State-dependent M/G/1 type queuing analysis for congestion control in data networks

Eitan Altman *Kostya Avrachenkov* ***Chadi Barakat*** *Rudesindo Nunez Queija*

INRIA - Sophia Antipolis, France CWI - The Netherlands

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Outline

- ☞ Congestion control modeling.
- ☞ Our model and its analysis.
- ☞ Numerical and experimental results.
- ☞ Concluding remarks.

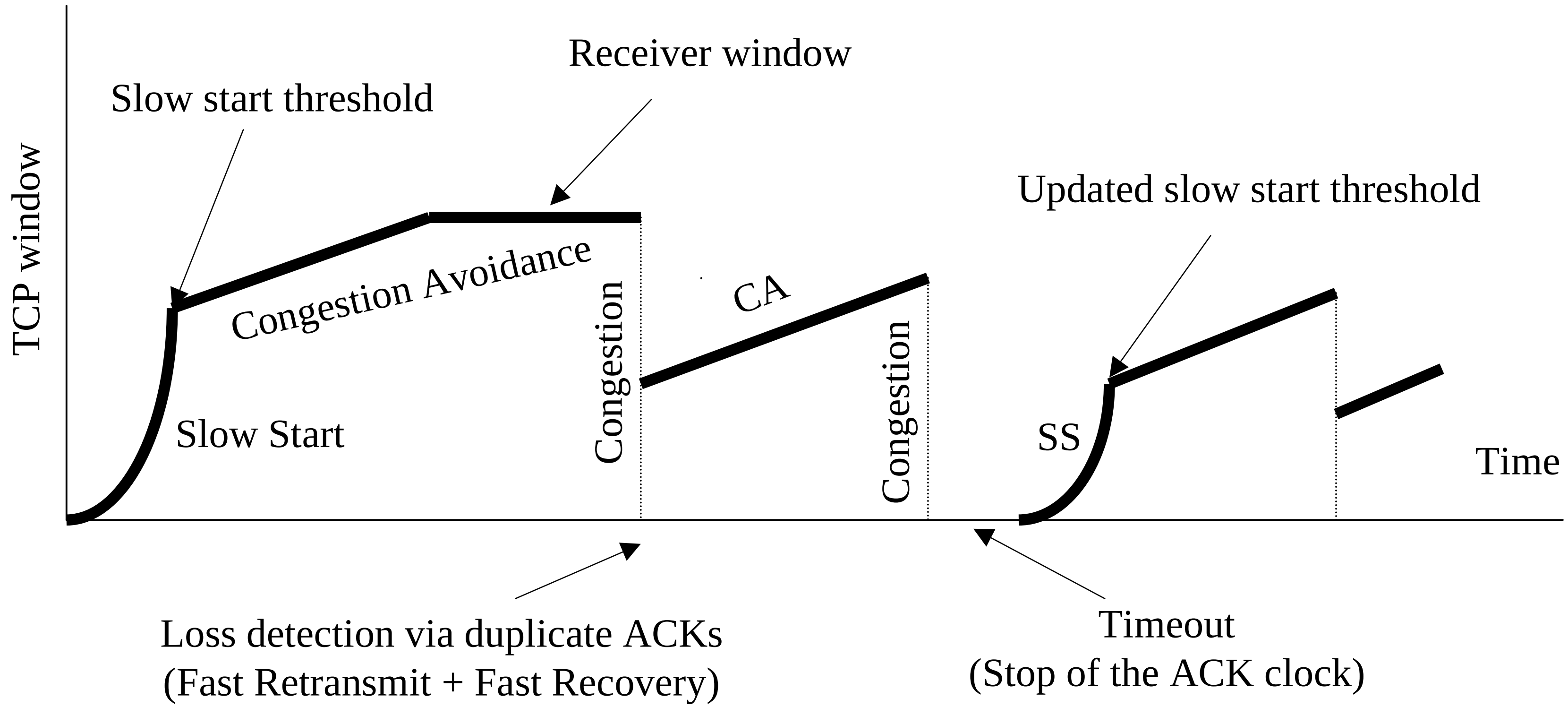
Congestion control in data networks

- **Objective:** Efficient use and fair sharing of network resources.
- Widely used algorithm:
  - Probe the network by linearly increasing the transmission rate.
  - Divide the transmission rate by a constant factor (*typically 2*) when the network becomes congested.
  - Congestion is detected either by an explicit signal sent by the network, or by inferring mechanisms at the sources.
- Henceforth, consider as a reference the TCP protocol, widely used for congestion control in the Internet …

TCP mechanisms

☞ A window-based flow control protocol.

☞ Use the loss of packets to detect network congestion.



Congestion control modeling

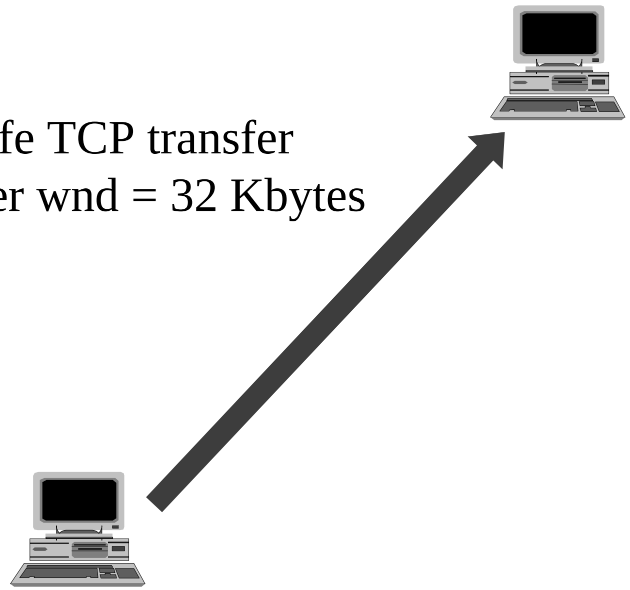
- **Main objective:** Calculation of the average transmission rate (*the throughput*).
  - Useful for understanding the behavior of the congestion control and for the design of new applications and protocols (e.g., mechanisms for routers).
- **Requirements:**
  - Model for the variation of the transmission rate between congestion events and during congestion.
  - Model for the appearance of congestion events.
- **Literature:**
  - General models for congestion events, but
  - No exact expressions of the throughput when the rate is limited ...

Limitation of the transmission rate

Caused by the receiver window in case of TCP ...

Michigan Univ.

Long-life TCP transfer
Receiver wnd = 32 Kbytes



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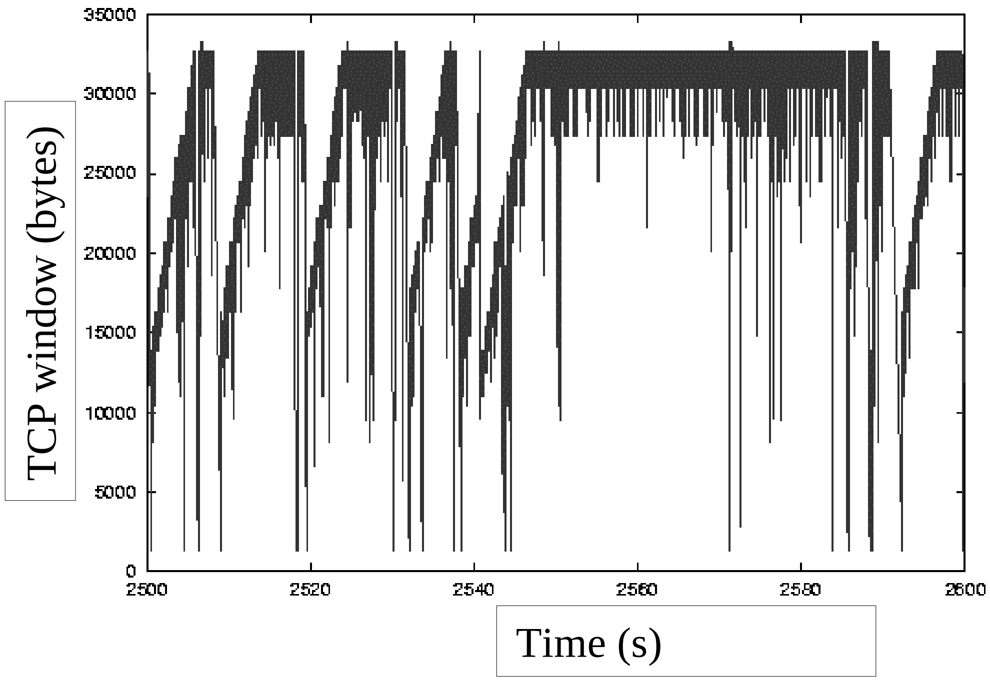
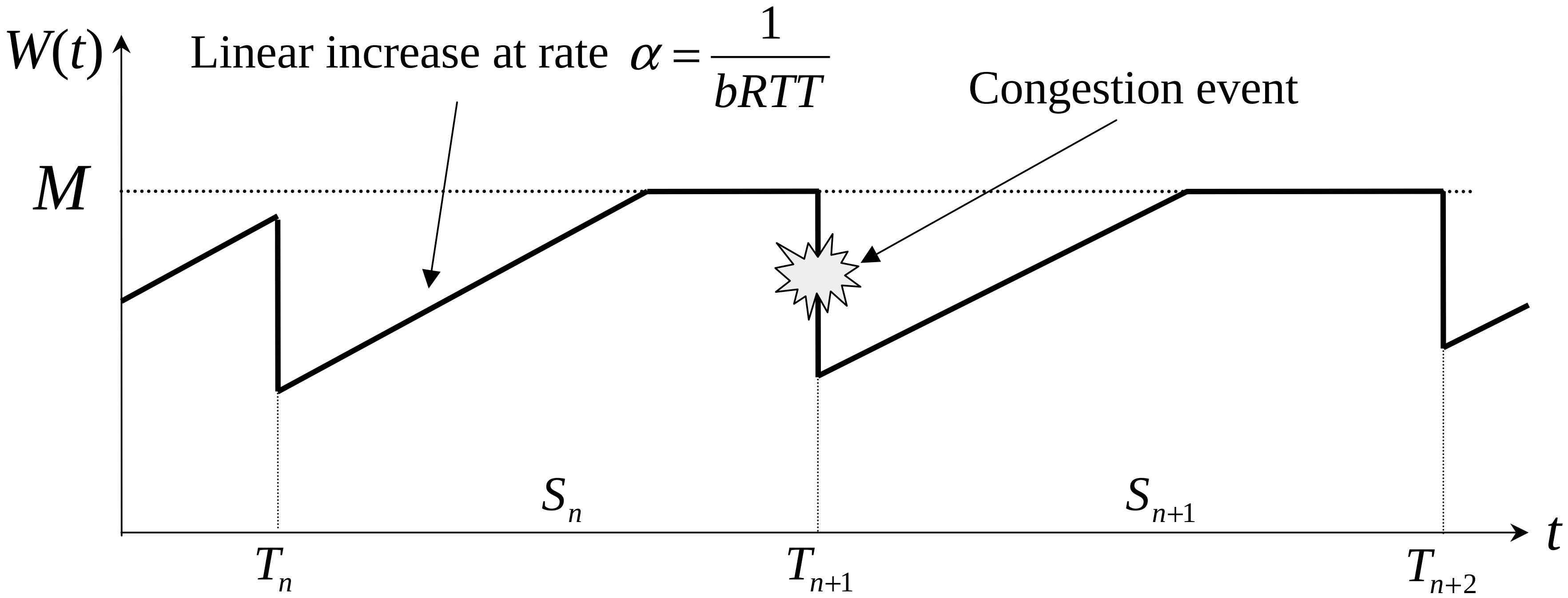


Our model for congestion control

☞A fluid model for the variation of TCP window:



☞Use the techniques in [Altman,Avrachenkov,Barakat,SIGCOMM'00] to account for timeouts and the discrete nature of TCP.

Our model for congestion control (Ctd.)

☞At the n-th congestion event, N_n divisions of the window by a factor $\gamma > 1$

$$W(T_n+) = \frac{W(T_n-)}{\gamma^{N_n}}$$

$\{N_n\}$ form an i.i.d. sequence with,

$$Q(z) = \sum_{k=1}^{\infty} z^k \mathrm{P}\{N_n = k\}$$

☞Useful when the congestion lasts for multiple consecutive RTTs, or

☞For future versions of TCP that reduce the window as a function of the congestion level (e.g., number of packets dropped during congestion).

☞Congestion events (i.e., the instants $\{T_n\}$) occur according to a homogenous Poisson process of intensity λ.

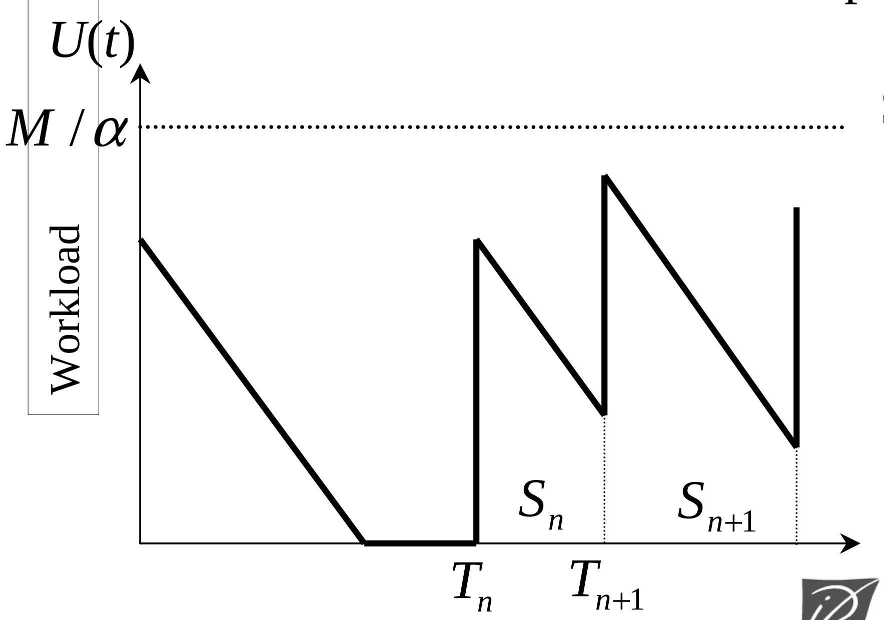
The analysis

- **Theorem:** The window of the TCP connection converges to the same stationary regime for any initial state.
- **Outputs of the analysis:**
  - $F(x)$ = PDF of $W(t)$ in the stationary regime.
  - $\mathrm{E}[W^k(t)],\ k \geq 1$ = Moments of $W(t)$ in the stationary regime.
  - In particular:

TCP throughput: $$\overline{X} = \lim_{t\to\infty} \frac{1}{t} \int_0^t \frac{W(\tau)}{RTT} d\tau = \frac{\mathrm{E}[W(t)]}{RTT}$$

Dual M/G/1 queuing model *

- The problem of congestion control can be seen as a queuing model:
  - The window corresponds to system workload ($U(t) = \dfrac{M - W(t)}{\alpha}$).
  - Congestion events correspond to customer arrivals.
  - Reduction of window corresponds to increase in workload.



Service time of a customer in the dual model:

$$x_n = \left(\frac{M}{\alpha} - U_n \right) \left(1 - \frac{1}{\gamma^{N_n}} \right)$$

Dependent of the workload.

* [Misra, Gong, Towsley, 1999]

Recursive equations for moments

☞ **Steady state Kolmogorov equation:** Relation between $F(x+dx)$ and $F(x)$

$$\alpha \frac{dF(x)}{dx} = \lambda \sum_{k=1}^{\infty} \mathrm{P}\{N=k\} F(\min(\gamma^k x, M)) - \lambda F(x)$$

☞Applying Laplace Stieltjes Transform, then differentiating, we get

$$\text{For } k = 1,2,\ldots, \qquad \mathrm{E}[W^k(t)] = \frac{k\alpha(\mathrm{E}[W^{k-1}(t)] - P_M M^{k-1})}{\lambda(1 - Q(\gamma^{-k}))}$$

which gives all the moments of W(t), as a function of $P_M = \mathrm{P}\{W(t) = M\}$ ...

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Window distribution

☞ $F(x)$ is continuous on the interval $(\ 0\ ,\ M\ )$ with a jump at M due to P_M .

☞ From the Kolmogorov equation we can write:

$$\text{For } x \in \left[\frac{M}{\gamma^k}, \frac{M}{\gamma^{k-1}}\right[, k \geq 1, \qquad \overline{F}(x) = 1 - F(x) = P_M \sum_{i=1}^{k} c_i^{(k)} e^{-\frac{\lambda}{\alpha}\gamma^{i-1}x}$$

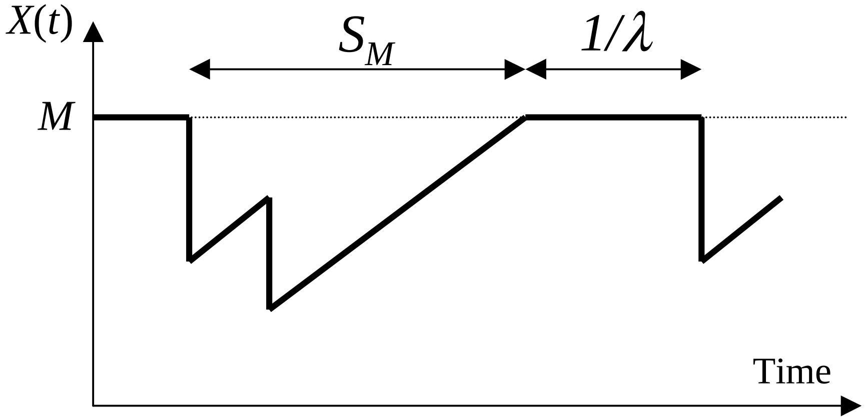
with $c_i^{(k)}$ some constants that can be recursively determined.

First method to calculate P_M :

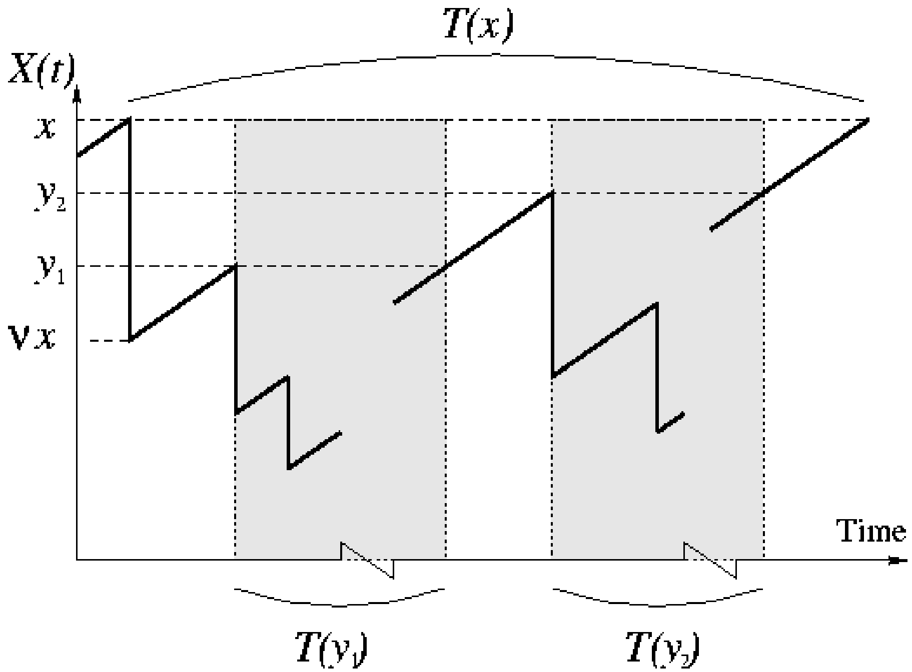
$$\lim_{x \to 0} F(x) = 0$$

But, not efficient since the calculation of $c_i^{(k)}$ for large k is not very accurate.

Efficient method to calculate P_M



$$P_M = \frac{1/\lambda}{1/\lambda + S_M}$$



☞ $\mathrm{E}[T(x)]$: Average time to return to x

$$S_M = \mathrm{E}[T(M)]$$

Efficient method to calculate P_M (Ctd.)

☞ Derive an integral equation for $\mathrm{E}[T(x)]$.

☞ Apply Laplace Transforms, solve the equation and invert back:

$$\mathrm{E}[T(x)] = \frac{1-Q(\gamma^{-1})}{\lambda} \sum_{i=0}^{\infty} d_i \left(e^{\gamma^{-i} \frac{\lambda}{\alpha} x} - 1 \right)$$

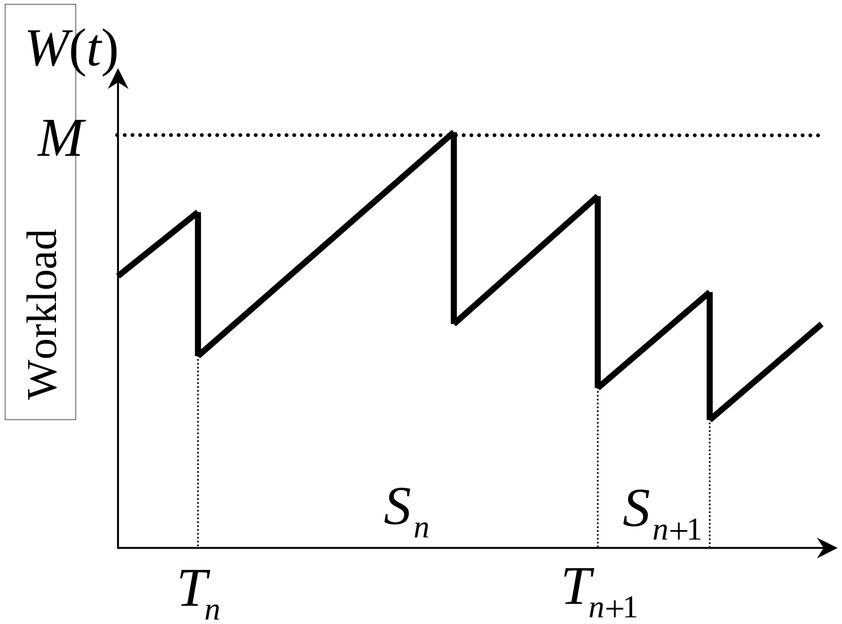
where d_i are constants that can be recursively determined.

➘ The algorithm is efficient since the infinite series converge very fast.

Extension to congestion limitation

☞ The model can be easily extended to the case when the window is reduced whenever it reaches M (*Congestion Limitation case*).

☞ **Example:** M corresponds to the available bandwidth.



$$F^{cl}(x) = \frac{F(x)}{1 - P_M}$$

$$\mathrm{E}[(W^{cl})^k] = \frac{\mathrm{E}[W^k] - P_M M^k}{1 - P_M}$$

Model validation: Throughput

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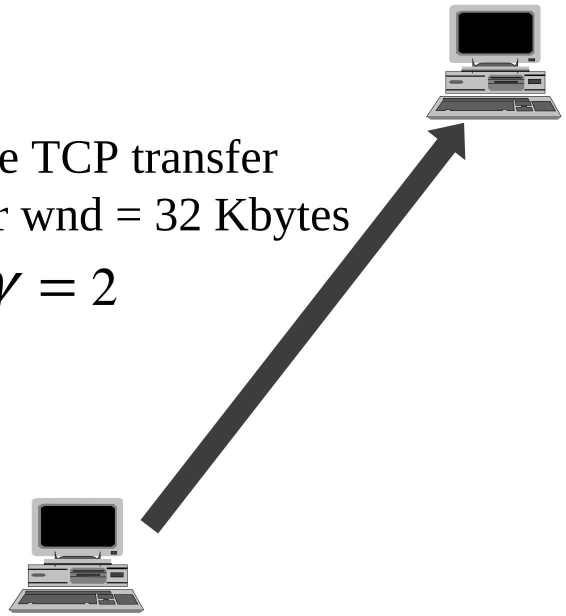
South Australia

Long-life TCP transfer
Receiver wnd = 32 Kbytes

$\gamma = 2$



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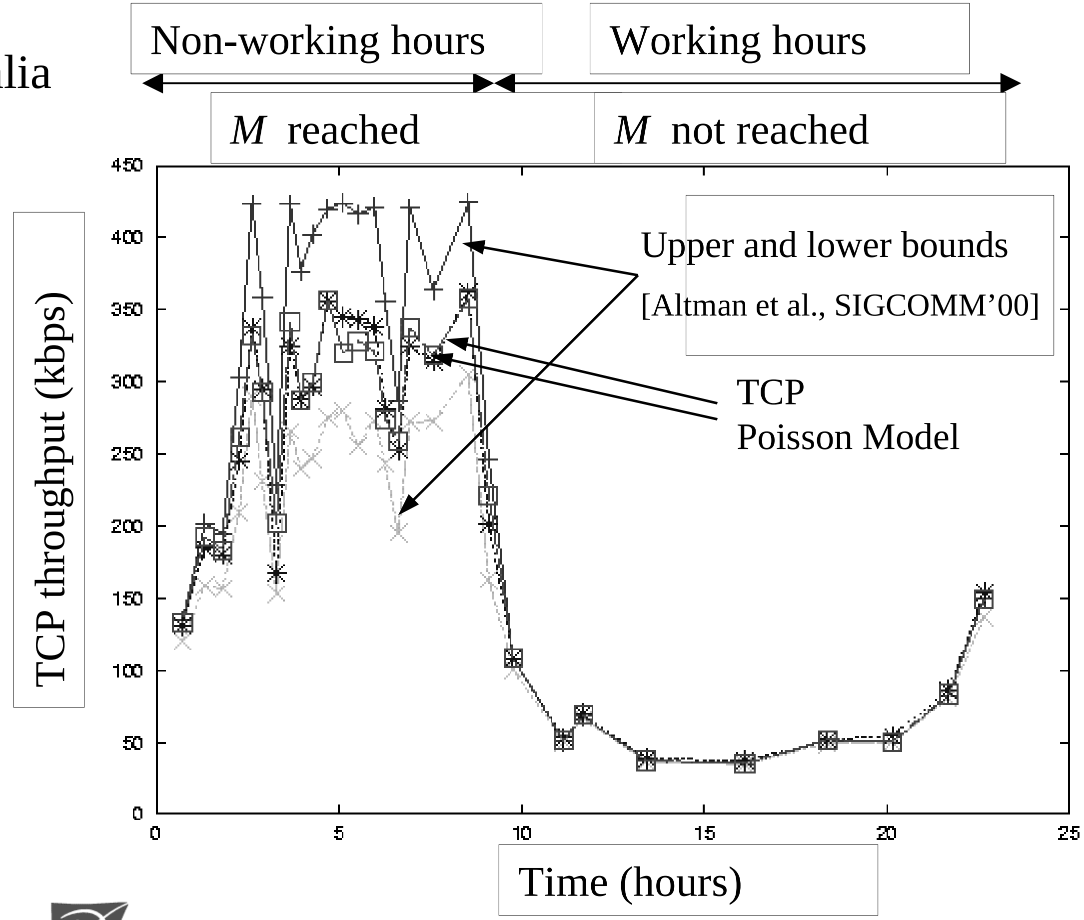
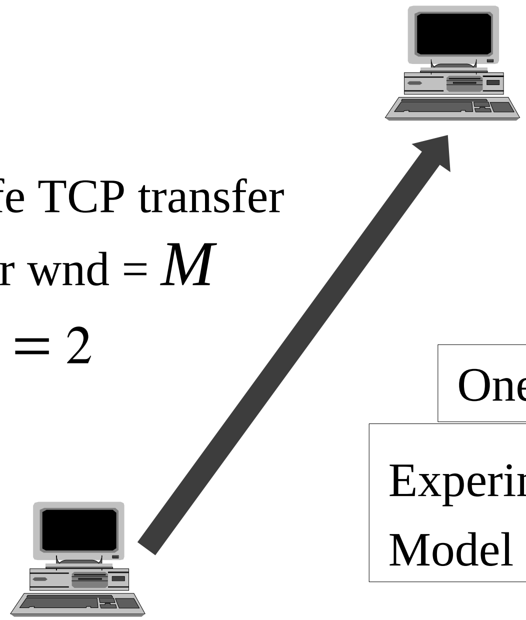


Model validation: Distribution

Michigan State Univ.



Long-life TCP transfer

Receiver wnd = M

$\gamma = 2$

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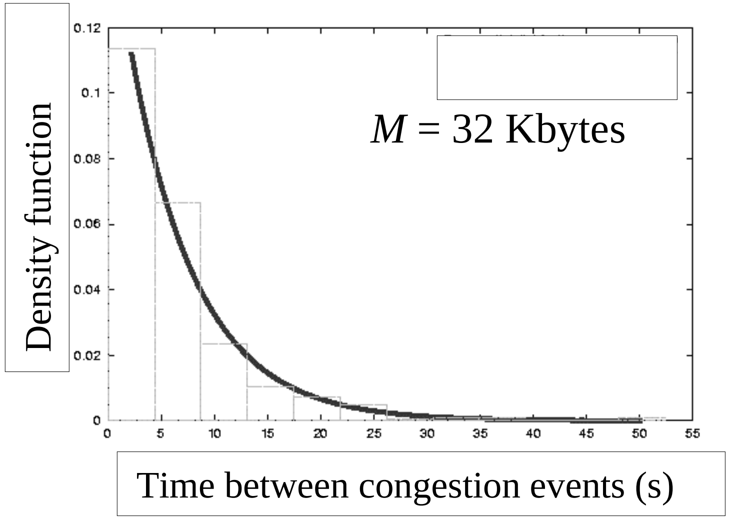
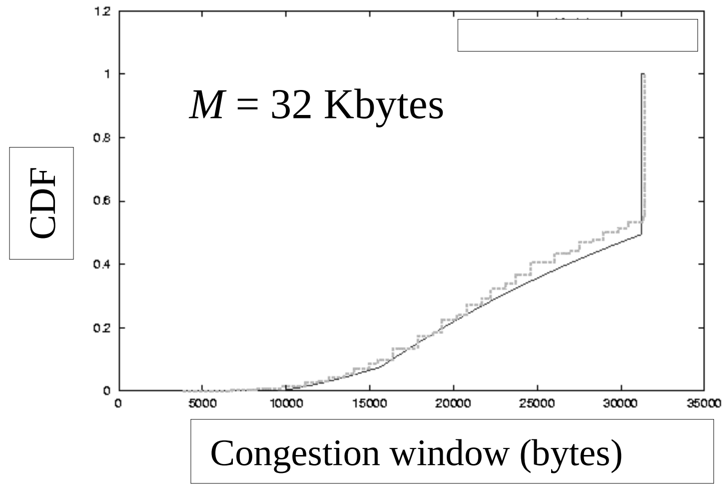
One-hour traces

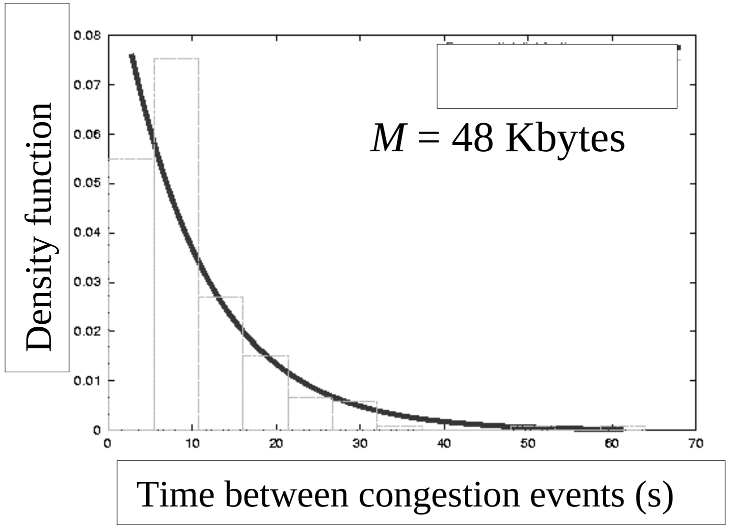
Experimentation

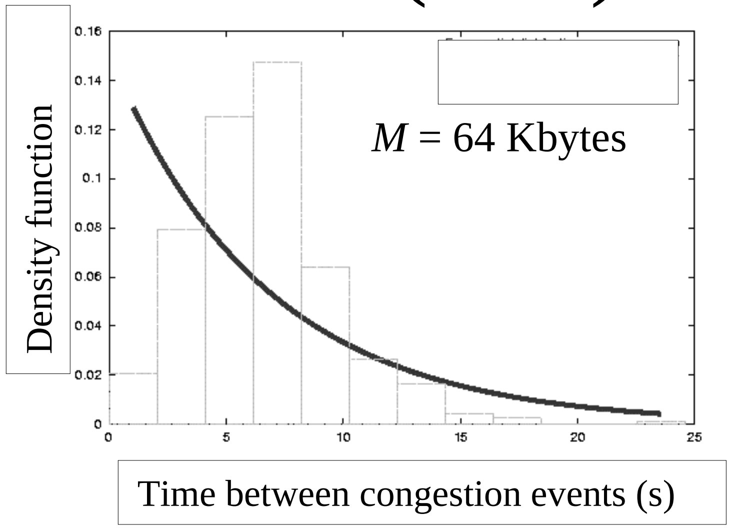
Model

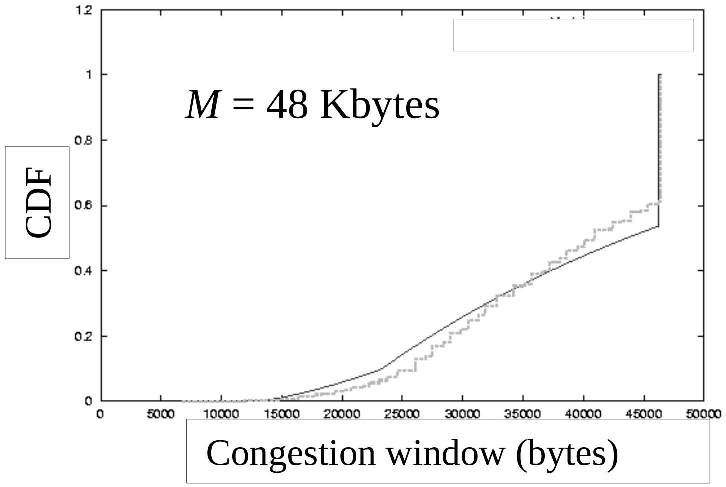


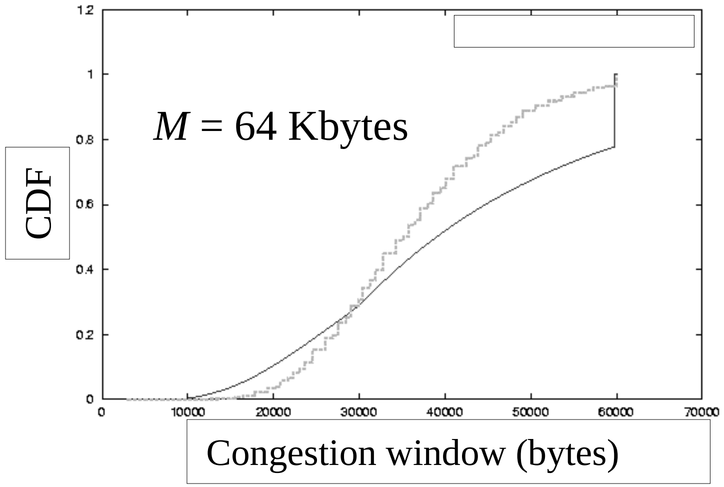


Model validation: Distribution (Ctd.)









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Conclusions and perspectives

☞ **Results:** Moments and distribution of transmission rate were obtained when there is a maximum limit and when the process of congestion events is close to Poisson.

Approximations of the throughput exist in the literature (e.g., the fixed-point approach by [Padhye, Firoiu, Towsley, and Kurose, SIGCOMM'98]).

☞ **Future work:**

- ☞ Consider more general processes for congestion events (e.g., MMPP).
- ☞ Validate the model on bursty paths where the congestion of the network lasts for multiple consecutive RTTs.
- ☞ Other rate increase policies (e.g., sub-linear increase).