Cooperation in Multiple Access Channels with States

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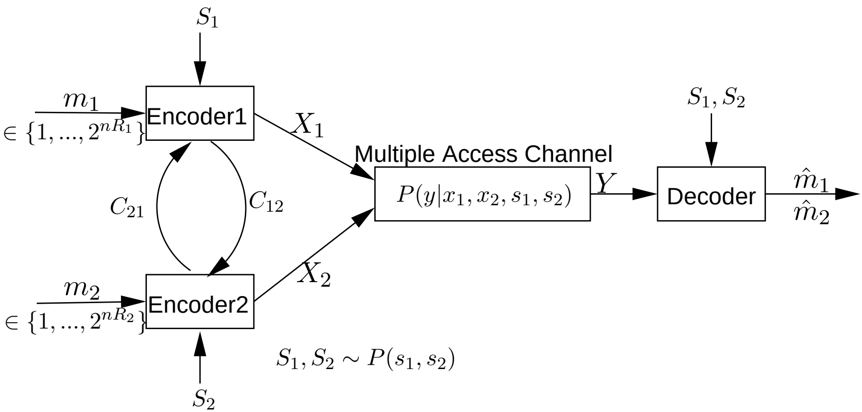
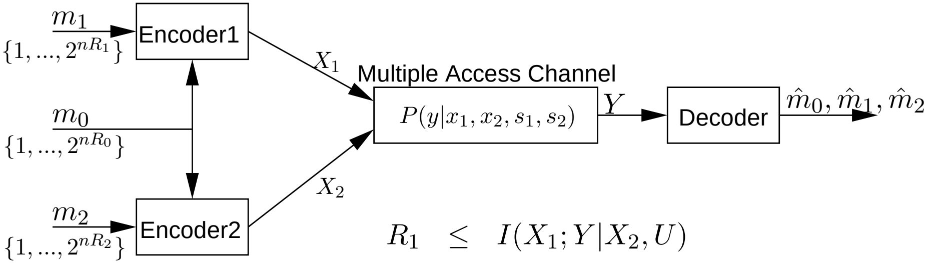
The communication setting considered in the talk



- Encoders have different partial state information
- This setting captures the idea of, simultaneously, sharing a part of the private messages m_1, m_2 and sharing the information on channel state (S_1, S_2).

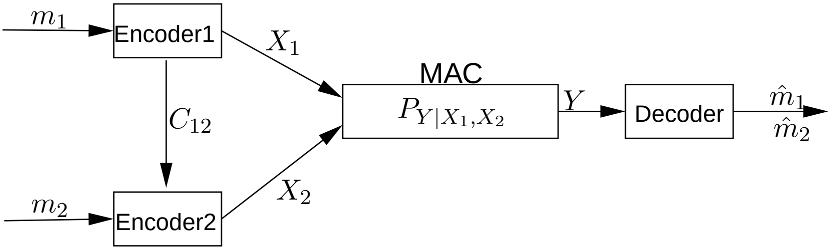
Background : Memoryless MAC (Mulitple Acess Channel) with common message



$$\begin{aligned} R_1 &\leq I(X_1; Y|X_2, U) \\ R_2 &\leq I(X_2; Y|X_1, U) \\ R_1 + R_2 &\leq I(X_1, X_2; Y|U) \\ R_0 + R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned}$$

for $P(u)P(x_1|u)P(x_2|u)P(y|x_1, x_2)$. [Slepien/Wolf73]

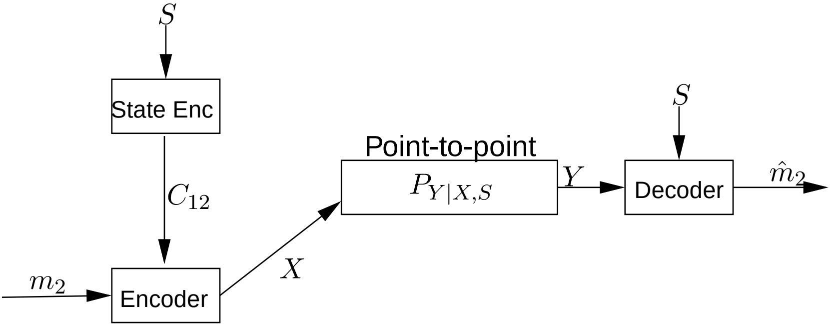
Memoryless MAC - Message cooperation



- Optimal coding scheme: Encoder 2 obtains part of the private message m_1 and use it as common message scheme m_0'. $R_0' = C_{12}$, $R_1' = R_1 - C_{12}$, $R_2' = R_2$.

[Willems82]

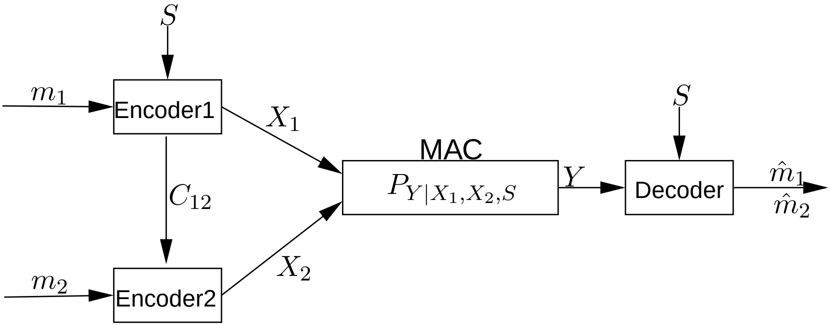
Special case: Point-to-point with encoded state information



$$
\begin{aligned}
C_{12} &\geq I(U;S), \\
R &\leq I(X;Y,S|U) = I(X;Y|S,U),
\end{aligned}
$$

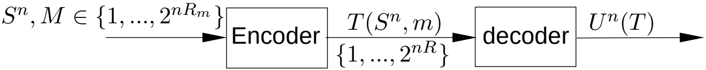
for $P(s)P(u|s)P(x|u)P(y|x,s)$. [Cemal/Steinberg07]

Message-state cooperation



- Encopder 1 and Decoder have state information
- This setting captures the idea of, simultaneously, sharing a part of the private message m_1 and sharing the information on channel state S.

Simplified problem



S^n is i.i.d., $S \sim P(s)$

Generate a sequence U^n such that

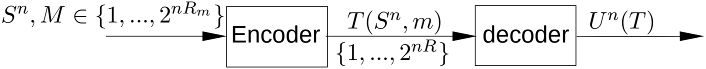
1. U^n is jointly typical with S^n, i.e.,

$$\lim_{n\to\infty} \Pr\{(U^n, S^n) \in T_\epsilon^{(n)}(U, S)\} = 1,$$

2. there exists a function $g(U^n)$ such that

$$\lim_{n\to\infty} \Pr\{g(U^n) \neq M\} = 0.$$

Simplified problem-solution



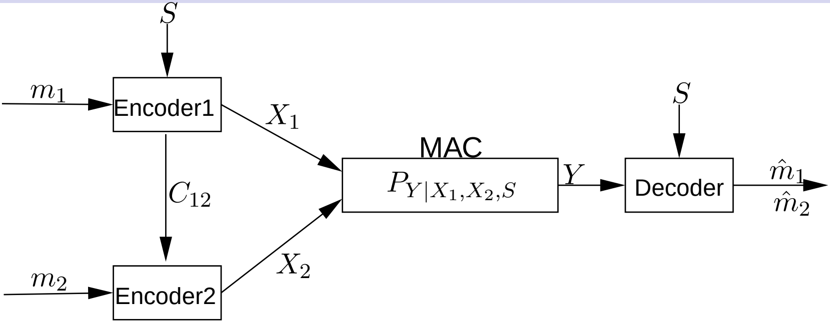
S^n is i.i.d., $S \sim P(s)$

Theorem

$R > R_m + I(U;S)$ *and* $H(U|S) \geq R_m$.

1. Generate 2^{nR_m} bins, in each bin $2^{n(I(U;S)+\epsilon)}$ codewords, generated i.i.d. $\sim P(u)$.
2. The message is associated with Bin.
3. A codeword is chosen from the bin to be jointly typical with S^n.

Message-state cooperation

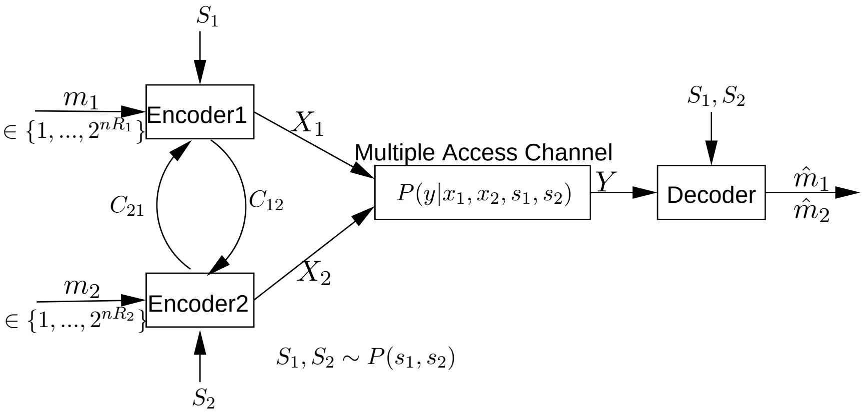


Theorem

$$
\begin{aligned}
C_{12} &\geq I(U;S) \\
R_1 &\leq I(X_1;Y|X_2,S,U) + C_{12} - I(U;S) \\
R_2 &\leq I(X_2;Y|X_1,S,U) \\
R_1 + R_2 &\leq \min \left\{ \begin{array}{c} I(X_1,X_2;Y|S,U) + C_{12} - I(U;S), \\ I(X_1,X_2;Y|S) \end{array} \right\},
\end{aligned}
$$

for distributions $P(s)P(u,x_1|s)P(x_2|u)P(y|x_1,x_2,s)$.

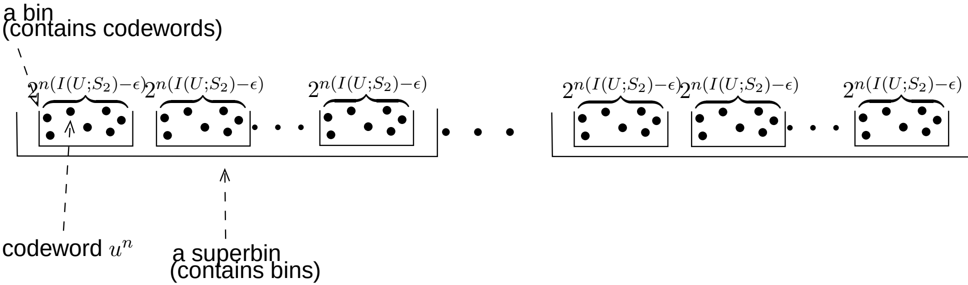
General case



Here a double binning is needed.

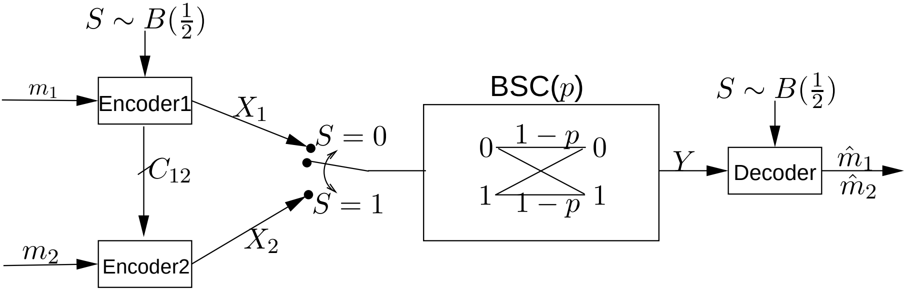
1. First layer for generating coordination, as in Wyner-Ziv.
2. Second layer for transmitting a message.

Combining message and state using new double binning

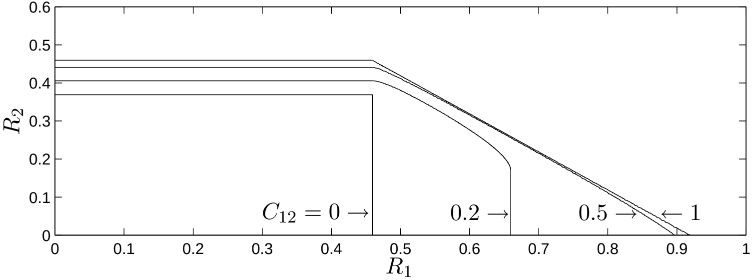


- consists of two-layer bins (codes), where in the first layer we have bins(codes) that contain codewords and in the second layer we have superbins that contain bins.

Example

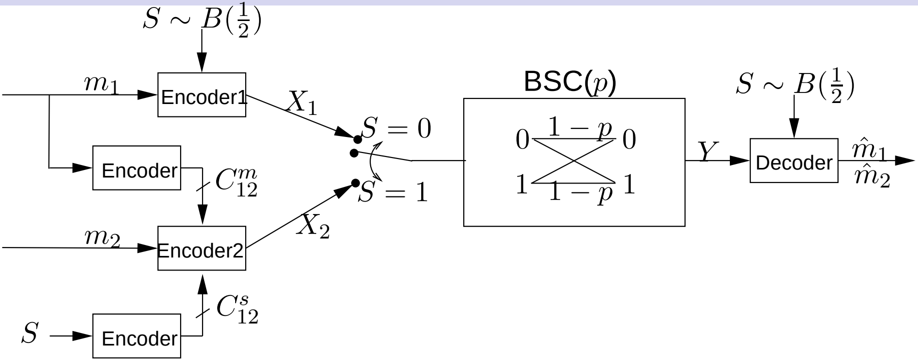


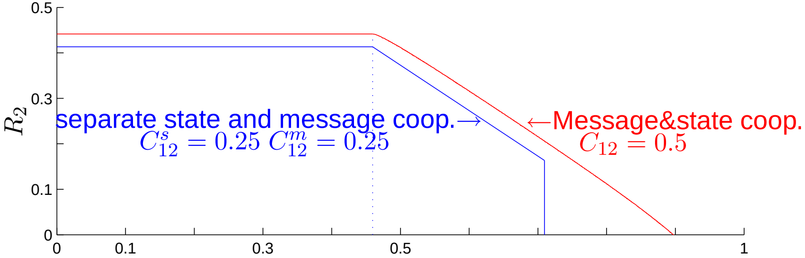
$S \sim B(\frac{1}{2})$
m_1
Encoder1
X_1
$S = 0$
C_{12}
$S = 1$
X_2
m_2
Encoder2
BSC(p)
0
$1 - p$
0
1
$1 - p$
1
Y
Decoder
$\hat{m}_1$
$\hat{m}_2$
$S \sim B(\frac{1}{2})$



0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R_1
$C_{12} = 0 \rightarrow$
$0.2 \rightarrow$
$0.5 \rightarrow$
$\leftarrow 1$

Separate links for state and for coordination





Summary

- We investigated MAC with cooperating encoders and partial state information.
- The cooperation has a two-fold purpose:
  - generating empirical state coordination
  - sharing the private messages
- Double binning- an optimal technique for combining state and message
- Message and state cooperation strictly increases the capacity.

Summary

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- The cooperation has a two-fold purpose:
  - generating empirical state coordination
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- Double binning- an optimal technique for combining state and message
- Message and state cooperation strictly increases the capacity.

Thank you very much !