

Digging Deeper Into the Secondary Mathematics Common Core State Standards

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for

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Overview

- Reminders from this morning's session
- Toward college and career readiness
- What is needed?
- A look inside the CCSS for High School mathematics
- A look inside the Model Pathways
- About serving all students



Mathematics Achievement Trends

- Achievement is up by many indicators
 - Significant growth in grades 4 and 8
 - High school diploma, math course taking
 - College attendance, college completion
- High school achievement is flat
 - U.S. 15-year-olds lag in applying math
 - Poor results on H.S. end-of-course exams
 - College remediation rates remain high
- **Today's world demands more**



Stagnancy Is not a Crisis!

- A crisis is a “turning point”
 - John Ewing, Math for America
- We have long-term structural problems
 - And many inadequate improvement initiatives
- We need to make fundamental long-term changes over decades

- The Common Core State Standards are a response to this challenge

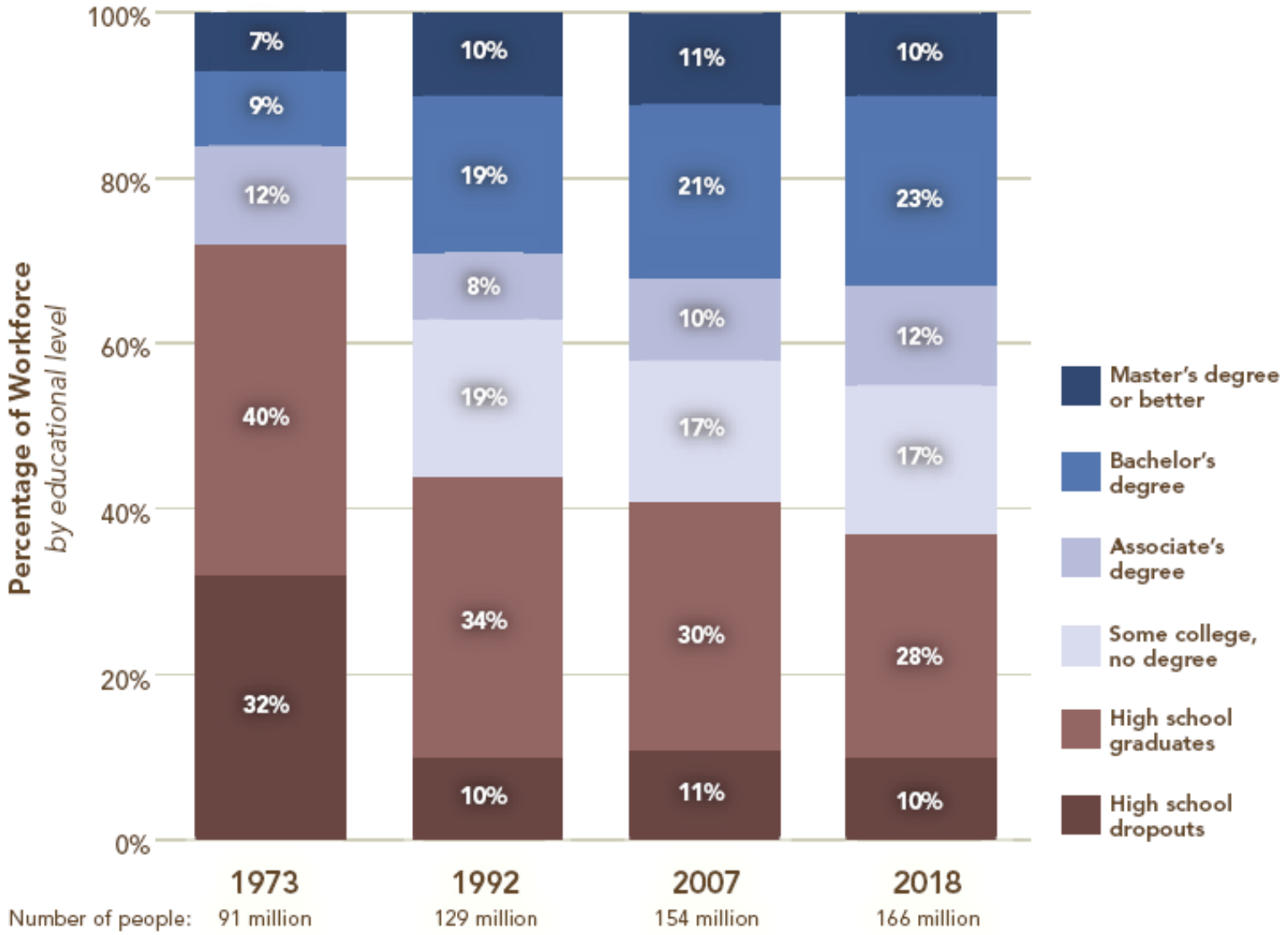


Toward College and Career Readiness

- More states are requiring Algebra 2 or its equivalent (A2E)
 - A proxy for college and career readiness
- CCSS provides a definition of college and career readiness: All standards not indicated by (+)
- We need to make A2E rigorous, relevant, and attainable
 - Your parents' Algebra 2 will not do
 - Is this feasible?
- What is the rationale?



More Jobs Require Some College



Not Enough College Graduates

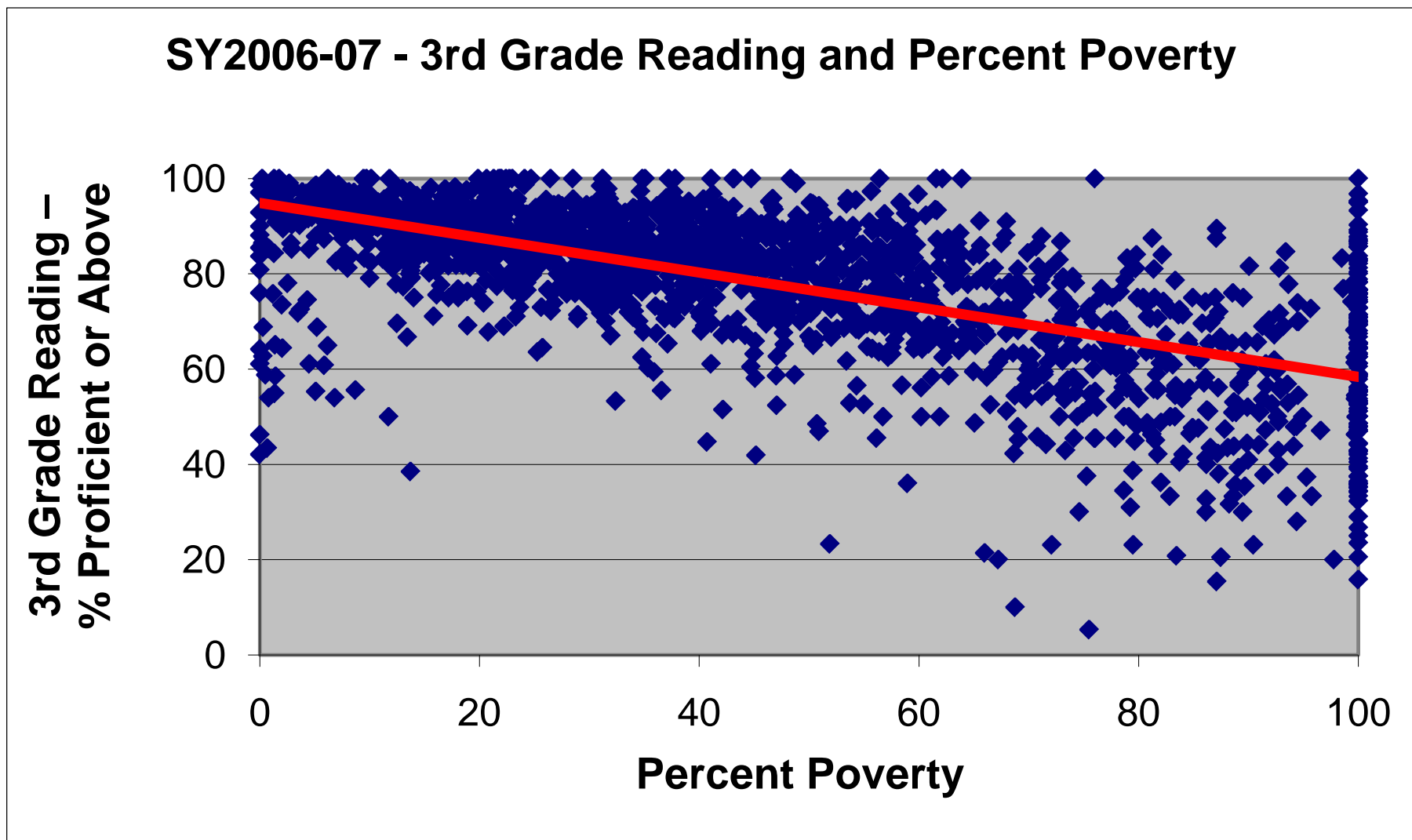
- “Demand for workers with college educations will outpace supply to the tune of 300,000 per year. By 2018, the postsecondary system will have produced 3 million fewer college graduates than demanded by the labor market.”

(Carnevale, Smith, Strohl, 2010, *Help Wanted*, p. 16)

See <http://cew.georgetown.edu/jobs2018/>



Who Can Interpret This?



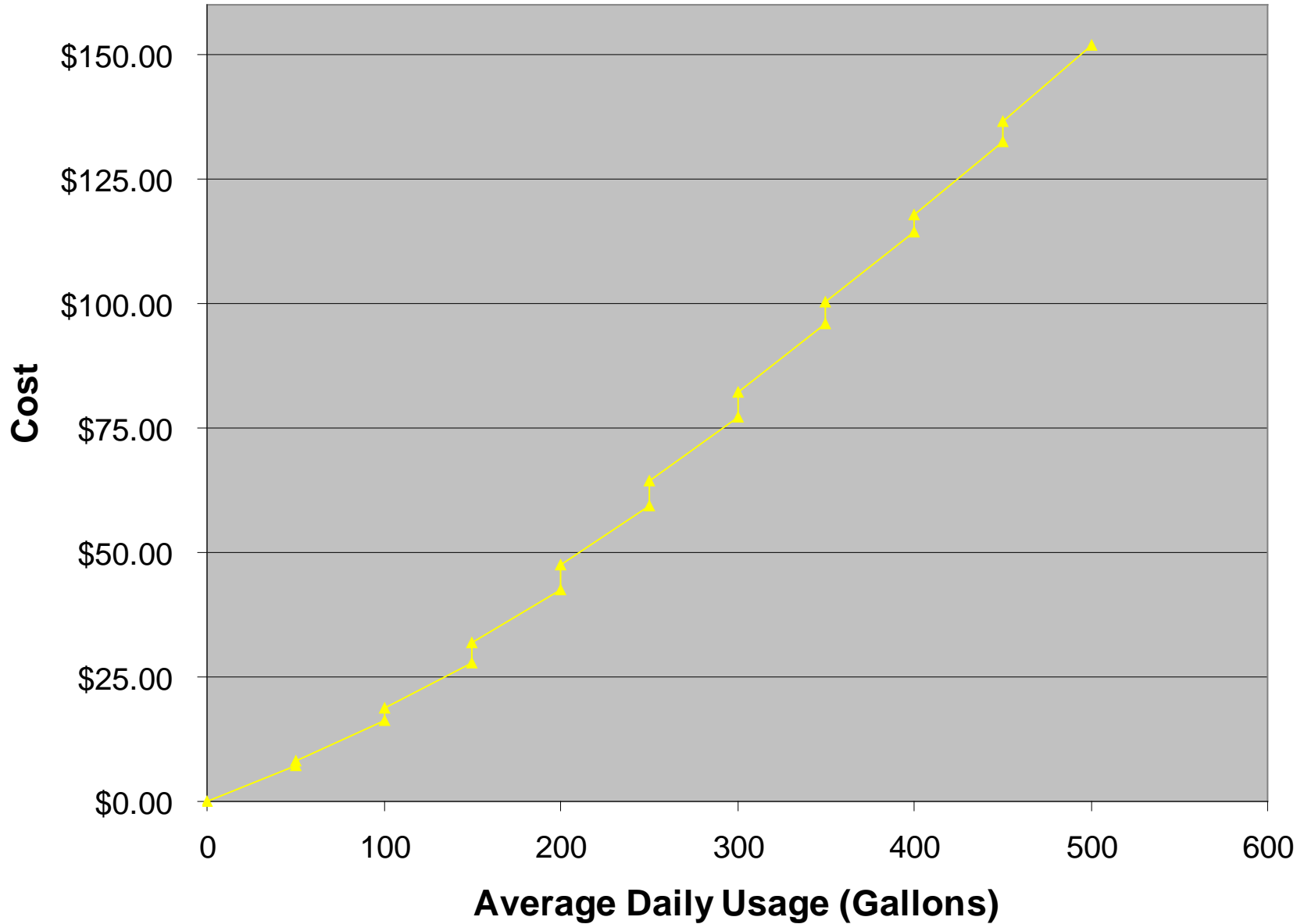
Washington Suburban Sanitary Commission

Rate Schedule, July 1, 2008

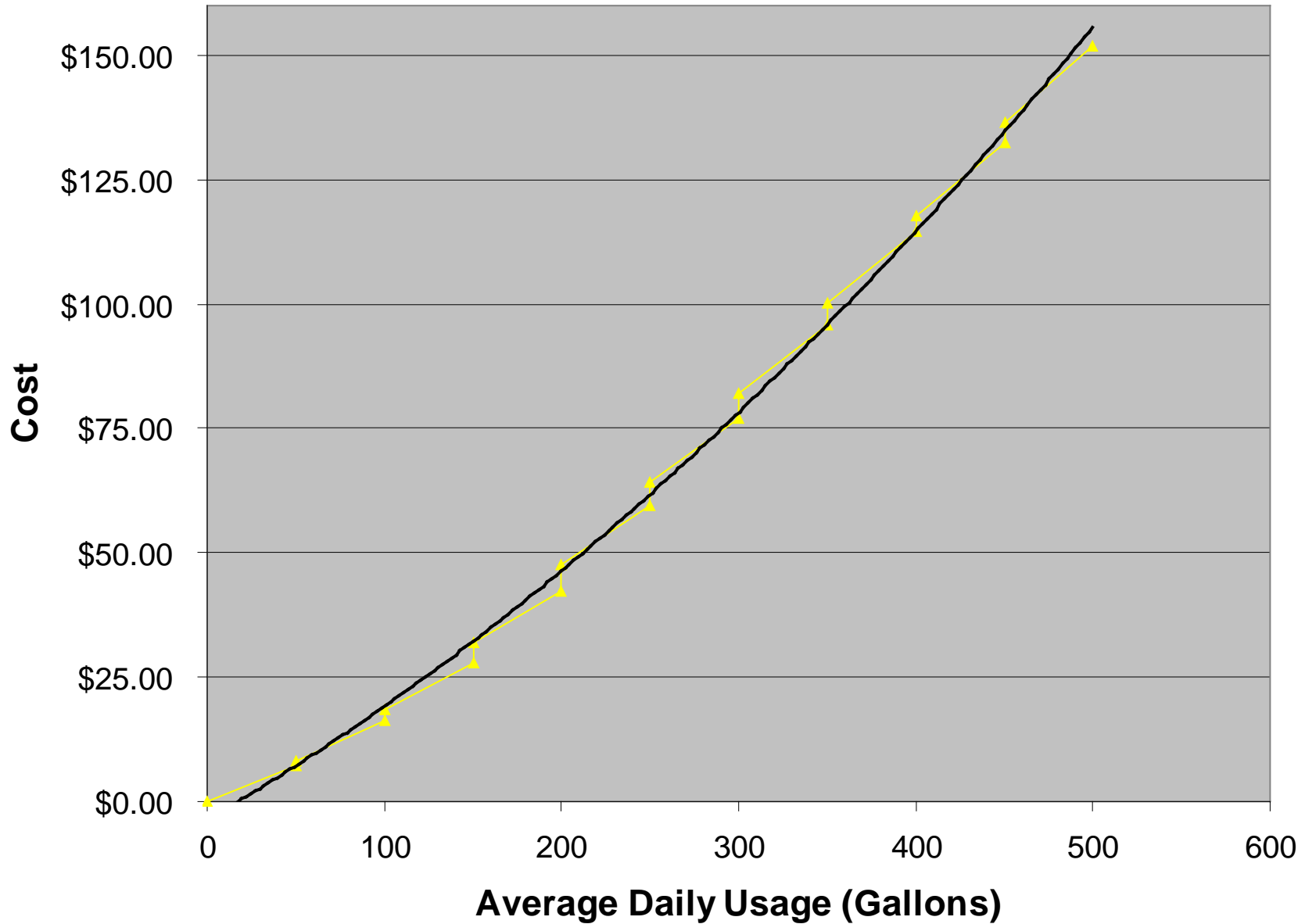
Average Daily Consumption (Gallons/Day)	Water Rate Per 1,000 Gallons	Sewer Rate Per 1,000 Gallons	Combined Rate Per 1,000 Gallons
0-49	\$1.97	\$2.77	\$4.74
50 - 99	2.21	3.22	5.43
100 - 149	2.42	3.79	6.21
150 - 199	2.71	4.36	7.07
200 - 249	3.17	4.76	7.93
250 - 299	3.43	5.14	8.57
300 - 349	3.63	5.50	9.13
350 - 399	3.79	5.75	9.54
400 - 449	3.94	5.88	9.82
...

Source: <http://www.wsscwater.com/service/rates.cfm>

Monthly Water and Sewer Cost



Monthly Water and Sewer Cost



Rationale for A2E for All

- Algebra 1 and Geometry provide insufficient readiness for college and most careers
- All students need proficiency in A2E for
 - Many careers, with or without college
 - Informed citizenship
 - Individual empowerment
- High school mathematics should open doors
 - But adult decisions often close doors for students
 - After students complete A2E, they have choices
- And again, not your parents' Algebra 2



What Is Needed?

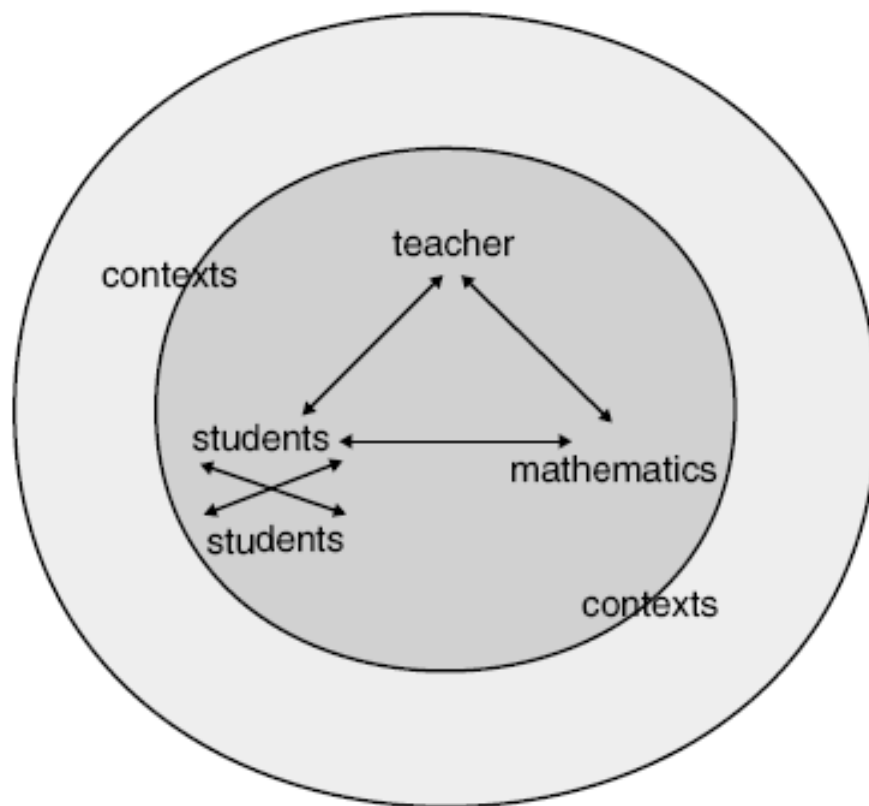
- Renewed curriculum and instruction
 - Across middle and high school toward a rigorous and relevant A2E
- “Focal Points” for high school
- Support for students are behind
 - To help them catch up

- The CCSS and the Model Pathways are foundational responses to these needs



Instruction as Interaction

What matters are the interactions, in classrooms, among the teacher, the students, and the mathematical ideas



Source: Cohen & Ball, 1999, 2000, as cited in NRC, 2001.

Secondary Mathematics Problems

- Three ways to improve achievement
 - Invest in the knowledge and skill of teacher
 - Change the level of content
 - Change the role of the student in the instructional process.
- Problem of *access*
- Problem of *teaching quality*
- Both of these problems are perpetuated and exacerbated by pervasive myths

Myth: Basic Skills First

- Myth: Students cannot engage in high-level thinking until they have mastered basic skills
- View is pervasive in high schools, which function primarily as sorting mechanisms
- Students are denied access to quality instruction because of adult judgments
- High schools and their curricula were not designed to teach high-level content to all students

Myth: Teachers Are Born

- Myth: Teaching ability is a natural predisposition
 - Teaching is an art that cannot be learned
 - The system does not learn; we rarely refine the wisdom of practice
- Teaching is a mass profession
 - Ordinary people doing extraordinary things (Japan)
- Teaching is a skill, with a knowledge base



High School Math Curriculum

- Algebra 1 and Geometry courses typically
 - Reteach much middle grades content
- Algebra 2 courses typically
 - Reteach Algebra 1
 - Include some statistics and probability
 - Include optional topics
 - Pre-teach Precalculus content
- ***Algebra 2 is two miles wide***
 - *And a quarter inch deep*



An Algebra Idea Across K–12

- Compare and contrast: patterns, functions, and sequences
- In grades K–8, students study patterns
- In grades 9–11, students study functions
- In grade 12, students might study sequences
- A sequence is a pattern
- A pattern suggests a function
- A sequence is a function with a domain consisting of whole numbers



A Look Inside the CCSS for High School Mathematics



CCSS for High School Mathematics

- Organized in “Conceptual Categories”
 - Number and Quantity
 - Algebra
 - Functions
 - Modeling
 - Geometry
 - Statistics and Probability
- Conceptual categories are not courses
- Additional mathematics for advanced courses indicated by (+)
- Standards with connections to modeling indicated by (★)



Conceptual Category Introduction

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here



Conceptual Category Overview

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Domain

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Cluster

Mathematical Practices

1. Apply geometric concepts in modeling situations
2. Mathematical Practices
3. Make sense of problems and persevere in solving them.
4. Reason abstractly and quantitatively.
5. Construct viable arguments and critique the reasoning of others.
6. Model with mathematics.
7. Use appropriate tools strategically.
8. Attend to precision.
9. Look for and make use of structure.
10. Look for and express regularity in repeated reasoning.



Standards for High School Math

Domain

The Complex Number System

N-CN

Cluster

Perform arithmetic operations with complex numbers.

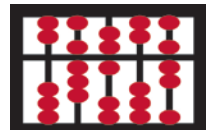
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Standard

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number

Advanced



ASSM

HS CCSS: Changing Content Emphases

- Number and quantity
 - Number systems, attention to units
- Modeling
 - Threaded throughout the standards
- Geometry
 - Proof for all, based on transformations
- Algebra and functions
 - Organized by mathematical practices
- Statistics and probability
 - Inference for all, based on simulation

Calls from Business

- Students need to solve interdisciplinary problems
 - So teachers need to work across disciplines
 - And across algebra, geometry, and data analysis
- Students need to learn to collaborate
 - So teachers need to collaborate
 - And take responsibility for **all** students



A Look Inside the Model Pathways



High School Mathematical Pathways

- Two main pathways:

- Traditional: Two algebra courses and a geometry course, with statistics and probability in each
- Integrated: Three courses, each of which includes algebra, geometry, statistics, and probability

*Typical
in U.S.*

- Both pathways:

- Complete the Core in the third year
- Include the same “critical areas”
- Require rethinking high school mathematics
- Prepare students for a menu of fourth-year courses

*Typical
outside U.S.*



Two Main Pathways

Courses in higher level mathematics: Precalculus, Calculus*, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.

Algebra II

Geometry

High School
Algebra I

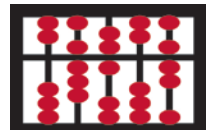
Traditional Pathway
Typical in U.S.

Mathematics III

Mathematics II

Mathematics I

Integrated Pathway
Typical outside of U.S.



ASSM

Pathway Overview

Overview of the Traditional Pathway for the Common Core State Mathematics

This table shows the domains and clusters in each course in the Traditional Pathway. Each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics.

Domain

Conceptual Category

Courses

Clusters, Notes, and Standards

Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses ¹
Number and Quantity	<ul style="list-style-type: none"> Extend the properties of exponents to rational exponents. N.RN.1, 2 Use properties of rational and irrational numbers. N.RN.3 			
Quantities	<ul style="list-style-type: none"> Reason quantitatively and use units to solve problems. <i>Foundation for work with expressions, equations and functions</i> N.Q.1, 2, 3 			
The Complex Number System			<ul style="list-style-type: none"> Perform arithmetic operations with complex numbers. N.CN.1, 2 Use complex numbers in polynomial identities and equations. 	<ul style="list-style-type: none"> Perform arithmetic operations with complex numbers. (+) N.CN.3 Represent complex numbers and their operations on the complex plane.

Course Overview: Critical Areas (units)

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe

Course Detail by Unit (critical area)

Unit 1: Relationships Between Quantities

By the end of eighth grade students have learned to solve linear equations in one variable and use graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit Title and Overview

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	Common Core Standards
<p>SKILLS TO MAINTAIN</p> <p><i>Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.*</i></p> <ul style="list-style-type: none">Reason quantitatively and use units to solve problems. <p><i>Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.</i></p>	<p>N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> <p>N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>

Standards Associated with Cluster

Cluster

Instructional Note

Algebra 1 in Eighth Grade?

- The Grade 8 CCSS includes much of Algebra 1 *for all students*
- Model Pathway H.S. Algebra 1 builds on it
 - *So do not skip* the Grade 8 CCSS
- Two “compacted” Pathways for grades 7–9 provide paths to Calculus in high school
- Offer “compacted” courses to students who are willing to do the extra work
 - And make sure students succeed

All Students Means ALL Students

- How well are you serving the following groups?
 - High-achieving students
 - Middle-achieving students
 - Low-achieving students
- District goals sometimes consider only the state assessments
- Do you spend time considering progress of and projections for individual students?



High-Achieving Students

- What percentage of your students take AP courses?
- How successful are your calculus offerings?
 - High school calculus should be AP Calculus.
- What happens to accelerated students?
 - Do they take mathematics every year?
 - If not, why not?
 - Are they successful?
- What about radically accelerated students?



Middle-Achieving Students

- How many of your seniors are taking significant (non-remedial) mathematics?
- Do you have fourth-year alternatives to Precalculus?
 - AP Statistics
 - Advanced Quantitative Reasoning
 - National work:

http://math.arizona.edu/~ime/2008-09/1018_retreat.html



Low-Achieving Students

- How many of your seniors are taking low-level mathematics?
- Does your program help low-achieving students get back on track?
 - You can't help students catch up by slowing them down
- A guiding principle for intervention:
 - Give all students access to the regular curriculum, and provide differentiated instruction and support
- How many tracks do you need?



Prealgebra at High School?

- Prealgebra should not count as high school mathematics
 - Preparation for Ohio Graduation Test
 - College admissions requirements (and NCAA)
 - Reaching college and career readiness
- When students are behind
 - Give them access to the regular curriculum *and* extra support
 - Response to Intervention

Implementation Questions for You

- Can we empower high school mathematics teachers to make necessary changes?
 - Curriculum, instruction, support, programs, ...
- Can we get the incentives right?
 - So that teachers will regularly work together to reach more students more of the time
 - So that we all learn from and with our best teachers
- Can we bring mathematics leadership to the decision-making table?
 - So that school-improvement efforts focus on long-term improvements not short-term fixes



Resources for H.S. Improvement

- NCTM's high school reports
 - [Focus on Reasoning and Sense Making](#)
- Use the Common Core State Standards
 - Identify A2E content for all students
- Use Pathways and Standards Progressions
 - Reduce redundancy and incoherence
 - Use previous mathematics in service of new ideas
- Ohio's [Model Curriculum](#)
 - To be adopted in March 2011



Closing Thought

- “These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.”
 - (CCSS, 2010, p. 5)

