

# STEM/The Muffin Problem

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# STEM

- ▶ **S:** Science
- ▶ **T:** Technology
- ▶ **E:** Engineering
- ▶ **M:** Mathematics

# What Should You Major In

1. Major in a subject you **enjoy** and **are good at**.
2. Job considerations?- Within STEM **its all good!**
3. Whataever you major in pick up Comp Science as programming is now in everything.

# UMCP

UMCP=University of Maryland At College Park

1. Cheap (relatively)
2. Scholarships!
3. Close to home (this is both good and bad)
4. Excellent Math, CS, Physics, Engineering
5. Very Good Chemistry and Biology

# UMCP

1. Big (This is both good and bad.)
2. Lots of clubs- Good!
3. Big Classes- Bad! but okay with recitations
4. Honors Programs- The Best of Both Worlds

# My Story

1. Ugrad: University of New York at Stonybrook
2. Graduated in 1980 with BS in Math and Applied Math
3. Grad: Harvard
4. Graduated in 1985 with PhD in Computer Science
5. Job: UMCP 1985-now.
6. What I do now: Mentor Students on Projects!
7. Today's talk is about one of them!

# Five Muffins, Three Students

At

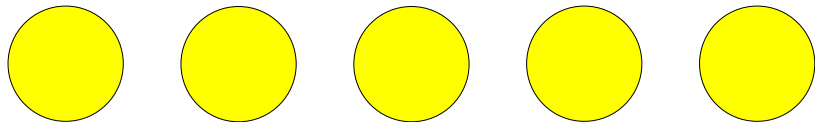
## A Recreational Math Conference (Gathering for Gardner)

I found a pamphlet advertising

### The Julia Robinson Mathematics Festival

which had this problem, proposed by Alan Frank:

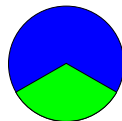
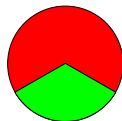
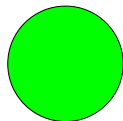
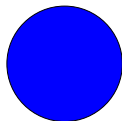
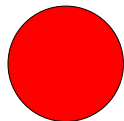
*How can you divide and distribute 5 muffins to 3 students so that every student gets  $\frac{5}{3}$  where nobody gets a tiny sliver?*



# Five Muffins, Three Students, Proc by Picture

| Person | Color | What they Get                                 |
|--------|-------|---|
| Alice  | RED   | $1 + \frac{2}{3} = \frac{5}{3}$               |
| Bob    | BLUE  | $1 + \frac{2}{3} = \frac{5}{3}$               |
| Carol  | GREEN | $1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$ |

Smallest Piece:  $\frac{1}{3}$





## Can We Do Better?

The smallest piece in the above solution is  $\frac{1}{3}$ .

**Is there a procedure with a larger smallest piece?**

**VOTE**

## Can We Do Better?

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**VOTE**

- ▶ **YES**
- ▶ **NO**

## Can We Do Better?

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**VOTE**

- ▶ **YES**
- ▶ **NO**

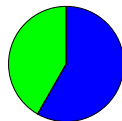
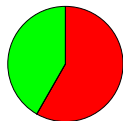
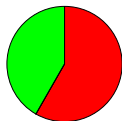
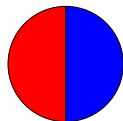
**YES WE CAN!**

We use **!** since we are excited that we can!

## Five Muffins, Three People—Proc by Picture

| Person | Color | What they Get   |
|--------|-------|---|
| Alice  | RED   | $\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$                |
| Bob    | BLUE  | $\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$                |
| Carol  | GREEN | $\frac{5}{12} + \frac{5}{12} + \frac{5}{12} + \frac{5}{12}$ |

Smallest Piece:  $\frac{5}{12}$



# Can We Do Better?

The smallest piece in the above solution is  $\frac{5}{12}$ .

**Is there a procedure with a larger smallest piece?**

**VOTE**

- ▶ **YES**
- ▶ **NO**

# Can We Do Better?

The smallest piece in the above solution is  $\frac{5}{12}$ .

**Is there a procedure with a larger smallest piece?**

**VOTE**

- ▶ **YES**
- ▶ **NO**

**NO WE CAN'T!**

We use **!** since we are excited to prove we can't do better!

## Assumption We Can Make

There is a procedure for 5 muffins, 3 students where each student gets  $\frac{5}{3}$  muffins, smallest piece  $N$ . We want  $N \leq \frac{5}{12}$ .

We **ASSUME** each muffin cut into **at least 2** pieces: If not then cut that muffin  $(\frac{1}{2}, \frac{1}{2})$ .

**THIS TALK** ALL proofs will be about opt being  $\leq 1/2$ . We assume each muffin is cut into **at least 2** pieces.

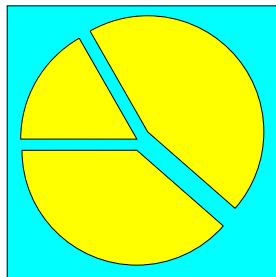
**PIECES VS SHARES:** They are the same.

- ▶ **PIECE** is muffin-view,
- ▶ **SHARE** is student-view.

# Muffin Principle

If a muffin is cut into  $\geq u$  pieces then there is a piece  $\leq \frac{1}{u}$

**Example:** If a Muffin cut into 3 pieces:



some piece is  $\leq \frac{1}{3}$ .

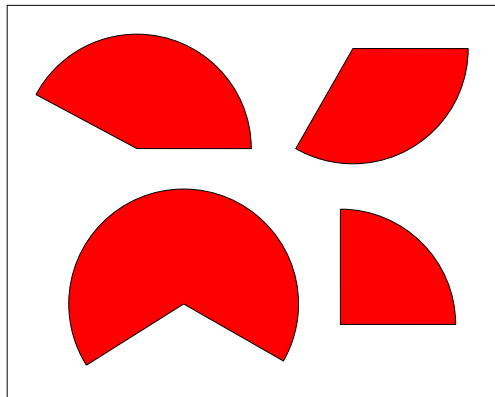


## Student Principle (not Principal)

If a student gets  $\geq u$  shares then there is a share  $\leq \frac{m}{s} \times \frac{1}{u}$

**Example:** 5 muffins, 3 students. All student gets  $\frac{5}{3}$ .

If some student gets  $\geq 4$  shares:



Then one of these pieces is  $\leq \frac{5}{3} \times \frac{1}{4}$

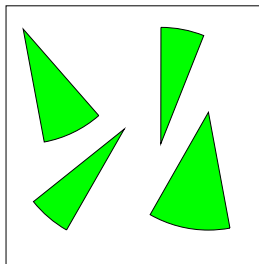
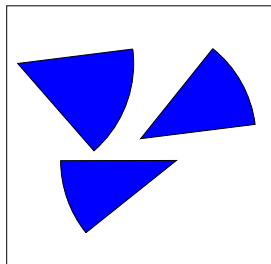
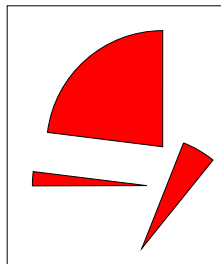
# Pieces Principle

If there are  $P$  pieces then:

Some student gets  $\geq \lceil P/s \rceil$

Some student gets  $\leq \lfloor P/s \rfloor$

**Example:** 5 muffins, 3 people. If there are 10 pieces:



Some student gets  $\geq \lceil \frac{10}{3} \rceil = 4$

Some student gets  $\leq \lfloor \frac{10}{3} \rfloor = 3$

## Five Muffins, Three People—Can't Do Better Than $\frac{5}{12}$

There is a procedure for 5 muffins, 3 students where each student gets  $\frac{5}{3}$  muffins, smallest piece  $N$ . We want  $N \leq \frac{5}{12}$ .

**Case 1:** Some muffin is cut into  $\geq 3$  pieces. Then  $N \leq \frac{1}{3} < \frac{5}{12}$ .  
(**Negation:** All muffins are cut into 2 pieces.)

**Case 2:** All muffins are cut into 2 pieces. 10 pieces, 3 students:  
**Someone** gets  $\geq 4$  pieces. He has some piece

$$\leq \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \quad \text{Great to see } \frac{5}{12}$$

## Be Amazed Now! And Later!

1. Procedure for 5 muffins, 3 people, smallest piece  $\frac{5}{12}$ .
2. NO Procedure for 5 muffins, 3 people, smallest piece  $> \frac{5}{12}$ .

**Amazing That Have Exact Result!**

# Be Amazed Now! And Later!

1. Procedure for 5 muffins, 3 people, smallest piece  $\frac{5}{12}$ .
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**Amazing That Have Exact Result!**

Prepare To Be More Amazed!

# Be Amazed Now! And Later!

1. Procedure for 5 muffins, 3 people, smallest piece  $\frac{5}{12}$ .
2. NO Procedure for 5 muffins, 3 people, smallest piece  $> \frac{5}{12}$ .

## Amazing That Have Exact Result!

Prepare To Be More Amazed!

We have **many** results like this!:

$$f(47, 9) = \frac{111}{234}$$

$$f(52, 11) = \frac{83}{176}$$

$$f(35, 13) = \frac{64}{143}$$

# General Problem

*How can you divide and distribute  $m$  muffins to  $s$  students so that each student gets  $\frac{m}{s}$  AND the MIN piece is MAXIMIZED?*

An  $(m, s)$ -*procedure* is a way to divide and distribute  $m$  muffins to  $s$  students so that each student gets  $\frac{m}{s}$  muffins.

An  $(m, s)$ -procedure is *optimal* if it has the largest smallest piece of any procedure.

$f(m, s)$  be the smallest piece in an optimal  $(m, s)$ -procedure.

We have shown  $f(5, 3) = \frac{5}{12}$ .

**Note:**  $f(m, s) \geq \frac{1}{s}$ : divide each  $M$  into  $s$  pieces of size  $\frac{1}{s}$  and give each  $S$   $m$  of them.

# Terminology Issue

Let  $m, s \in \mathbb{N}$ .

$m$  is the number of muffins.

$s$  is the number of students.

1.  $f(m, s) \geq \alpha$  means that there is a procedure with smallest piece  $\alpha$ . We call this *A Procedure*.
2.  $f(m, s) \leq \alpha$  means that there is NO procedure with smallest piece  $> \alpha$ . We call this *An Optimality Result* or *An Opt Result*.

**DO NOT** use terms **upper bound** and **lower bounds**:

1. Procedures are lower bounds, **opposite** of usual terminology.
2. Opt results are upper bounds, **opposite** of usual terminology.



## Floor-Ceiling Theorem

$$f(m, s) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s \lceil 2m/s \rceil}, 1 - \frac{m}{s \lfloor 2m/s \rfloor} \right\} \right\}.$$

**Proof:**

**Case 1:** Some muffin is cut into  $\geq 3$  pieces. Some piece  $\leq \frac{1}{3}$ .

**Case 2:** Every muffin is cut into 2 pieces, so  $2m$  pieces.

**Someone** gets  $\geq \lceil \frac{2m}{s} \rceil$  pieces. Some piece is  $\leq \frac{(m/s)}{\lceil 2m/s \rceil} = \frac{m}{s \lceil 2m/s \rceil}$ .

**Someone** gets  $\leq \lfloor \frac{2m}{s} \rfloor$  pieces. Some piece is  $\geq \frac{(m/s)}{\lfloor 2m/s \rfloor} = \frac{m}{s \lfloor 2m/s \rfloor}$ .

The other piece from that muffin is of size  $\leq 1 - \frac{m}{s \lfloor 2m/s \rfloor}$ .

## THREE Students

**CLEVERNESS, COMP PROGS** for the procedure.

**Floor-Ceiling Theorem** for optimality.

$$f(1, 3) = \frac{1}{3}$$

$$f(3k, 3) = 1.$$

$$f(3k + 1, 3) = \frac{3k-1}{6k}, k \geq 1.$$

$$f(3k + 2, 3) = \frac{3k+2}{6k+6}.$$

## FOUR Students

**CLEVERNESS, COMP PROGS** for procedures.

**Floor-Ceiling Theorem** for optimality.

$$f(4k, 4) = 1 \text{ (easy)}$$

$$f(1, 4) = \frac{1}{4} \text{ (easy)}$$

$$f(4k + 1, 4) = \frac{4k-1}{8k}, k \geq 1.$$

$$f(4k + 2, 4) = \frac{1}{2}.$$

$$f(4k + 3, 4) = \frac{4k+1}{8k+4}.$$

**Is FIVE student case a Mod 5 pattern?**

**VOTE YES or NO**

## FOUR Students

**CLEVERNESS, COMP PROGS** for procedures.

**Floor-Ceiling Theorem** for optimality.

$$f(4k, 4) = 1 \text{ (easy)}$$

$$f(1, 4) = \frac{1}{4} \text{ (easy)}$$

$$f(4k + 1, 4) = \frac{4k-1}{8k}, k \geq 1.$$

$$f(4k + 2, 4) = \frac{1}{2}.$$

$$f(4k + 3, 4) = \frac{4k+1}{8k+4}.$$

**Is FIVE student case a Mod 5 pattern?**

**VOTE YES or NO**

**NO!** (excited because **YES** would be boring)

## FIVE Students, $m = 1, 2, 3, 4, 7, 11, 10k$

$$f(1, 5) = \frac{1}{5} \text{ (easy)}$$

$$f(2, 5) = \frac{1}{5} \text{ (easy)}$$

$$f(3, 5) = \frac{1}{4} \text{ (Will discuss briefly later)}$$

$$f(4, 5) = \frac{3}{10} \text{ (Will not discuss later)}$$

$$f(7, 5) = \frac{1}{3} \text{ (Use Floor-Ceiling Thm)}$$

$$f(11, 5) = \text{(Will come back to this later)}$$

$$f(10k, 5) = 1 \text{ (Trivial)}$$

# FIVE Students

Results on the next few slides:

**CLEVERNESS, COMP PROGS** for the procedure.

**Floor-Ceiling Theorem** for optimality.

## FIVE Students $m = 10k + 1, 10k + 2, 10k + 3$

If  $k$  not specified then  $k \geq 0$ .

$m = 10k + 1$ :

$$f(30k + 1, 5) = \frac{30k+1}{60k+5}$$

$$f(30k + 11, 5) = \frac{30k+11}{60k+25} \quad (k \geq 1)$$

$$f(30k + 21, 5) = \frac{10k+7}{20k+15}$$

$$f(10k + 2, 5) = \frac{10k-2}{20k} \quad (k \geq 1)$$

$$f(10k + 3, 5) = \frac{10k+3}{20k+10} \quad (k \geq 1)$$

## FIVE Students $m = 10k + 4, 10k + 5, 10k + 6$

$$m = 10k + 4$$

$$f(30k + 4, 5) = \frac{30k+1}{60k+5}$$

$$f(30k + 14, 5) = \frac{30k+11}{60k+25}$$

$$f(30k + 24, 5) = \frac{10k+7}{20k+15}$$

$$f(10k + 5, 5) = 1$$

$$m = 10k + 6:$$

$$f(30k + 6, 5) = \frac{10k+2}{20k+5}$$

$$f(30k + 16, 5) = \frac{30k+16}{60k+35}$$

$$f(30k + 26, 5) = \frac{30k+26}{60k+55}$$



## FIVE Students $m = 10k + 7, 10k + 8, 10k + 9$

$$f(10k + 7, 5) = \frac{10k+3}{20k+10}$$

$$f(10k + 8, 5) = \frac{5k+4}{10k+10}$$

$$m = 10k + 9$$

$$f(30k + 9, 5) = \frac{10k+2}{20k+5}$$

$$f(30k + 19, 5) = \frac{30k+16}{60k+35}$$

$$f(30k + 29, 5) = \frac{30k+26}{60k+55}$$

# What About FIVE students, ELEVEN muffins?

## Procedure:

### Divide the Muffins in to Pieces:

1. Divide 6 muffins into  $(\frac{13}{30}, \frac{17}{30})$ .
2. Divide 4 muffins into  $(\frac{9}{20}, \frac{11}{20})$ .
3. Divide 1 muffin into  $(\frac{1}{2}, \frac{1}{2})$ .

### Distribute the Shares to Students:

1. Give 2 students  $[\frac{17}{30}, \frac{17}{30}, \frac{17}{30}, \frac{1}{2}]$ .
2. Give 2 students  $[\frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{9}{20}, \frac{9}{20}]$
3. Give 1 student  $[\frac{11}{20}, \frac{11}{20}, \frac{11}{20}, \frac{11}{20}]$

So

$$f(11, 5) \geq \frac{13}{30}$$

# What About FIVE students, ELEVEN muffins? Opt

Recall: **Floor-Ceiling Theorem:**

$$f(m, s) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s \lceil 2m/s \rceil}, 1 - \frac{m}{s \lfloor 2m/s \rfloor} \right\} \right\}.$$

$$f(11, 5) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{11}{5 \lceil 22/5 \rceil}, 1 - \frac{11}{5 \lfloor 22/5 \rfloor} \right\} \right\}.$$

$$f(11, 5) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{11}{5 \times 5}, 1 - \frac{11}{5 \times 4} \right\} \right\}.$$

$$f(11, 5) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{11}{25}, \frac{9}{20} \right\} \right\}.$$

$$f(11, 5) \leq \max \left\{ \frac{1}{3}, \frac{11}{25} \right\} = \frac{11}{25}.$$

## Where Are We On FIVE students, ELEVEN muffins?

- ▶ By **Procedure**  $\frac{13}{30} \leq f(11, 5)$ .
- ▶ By **Floor-Ceiling**  $f(11, 5) \leq \frac{11}{25}$ .

So

$$\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff} = 0.006666\dots$$

## Where Are We On FIVE students, ELEVEN muffins?

- ▶ By **Procedure**  $\frac{13}{30} \leq f(11, 5)$ .
- ▶ By **Floor-Ceiling**  $f(11, 5) \leq \frac{11}{25}$ .

So

$$\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff} = 0.006666 \dots$$

### VOTE:

1. **KNOWN:**  $f(11, 5) = \frac{13}{30}$ : New opt technique.
2. **KNOWN:**  $f(11, 5) = \frac{11}{25}$ : New procedure.
3. **KNOWN:**  $\frac{13}{30} < f(11, 5) < \frac{11}{25}$ : New opt and new proc.
4. **UNKNOWN TO SCIENCE!**
5. **HARAMBE THE GORILLA!**

(In Poll of Discrete Math Students for Presidential Election 3 wrote in Harambe.)

## Where Are We On FIVE students, ELEVEN muffins?

- ▶ By **Procedure**  $\frac{13}{30} \leq f(11, 5)$ .
- ▶ By **Floor-Ceiling**  $f(11, 5) \leq \frac{11}{25}$ .

So

$$\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff} = 0.006666 \dots$$

### VOTE:

1. **KNOWN:**  $f(11, 5) = \frac{13}{30}$ : New opt technique.
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3. **KNOWN:**  $\frac{13}{30} < f(11, 5) < \frac{11}{25}$ : New opt and new proc.
4. **UNKNOWN TO SCIENCE!**
5. **HARAMBE THE GORILLA!**

(In Poll of Discrete Math Students for Presidential Election 3 wrote in Harambe.)

$$\text{KNOWN: } f(11, 5) = \frac{13}{30}$$

**HAPPY:** New opt tech more interesting than new proc.

## $f(11, 5) = \frac{13}{30}$ , Easy Case Based on Muffins

$N$  is smallest piece.

**Case 1:** Some muffin is cut into  $\geq 3$  pieces.  $N \leq \frac{1}{3} < \frac{13}{30}$ .

(**Negation:** All muffins cut into 2 pieces.)

## $f(11, 5) = \frac{13}{30}$ , Easy Case Based on Students

**Case 2:** Some student gets  $\geq 6$  pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

**Case 3:** Some student gets  $\leq 3$  pieces.

One of the shares is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

Look at the muffin it came from to find a piece that is

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

**(Negation of Cases 2 and 3:** Every student gets 4 or 5 shares.)



## $f(11, 5) = \frac{13}{30}$ , Fun Cases

**Case 4:** Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note  $\leq 11$  pieces are  $> \frac{1}{2}$ .

- ▶  $s_4$  is number of students who get 4 shares
- ▶  $s_5$  is number of students who get 5 shares

$$4s_4 + 5s_5 = 22$$

$$s_4 + s_5 = 5$$

$s_4 = 3$ : There are 3 students who have 4 shares.

$s_5 = 2$ : There are 2 students who have 5 shares.

## $f(11, 5) = \frac{13}{30}$ , Fun Cases

◇ and ○ are shares.

◇ ◇ ◇ ◇ ◇ (Sums to 11/5)

◇ ◇ ◇ ◇ ◇ (Sums to 11/5)

○ ○ ○ ○ (Sums to 11/5)

○ ○ ○ ○ (Sums to 11/5)

○ ○ ○ ○ (Sums to 11/5)

**Case 3.1:** One of (say)

○ ○ ○ ○ (Sums to 11/5)

is  $\leq \frac{1}{2}$ . Then there is a share

$$\geq \frac{(11/5) - (1/2)}{3} = \frac{17}{30}.$$

The other piece from the muffin is

$$\leq 1 - \frac{17}{30} = \frac{13}{30} \quad \text{Great to see } \frac{13}{30}.$$

$$f(11, 5) = \frac{13}{30}, \text{ Fun Cases}$$

### Case 3.2: All

- ○ ○ ○ (Sums to 11/5)
- ○ ○ ○ (Sums to 11/5)
- ○ ○ ○ (Sums to 11/5)

are  $> \frac{1}{2}$ .

There are  $\geq 12$  shares  $> \frac{1}{2}$ . Can't occur.

# The Techniques Generalizes!

## Good News!

The technique used to get  $f(11, 5) \leq \frac{13}{30}$  lead to a theorem that apply to other cases! We call it **The Interval Theorem**

## Bad News!

**Interval Theorem** is hard to state, so you don't **get** to see it.

## Good News!

**Interval Theorem** is hard to state, so you don't **have** to see it.

## Does $f(m, s)$ Always Exist? Rational? Computable?

Let  $x_{ij}$  be the fraction of Muffin  $i$  that Student  $j$  gets.

Each Muffin adds to 1:

$$(\forall i) \left[ \sum_{j=1}^s x_{ij} = 1 \right].$$

Each Student gets  $\frac{m}{s}$ :

$$(\forall j) \left[ \sum_{i=1}^m x_{ij} = \frac{m}{s} \right].$$

Each Piece is of size between 0 and 1:

$$(\forall i, j) [0 \leq x_{ij} \leq 1].$$

$$\text{Maximize } \min_{1 \leq i \leq m, 1 \leq j \leq s} x_{ij}$$

relative to the constraints above.

## Rephrase the Problem

Maximize  $z$

Relative to constraints:

$$(\forall i) \left[ \sum_{j=1}^s x_{ij} = 1 \right]$$

$$(\forall j) \left[ \sum_{i=1}^m x_{ij} = \frac{m}{s} \right]$$

$$(\forall i, j) [z \leq x_{ij} \leq 1]$$

This is a standard **Linear Programming Problem!**

There are very fast **packages** for it!

And Linear Programming is in P.

## Rephrase the Problem

Maximize  $z$

Relative to constraints:

$$(\forall i) \left[ \sum_{j=1}^s x_{ij} = 1 \right]$$

$$(\forall j) \left[ \sum_{i=1}^m x_{ij} = \frac{m}{s} \right]$$

$$(\forall i, j) [z \leq x_{ij} \leq 1]$$

This is a standard **Linear Programming Problem!**

There are very fast **packages** for it!

And Linear Programming is in P.

Does not work. Could have some  $x_{ij} = 0$ .

If NONE of Muffin 1's goes to Student 3, so  $x_{13} = 0$ .

Get  $z = 0$ . Not what we want.

# Plan for Correct Version of the Problem

For each  $i, j$  introduce variable  $y_{ij} \in \{0, 1\}$  (0 OR 1).

## Plan:

1. Will ensure that  $x_{ij} = 0 \implies y_{ij} = 1$
2. Will ensure that  $x_{ij} > 0 \implies y_{ij} = 0$
3. Will constrain  $z$  by  $z \leq x_{ij} + y_{ij}$ 
  - 3.1 If  $x_{ij} = 0$  then constraint is  $z \leq 1$ , NO EFFECT.
  - 3.2 If  $x_{ij} > 0$  then constraint is  $z \leq x_{ij}$ . WHAT WE WANT.



## Correct Version of the Problem

Add to the constraints:

1. Add variable  $y_{ij}$  which is in  $\{0, 1\}$ .
2. Add the constraint  $x_{ij} + y_{ij} \leq 1$ . Note that
  - ▶  $x_{ij} = 0 \implies x_{ij} + y_{ij} \leq 1$  (no constraint on  $y_{ij}$ )
  - ▶  $x_{ij} > 0 \implies y_{ij} < 1 \implies y_{ij} = 0$
3. Add the constraint  $x_{ij} + y_{ij} \geq \frac{1}{s}$ . Note that
  - ▶  $x_{ij} = 0 \implies y_{ij} \geq \frac{1}{s} \implies y_{ij} = 1 \implies x_{ij} + y_{ij} = 1$
  - ▶  $x_{ij} > 0 \implies x_{ij} \geq \frac{1}{s} \implies x_{ij} + y_{ij} \geq \frac{1}{s}$  (no constraint on  $y_{ij}$ )
4. Replace the constraint  $z \leq x_{ij}$  with  $z \leq x_{ij} + y_{ij}$ .

# $f(m, s)$ Rational! $f(m, s)$ Computable!

**Definition:** A **Mixed Integer Problem** is defined by

1. linear constraints on the variables,
2. want to maximize (or minimize) a linear function,
3. some of the variables are constrained to be integers, the rest reals.

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**Known:**

1. All MIP's with integer coefficients have rational solutions.
2. There is an algorithm to FIND the solutions to an MIP.
3. The problem is NP-complete (so thought to be hard to compute).

We have an MIP for  $f(m, s)$  hence  $f(m, s)$  is **rational!**

**Computable!**

# Not Just Theoretical

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**Good News:** We HAVE coded it up and we HAVE gotten some results this way.



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Pure Math, Applied Math, Computer Science, Physics, all play off each other!

# How Research Works

## History:

1. Obtain particular results.
2. Prove a general theorem based on those results.
3. Run into a case we cannot solve (e.g.,  $(11,5)$  and  $(35,13)$ ).
4. Lather, Rinse, Repeat.

## What Else Have We Accomplished?

1. A formula for  $f(s + 1, s)$ .
2. A computer program that helps us get procedures- used MIP
3. For  $1 \leq s \leq 15$ , for all  $m$ , know  $f(m, s)$ .
4. Convinced 4 High School students, 1 college student, and one professor that the most important field of Mathematics is **Muffinry.**

# Open Questions

1. For all  $s$  there is a pattern for  $f(m, s)$  that depends on  $m \bmod T$  where  $s$  divides  $T$ .
2.  $f(m, s) = \frac{a}{b}$  (lowest terms) where  $s$  divides  $b$ .
3. For all  $m \geq s$ ,  $f(m, s)$  is always determined by either
  - ▶ Floor Ceiling Theorem
  - ▶ Interval Theorem
  - ▶  $f(s + 1, s)$  Theorem.