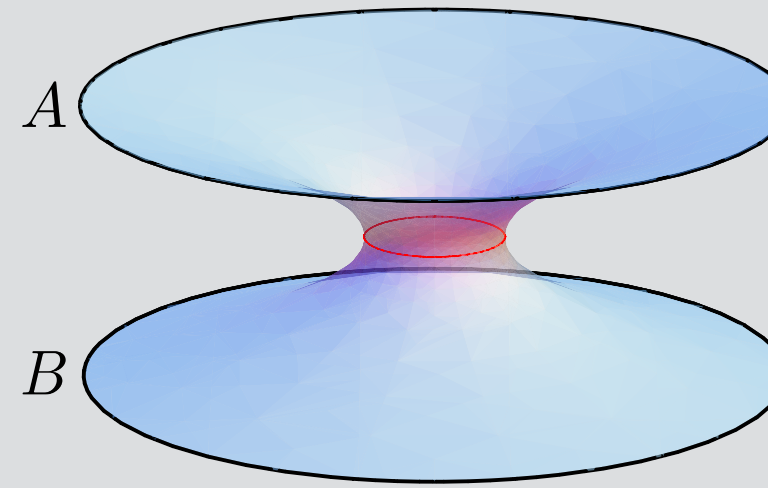
HOLOGRAPHIC ENTANGLEMENT ENTROPY

Black hole = thermal state



Maldacena '01:
2 black holes joined by Einstein-Rosen bridge
= 2 entangled QFTs

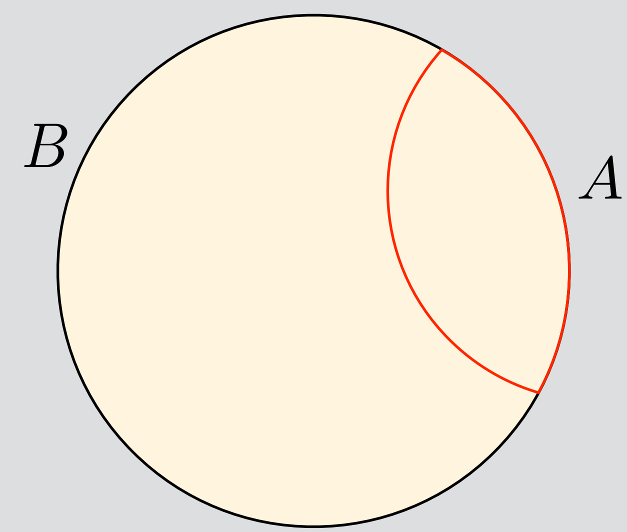
$$S_A = \frac{a}{4G_{\mathrm{N}}\hbar}$$



Ryu, Takayanagi '06 proposed that, in general,

$$S_A = \frac{a}{4G_{\mathrm{N}}\hbar}$$

area of *minimal surface* between A and B



"*Entanglement is the fabric of spacetime*"

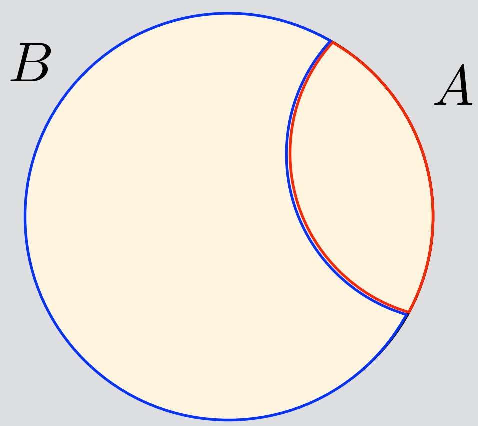
Simple & beautiful . . . widely applied . . . but is it right?

Holographic Entanglement Entropy

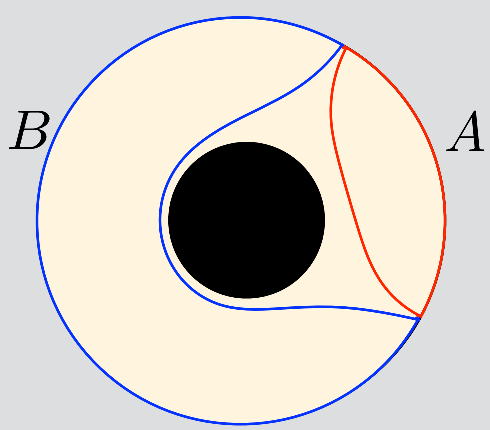
MH, Takayanagi ’07; Hayden, MH, Maloney ’11; MH ’13:
Holographic formula obeys all general properties of entanglement entropies.

Examples:

If full system is pure then $S_A = S_B$



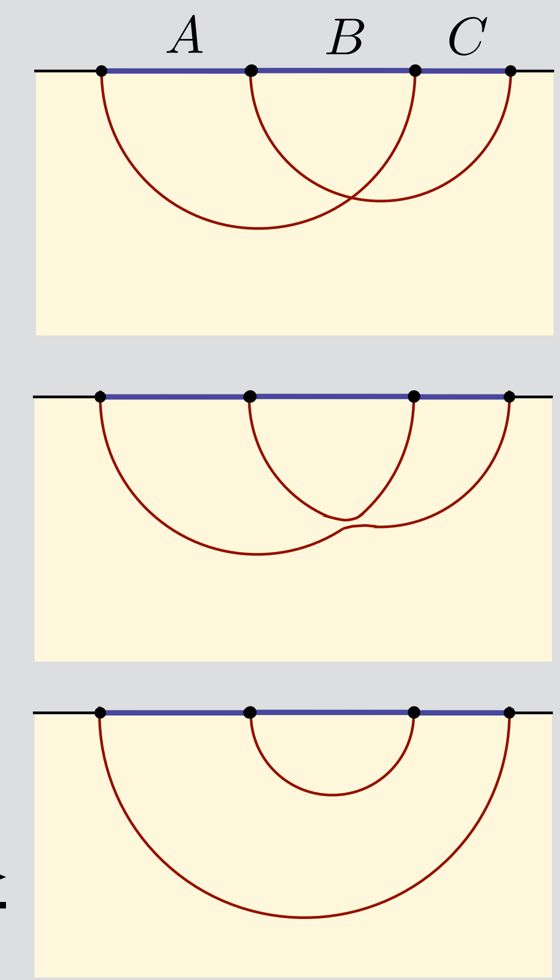
Otherwise,



$S_A \neq S_B$

Strong subadditivity: $S_{AB} + S_{BC} \geq S_{ABC} + S_B$

$S_{AB} + S_{BC}$ = = ≥



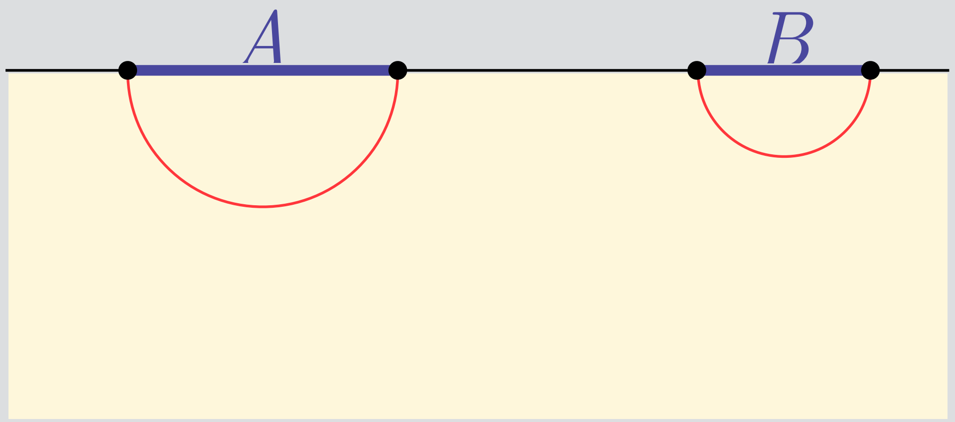
$= S_{ABC} + S_B$

Quantum information theory is built into *spacetime geometry*.
Holographic entanglement also has a *special* property: “monogamy of mutual information”.

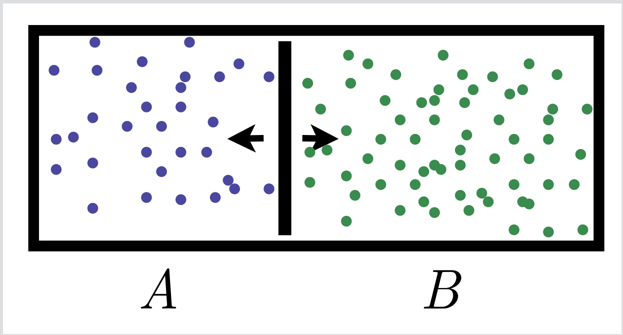
Holographic Entanglement Entropy

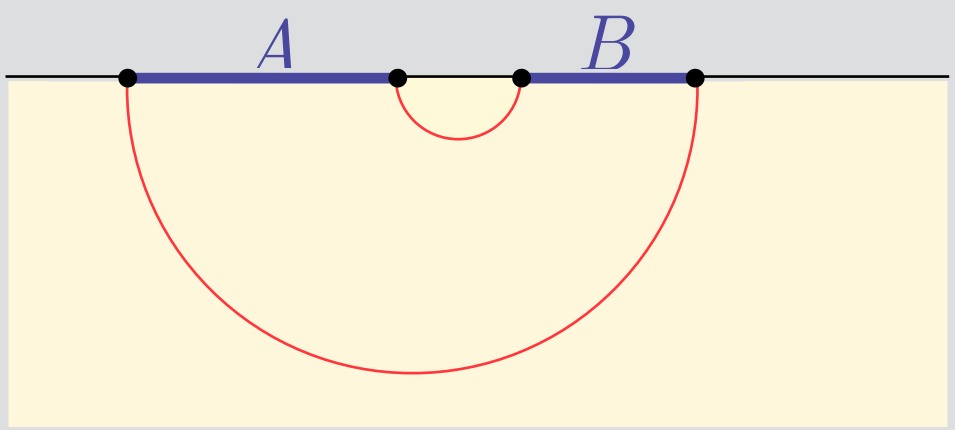
MH '10:
- Explained how to apply replica trick to holographic theories.
- Debunked previous "derivation" of holographic formula.
- Showed that holographic formula predicts phase transition for separated regions.
- Confirmed using Euclidean quantum gravity & orbifold CFT techniques.

Lewkowycz, Maldacena '13: General "derivation" of holographic formula.



$S_{AB} = S_A + S_B$
correlations mediated by $O(1)$ confined degrees of freedom





$S_{AB} < S_A + S_B$
correlations mediated by $O(N)$ elementary degrees of freedom

Time-Dependent Holographic Entanglement Entropy

What about *time*?

Original (Ryu-Takayanagi) holographic formula assumes state is *static*.

Hubeny, Rangamani, Takayanagi '07:
For non-static states, replace *minimal* surface in bulk space with *extremal* surface in bulk spacetime.

Simple & beautiful . . . widely applied . . . but is it right?

Callan, He, MH '12: Obeys strong subadditivity in examples.

Extremal surface goes behind horizons!
MH, Hubeny, Lawrence, Rangamani '14: Nonetheless obeys causality.
Implies that QFT state is encoded by (part of) spacetime behind horizon.

MH, Myers, Wien '14: Proved that area of a *general* surface (not just extremal) is given by *differential entropy* in QFT.

Summary

Entanglement entropy in holographic theories:

- Enormous progress in recent years.
- Still many mysteries.
- Suggests a deep and general connection between entanglement and the geometry of spacetime.

(How) does spacetime itself emerge from quantum mechanics?

Stay tuned . . .