# Spatially-Localized Compressed Sensing and Routing in Multi-Hop Sensor Networks [1]

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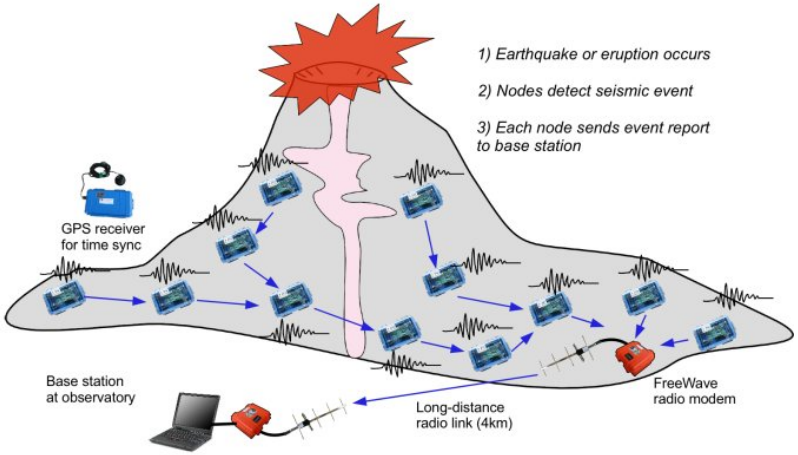
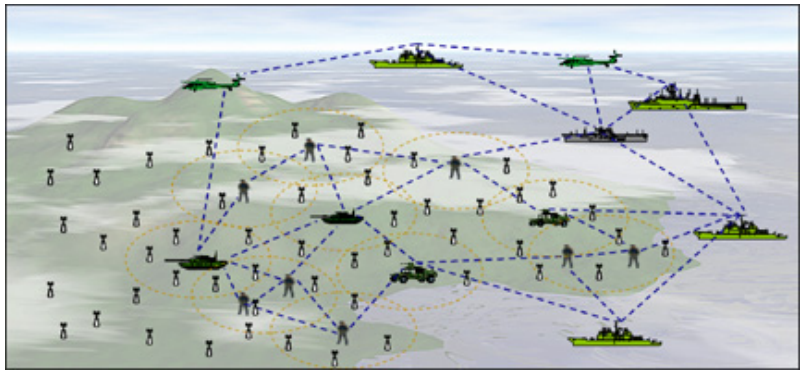
# Table of Contents

# Correlated data gathering



- Continuous data gathering using a wireless sensor network
- Spatially distributed data has spatio-temporal correlations
- Compression is required for energy efficiency and longevity
- Many joint routing and compression techniques are proposed [2] [3] [4]

[2] Cristescu, R., Beferull-Lozano, B., Vetterli, M.: On network correlated data gathering. In: INFOCOM. (March 2004)

[3] Pattem, S., Krishnamachari, B., Govindan, R.: The impact of spatial correlation on routing with compression in wireless sensor networks. In: IPSN. (April 2004)

[4] von Rickenbach, P., Wattenhofer, R.: Gathering correlated data in sensor networks. In: DIALM-POMC, ACM (October 2004)

# Joint Routing and Compression

- Transform-based approaches
  - Wavelet-based approaches [5] [6] [7] and distributed KLT [8]
  - Exploit spatial correlation to reduce the number of bits to be transmitted to the sink

[5] Ciancio, A., Pattem, S., Ortega, A., Krishnamachari, B.: Energy-efficient data representation and routing for wireless sensor networks based on a distributed wavelet compression algorithm. In: IPSN. (April 2006)

[6] Shen, G., Ortega, A.: Joint routing and 2d transform optimization for irregular sensor network grids using wavelet lifting. In: IPSN. (April 2008)

[7] Wagner, R., Choi, H., Baraniuk, R., Delouille, V.: Distributed wavelet transform for irregular sensor network grids. In: SSP. (July 2005)

[8] Gastpar, M., Dragotti, P., Vetterli, M.: The distributed karhunen-loeve transform. In: MMSP. (December 2002)

# Joint Routing and Compression

- Transform-based approaches
  - Wavelet-based approaches [5] [6] [7] and distributed KLT [8]
  - Exploit spatial correlation to reduce the number of bits to be transmitted to the sink
  - Critically sampled approaches
    ⇒ cost of gathering scales up with the number of sensors
    ⇒ undesirable for large deployments of sensors

[5] Ciancio, A., Pattem, S., Ortega, A., Krishnamachari, B.: Energy-efficient data representation and routing for wireless sensor networks based on a distributed wavelet compression algorithm. In: IPSN. (April 2006)

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- Compressed Sensing approach
  - Potential alternative for large-scaled data gathering
  - Number of samples required depends on data characteristics (sparseness).
  - Potentially attractive for wireless sensor network
    - Most computations take place at the sink rather than sensors $\Rightarrow$ sensors can encode data with minimal computational power
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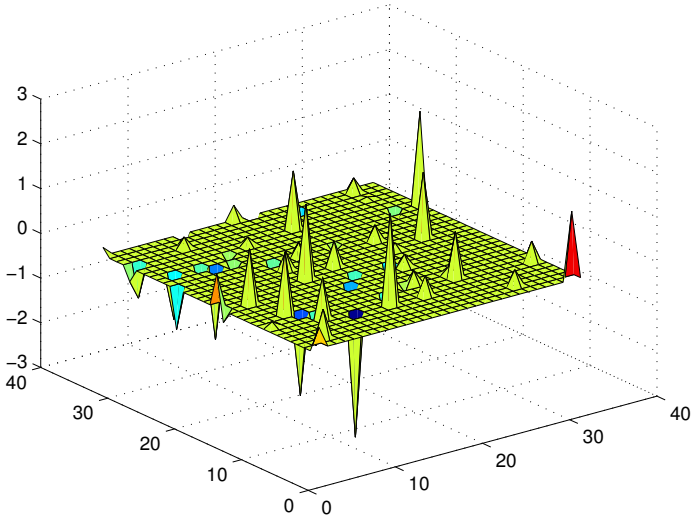
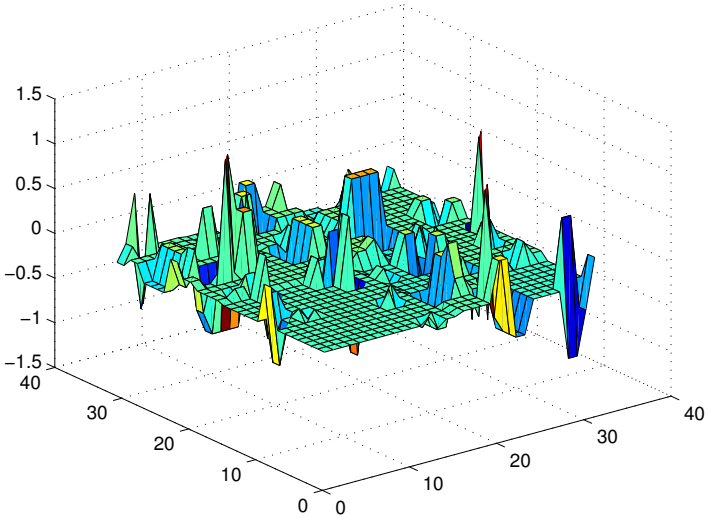
    - Most computations take place at the sink rather than sensors $\Rightarrow$ sensors can encode data with minimal computational power
    - Secured data transmission due to random linear combination of signal
  - Challenges
    - Focus on minimizing the cost of each measurement rather than number of measurements
    - Localized aggregation (projection) to reduce the cost

# Compressed Sensing Basics [9] [10] [11]

- Assume that a signal, $\mathbf{x} \in \Re^n$, is $k$-sparse in a given basis: $\mathbf{\Psi}$
  $\mathbf{x} = \mathbf{\Psi}\mathbf{a}, |\mathbf{a}|_0 = k$, where $k \ll n$




(a) 55 non-zero bases out of 1024



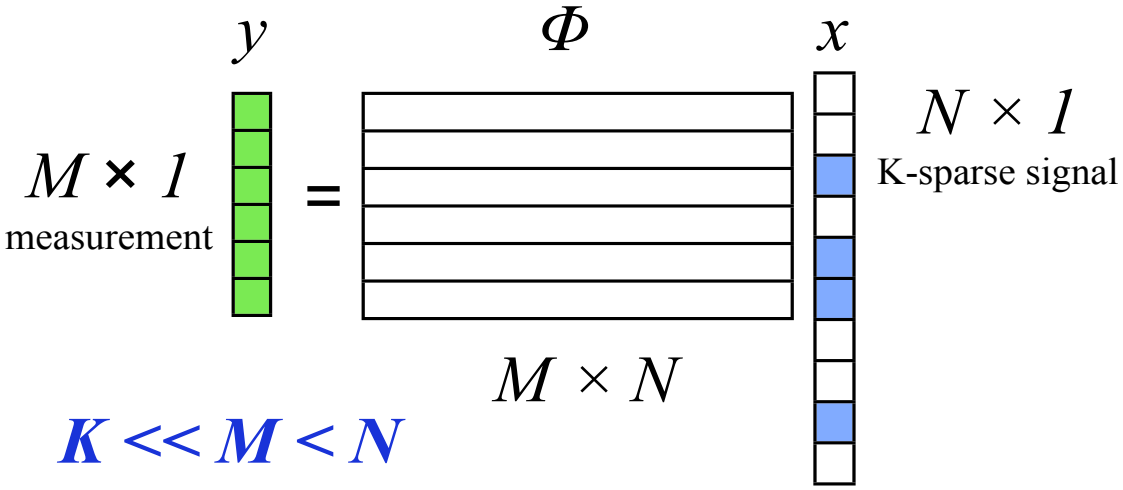

(b) 55-sparse signal in Haar basis

[9] Donoho, D.L.: Compressed sensing. In: IEEE Transactions on Information Theory. (April 2006)

[10] Candes, E., Romberg, J., Tao, T.: Robust uncertainity principles : exact signal reconstruction from highly incomplete frequency information. In: IEEE Transactions on Information Theory. (February 2006)

[11] Candes, E., Romberg, J.: Sparsity and incoherence in compressive sampling. In:Inverse Problems. (June 2007)

- Replace data samples with few linear projections, $y = \mathbf{\Phi} x$.




- Reconstruct original signal with $O(klogn)$ measurements.
  - Find sparse solutions to under-determined linear systems of equations
  - Solve convex unconstrained optimization problem $\min_x \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \gamma\|\mathbf{x}\|_1$, where $\mathbf{H} = \mathbf{\Phi}\mathbf{\Psi}$

# Compressed Sensing Extension

- Traditional CS
  - Assume **DENSE** random projection matrix
  - Focus on minimizing the number of measurements (i.e., the number of samples captured), rather than on minimizing the cost of each measurement.
    $\Rightarrow$ every sensor is required to transmit its data once for each measurement
    $\Rightarrow$ high energy consumption (higher than raw data transmission)

# Compressed Sensing Extension

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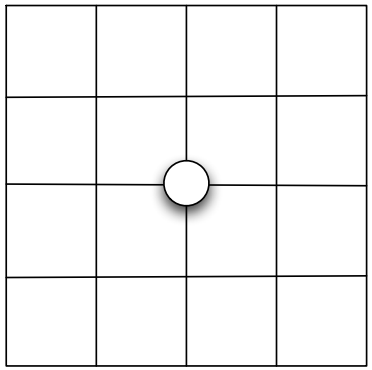
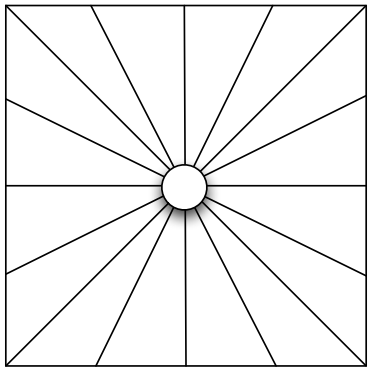
- Spatially-localized Sparse CS
  - Cost depends on
    - Sparsity of the measurement matrix:
      the number of samples contributing to each measurement
    - Position of the sensors whose samples are aggregated in the measurements
  - Thus, sparse aggregation of neighboring sensors is energy-efficient

# Low-cost sparse projection based on clustering

- To reduce transmission cost,
  - Reduce the number of samples for each measurement
  - Aggregate samples of sensors close to each other
- Sparse projection based on clustering
  1. Divide network into clusters of adjacent sensors
  2. Force projections to be obtained only from nodes within a cluster
  3. Localized measurements are transmitted along the shortest path to the sink
- Consider two basic clustering schemes

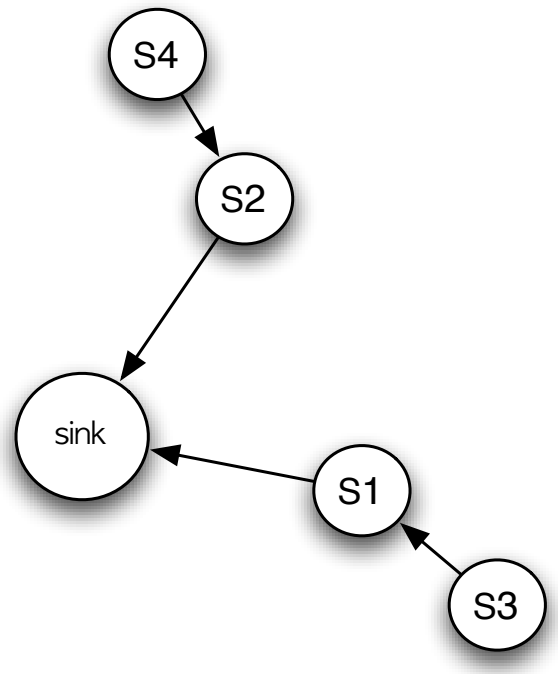


Square clustering



SPT-based clustering

- Example
  Four sensors ($S_1 \sim S_4$) with two clusters ($\{S_1, S_3\}$ *and* $\{S_2, S_4\}$)



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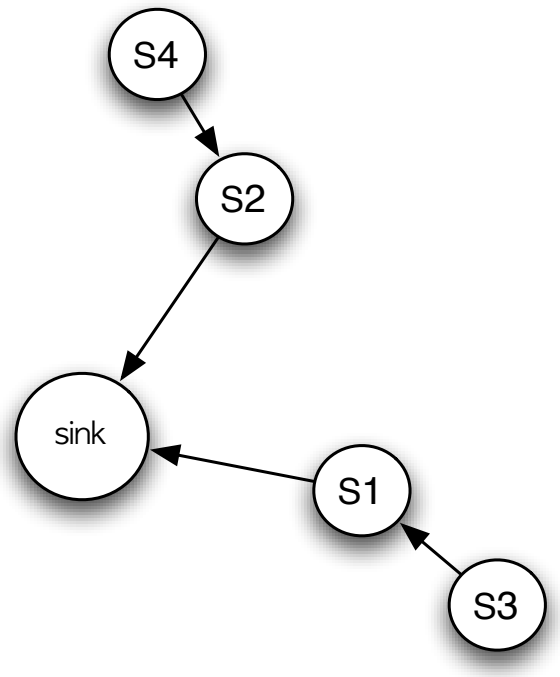


- Let $x_i$ be data sample in $S_i$
- Let $w_i$ be random variables for $S_i$
- Two data aggregation paths
  1. $S_3 \rightarrow S_1 \rightarrow Sink$
  2. $S_4 \rightarrow S_2 \rightarrow Sink$
- Generate measurements based on the paths
  1. $y_1 = w_1 x_1 + w_3 x_3$
  2. $y_2 = w_2 x_2 + w_4 x_4$

- Matrix formulation

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} w_1 & w_3 & 0 & 0 \\ 0 & 0 & w_2 & w_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 & w_3 & 0 & 0 \\ w_1' & w_3' & 0 & 0 \\ 0 & 0 & w_2 & w_4 \\ 0 & 0 & w_2' & w_4' \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$

- Leads to a sparse block-diagonal structure for $\Phi$
  - Similar to recent work [12] [13] proposed for fast CS computation
  - Showed comparable results to dense random projection matrices

[12] Gan, L., Do, T.T., Tran, T.D.: Fast compressive imaging using scrambled block hadamard ensemble. In: preprint. (2008)

[13] Do, T., Tran, T., Gan, L.: Fast compressive sampling with structurally random matrices. In: ICASSP. (April 2008)

# Interaction between localized gathering and reconstruction

- Not obvious how localized gathering impacts reconstruction quality
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# Interaction between localized gathering and reconstruction

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- Two extreme cases
  1. DCT ("global") basis
     $\Rightarrow$ Optimally incoherent $\mathbf{\Phi} = \mathbf{I}$
     $\Rightarrow$ Sample *klogn* randomly chosen sensors and then forward each measurement directly to the sink (no aggregation)
  2. $\mathbf{I}$ ("completely localized") basis
     $\Rightarrow$ Dense projection may be best
     $\Rightarrow$ Better to have sensors transmit data to the sink via the SPT whenever they sense something new (e.g., when measurements exceed a threshold)

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  $\Rightarrow$ Direct choice of incoherent $\Phi$ may NOT lead to efficient routing
- Focus on intermediate cases, localized bases with different spatial resolutions (e.g., wavelets)

# Joint vs Independent reconstruction

- Compare two types of reconstruction:
  1. Joint reconstruction:
     performed with the basis, $\mathbf{\Psi}$, where signal is originally dened
  2. Independent reconstruction:
     performed with truncated basis functions corresponding to each cluster

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- Example with two clusters (represented by $\phi_1$ and $\phi_2$)
  - Joint reconstruction with original $\mathbf{H}$
  - Independent reconstruction with $\mathbf{H_1}$ and $\mathbf{H_2}$ separately

$$\mathbf{H} = \mathbf{\Phi\Psi} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix} \begin{bmatrix} \psi_1 & \psi_2 \\ \psi_3 & \psi_4 \end{bmatrix} \Rightarrow \left\{ \begin{array}{l} \mathbf{H_1} = \begin{bmatrix} \phi_1\psi_1, & \phi_1\psi_2 \end{bmatrix} \\ \\ \mathbf{H_2} = \begin{bmatrix} \phi_2\psi_3, & \phi_2\psi_4 \end{bmatrix} \end{array} \right\}$$

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$\Rightarrow$ With joint reconstruction, basis functions overlapped with more than one clusters can be identied with measurements from those clusters

# Energy overlap analysis

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  $\Rightarrow$ Higher chance to reconstruct signal correctly with joint reconstruction
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- Definition
  - $N_c$: number of clusters
  - $N_o(i)$: number of basis vectors overlapped with $i^{th}$ cluster
  - $C_i$: set of nodes in $i^{th}$ cluster
  - $\Psi(i,j)$: $j^{th}$ entry in the normalized $i^{th}$ column of $\Psi$
  - $E_o(i,j)$: Energy overlap between $i^{th}$ cluster and $j^{th}$ basis vector

- Energy overlap per overlapped basis, $E_{oa}$
  - Shows how much energy of basis functions are captured by each cluster
  - More evenly distributed energy over overlapped clusters
    $\Rightarrow E_{oa}$ decreases
    $\Rightarrow$ Better reconstruction performance with joint reconstruction.

For each cluster,

$$E_{oa}(i) = \frac{1}{N_o(i)} \sum_{j=1}^{N} E_o(i,j),\ \forall i \in \{1, 2, \cdots, N_c\}, \textit{where}$$

$$E_o(i,j) = \sum_{k \in C_i} \psi(j,k)^2$$

$\Rightarrow E_{oa} = \frac{1}{N_c} \sum_{i=1}^{N_c} E_{oa}(i)$

# Simulation environment

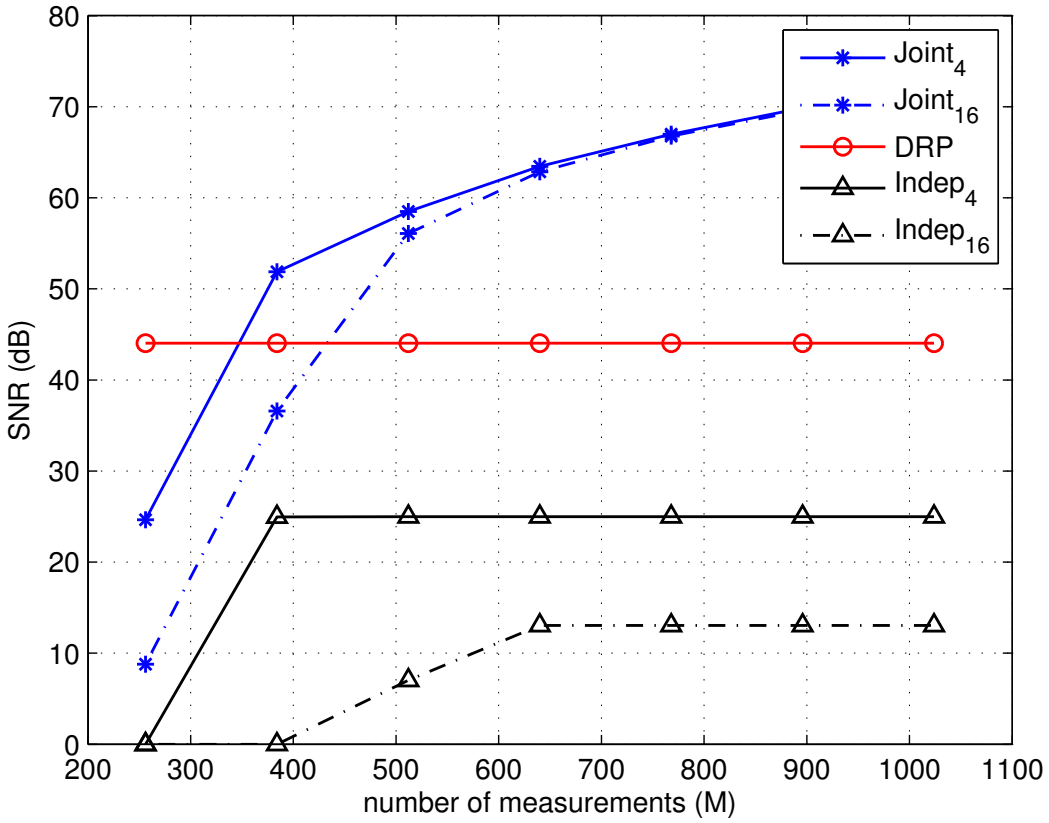
- 500 data generated with 55 random coefficients in different basis
- 1024 sensors on the square grid
- Error free communication
- Reconstruction with Gradient Pursuit for Sparse Reconstruction (GPSR) [14]
- Evaluation
  - Reconstruction evaluation by SNR $= 10 \times log_{10} \frac{\sum signal^2}{\sum error^2}$
  - Cost evaluation by $\sum (bit) \times (distance)^2$
  - Cost ratio is the ratio to the cost for raw data gathering without compression.

[14] Figueiredo, M., Nowak, R., Wright, S.: Gradient pro jection for sparse reconstruction:application to compressed sensing and other inverse problems. In: IEEE Journal of Selected Topics in Signal Processing. (2007)

# Joint vs Independent reconstruction

- Joint reconstruction outperforms independent reconstruction
- Measurements from other clusters overlapped with basis functions in the data support

  $\Rightarrow$ Joint reconstruction can alleviate the worst situation

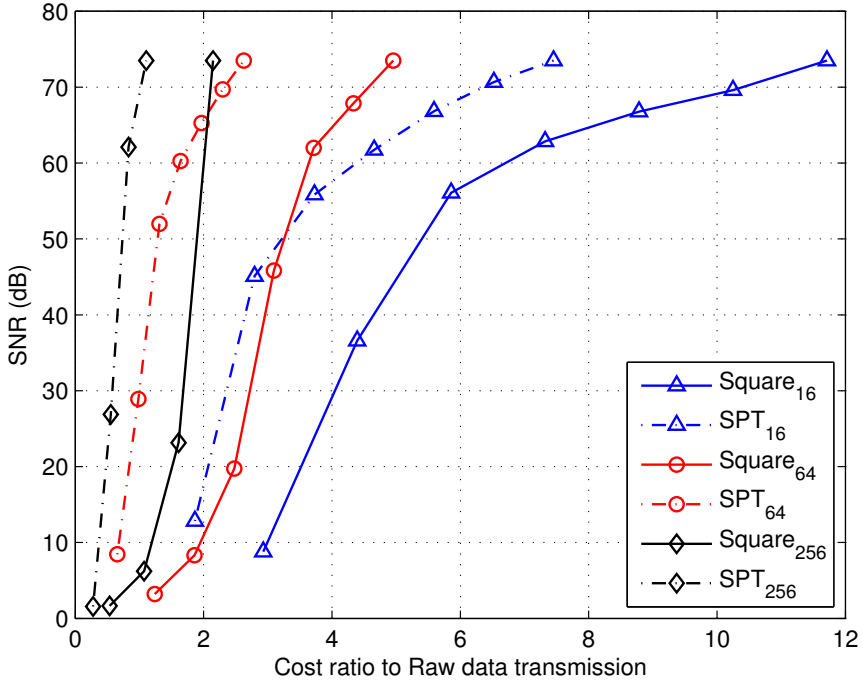


# Square clustering vs. SPT-based clustering

- SPT-based clustering outperforms square clustering for different $N_c$
- With larger $N_c$,
  - reconstruction accuracy decreases
  - cost per each measurement decreases

$\Rightarrow$ Less cost compensates worse reconstruction

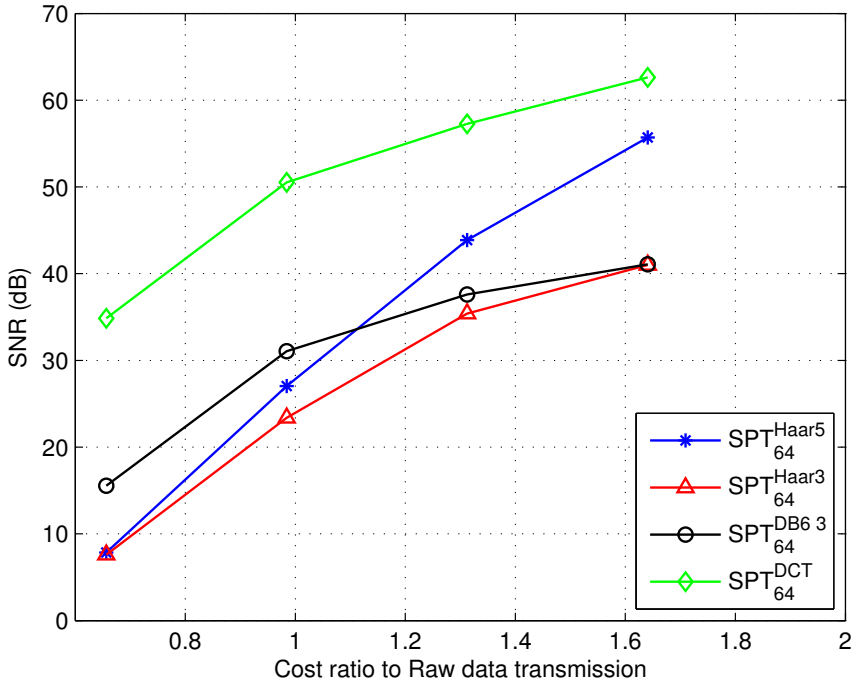
$\Rightarrow$ Better performance in terms of reconstruction and cost



# Effects of different bases

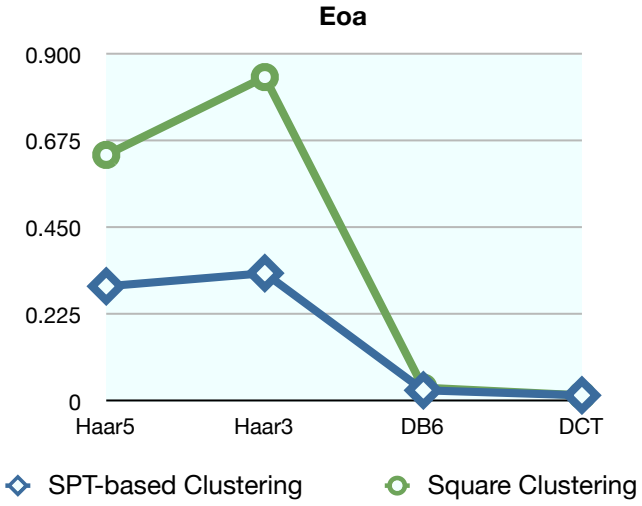
- Fix SPT-based clustering with 64 clusters
- Three different kinds of bases
  1. DCT basis: high overlaps in energy
  2. Haar basis: less overlap and variant energy distribution
  3. Daubechies (DB6) basis: intermediate to DCT and Haar

$\Rightarrow$ Depends on how well-spread the energy in basis vectors is.

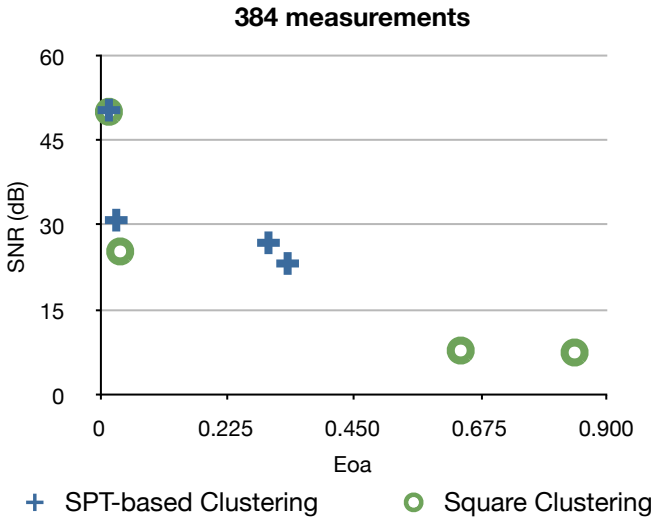


# $E_{oa}$ evaluation

- Compute $E_{oa}$ with different clusterings for different bases
- Accurate indicator for performance (lower $E_{oa}$, better reconstruction)
  - SPT-based clusters capture more energy of basis functions than square clusters
  - lower overlap energy as basis functions are more spread over in spatial domain



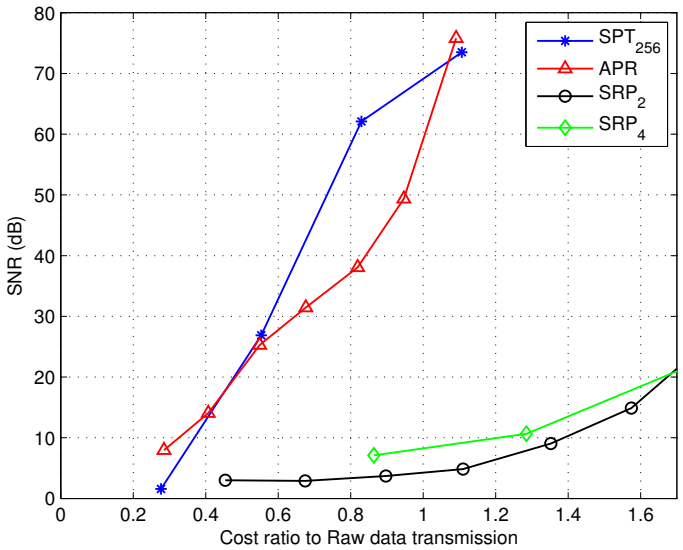

$E_{oa}$




$E_{oa}$ vs. SNR (dB)

# Comparison with existing CS approaches

- Our approach outperforms SRP totally
- Our approach achieves reasonable reconstruction quality from 28dB to 70dB with less cost than APR
  - *APR* [15] : aggregate data of all the sensors along SPT to the sink
  - $SRP_s$ [16] : randomly chooses $s$ nodes without considering routing $\longrightarrow$ transmit data to the sink via SPT with opportunistic aggregation.




[15] Quer, G., Masierto, R., Munaretto, D., Rossi, M., Widmer, J., Zorzi, M.: On the interplay between routing and signal representation for compressive sensing in wireless sensor network. In: ITA. (February 2009)

# Conclusion

- Proposed a framework using spatially-localized compressed sensing
- Joint reconstruction showed better reconstruction than independent reconstruction
- Evenly distributed energy of basis functions show better performance
- Quantified the level of energy overlap between data gathering clusters and basis functions
- Our proposed approach outperforms state of the art CS techniques for sensor network

Thanks !