

# Dynamic texture modeling and synthesis using Multi-kernel Gaussian process Dynamic model



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Master 1 MLDM 2015/2016

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# DYNAMIC TEXTURES





# What is a definition of texture?

- For a single image

Realization from stationary stochastic process



# What is a definition of texture?

- For a single image  
Realization from stationary stochastic process
- For a sequence of images  
Stochastic process of interest defined over space and time.



# Why is it important?

Applications in Computer vision:

- video surveillance
- object tracking
- etc.



Application in video processing fields:

- video indexing
- video animation
- etc.



# What we are doing

Input – video  $t_1$  seconds

Output – new videos  $t_2$  seconds,  $t_2 > 0$

- Dynamic texture can be captured from a video
- It can be used then to synthesize new videos with necessary length of time

Existing dynamic textures modeling and synthesis methods:

- *physics based* – costly, not universal
- *sampling based* – memory demanding, mostly manual
- ***learning based methods*** – more universal, automatic



**CONTRIBUTION**



# Contribution

First mission – understand the model

Reimplementation of the method in Python – understand things:

- Gaussian process
- MK-GPDM
- sequence of steps of learning algorithm
- functions which must be optimized
- functions gradients derivation
- dimensionality analysis



# Learning based methods

- high dimensionality – curse of dimensionality

**dimensionality reduction process must be applied** - infeasible by Gaussian process

- most of dynamic textures are not linear – learning algorithm cannot be linear

**more flexible model must be found** – based on Gaussian process

- big computational cost of optimization – necessary to have a good performance

**first-order Markov model** - based on Gaussian process

Why Gaussian process? Not many approaches can be used for modeling and synthesis



# GAUSSIAN PROCESS



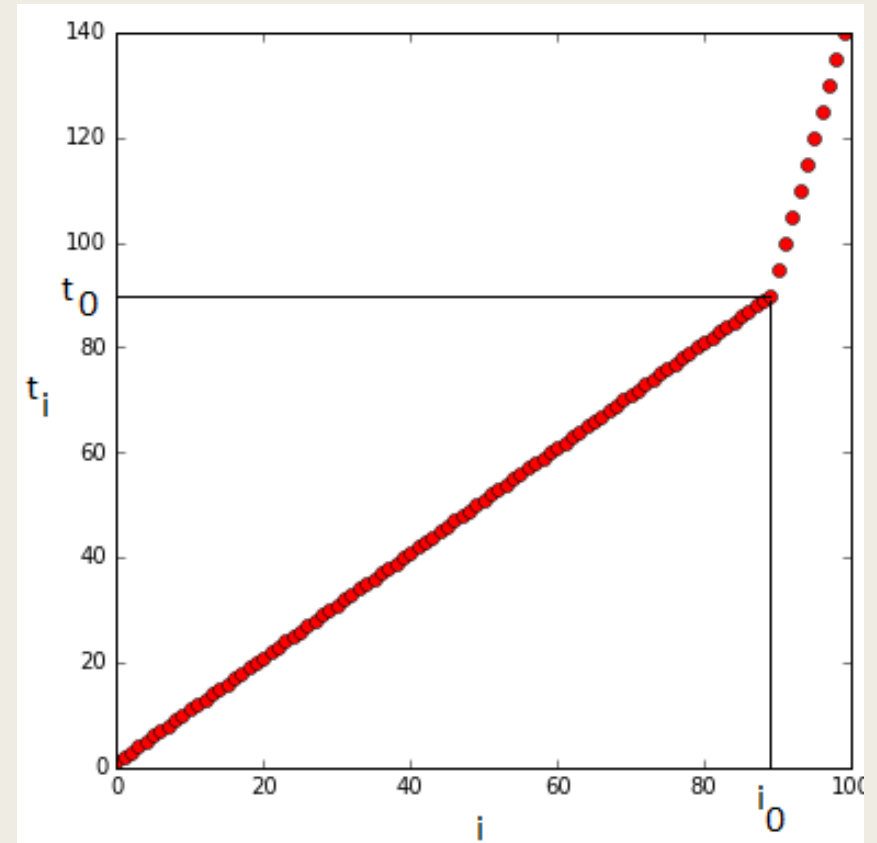
# Gaussian process

## Definition

Gaussian processes – infinite-dimensional generalization of multivariate normal distributions

Initially we have:

- $d$  random variables  $(X_{t_1}, \dots, X_{t_d})$  not equally spaced
- timeline  $t_i$
- $d = 100$
- 100 time indexes  $t_i$
- $i_0$  – index when space is increased
- not regular sampling



# Gaussian process

Characterized by mean and covariance

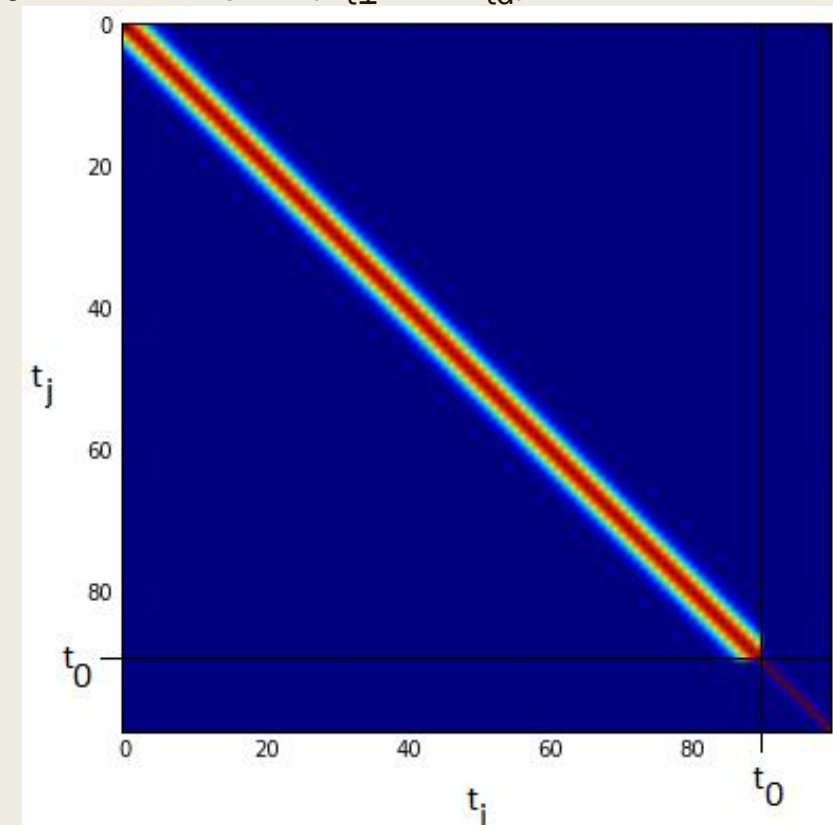
- all the vector of interest are centered – zero mean initially
- assume a specific form for the covariance of any subsample  $(X_{t_1}, \dots, X_{t_d})$

$$k(t_i, t_j) = \text{cov}(X_{t_i}, X_{t_j}) = \alpha \exp\left(-\frac{\|t_i - t_j\|^2}{2l^2}\right)$$

Parameters:

$\alpha$  - scale parameter,

$l$  - dispersion parameter





# Gaussian process

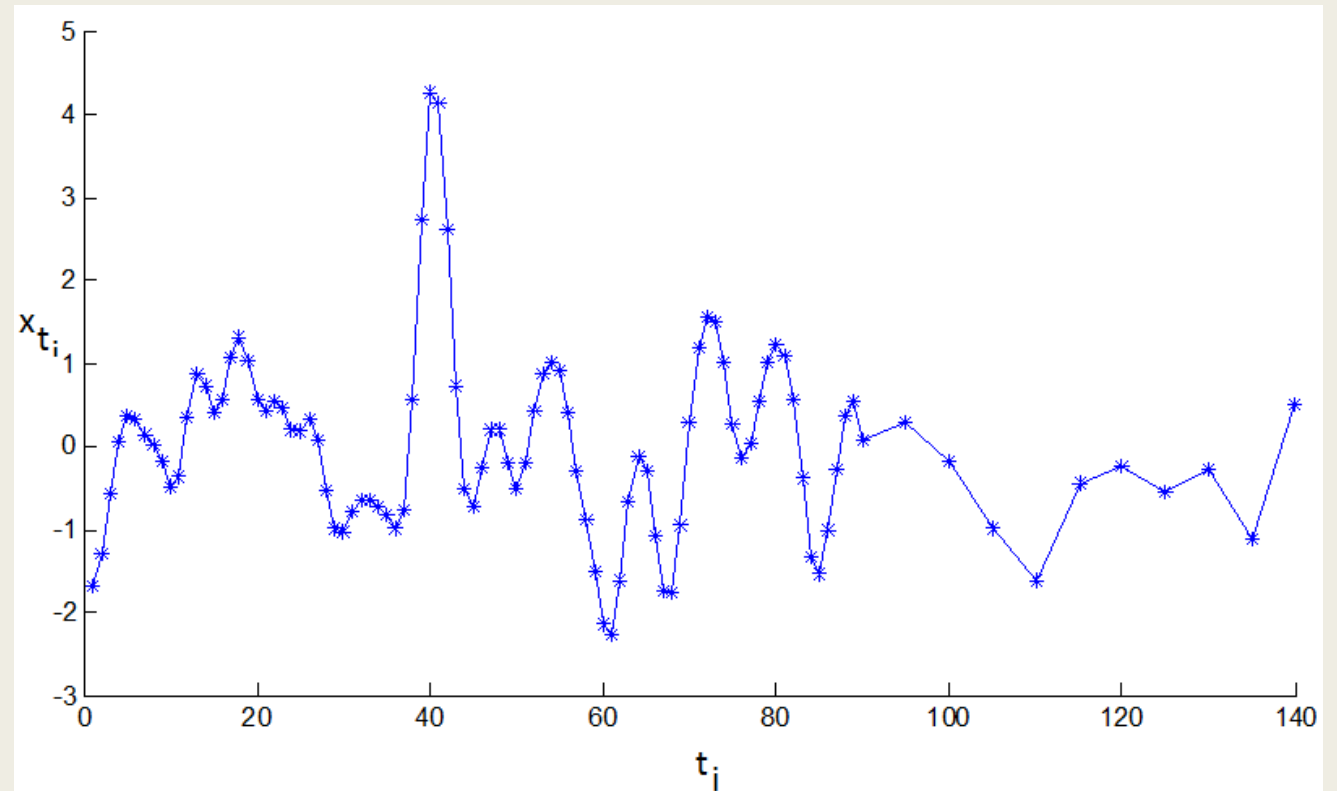
From one covariance matrix and one sequence of time  $(t_1, \dots, t_k)$  possible Gaussian distribution  $(X_{t_1}, \dots, X_{t_k})$  with zero mean and covariance  $C$

On the axis:

$x_{t_i}$  – values of random variables

$t_i$  – time indexes

Trend of samples is not spoiled  
by not equally spaced time indexes  
and can be easily seen



# Gaussian process

Now assume that we are conditioning our stochastic process for example:

$x_{t_1} = 1, x_{t_2} = 1$  with  $t_1 = 40, t_2 = 80$

Compute new mean vector and covariance matrix (multivariate conditional density):

$$p(\mathbf{f}_* | \mathbf{f}) = \mathcal{N}(\mathbf{f}_* | \boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad \boldsymbol{\mu} = \mathbf{K}_{*,f} \mathbf{K}_{f,f}^{-1} \mathbf{f}, \quad \boldsymbol{\Sigma} = \mathbf{K}_{*,*} - \mathbf{K}_{*,f} \mathbf{K}_{f,f}^{-1} \mathbf{K}_{f,*}$$

$\mathbf{f}_*$  – Gaussian vector  $\mathbf{x}_* = \mathbf{x}_{\{t \in T \setminus T'\}}$

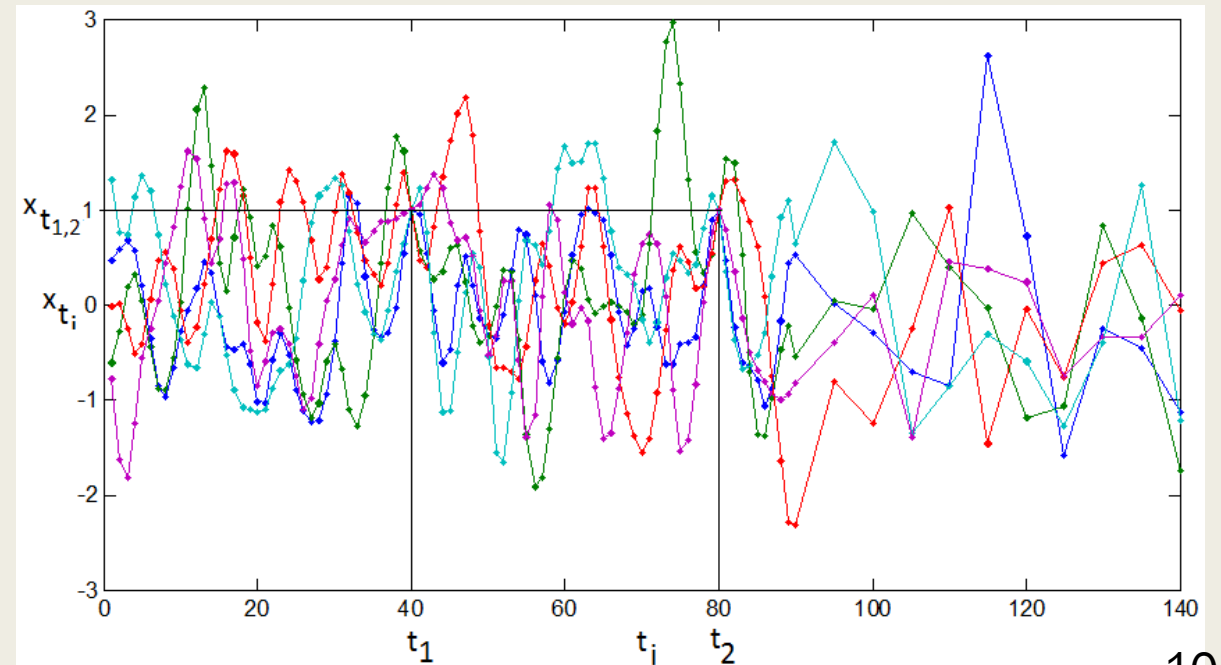
$\mathbf{f}$  – Gaussian vector  $\mathbf{x}_f = \mathbf{x}_{\{t \in T'\}}$

$\mathbf{K}$  – covariance matrices associated to two random variable sets  $\mathbf{X}_* = \mathbf{X}_{\{t \in T \setminus T'\}}, \mathbf{X}_f = \mathbf{X}_{\{t \in T'\}}$

New set of realizations of multivariate conditional distribution:

Important property of Gaussian processes

Interpolation can be performed using this approach



# GP with latent variables

- new class of models
- mappings from a latent space  $X \in \mathbb{R}^Q$  to an observed space  $Y \in \mathbb{R}^D$ ,  $Q \ll D$
- through a set of parameters  $W$
- one kernel function is used

$$K_Y = k_Y(x_i, x_j) = \sum_{l=1}^M w_l k_l(x_i, x_j) + w_\delta \delta_{x_i, x_j}$$

- $X, Y$  – multivariate Gaussian processes
- In a simplest case – probabilistic version of PCA
- likelihood of the full data set is given by:
- $W$  can then be found through maximizing:

$$p(\mathbf{Y}|\mathbf{W}, \beta) = \prod_{n=1}^N p(\mathbf{y}_n|\mathbf{W}, \beta)$$

# Dynamic system

In this work dynamical texture modeling consists of two steps:

- dimensionality reduction
- dynamic texture learning

They can be expressed as:

$$\begin{aligned}x_{t+1} &= f(x_t, A) + n_{x,t} && \text{– dynamic model} \\y_t &= g(x_t, B) + n_{y,t} && \text{– dimensionality reduction}\end{aligned}$$

$x_t$  - latent variable which affects dynamic behavior,  $x_t \in \mathbb{R}^Q$

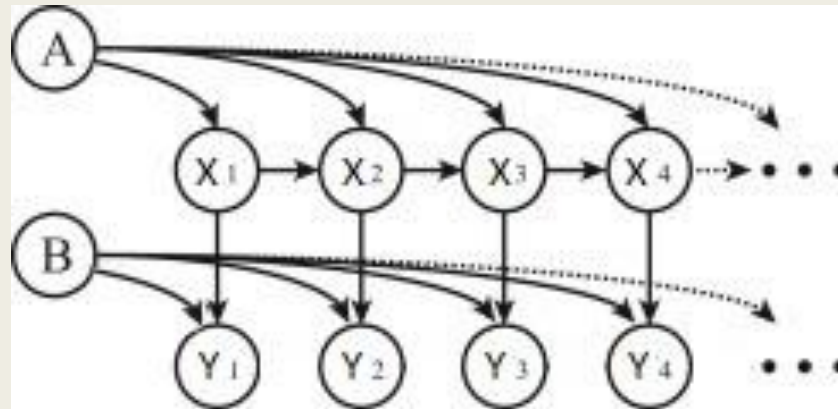
$y_t$  - column vector unfolded from the frame at time  $t$ ,  $y_t \in \mathbb{R}^D$ ,  $D$  – large,  $Q \ll D$

$n_{x,t}$ ,  $n_{y,t}$  - represent the noise



# GP dynamic model

- latent variable model
- 2 mappings: from a latent space  $X$  to the observation space  $Y$  and dynamic behavior of latent variables
- 2 kernel functions
- $X$  – sequence of row-vectors of artificial frames,  $Y$  – sequence of row-vectors of observed frames
- $A$  –  $K_Y$  parameters  $\theta$ ,  $B$  –  $K_X$  kernel weights  $W$



$Y_i$  is a multivariate Gaussian process indexed by  $X_i$  expressed by likelihood

$$p(\mathbf{Y}|\mathbf{X}, \boldsymbol{\theta}) = \prod_{t=1}^N p(y_t|x_t, \boldsymbol{\theta}) = \frac{1}{(2\pi)^{DN/2} |\mathbf{K}_Y|^{D/2}} \exp\left(-\frac{1}{2} \text{tr}(\mathbf{K}_Y^{-1} \mathbf{Y} \mathbf{Y}^T)\right)$$

# Multi-kernel GP dynamic model<sup>1</sup>

The likelihood extends to the following equation:

$$p(\mathbf{X}|\lambda, \mathbf{W}) = p(x_1) \prod_{t=2}^N p(x_t|x_{t-1}, \lambda, \mathbf{W}) = p(x_1) \frac{1}{(2\pi)^{Q(N-1)/2} |\mathbf{K}_X|^{Q/2}} \exp\left(-\frac{1}{2}\text{tr}\left(\mathbf{K}_X^{-1} \mathbf{X}_{2:N} \mathbf{X}_{2:N}^T\right)\right)$$

X – latent variable vector

Y – observed dynamic texture sequence vector

N – number of frames in original sample

Q – latent dimensionality

W – vector of weights of kernel functions  $K = k_l, l \in [1, M]$

M – number of different kernel functions used

Reference [1]:

Z. Zhu, et al., Dynamic texture modeling and synthesis using multi-kernel Gaussian process dynamic model, Signal Processing (2015)  
<http://dx.doi.org/10.1016/j.sigpro.2015.10.025>

# Multi-kernel GP dynamic model<sup>1</sup>

To achieve nonlinear mapping from a latent space X to the observation space Y special squared exponential covariance function is used

$$K_Y = k_Y(x_i, x_j) = \theta_1 \exp\left(-\frac{\theta_2}{2} (x_i - x_j)(x_i - x_j)^T\right) + \theta_3 \delta_{x_i, x_j}$$

To map dynamic behavior of latent variables

- Latent dynamic behavior varies greatly among different types of dynamic textures
- Difficult to design the most suitable kernel for a dynamic texture empirically

Multi-kernel dynamic model for dynamic texture modeling is proposed

$$K_X = k_X(x_i, x_j) = \sum_{l=1}^M w_l k_l(x_i, x_j) + w_\delta \delta_{x_i, x_j}$$

Reference [1]:

Z. Zhu, et al., Dynamic texture modeling and synthesis using multi-kernel Gaussian process dynamic model, Signal Processing (2015)  
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# Algorithm understanding

**Maximum a posteriori estimation** – main method. In Matlab specific library for optimization is used.

In general two main steps:

- fix  $W$  and perform optimization with respect to  $X$ ,  $\theta$  and  $\lambda$  using SCG (Scaled conjugate gradient)

$$F(X, \theta, \lambda) = -\ln P(X, \theta, \lambda | Y) = \frac{D}{2} \ln |K_Y| + \frac{1}{2} \text{tr}(K_Y^{-1} Y Y^T) + \frac{Q}{2} \ln |K_X| + \frac{1}{2} \text{tr}(K_X^{-1} X_{2:N} X_{2:N}^T) + \sum_i \theta_i + \sum_{i,j} (\lambda_i)_j + C$$

$W$  – weights vector

$X$  – latent variable

$\theta$  – vector of hyperparameters of kernel  $K_Y$

$\Lambda$  – vector of parameters of kernel  $K_X$

- fix obtained  $X$ ,  $\theta$  and  $\lambda$  and perform optimization with respect to  $W$  using gradient descent

$$F(W) = \frac{Q}{2} \ln |K_X^{-1}| + \frac{1}{2} \text{tr}(K_X^{-1} X_{2:N} X_{2:N}^T) + \alpha \|W\|_2$$

Functions are not convex – repeat these two main steps for  $I$  times,  $I$  – fixed





# RESULTS



# Matlab implementation properties

Implementation given in Matlab has problems:

- Inexplicable instability – often crashes with SVD computation error
- Static result – sometimes generated dynamic texture does not move at all
- Repetition of original sequences of frames

Attempt	Linear	RBF	Poly	RatQuad	MLP	Matern32
1	0.040278	0.682626	0	0	0	0.277096
3	0.067891	0.720069	0	0	0.212038	0
7	0.003638	0.368365	0	0.345235	0.069727	0.213032

# Python implementation properties

Implementation in Python:

- GPLVM does not have specific kernel function to capture the dynamic behavior
- Temporal structure is included in covariance

The results are quite surprising:

- it is stable
- it is able to generate new sequences of dynamic textures without visible repetitions
- it takes a lot of time to perform optimization
- visible result is still not good due to some random noise



**EXAMPLE**





# Examples

Matlab implementation:

- Input videos ***sunshade.avi*** and ***straw.avi***
- Example of a good result (sunshade.avi)
- Example of a bad result (straw.avi)

Python implementation

- Example of a good result (straw.avi)



**PERSPECTIVES**



# Perspectives

- MK-GPLVM reimplementation in Python is not finished – problems with gradients
- no numerical criteria for quality measuring – implement an evaluation method
- GPLVM implementation in Python is done and provides quite interesting result – improve the results
- try wavelets to reduce dimensionality of original input to decrease time for optimization



**THANK YOU !**