# Beamforming

A brief introduction

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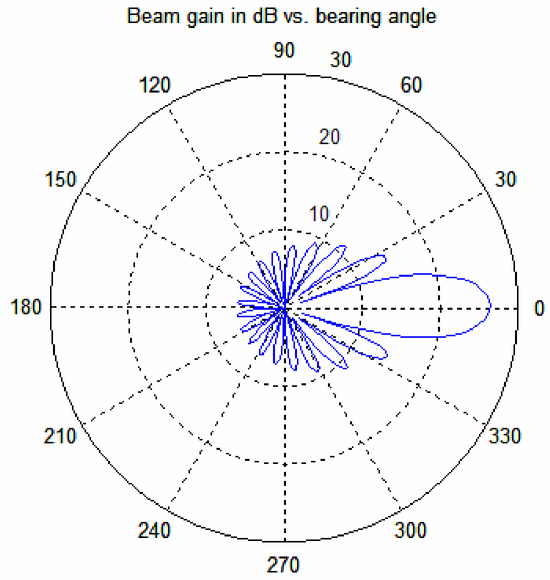
# References

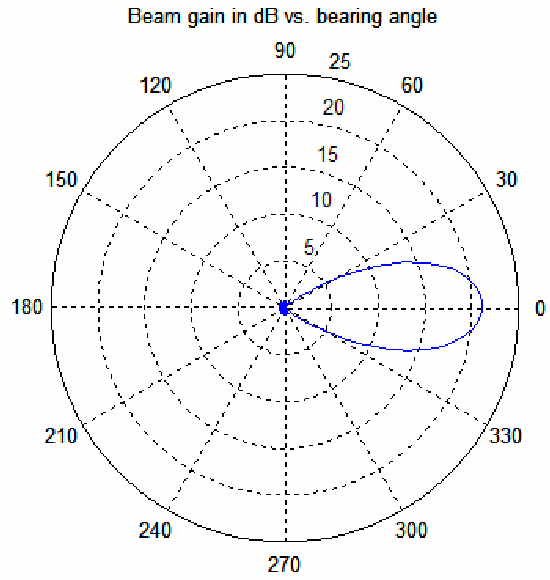

- Barry D. Van Veen and Kevin Buckley, “Beamforming: A Versatile Approach to Spatial Filtering,” IEEE ASSP Magazine, April 1988, pp. 4-24,
  - This is a nice tutorial. Good introduction to the topic, including several classical adaptive beamforming techniques.
- Harry L. Van Trees, *Optimum Array Processing. Part IV of Detection, Estimation and Modulation Theory, Wiley Interscience, New York, 2002.*
  - An exhaustive and thorough reference book.

# Beamforming is Spatial Filtering

- Sensors in any wave propagation medium (acoustic, electromagnetic) can form a response pattern with higher sensitivity in desired directions.




Pencil beam response, no windowing




Pencil beam response, Hamming window

# Two Types of Beamformers

- Method 1: Single sensor with directional response due to reflector, aperture size, baffles, pipes, etc.

Green Bank Telescope, National Radio Astronomy Observatory, West Virginia.

100 m clear aperture. Largest fully steerable antenna in the world.

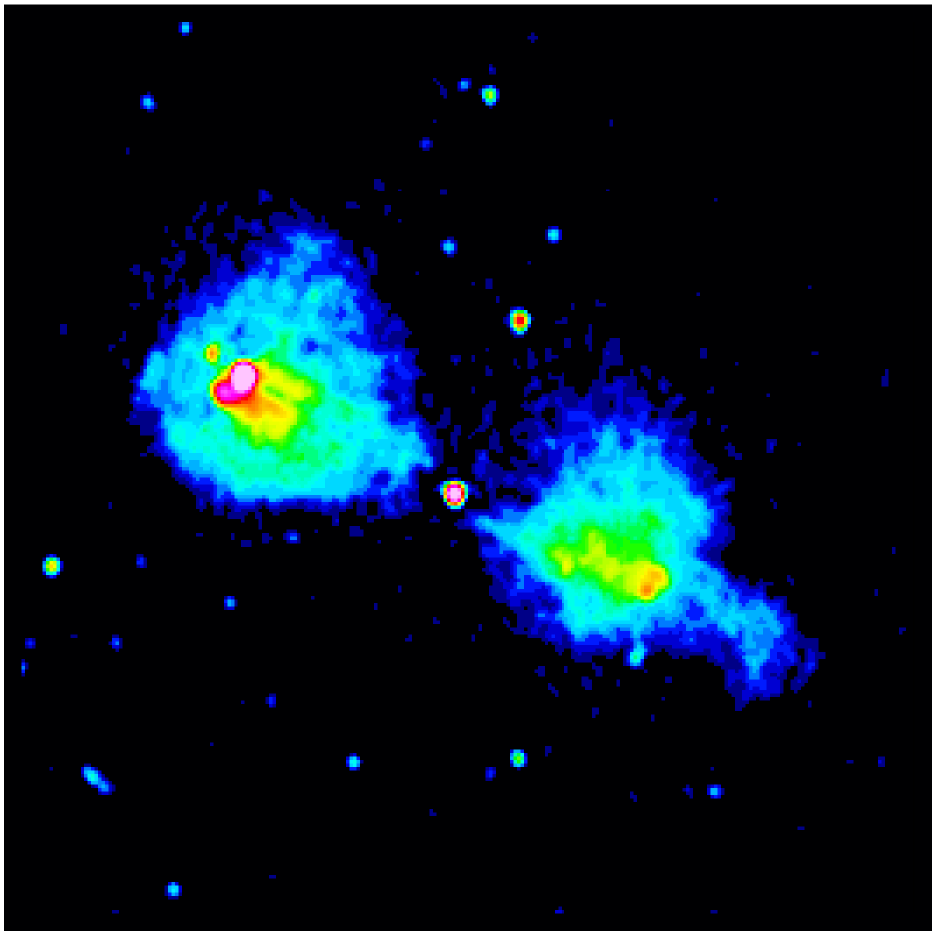
# Two Types of Beamformers (cont.)

- Method 2: Sensor arrays.  Used in SONAR, RADAR, communications, medical imaging, radio astronomy, etc.

Line array of directional sensors
Westerbork Synthesis Array Radio Telescope, (WSRT) the Neterlands.

(Thanks to ASTRON for these images)

# A WSRT Image Made with Beamforming and Array Processing Techniques

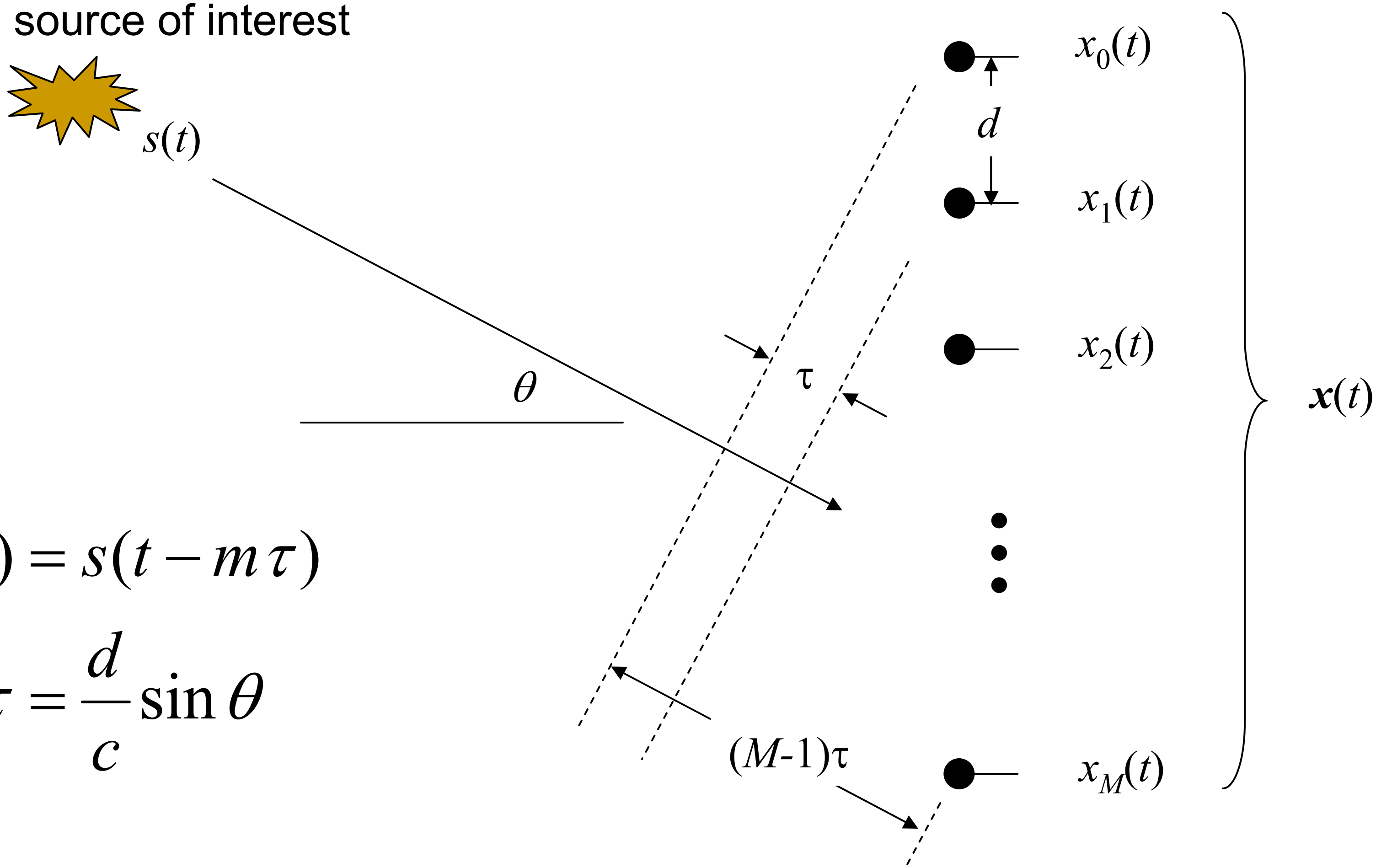


- WSRT 49 cm (612 MHz) image of 2 Mpc radio galaxy DA240
- Symmetric ionized gas jets ejected from black hole in central core.

(Thanks to ASTRON for these images)

# The Uniform Line Array

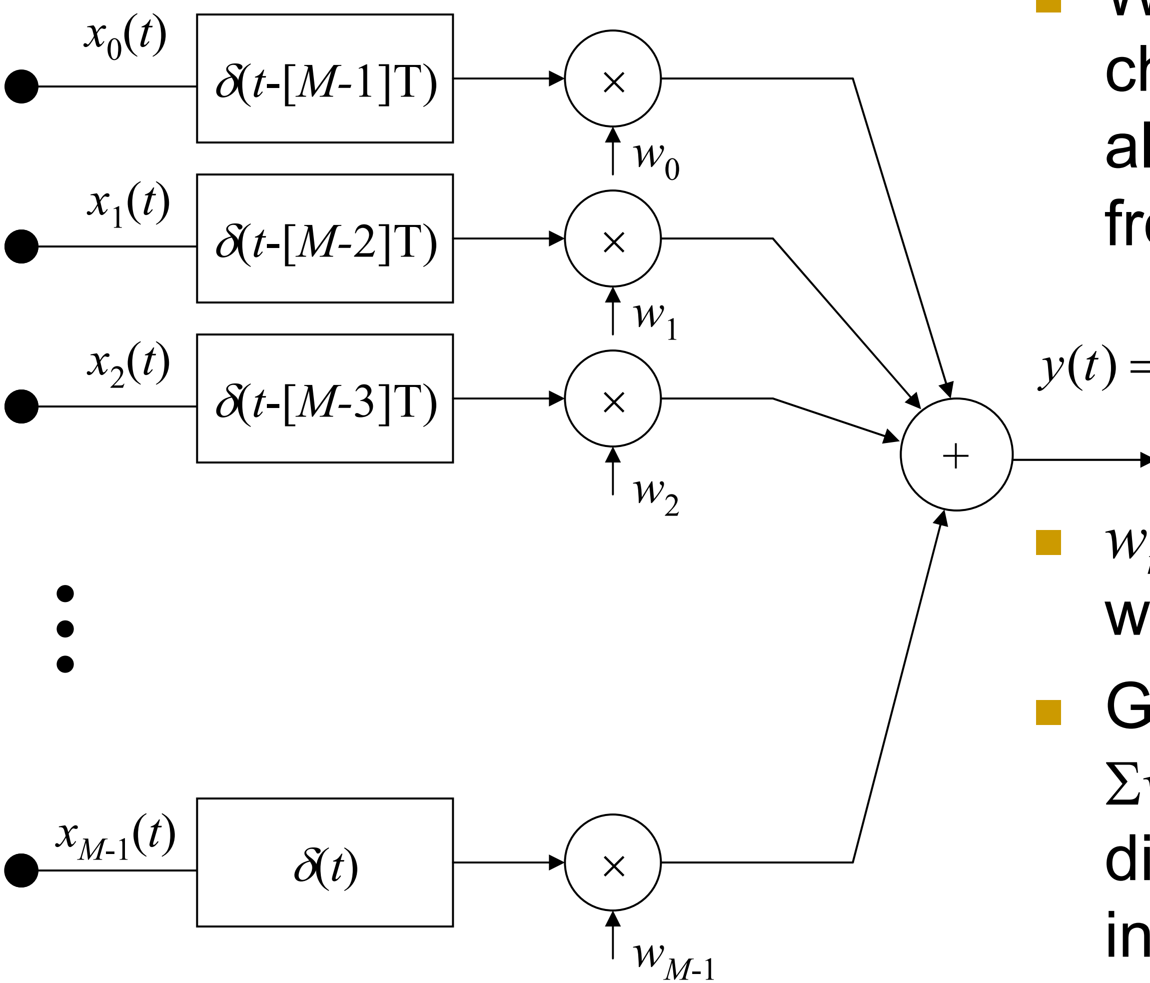
Signal source of interest




$$x_m(t) = s(t - m\tau)$$

$$\tau = \frac{d}{c}\sin\theta$$

# Delay-Sum Beamformer (broadband)

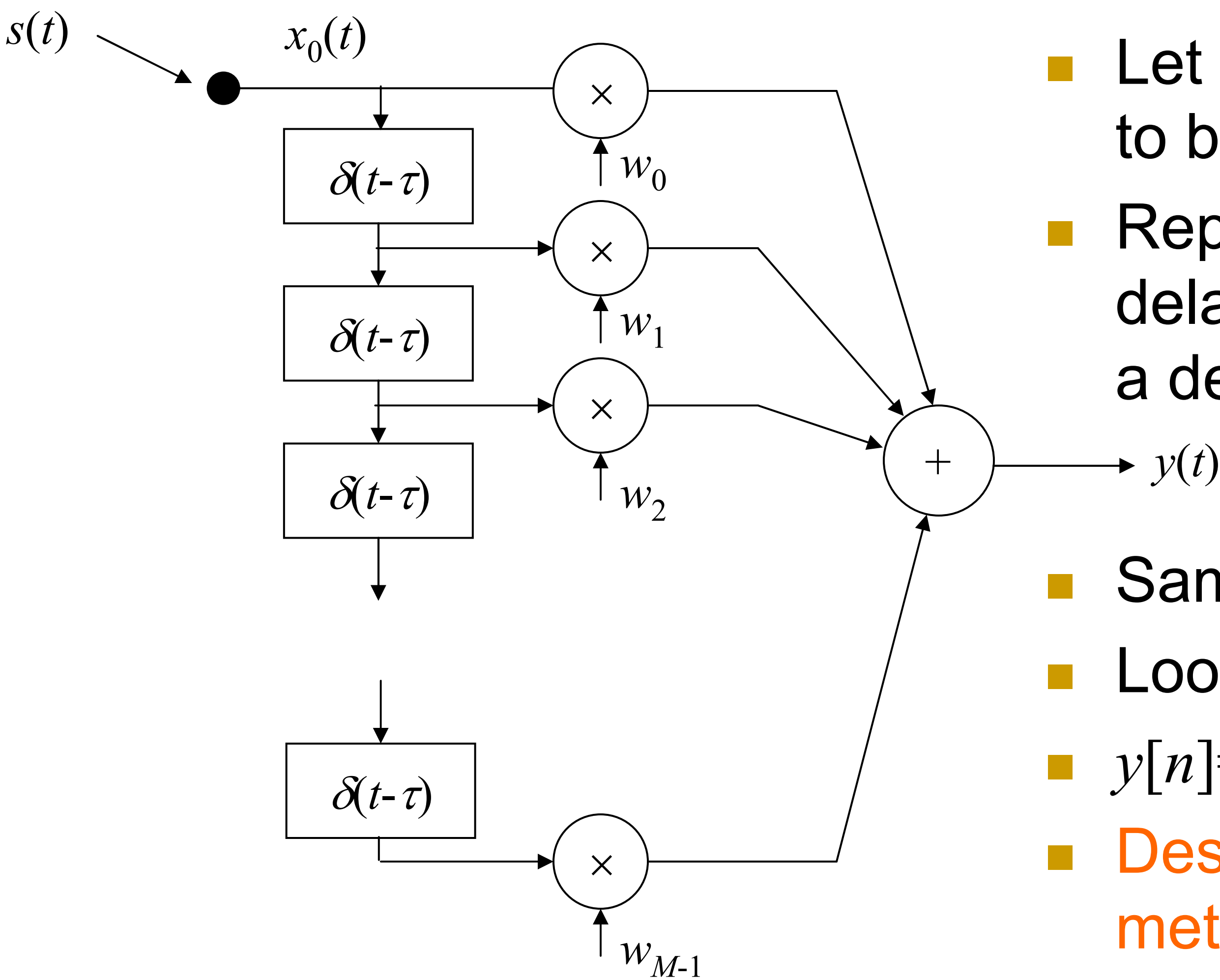



- When T = $\tau$, the channels are all time aligned for a signal from direction $\theta$.

$$y(t) = \sum_{m=0}^{M-1} w_m x_m(t - [M - m - 1]T)$$

- $w_m$ are beamformer weights.
- Gain in direction $\theta$ is $\Sigma w_m$. Less in other directions due to incoherent addition.

# Similarity to FIR filter




- Let T=0 (steer beam to broadside).
- Represent signal delay across array as a delay line.
- Sample: $x[\mathrm{n}] = x_0(nT)$ .
- Looks like an FIR filter!
- $y[n]=x[n] * w[n]$
- Design $\boldsymbol{w}$ with FIR methods!

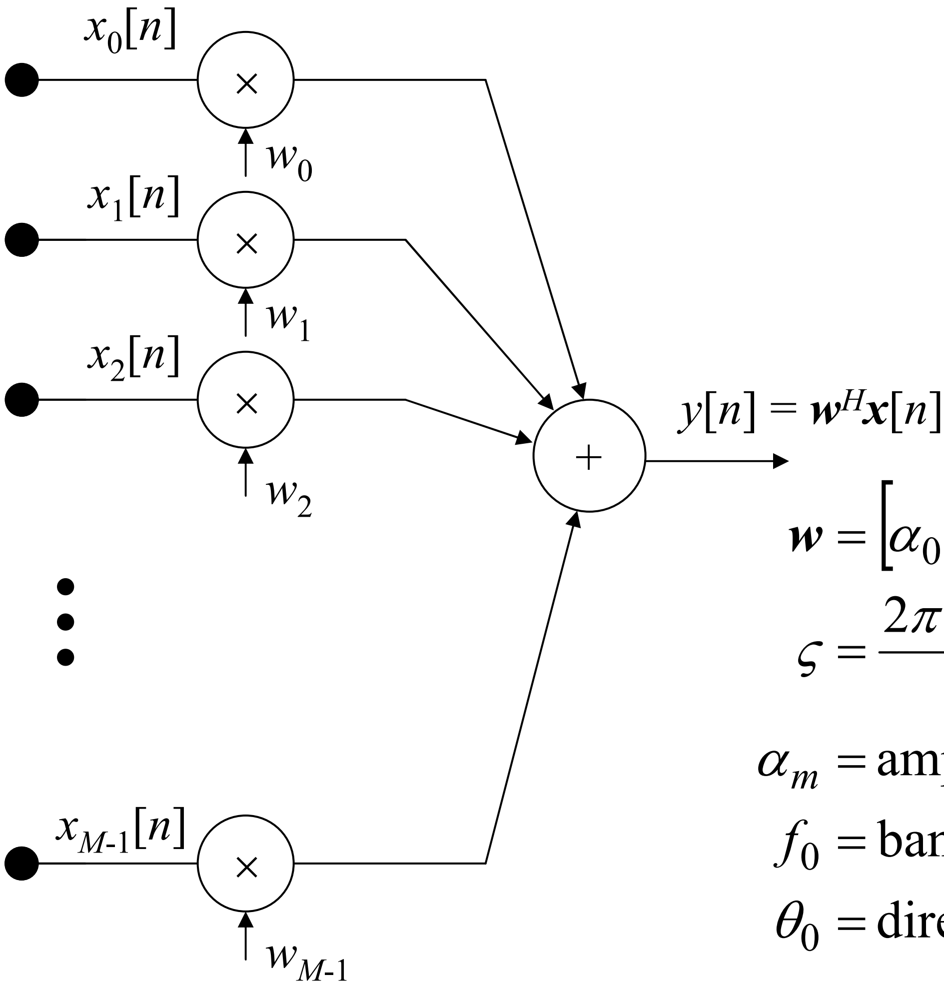
# Narrowband Beamformer

- Narrowband assumption: Let $s(t)$ be bandpass with $BW << c / (M\text{-}1)d$ Hz.
- This means the phase difference between upper and lower band edges for propagation across the entire array is small, e.g. < $\pi/10$ radians.
- Most communications signals fit this model.
- If signal is not narrowband, bandpass filter it and build a new beamformer for each subband.
- Sample the array $\boldsymbol{x}[\mathrm{n}] = \boldsymbol{x}(nT)$,

$$\boldsymbol{x}[n] = \left[1, e^{-j\xi}, \cdots, e^{-j(M-1)\xi}\right]^T s[n], \quad \xi = \frac{2\pi f_0 d}{c}\sin\theta.$$

- We can now eliminate time delays and use complex weights, $\boldsymbol{w} = [w_0, \ldots, w_{M\text{-}1}]^T$, to both steer (phase align) and weight (control beam shape).

# Narrowband Phased Array




$$\mathbf{w} = \left[\alpha_0, \alpha_1 e^{-j\varsigma}, \cdots, \alpha_{M-1} e^{-j(M-1)\varsigma}\right]^T,$$

$$\varsigma = \frac{2\pi f_0 d}{c} \sin\theta_0,$$

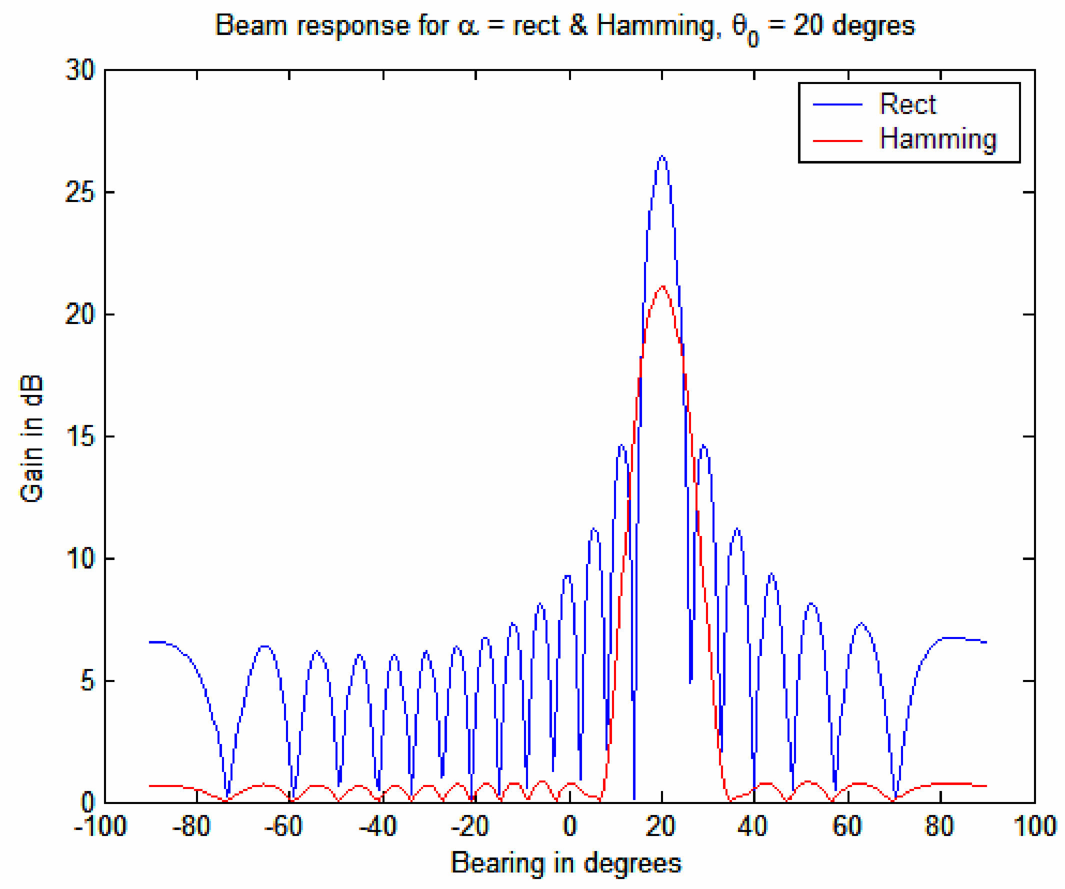
$\alpha_m$ = amplitude weight for sensor $m$,

$f_0$ = bandpass center frequency, Hz,

$\theta_0$ = direction of max response.

# Beam Response Plots

- Fix $w$ and plot $|y[n]|$ as a function of signal arrival angle, $\theta$.
- Design $\alpha$ to control sidelobe levels.



# FFT Implementation

- Suppose you want to form many beams at once, in different directions.
- Example: SONOR towed line array forms beams to look in many directions at once for simple direction finding. If beam $k$ steered to $\theta_k$, has strongest signal, we assume source is in that direction.

$$y_k[n] = \boldsymbol{w}_k^H \boldsymbol{x}[n],$$

$$\boldsymbol{w}_k = \left[\alpha_0, \alpha_1 e^{-j\varsigma_k}, \cdots, \alpha_{M-1} e^{-j(M-1)\varsigma_k}\right]^T,$$

$$\varsigma_k = \frac{2\pi f_0 d}{c} \sin\theta_k. \text{ This can be written :}$$

$$y_k[n] = \sum_{m=0}^{M-1} \alpha_m x_m[n] e^{-jm\varsigma_k}.$$

- Now let $\zeta_k = k2\pi/M$ and solve for $\theta_k$.
- This looks like a DFT across sensor channels!
- Frequency = Direction!

# FFT Multiple Beamformer Diagram