NP-Complete Problems

Satisfiability (SAT)

Given a Boolean expression over n variables in conjunctive normal form, is there a way to assign values to the n variables that will make the entire expression true?

This general problem is NP-Complete.

What does that mean?

NP-Complete

I will leave the finer details of the definition of NP-Completeness to CMSC 451, but we can discuss the relationship between NPC problems.

You can convert/reduce ***any*** NPC problem to ***any other*** NPC problem in polynomial time.

This means that if ***any*** NPC problem is ever solved in polynomial time, then EVERY NP-Complete problem can be solved in polynomial time!

Reductions

We often talk about showing that some "Problem B is ***at least as hard as*** "Problem A" as a runtime metric.

In our context, if:

- A is a well-understood problem
- B is a new problem

then if we can restate ("reduce") **A** as **B**, doing that conversion process in polynomial time, that means that we could use the solution to **B** as a way to solve **A** with polynomial overhead.

That means that if we could solve **B** in polynomial time them we'd also be able to solve **A** in polynomial time.

NPC Reductions

So, if A is a problem known to be in NPC and B is a new problem that looks like it will be computationally hard…

…if we can reduce a question about **A** to being a question about **B** (in only polynomial time) that means that if we could solve **B** in polynomial time them we'd also be able to solve **A** (and by transitivity ***ANYTHING*** in NP) in polynomial time.

This is half of what we need to show that a new problem **B** is also in NPC.

Hamiltonian Cycles

Given an undirected graph, is there a cycle that visits every vertex exactly once?

If I present you with a graph and a cycle through it (the certificate), it is "easy" (polynomial runtime) to tell whether it is a Hamiltonian Cycle.

How can I convince you that there is ***not*** a Hamiltonian Cycle?

Hamiltonian Path U→V

Consider the problem of being given a graph, and two distinct vertices in the graph, and having to say whether there is a Hamiltonian Path (every vertex visited exactly once) between the two points.

Is this in NP?

Given the fact that Hamiltonian Cycle is in NPC, is this in NPC?