

NP-Complete Problems

Satisfiability (SAT)

Given a Boolean expression over n variables in conjunctive normal form, is there a way to assign values to the n variables that will make the entire expression true?

This general problem is NP-Complete.

What does that mean?

NP-Complete

I will leave the finer details of the definition of NP-Completeness to CMSC 451, but we can discuss the relationship between NPC problems.

You can convert/reduce *any* NPC problem to *any other* NPC problem in polynomial time.

This means that if *any* NPC problem is ever solved in polynomial time, then EVERY NP-Complete problem can be solved in polynomial time!

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Reductions

We often talk about showing that some “Problem B is *at least as hard as* “Problem A” as a runtime metric.

In our context, if:

- A is a well-understood problem
- B is a new problem

then if we can restate (“reduce”) A as B, doing that conversion process in polynomial time, that means that we could use the solution to B as a way to solve A with polynomial overhead.

That means that if we could solve B in polynomial time then we’d also be able to solve A in polynomial time.

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NPC Reductions

So, if **A** is a problem known to be in **NPC** and **B** is a new problem that looks like it will be computationally hard...

...if we can reduce a question about **A** to being a question about **B** (in only polynomial time) that means that if we could solve **B** in polynomial time then we'd also be able to solve **A** (and by transitivity **ANYTHING** in NP) in polynomial time.

This is half of what we need to show that a new problem **B** is also in NPC.

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Hamiltonian Cycles

Given an undirected graph, is there a cycle that visits every vertex exactly once?

If I present you with a graph and a cycle through it (the certificate), it is “easy” (polynomial runtime) to tell whether it is a Hamiltonian Cycle.

How can I convince you that there is not a Hamiltonian Cycle?

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Hamiltonian Path $U \rightarrow V$

Consider the problem of being given a graph, and two distinct vertices in the graph, and having to say whether there is a Hamiltonian Path (every vertex visited exactly once) between the two points.

Is this in NP?

Given the fact that Hamiltonian Cycle is in NPC, is this in NPC?