Fluid approximation to controlled Markov chains with local transitions

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Fluid scaling is widely used in queuing theory, inventory, population dynamics etc because it usually leads to relatively simple deterministic models which describe the underlying stochastic systems accurately enough. In this talk, the idea of this approach will be illustrated on simple examples:
– firstly, for a fixed control strategy, the trajectories of the stochastic process converge almost surely to the (deterministic) solution of an ordinary differential equation;
– secondly, the solution of the original Bellman equation converges to the Bellman function of the corresponding deterministic optimal control problem.

We shall discuss the rates of convergence and estimate the error terms for a special case of absorbing model.

It should be emphasized that the straightforward fluid approximation sometimes fails to work. A concrete example will show that the deterministic Bellman fucntion can be finite and well defined, whereas in the underlying stochastic model the Bellman function equals infinity. To avoid such situations, we propose the 'refined' fluid approximation and show that it works for a wider class of models.

If time permits, we can also discuss applications of this theory to information transmission, mathematical epidemiology and inventory.

Several recent publications of mine on the topic:

A.Piunovskiy. Controlled jump Markov processes with local transitions and their fluid approximation. WSEAS Trans. On Systems and Control, 2009, V.4, N.8, p.399-412.

A.Piunovskiy. Random walk, birth-and-death process and their fluid approximations: absorbing case. Math. Meth. Oper. Res., 2009, V.70, N.2, p.285-312.

A.Piunovskiy and Y.Zhang. Accuracy of fluid approximation to controlled birth-and-death processes: absorbing case. Math. Meth. Oper. Res., 2011, V.73, N.2, p.159-187

K.Avrachenkov, A.Piunovskiy and Y.Zhang. Asymptotic fluid optimality and efficiency of tracking policy for bandwidth-sharing networks. J. Appl. Prob., 2011, V.48, N.1, p.90-113.

A.Piunovskiy and Y.Zhang. On the accuracy of fluid approximations to a class of inventory-level dependent EOQ and EPQ models. Adv. Oper. Res., 2011 (to appear).

Of course, I would be happy to provide these articles to the interested people.