

Stereo and Epipolar geometry

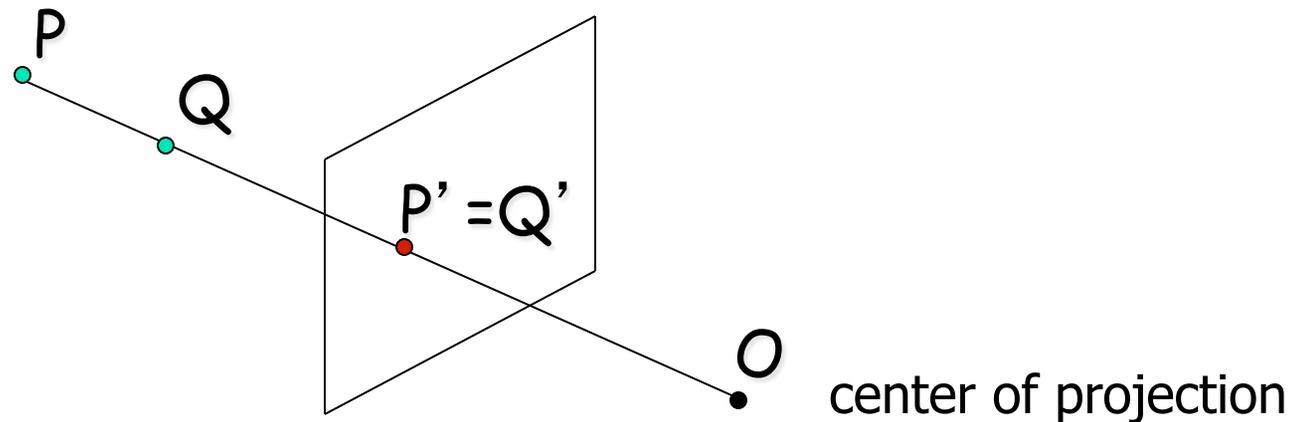
Jana Kosecka

Previously

- Image Primitives (feature points, lines, contours)
- Today:
 - How to match primitives between two (multiple) views)
 - Goals: 3D reconstruction, recognition
- Stereo matching and reconstruction (canonical configuration)
- Epipolar Geometry (general two view setting)

Why Stereo Vision?

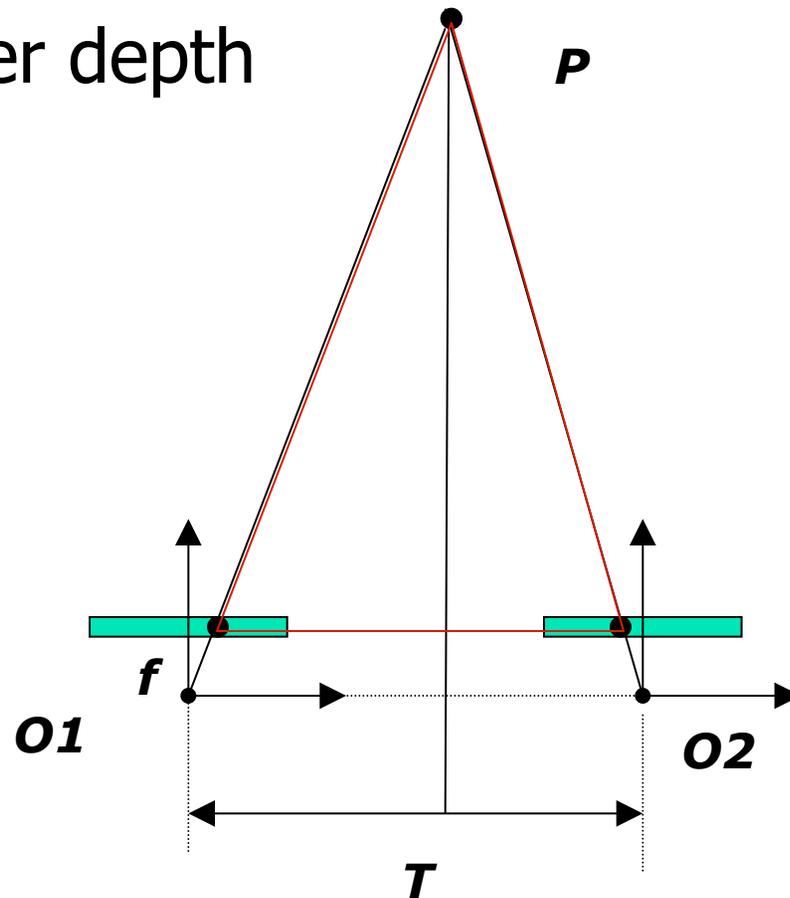
- 2D images project 3D points into 2D:



- 3D points on the same viewing line have the same 2D image:
 - 2D imaging results in depth information loss

Canonical Stereo Configuration

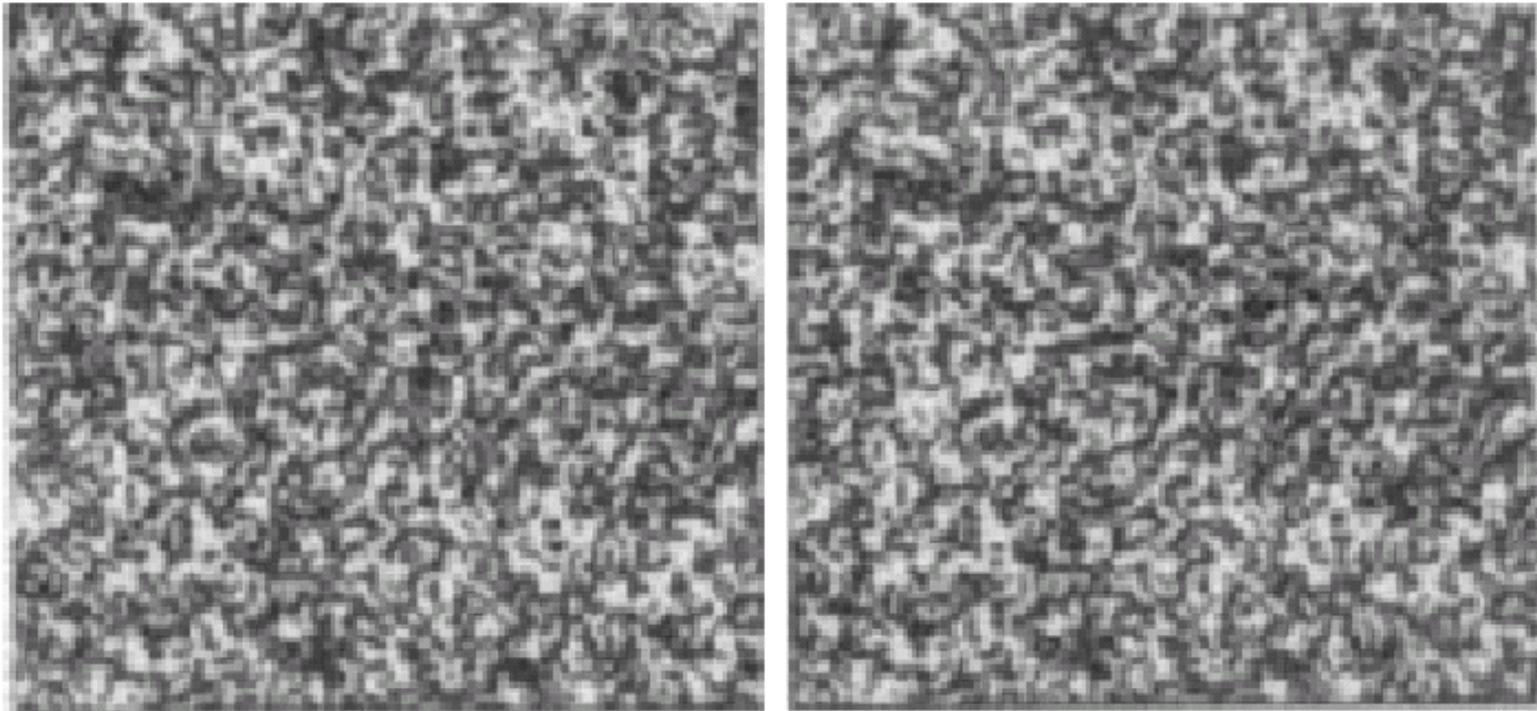
- Assumes (two) cameras
- Known positions and focal lengths
- Recover depth



$$\frac{Z}{T} = \frac{Z-f}{T-x_l-x_r}$$

$$Z = \frac{fT}{\text{disparity}}$$

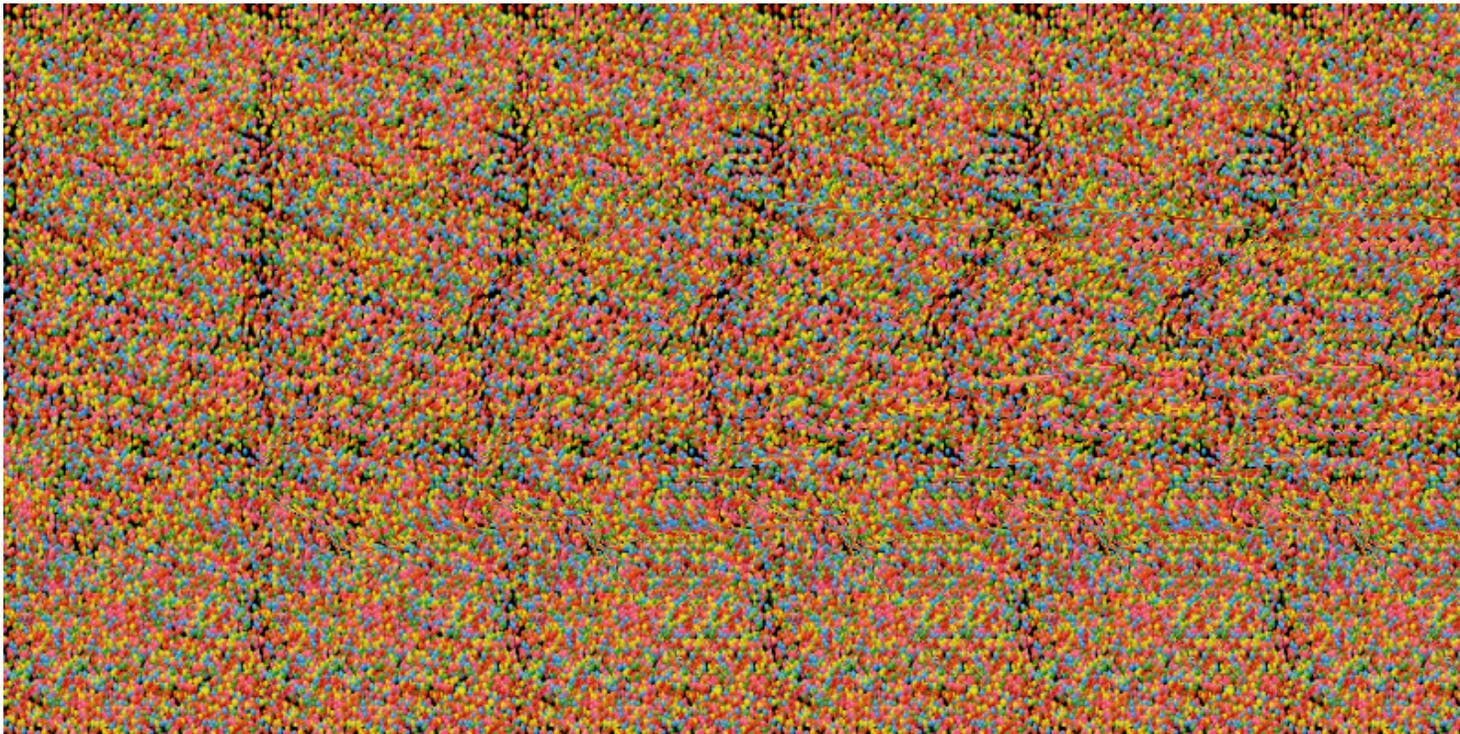
Random Dot Stereo-grams



B. Julesz: showed that the depth can be perceived in the absence of any identifiable objects in correspondence

Autostereograms

- Depth perception from one image



- Viewing trick the brain by focusing at the plane behind - match can be established perception of 3D

Correspondence Problem

- Two classes of algorithms:
 - Correlation-based algorithms
 - Produce a DENSE set of correspondences
 - Feature-based algorithms
 - Produce a SPARSE set of correspondences

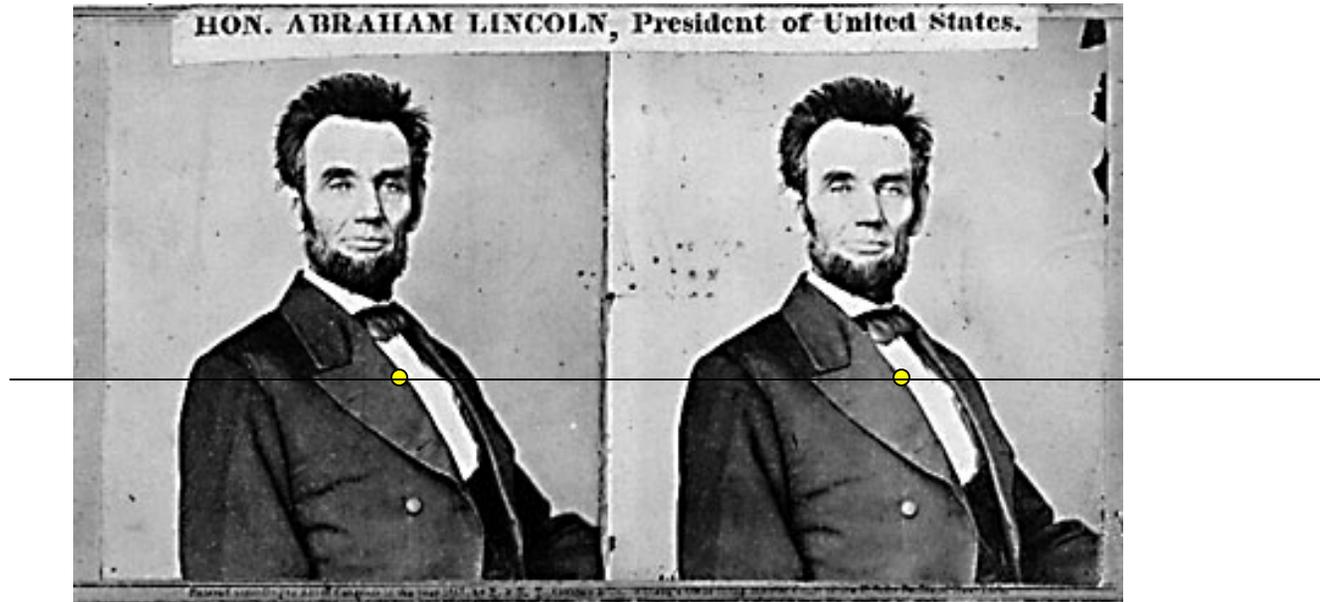
Stereo – Photometric Constraint

- Same world point has same intensity in both images.
 - Lambertian fronto-parallel
 - Issues (noise, specularities, foreshortening)



- Difficulties – ambiguities, large changes of appearance, due to change Of viewpoint, non-uniquess

Stereo Matching

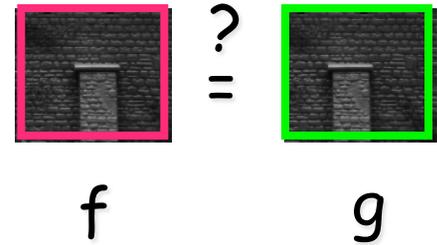


What if ?

For each scanline , for each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost
- This will never work, so: **improvement match windows**

Comparing Windows:



$$SSD = \sum_{[i,j] \in R} (f(i,j) - g(i,j))^2$$
$$C_{fg} = \sum_{[i,j] \in R} f(i,j)g(i,j)$$

Most popular

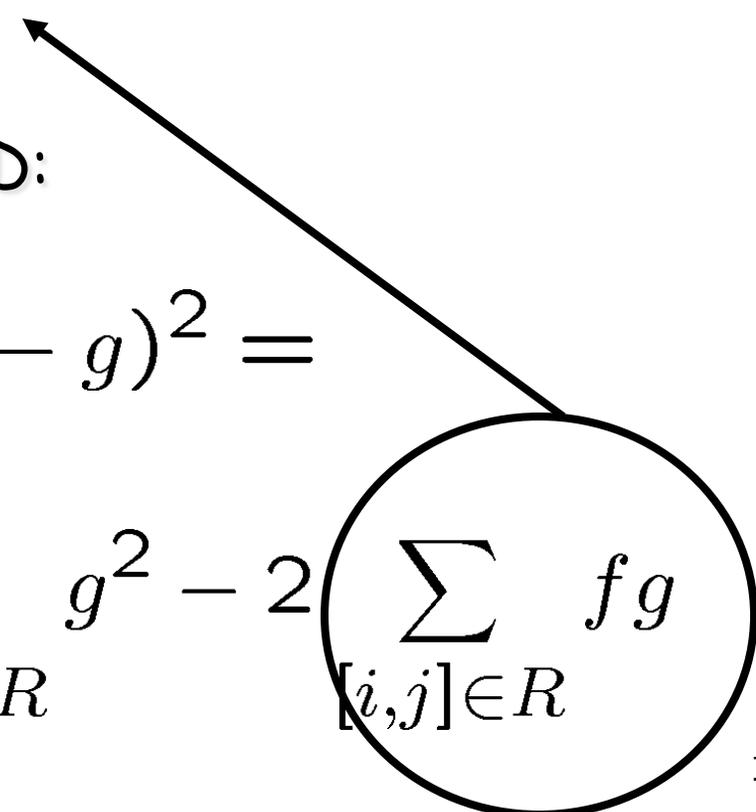
For each window, match to closest window on epipolar line in other image.

Comparing Windows:

Minimize $\sum_{[i,j] \in R} (f(i,j) - g(i,j))^2$ Sum of Squared Differences

Maximize $C_{fg} = \sum_{[i,j] \in R} f(i,j)g(i,j)$ Cross correlation

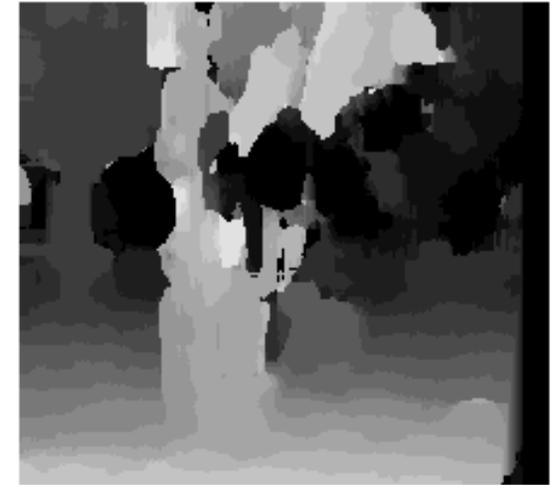
It is closely related to the SSD:

$$\begin{aligned} SSD &= \sum_{[i,j] \in R} (f - g)^2 = \\ &= \sum_{[i,j] \in R} f^2 + \sum_{[i,j] \in R} g^2 - 2 \sum_{[i,j] \in R} fg \end{aligned}$$


Window size



$W = 3$



$W = 20$

■ Effect of window size

Better results with *adaptive window*

- T. Kanade and M. Okutomi, [*A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment*](#), Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski. [*Stereo matching with nonlinear diffusion*](#). International Journal of Computer Vision, 28(2):155-174, July 1998

(S. Seitz)

Stereo results

- Data from University of Tsukuba



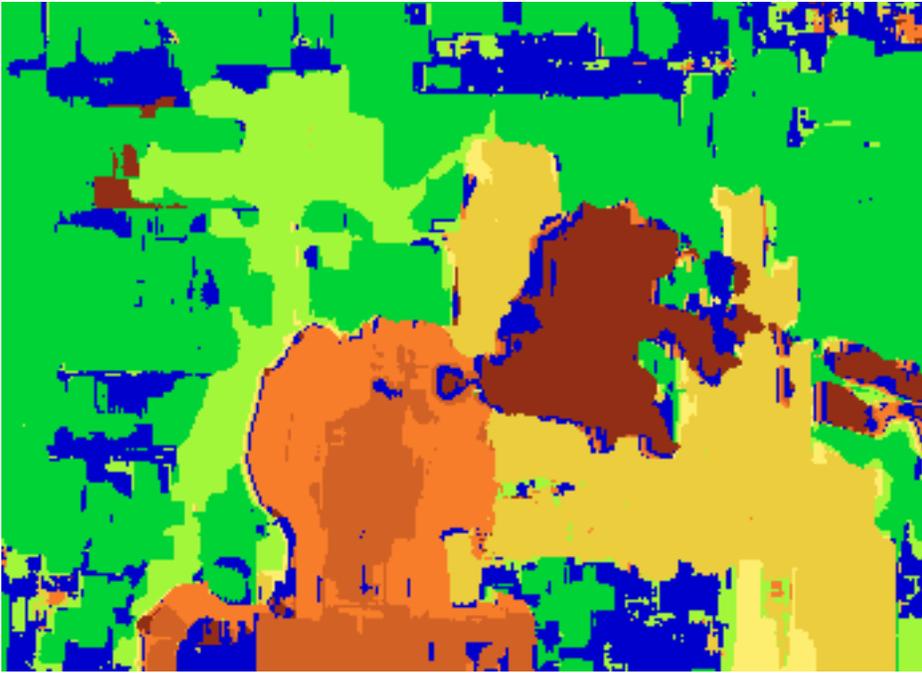
Scene



Ground truth

(Seitz)

Results with window correlation



Window-based matching
(best window size)



Ground truth

(Seitz)

Results with better method



State of the art

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),
International Conference on Computer Vision, September 1999.



Ground truth

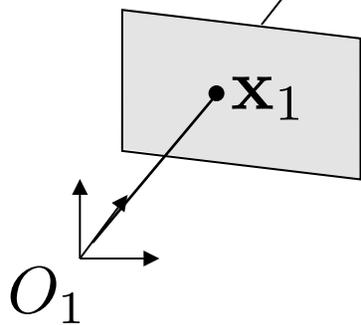
(Seitz)

More of advanced stereo (later)

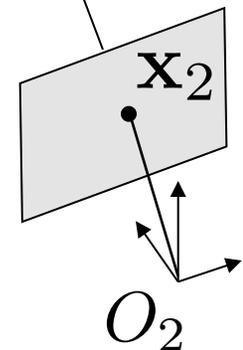
- Ordering constraint
- Dynamic programming
- Global optimization

Two view – General Configuration

- Motion between the two views is not known



Given two views of the scene
recover the unknown camera
displacement and 3D scene
structure



Pinhole Camera Imaging Model

- 3D points $\mathbf{X} = [X, Y, Z, W]^T \in \mathbb{R}^4$, ($W = 1$)

- Image points $\mathbf{x} = [x, y, z]^T \in \mathbb{R}^3$, ($z = 1$)

- Perspective Projection $\lambda \mathbf{x} = \mathbf{X}$

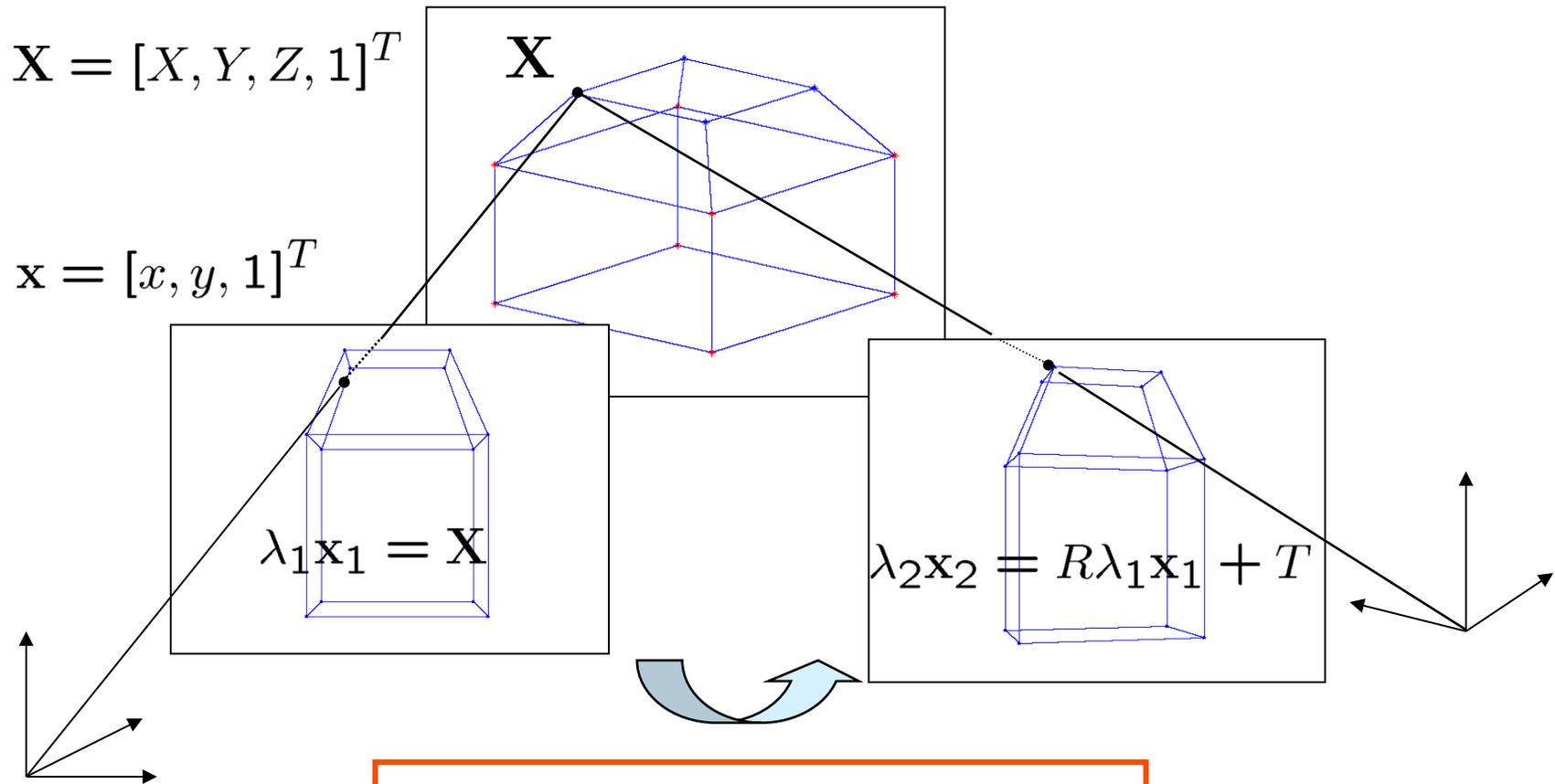
$$\lambda = Z \quad x = \frac{X}{Z} \quad y = \frac{Y}{Z}$$

- Rigid Body Motion $\Pi = [R, T] \in \mathbb{R}^{3 \times 4}$

- Rigid Body Motion + Persp. projection $\lambda \mathbf{x} = \Pi \mathbf{X} = [R, T] \mathbf{X}$

$$\lambda \mathbf{x}' = K \Pi_0 \mathbf{X} = \begin{bmatrix} f s_x & f s_\theta & o_x \\ 0 & f s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R & T \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Rigid Body Motion – Two Views



$$\lambda_2 \mathbf{x}_2 = R \lambda_1 \mathbf{x}_1 + T$$

$$\lambda \mathbf{x} = \Pi \mathbf{X} = [R, T] \mathbf{X} \quad \Pi = [R, T] \in \mathbb{R}^{3 \times 4}$$

Notation

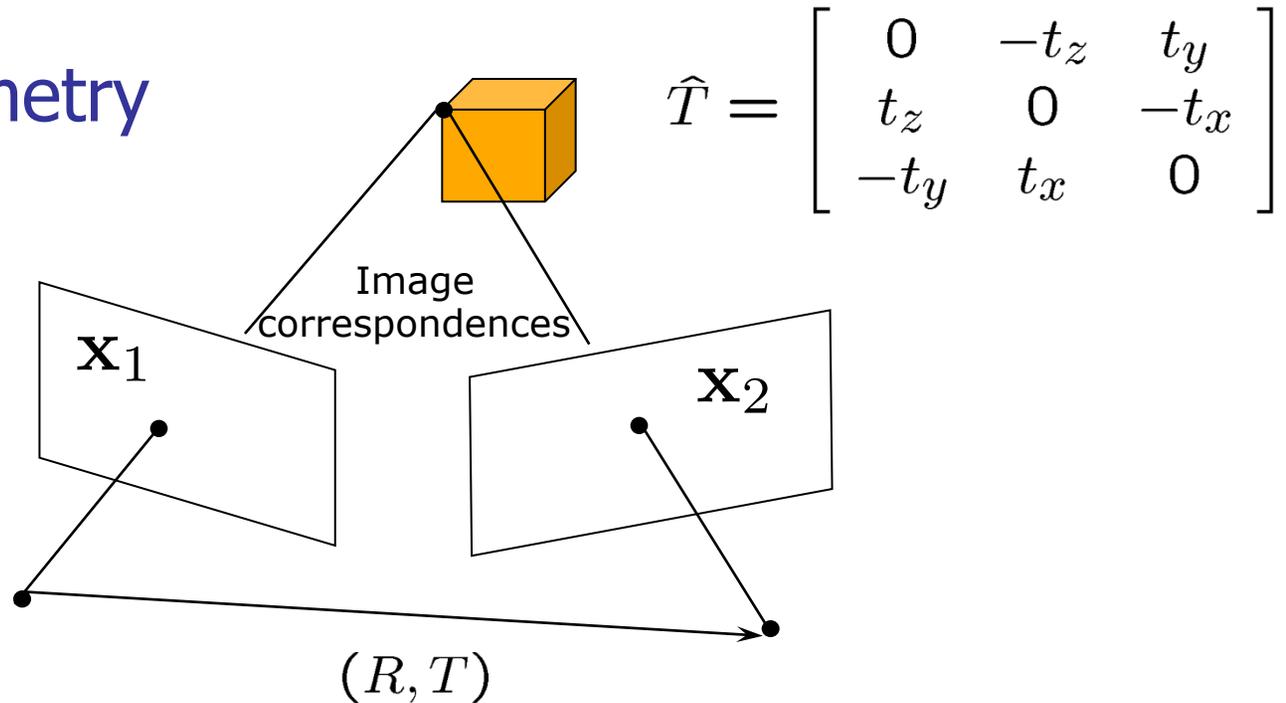
- Cross product between two vectors in

$$c = a \times b \quad c = \begin{bmatrix} -a_3b_2 + a_2b_3 \\ a_3b_1 - a_1b_3 \\ -a_2b_1 + a_1b_2 \end{bmatrix}$$

where

$$\hat{a} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Epipolar Geometry



$$\lambda_2 \mathbf{x}_2 = R \lambda_1 \mathbf{x}_1 + T \quad / \widehat{\mathbf{x}}_2^T T$$

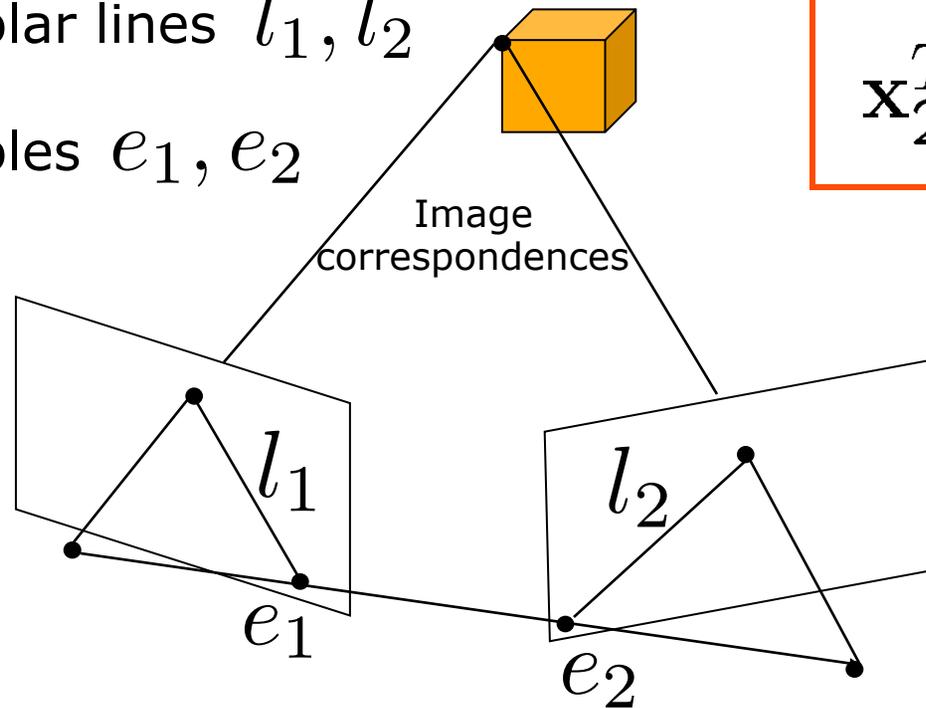
- Algebraic Elimination of Depth [Longuet-Higgins '81]:

$$\mathbf{x}_2^T \underbrace{\hat{T} R}_E \mathbf{x}_1 = 0$$

- Essential matrix $E = \hat{T} R$

Epipolar Geometry

- Epipolar lines l_1, l_2
- Epipoles e_1, e_2



$$\mathbf{x}_2^T E \mathbf{x}_1 = 0$$

$$E = \hat{T} R$$

- Additional constraints

$$l_1 \sim E^T \mathbf{x}_2$$

$$l_i^T \mathbf{x}_i = 0$$

$$l_2 \sim E \mathbf{x}_1$$

$$E \mathbf{e}_1 = 0$$

$$l_i^T \mathbf{e}_i = 0$$

$$\mathbf{e}_2 E^T = 0$$

Epipolar transfer

Characterization of Essential Matrix

$$\mathbf{x}_2^T \hat{T} R \mathbf{x}_1 = 0$$

Essential matrix $E = \hat{T} R$ special 3x3 matrix

$$\mathbf{x}_2^T \begin{bmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{bmatrix} \mathbf{x}_1 = 0$$

(Essential Matrix Characterization)

A non-zero matrix E is an essential matrix iff its SVD: $E = U \Sigma V^T$ satisfies: $\Sigma = \text{diag}([\sigma_1, \sigma_2, \sigma_3])$ with $\sigma_1 = \sigma_2 \neq 0$ and $\sigma_3 = 0$ and $U, V \in SO(3)$

Estimating Essential Matrix

- Find such **Rotation** and **Translation** that the epipolar error is minimized

$$\min_E \sum_{j=1}^n (\mathbf{x}_2^{jT} E \mathbf{x}_1^j)^2$$

- Space of all **Essential Matrices** is 5 dimensional
- 3 DOF Rotation, 2 DOF – Translation (**up to scale !**)
- Denote $\mathbf{a} = \mathbf{x}_1 \otimes \mathbf{x}_2$

$$\mathbf{a} = [x_1x_2, x_1y_2, x_1z_2, y_1x_2, y_1y_2, y_1z_2, z_1x_2, z_1y_2, z_1z_2]^T$$

$$E^s = [e_1, e_4, e_7, e_2, e_5, e_8, e_3, e_6, e_9]^T$$

- Rewrite $\mathbf{a}^T E^s = 0$

- Collect constraints from all points

$$\chi E^s = 0$$

$$\min_E \sum_{j=1}^n \mathbf{x}_2^{jT} E \mathbf{x}_1^j \quad \longrightarrow \quad \min_{E^s} \|\chi E^s\|^2$$

Estimating Essential Matrix

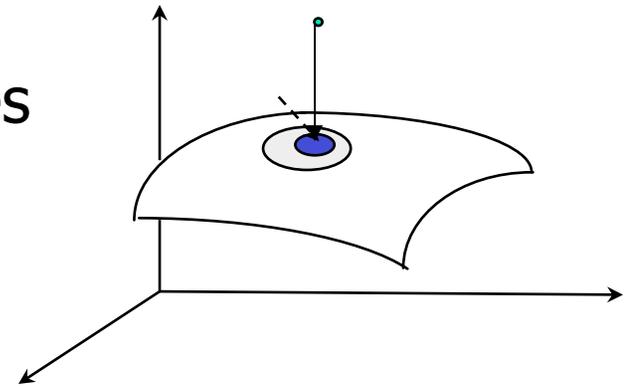
$$\min_E \sum_{j=1}^n \mathbf{x}_2^{jT} E \mathbf{x}_1^j \quad \longrightarrow \quad \min_{E^s} \|\chi E^s\|^2$$

Solution is

- Eigenvector associated with the smallest eigenvalue of $\chi^T \chi$
- If $\text{rank}(\chi^T \chi) < 8$ degenerate configuration

E_s estimated using linear least squares
 unstack $E_s \rightarrow F$

Projection on to Essential Space



(Project onto a space of Essential Matrices)

If the SVD of a matrix $F \in \mathcal{R}^{3 \times 3}$ is given by $F = U \text{diag}(\sigma_1, \sigma_2, \sigma_3) V^T$ then the essential matrix which minimizes the Frobenius distance $\|E - F\|_f^2$ is given by $E = U \text{diag}(\sigma, \sigma, 0) V^T$ with $\sigma = \frac{\sigma_1 + \sigma_2}{2}$

Pose Recovery from Essential Matrix

Essential matrix $E = \hat{T}R$

(Pose Recovery)

There are two relative poses (R, T) with $T \in \mathcal{R}^3$ and $R \in SO(3)$ corresponding to a non-zero matrix essential matrix.

$$E = U\Sigma V^T$$

$$\begin{aligned}(\hat{T}_1, R_1) &= (UR_Z(+\frac{\pi}{2})\Sigma U^T, UR_Z^T(+\frac{\pi}{2})V^T) \\(\hat{T}_2, R_2) &= (UR_Z(-\frac{\pi}{2})\Sigma U^T, UR_Z^T(-\frac{\pi}{2})V^T)\end{aligned}$$

$$\Sigma = \text{diag}([1, 1, 0]) \quad R_z(+\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Twisted pair ambiguity $(R_2, T_2) = (e^{\hat{u}\pi}R_1, -T_1)$

Two view linear algorithm - summary

$$E = \{\hat{T}R | R \in SO(2), T \in S^2\}$$

- Solve the **LLSE** problem:

$$\min_E \sum_{j=1}^n (\mathbf{x}_2^{jT} E \mathbf{x}_1^j)^2 \rightarrow \chi E^s = 0$$

- Solution eigenvector associated with smallest eigenvalue of $\chi^T \chi$

- Compute SVD of F recovered from data

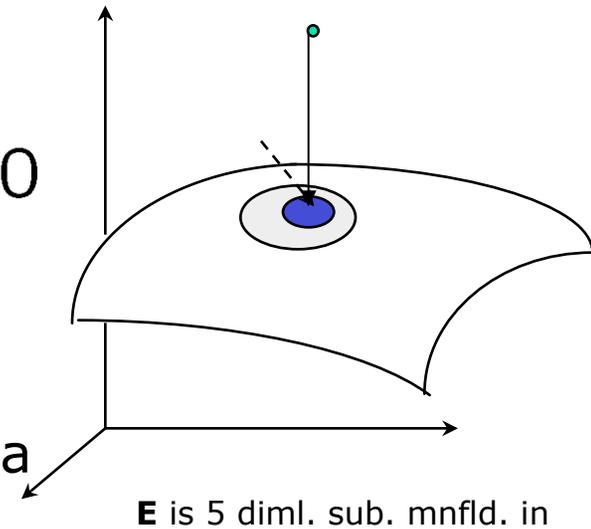
$$E^s \rightarrow F \quad F = U \Sigma V^T$$

- **Project** onto the essential manifold:

$$\Sigma' = \text{diag}(1, 1, 0) \quad E = U \Sigma' V^T$$

- **Recover** the unknown pose:

$$(\hat{T}, R) = (UR_Z(\pm \frac{\pi}{2}) \Sigma U^T, UR_Z^T(\pm \frac{\pi}{2}) V^T)$$



- 8-point linear algorithm

Pose Recovery

- There are **two** pairs (R, T) corresponding to essential matrix .
- There are **two** pairs (R, T) corresponding to essential matrix .
- Positive depth constraint disambiguates the impossible solutions
- Translation has to be non-zero
- Points have to be in general position
 - degenerate configurations – planar points
 - quadratic surface
- Linear 8-point algorithm
- Nonlinear 5-point algorithms yields up to 10 solutions

3D Structure Recovery

$$\underline{\lambda_2} \mathbf{x}_2 = \underline{R} \underline{\lambda_1} \mathbf{x}_1 + \underline{\gamma T} \quad \text{unknowns}$$

- Eliminate one of the scale's

$$\lambda_1^j \widehat{\mathbf{x}}_2^j R \mathbf{x}_1^j + \gamma \widehat{\mathbf{x}}_2^j T = 0, \quad j = 1, 2, \dots, n$$

- Solve LLSE problem

$$M^j \bar{\lambda}^j \doteq \begin{bmatrix} \widehat{\mathbf{x}}_2^j R \mathbf{x}_1^j, & \widehat{\mathbf{x}}_2^j T \end{bmatrix} \begin{bmatrix} \lambda_1^j \\ \gamma \end{bmatrix} = 0$$

If the configuration is non-critical, the Euclidean structure of the points and motion of the camera can be reconstructed up to a universal scale.

- Alternatively recover each point depth separately

Example

Two views

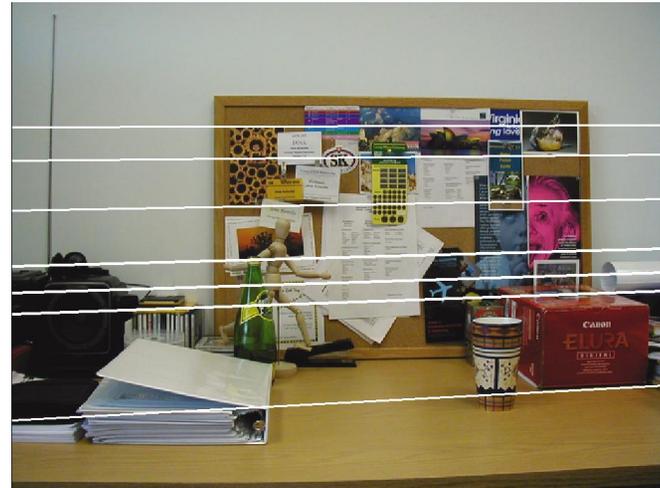
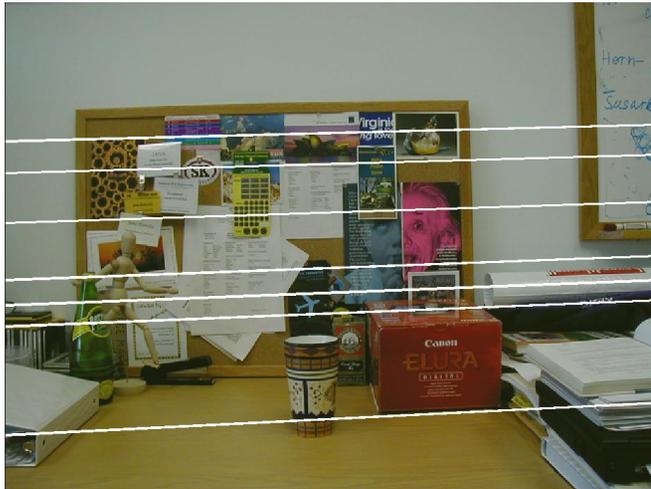


Point Feature Matching



Example

Epipolar Geometry



Camera Pose
and
Sparse Structure Recovery



Epipolar Geometry - Planar case

- Plane in first camera coordinate frame

$$aX + bY + cZ + d = 0$$

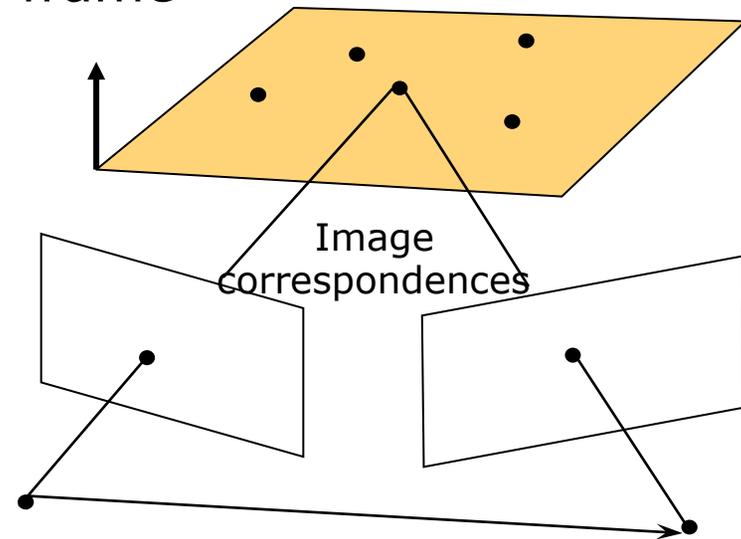
$$\frac{1}{d}N^T \mathbf{X} = 1$$

$$\lambda_2 \mathbf{x}_2 = R\lambda_1 \mathbf{x}_1 + T$$

$$\lambda_2 \mathbf{x}_2 = \left(R + \frac{1}{d}TN^T\right)\lambda_1 \mathbf{x}_1$$

$$\mathbf{x}_2 \sim H\mathbf{x}_1$$

$$H = \left(R + \frac{1}{d}TN^T\right)$$



Planar homography

Linear mapping relating two corresponding planar points in two views

Decomposition of H (into motion and plane normal)

- Algebraic elimination of depth $\widehat{x}_2 H x_1 = 0$
- can be estimated linearly $H_L = \lambda H$
- Normalization of $H = H_L / \sigma_3$
- Decomposition of H into 4 solutions $H = (R + \frac{1}{d} T N^T)$

$R_1 = W_1 U_1^T$ $N_1 = \widehat{v}_2 u_1$ $\frac{1}{d} T_1 = (H - R_1) N_1$	$R_3 = R_1$ $N_3 = -N_1$ $\frac{1}{d} T_3 = -\frac{1}{d} T_1$	$R_2 = W_2 U_2^T$ $N_2 = \widehat{v}_2 u_2$ $\frac{1}{d} T_2 = (H - R_2) N_2$	$R_4 = R_2$ $N_4 = -N_2$ $\frac{1}{d} T_4 = -\frac{1}{d} T_2$
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- $$H^T H = V \Sigma V^T \quad V = [v_1, v_2, v_3] \quad \Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2)$$

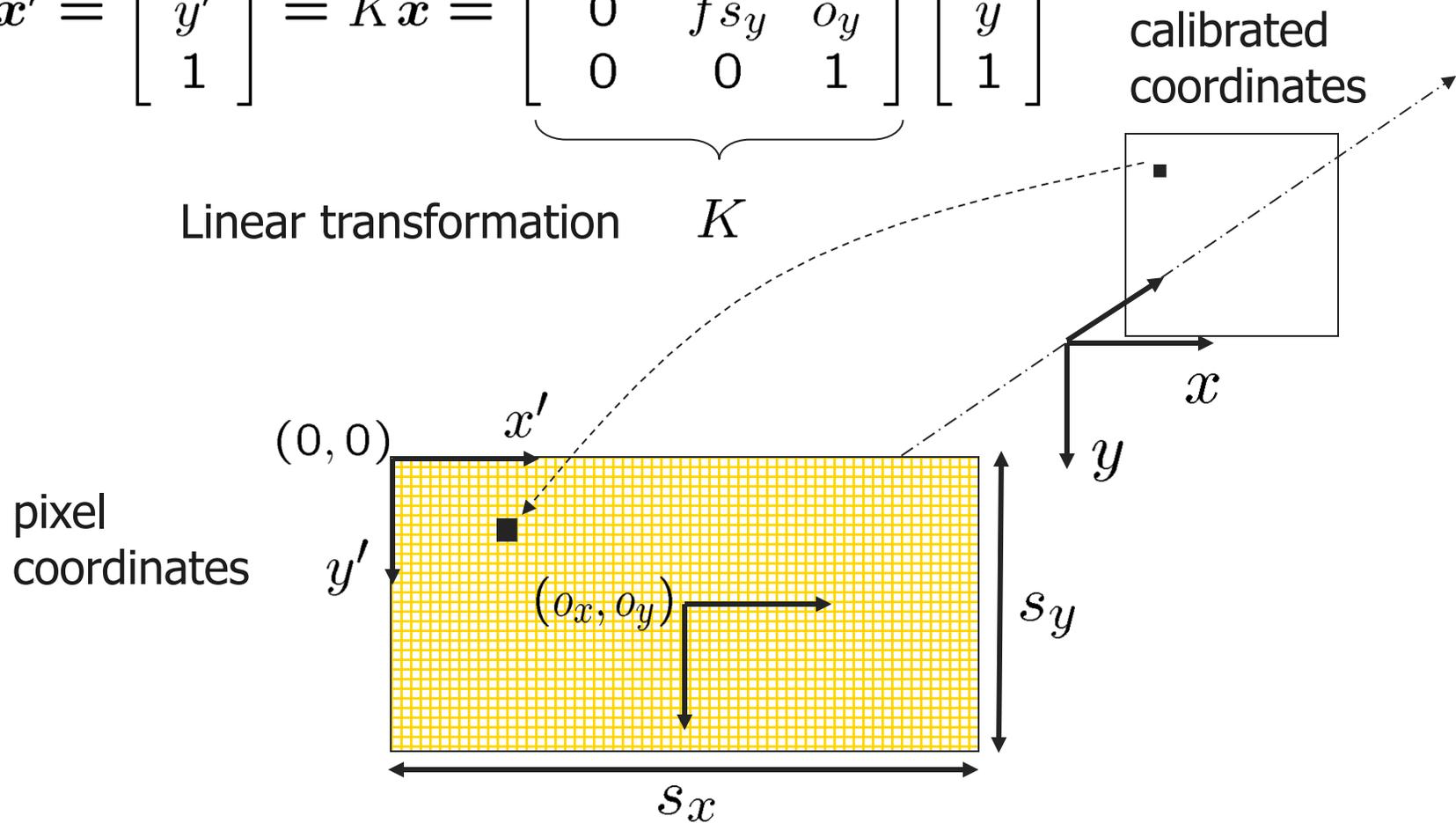
$$u_1 \doteq \frac{\sqrt{1-\sigma_3^2} v_1 + \sqrt{\sigma_1^2 - 1} v_3}{\sqrt{\sigma_1^2 - \sigma_3^2}} \quad u_2 \doteq \frac{\sqrt{1-\sigma_3^2} v_1 - \sqrt{\sigma_1^2 - 1} v_3}{\sqrt{\sigma_1^2 - \sigma_3^2}}$$

$$U_1 = [v_2, u_1, \widehat{v}_2 u_1], \quad W_1 = [H v_2, H u_1, \widehat{H} v_2 H u_1];$$

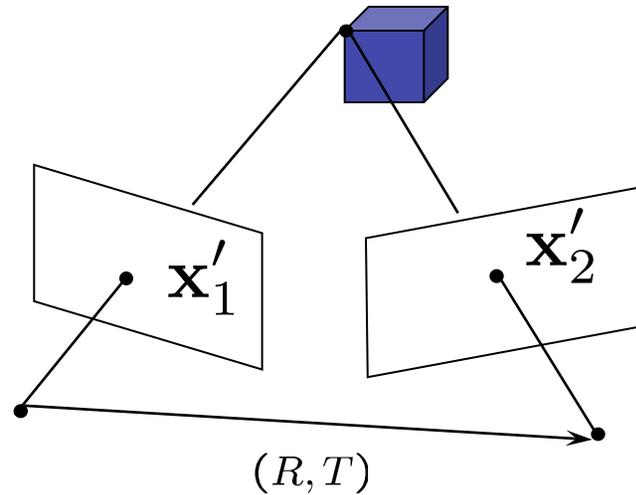
$$U_2 = [v_2, u_2, \widehat{v}_2 u_2], \quad W_2 = [H v_2, H u_2, \widehat{H} v_2 H u_2].$$

Uncalibrated Camera

$$\mathbf{x}' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K \mathbf{x} = \underbrace{\begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_K \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Uncalibrated Epipolar Geometry

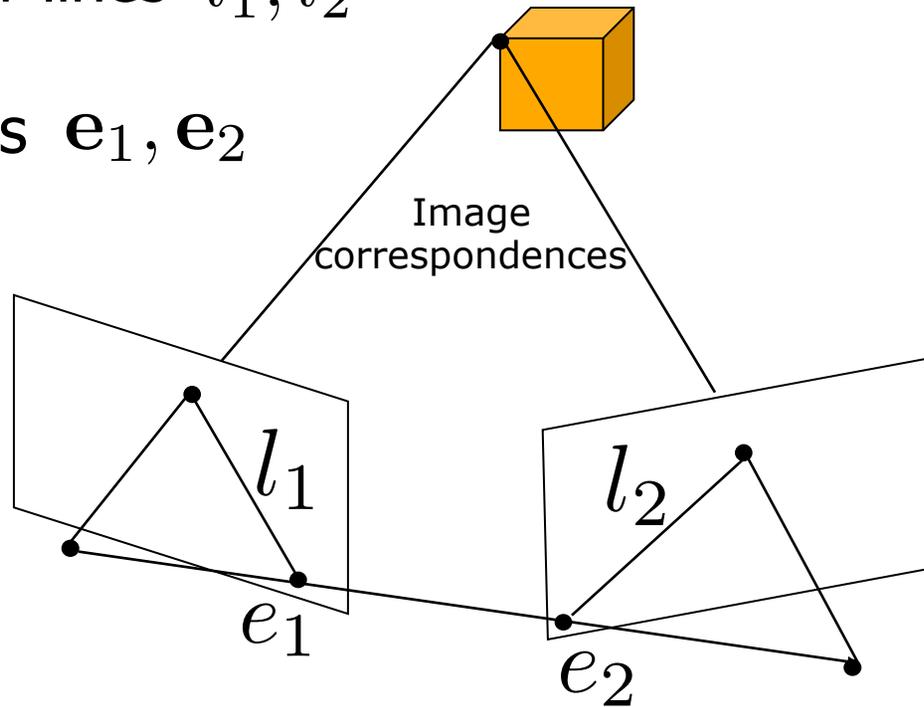


- Epipolar constraint $\mathbf{x}'_2{}^T \underbrace{K^{-T} \hat{T} R K^{-1}} \mathbf{x}'_1 = 0$
- Fundamental matrix $F = K^{-T} \hat{T} R K^{-1}$

Properties of the Fundamental Matrix

$$\mathbf{x}'_2{}^T F \mathbf{x}'_1 = 0$$

- Epipolar lines l_1, l_2
- Epipoles $\mathbf{e}_1, \mathbf{e}_2$



$$l_1 \sim F^T \mathbf{x}'_2$$

$$F \mathbf{e}_1 = 0$$

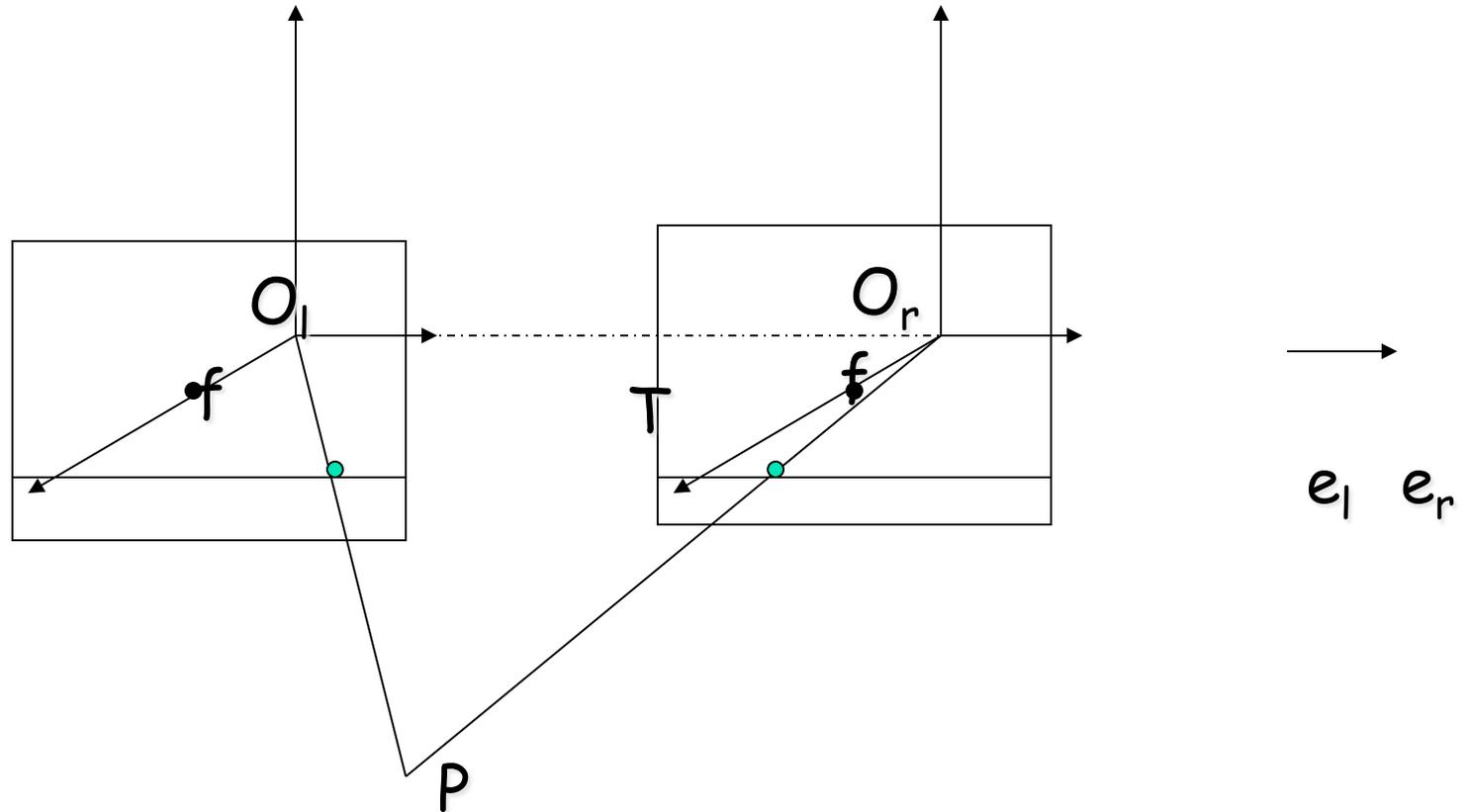
$$l_i^T \mathbf{x}'_i = 0$$

$$l_i^T \mathbf{e}_i = 0$$

$$l_2 \sim F \mathbf{x}'_1$$

$$\mathbf{e}_2^T F = 0$$

Epipolar Geometry for Parallel Cameras



Epipoles are at infinite
Epipolar lines are parallel to the baseline

Properties of the Fundamental Matrix

A nonzero matrix $F \in \mathbb{R}^{3 \times 3}$ is a fundamental matrix if it has a singular value decomposition (SVD) $F = U\Sigma V^T$ with

$$\Sigma = \text{diag}\{\sigma_1, \sigma_2, 0\}$$

for some $\sigma_1, \sigma_2 \in \mathbb{R}_+$.

There is little structure in the matrix F except that

$$\det(F) = 0$$

Estimating Fundamental Matrix

- Find such F that the epipolar error is minimized

$$\min_F \sum_{j=1}^n (\mathbf{x}'_2{}^T F \mathbf{x}'_1)^2 \leftarrow \text{Pixel coordinates}$$

- Fundamental matrix can be estimated up to scale

- Denote $\mathbf{a} = \mathbf{x}'_1 \otimes \mathbf{x}'_2$

$$\mathbf{a} = [x_1 x_2, x_1 y_2, x_1 z_2, y_1 x_2, y_1 y_2, y_1 z_2, z_1 x_2, z_1 y_2, z_1 z_2]^T$$

$$F^s = [f_1, f_4, f_7, f_2, f_5, f_8, f_3, f_6, f_9]^T$$

- Rewrite $\mathbf{a}^T F^s = 0$

- Collect constraints from all points

$$\chi F^s = 0$$

$$\min_F \sum_{j=1}^n (\mathbf{x}_2^{jT} F \mathbf{x}_1^j)^2 \quad \longrightarrow \quad \min_{F^s} \|\chi F^s\|^2$$

Two view linear algorithm – 8-point algorithm

- Solve the **LLSE** problem:

$$\min_F \sum_{j=1}^n (\mathbf{x}_2'^j T F \mathbf{x}_1'^j)^2 \rightarrow \min_{F_s} \|\chi F^s\|^2 = 0$$

- Solution eigenvector associated with smallest eigenvalue of $\chi^T \chi$

- Compute SVD of F recovered from data

$$F = U \Sigma V^T \quad \Sigma = \text{diag}(\sigma_1, \sigma_2, \sigma_3)$$

- **Project** onto the essential manifold:

$$\Sigma' = \text{diag}(\sigma_1, \sigma_2, 0) \quad F = U \Sigma' V^T$$

- cannot be unambiguously decomposed into pose and calibration

$$F = K^{-T} \hat{T} R K^{-1}$$

Dealing with correspondences

- Previous methods assumed that we have exact correspondences
- Followed by linear least squares estimation
- Correspondences established either by tracking (using affine or translational flow models)
- Or wide-baseline matching (using scale/rotation invariant features and their descriptors)
- In many cases we get incorrect matches/tracks

Robust estimators for dealing with outliers

- Use robust objective function
 - The M-estimator and Least Median of Squares (LMedS) Estimator (neither of them can tolerate more than 50% outliers)
- The RANSAC (RANdom SAMple Consensus) algorithm
 - Proposed by Fischler and Bolles
 - Popular technique used in Computer Vision community (and else where for robust estimation problems)
- It can tolerate more than 50% outliers

The RANSAC algorithm

- Generate M (a predetermined number) model hypotheses, each of them is computed using a minimal subset of points
- Evaluate each hypothesis
- Compute its residuals with respect to all data points.
- Points with residuals less than some threshold are classified as its inliers
- The hypothesis with the maximal number of inliers is chosen. Then re-estimate the model parameter using its identified inliers.

RANSAC – Practice

- The theoretical number of samples needed to ensure 95% confidence that at least one outlier free sample could be obtained.

$$\rho = 1 - (1 - (1 - \epsilon)^k)^s$$

- Probability that a point is an outlier $1 - \epsilon$
- Number of points per sample k
- Probability of at least one outlier free sample ρ
- Then number of samples needed to get an outlier free sample with probability ρ

$$s = \frac{\log(1 - \rho)}{\log(1 - (1 - \epsilon)^k)}$$

RANSAC – Practice

- The theoretical number of samples needed to ensure 95% confidence that at least one outlier free sample could be obtained.
- Example for estimation of essential/fundamental matrix
- Need at least 7 or 8 points in one sample i.e. $k = 7$, probability is
- 0.95 then the number of samples for different outlier ratio ϵ

Outlier ratio	20%	30%	40%	50%	60%	70%
seven-point algorithm	13	35	106	382	1827	13696
eight-point algorithm	17	51	177	766	4570	45658

- In practice we do not know the outlier ratio
- Solution adaptively adjust number of samples as you go along
- While estimating the outlier ratio

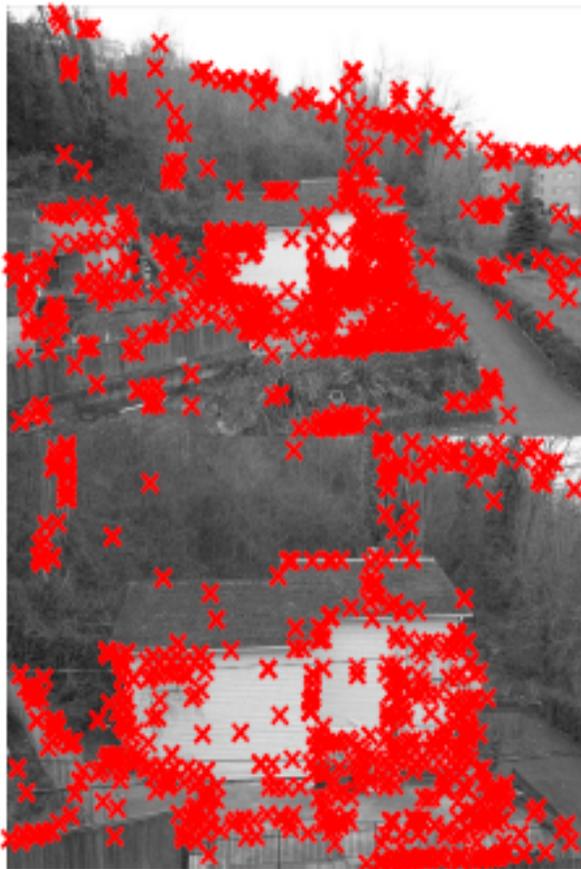
The difficulty in applying RANSAC

- Drawbacks of the standard RANSAC algorithm
 - Requires a large number of samples for data with many outliers (exactly the data that we are dealing with)
 - Needs to know the outlier ratio to estimate the number of samples
 - Requires a threshold for determining whether points are inliers
- Various improvements to standard approaches [Torr'99, Murray'02, Nister'04, Matas'05, Sutter'05 and many others]

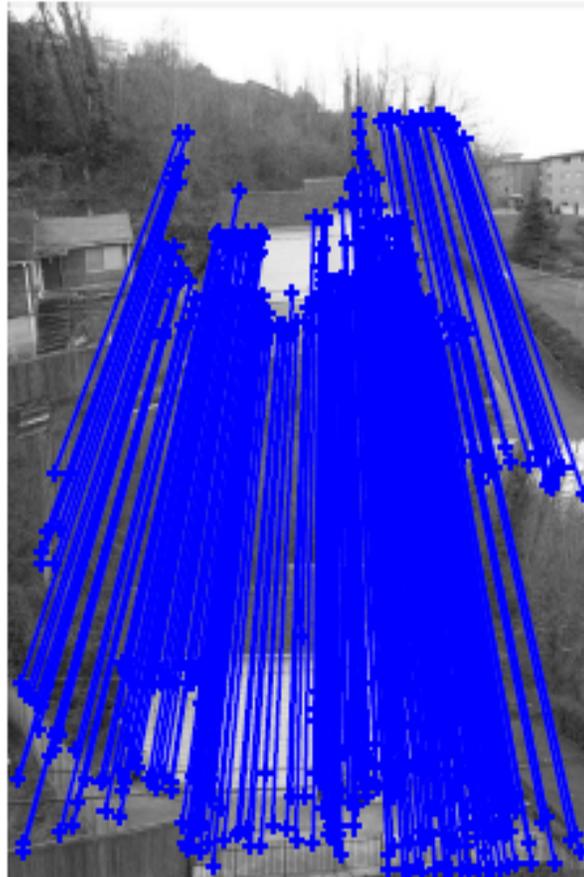
Adaptive RANSAC

- $s = \text{infinity}$, $\text{sample_count} = 0$;
- While $s > \text{sample_count}$ repeat
 - choose a sample and count the number of inliers
 - set $\epsilon = 1 - (\text{number_of_inliers}/\text{total_number_of_points})$
 - set s from ϵ and $\rho = 0.99$
 - increment sample_count by 1
- terminate

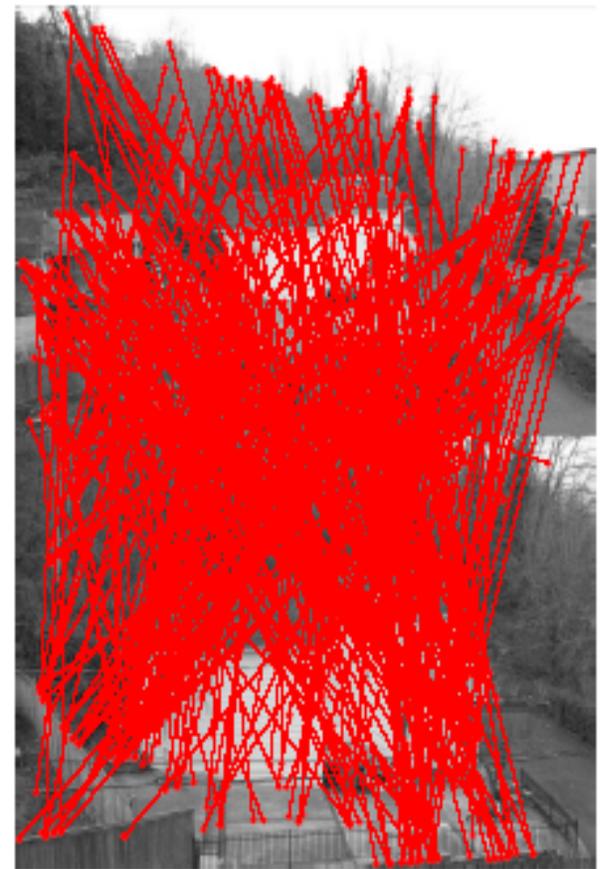
Robust technique



(a) correspondences.



(b) identified inliers.



(c) identified outliers.

More correspondences and Robust matching

- Select set of putative correspondences $\mathbf{x}_1^j, \mathbf{x}_2^j$

$$\mathbf{x}_2^T F \mathbf{x}_1 = 0$$

- Repeat

1. Select at random a set of 8 successful matches
2. Compute fundamental matrix
3. Determine the subset of inliers, compute distance to epipolar line

$$d_j^2 \doteq \frac{(\mathbf{x}_2^{jT} F \mathbf{x}_1^j)^2}{\|\hat{\mathbf{e}}_3 F \mathbf{x}_1^j\|^2 + \|\mathbf{x}_2^{jT} F \hat{\mathbf{e}}_3\|^2} \quad d_j \leq \tau_d$$

4. Count the number of points in the consensus set

RANSAC in action



$$d_j \leq \tau_d$$

Inliers



$$d_j > \tau_d$$

Outliers

Epipolar Geometry



- Epipolar geometry in two views
- **Refined epipolar geometry using nonlinear estimation of F**
- The techniques mentioned so far simple linear least-squares estimation methods. The obtained estimates are used as initialization for non-linear optimization methods

Special Motions – Pure Rotation

- Calibrated Two views related by rotation only $\hat{\mathbf{x}}_2 R \mathbf{x}_1 = 0$
 $\lambda_2 \mathbf{x}_2 = R \lambda_1 \mathbf{x}_1$
- Uncalibrated Case $\mathbf{x}' = K \mathbf{x}$ $\mathbf{x} = K^{-1} \mathbf{x}'$
 $\hat{\mathbf{x}}'_2 H \mathbf{x}'_1 = \hat{\mathbf{x}}'_2 K R K^{-1} \mathbf{x}'_1 = 0$
- Mapping to a reference view – H can be estimated

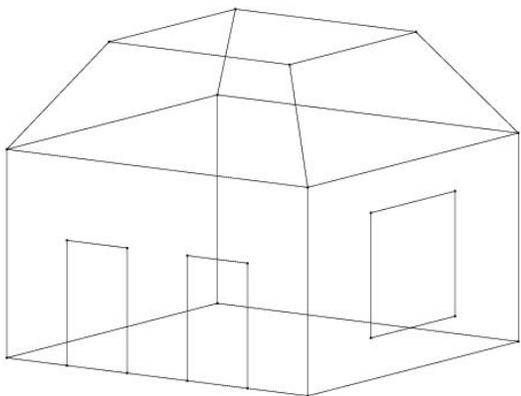


- Mapping to a cylindrical surface - applications – image mosaics

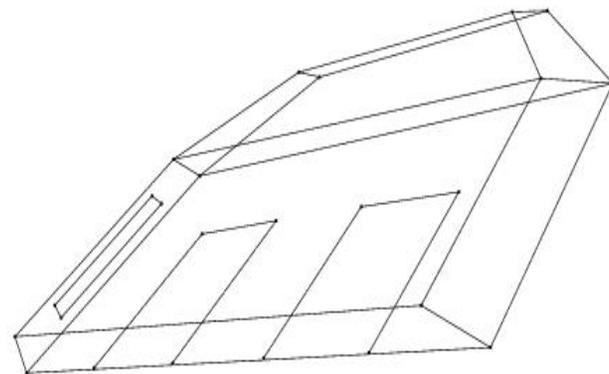


Projective Reconstruction

- Euclidean Motion Cannot be obtained in uncalibrated setting (F cannot be uniquely decomposed into R, T and K matrix)
- Can we still say something about 3D ?
- Notion of the projective 3D structure (study of projective geometry)



Euclidean reconstruction



Projective reconstruction

Euclidean vs Projective reconstruction

- **Euclidean reconstruction** – true metric properties of objects lengths (distances), angles, parallelism are preserved
- Unchanged under rigid body transformations
- => Euclidean Geometry – properties of rigid bodies under rigid body transformations, similarity transformation

- **Projective reconstruction** – lengths, angles, parallelism are **NOT** preserved – we get distorted images of objects – their distorted 3D counterparts --> 3D projective reconstruction
- => Projective Geometry