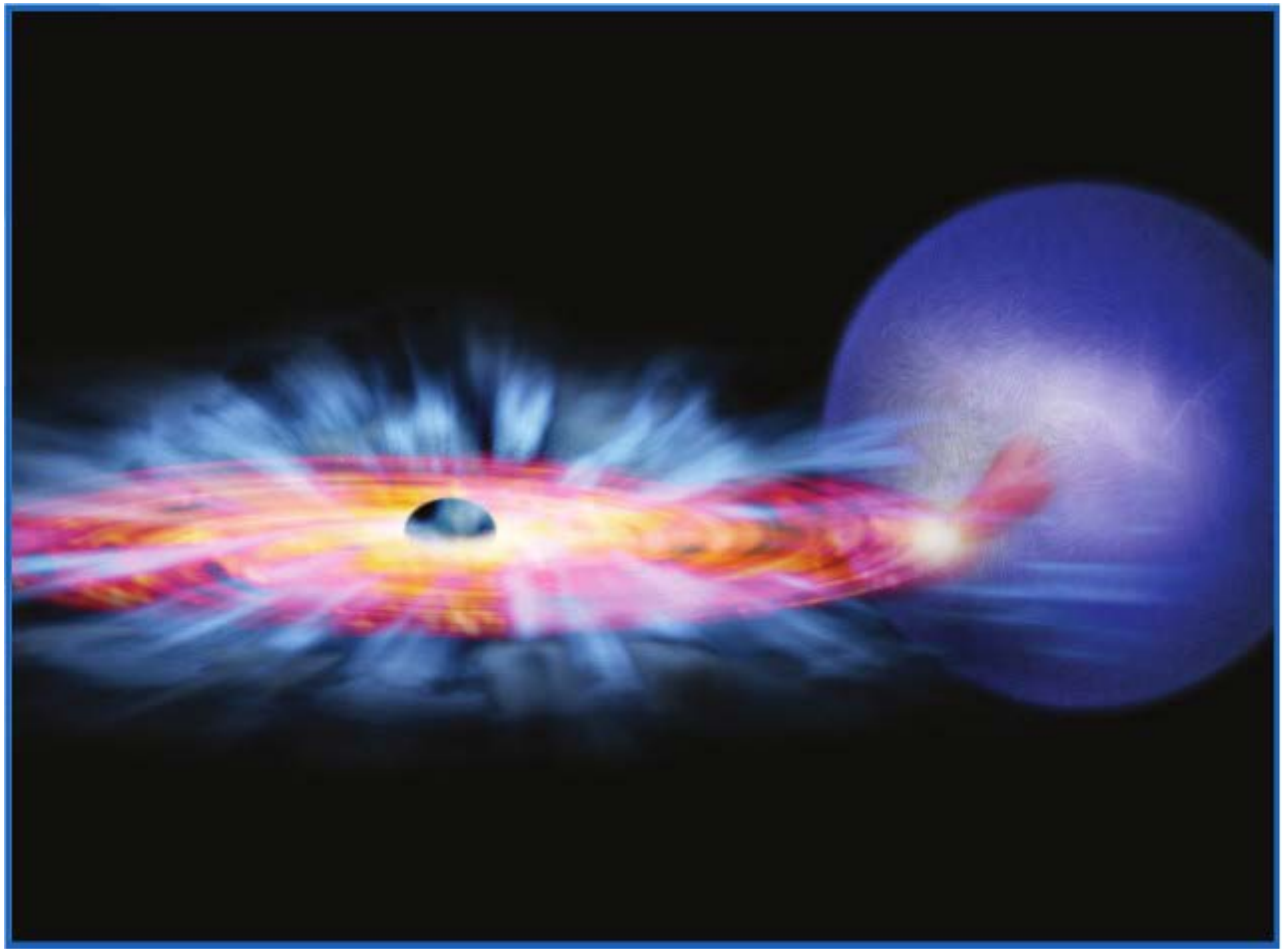
Black Holes



ASTRO 410: Black Holes
Spring 2015

Lecture: Thursday
Location: J. C. Long, room 219
Time: 4:00 – 5:15 PM

Instructor: Dr. George Chartas
Office: J. C. Long, room 206
Office hours: T 3:00 – 4:00 pm
Phone: (843) 953-3609
Email: chartasg@cofc.edu

Suggested Projects

1. Determine the mass of a black hole in a spectroscopic binary system.

2. Determine the mass of a supermassive black hole

3. Determine the spin of a supermassive black hole from X-ray observations of a bright AGN.

4. Calculate the distortion of an emission line produced close to the event horizon of a supermassive black hole due to special and general relativistic effects.

5. Simulate the emission spectrum of an AGNs accretion disk.

6. Use Cactus to simulate gravitational waves from two colliding black holes.

7. Calculate the precession rate for a binary pulsar following exercises from the book : **Exploring Black Holes by E.F. Taylor and J.A. Wheeler**.

Suggested Projects

9. Review of current methods used to image black holes.

10. Review past, current and planned experiments of measuring frame dragging.

11. The effects of general relativity on GPS satellites.

12. Black hole entropy and Hawking radiation. Does information disappear forever when in crosses the event horizon of a black hole?

13. How to build a time machine.

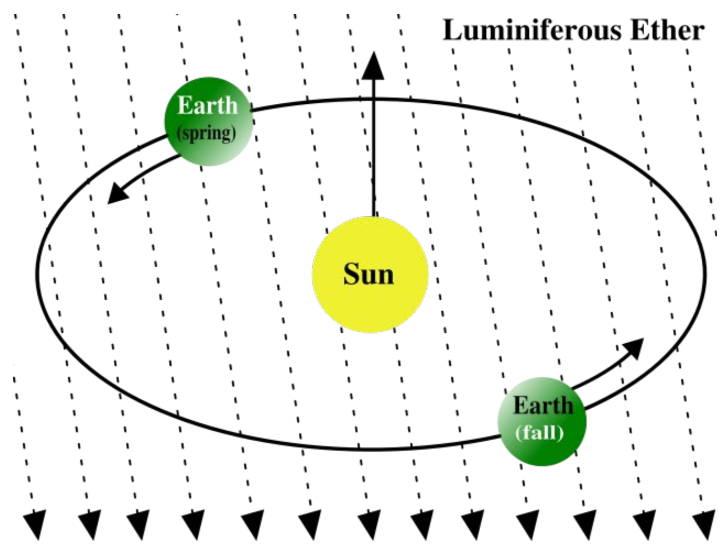
14. Review of theories describing the BH singularity, wormholes, quantum physics and General Relativity and String Theory

Special Relativity

A wave is a disturbance that travels through a medium. Sound waves is an example of a wave propagating through air. James Clerk Maxwell found that light consists of periodic modulations of electromagnetic waves.

At the time some scientists including Maxwell thought that there was a medium (ether) through which EM waves traveled.

The Michelson Morley experiment proved the non-existence of an ether.



Special Relativity

Principles of Special Relativity:
1. The laws of physics are the same for all inertial observers.

2. The speed of light is the same for all **inertial** observers regardless of the state of motion of the source.

An **inertial observer** is one that is not accelerating.

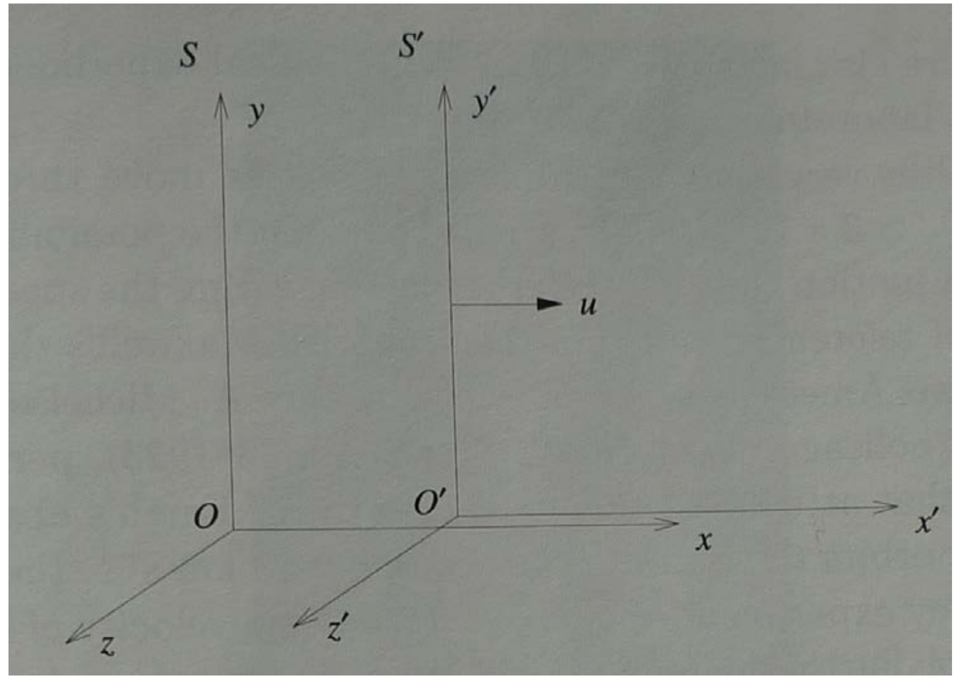
Lorentz Transformations

$$t' = \frac{t - \frac{ux}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma\left(t - ux/c^2\right)$$

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$



The factor : $\gamma = \dfrac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$ is called the Lorentz factor.

Note the intertwining roles of space and time. Events are identified by their **spacetime** coordinates (*x, y, z, t*).

Lorentz Transformations (matrix notation)

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

A boost in the x direction.

The downfall of simultaneity:
If two flashbulbs at locations x_1 and x_2 go off at the same time in frame S what is the time interval between these 2 events for an observer in frame S'?

Special Relativity

The length you measure an object to have depends on how that object is moving; the faster it moves, the shorter its length along its direction of motion. This phenomenon is called **length contraction.**

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2}$$

L = observed length of object along direction of motion

L_0 = length of object at rest (proper length)

v = speed of object with respect to observer

c = speed of light

Special Relativity

A clock runs slower when observed by someone moving relative to the clock than someone not moving relative to the clock. This phenomenon is called **time dilation.**

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Δt = time interval measured by an observer moving relative to the phenomenon

Δt_0 = time interval measured by an observer not moving relative to the phenomenon

v = speed of phenomenon relative to observer

c = speed of light

Special Relativity

Example: When unstable particles called **muons** are produced in experiments on Earth, they **decay** into other particles **in an average time of 2.2 × 10^{-6} s**.

Muons are also produced by fast-moving protons from interstellar space when they collide with atoms in Earth's upper atmosphere. These **muons** typically move at 99.9% of the speed of light and are formed at an altitude of 10 km.

How long does it take for a muon to reach the Earth from 10 km (as observed from a non-moving observer)?

How long does it take for a muon to reach the Earth from 10 km (as observed from the muon)? Does the muon reach the Earth before decaying and why?

Doppler Shift of Sound Waves

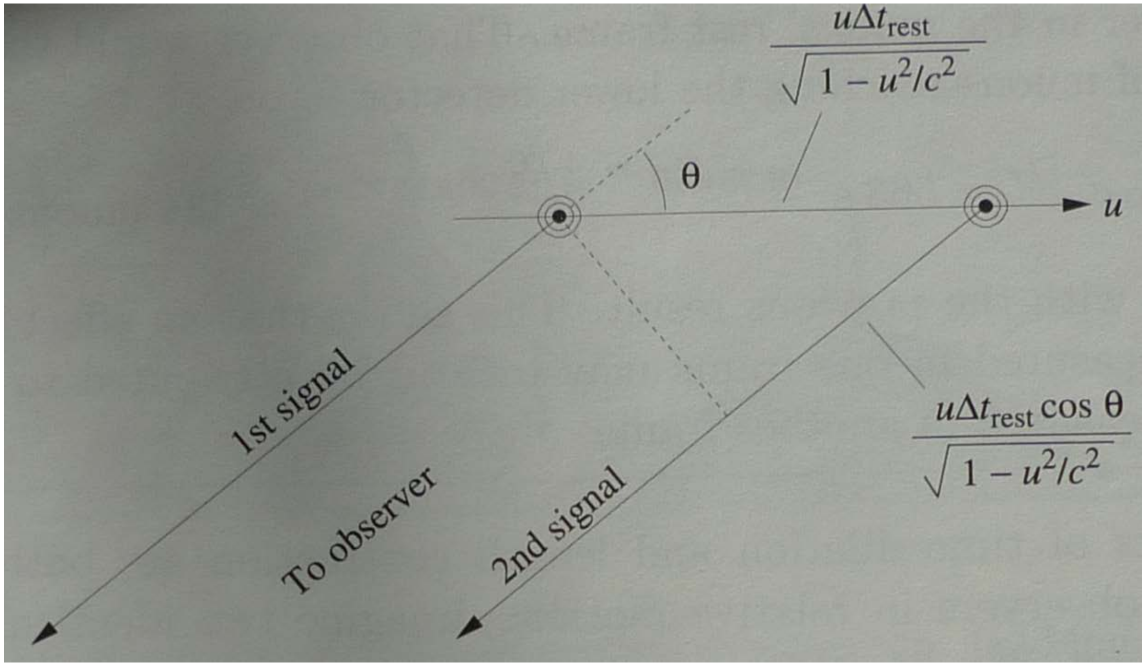
In 1842 Austrian physicist Christian Doppler showed that as a source of sound moves through a medium the wavelength is compressed in the direction of motion of the source and expanded in the direction opposite to the motion.

The relationship between the observed wavelength and the restframe wavelength is :

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{v_{sound}}$$

where v_r is the radial component of the velocity of the moving object and v_{sound} is the speed of sound in the medium.

Doppler Shift of Light



$$\Delta t_{obs} = \gamma \Delta t_{rest} + \frac{\gamma u \Delta t_{rest} \cos\theta}{c}$$

The second term represents the observed time for light to cover the difference in the two paths.

$$\Delta t_{obs} = \gamma \Delta t_{rest}\left[1 + \frac{u\cos\theta}{c}\right] \Rightarrow \nu_{rest} = \gamma \nu_{obs}\left[1 + \frac{u\cos\theta}{c}\right] \Rightarrow \nu_{obs} = \frac{\nu_{rest}}{\gamma\left[1 + \frac{u_r}{c}\right]}$$

Doppler Shift of Light

$$\text{Doppler Shift of Light}: \quad \frac{\lambda_{rest}}{\lambda_{obs}} = \frac{\sqrt{1-\left(\frac{u}{c}\right)^2}}{\left[1+\frac{u\cos\theta}{c}\right]}$$

Two particular cases of interest are when
(α) $\theta = 0°$ (b) $\theta = 90°$ (c) $u/c \ll 1$ and $\theta = 0°$

Derive the Doppler shift formulas for these 3 cases.

Cosmological Redshift of Light

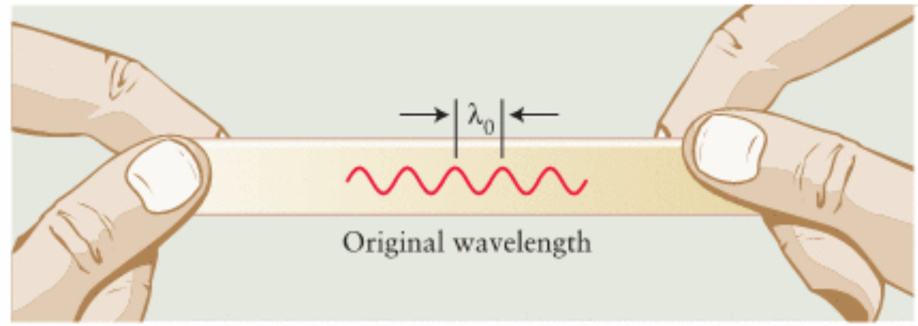
A redshift caused by the expansion of the universe is called cosmological redshift.

We can easily calculate the factor by which the Universe has expanded from some previous time as follows:

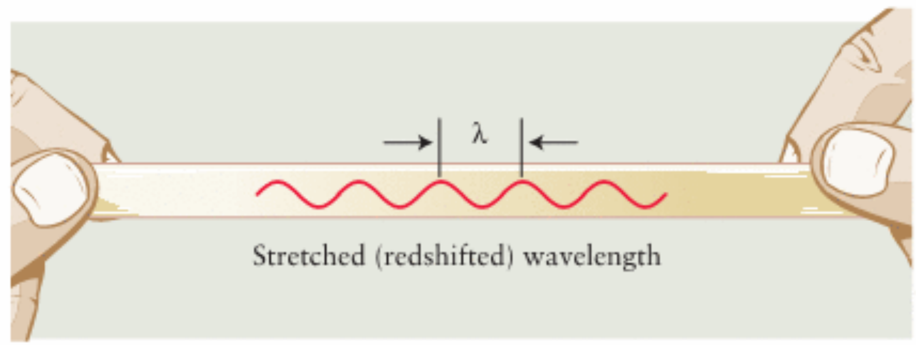
$z = (\lambda_{obs} - \lambda_0)/\lambda_0 \rightarrow \lambda_{obs}/\lambda_0 = (1+z)$

This means that if you observe an object to have a redshift of $z = 1$ the distance between us and the object has increased by a factor of 2 from the time the photon left that object and arrived to Earth.

How does the volume and density change?



(a) A wave drawn on a rubber band ...



(b) ... increases in wavelength as the rubber band is stretched.

Distances

lookback time (or light travel time) indicates how far into the past we are looking when we see a particular object.

comoving radial distance (which goes into the Hubble law : $v = H_0 d$) is the distance now between the object and us. During the time that it takes a photon to reach us from a distant object, that object has moved farther away due to the expansion of the universe.

luminosity distance (which goes into the inverse square law)

Several online cosmology calculators can be found at:

http://lambda.gsfc.nasa.gov/toolbox/tb_calclinks.cfm

Relativistic Velocity Transformations

Write the Lorentz transformations as differentials and divide the *dx'*, *dy',* and *dz'* equations by the *dt'* equation.

$$v_x' = \frac{(v_x - u)}{1 - uv_x/c^2}$$

$$v_y' = \frac{v_y}{\gamma(1 - uv_x/c^2)}$$

$$v_z' = \frac{v_z}{\gamma(1 - uv_x/c^2)}$$

You can infer the inverse transformations by substituting u➔-u

Now assume frame *S'* is moving with respect to frame *S* along the *x* axis with a velocity *u* and a photon is moving in frame *S'* along the *y'* axis.

$v_x' = 0,\ v_y' = c,\ v_z' = 0$

Use the inverse velocity transformations to infer v_x, v_y, v_z

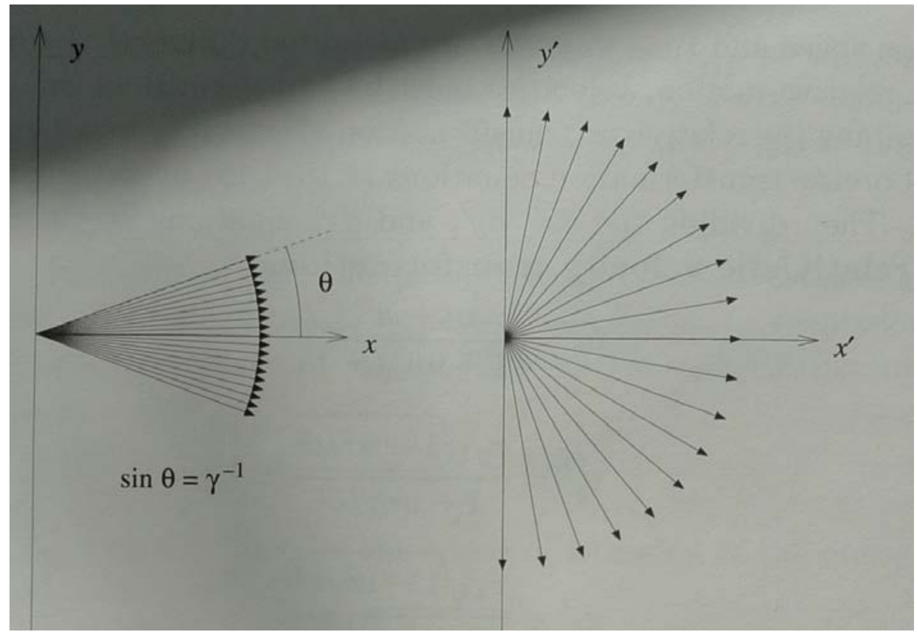
Relativistic Beaming

$v_x = u,\ v_y = c/\gamma,\ v_z = 0$

For u/c~1

$\sin\theta = v_y/v = \gamma^{-1}$

The **beaming angle** θ is half of the opening angle and the inverse of the Lorentz factor.



Examples: relativistic electrons spiraling around magnetic field lines emit synchrotron radiation collimated in beams pointed in their direction of motion.

Relativistic Momentum and Energy

Relativistic momentum: $P = \gamma m v$

Derive the relativistic kinetic energy from Newton's second law:
$F = dP/dt$

Relativistic Kinetic Energy: $E_k = mc^2(\gamma-1)$

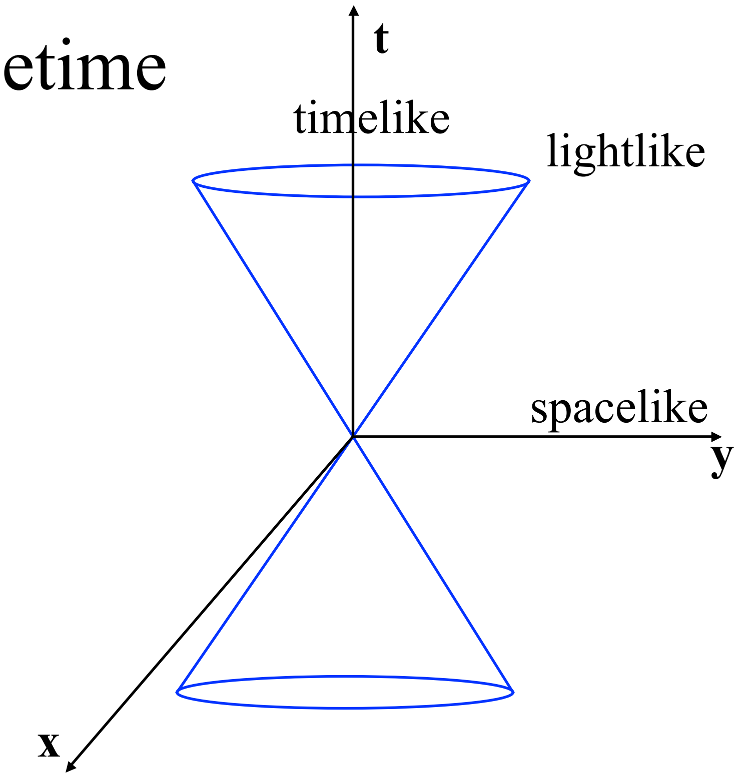
Metrics in Spacetime

An event in spacetime is specified by a 4-coordinate x_a (a=0,1,2,3) where a=0 corresponds to time.

The path followed by an object as it moves through spacetime is called a **worldline**.

The worldline of photons in a flat spacetime is a light cone.

The spacetime interval ds^2 is invariant under a Lorentz transformation.



The differential interval **along a worldline** for a flat-spacetime (no matter present) is:

$$ds^2 = cdt^2 - dx^2 - dy^2 - dz^2$$
$$ds^2 = g_{ab} dx^a dx^b$$

$ds^2 > 0$, the interval in timelike

$ds^2 = 0$, the interval is lightlike

$ds^2 < 0$, the interval is spacelike

Metrics in Spacetime

In flat spacetime the interval ds^2 between two events A and B =
(distance travelled by light in a time of t_B-t_A) −
(distance between events A and B)

g_{ab} is referred to as the metric tensor.

$$ds^2 = g_{ab}dx^a dx^b, \text{where } g_{ab} = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In general relativity g_{ab} is usually a function of position and is often non-diagonal.

Proper Time and Proper Length

The time between two events A and B that occur at the same location is called **proper time ($\Delta\tau$)**. An observer moving along the worldline from A to B will measure proper time.

$$\Delta\tau = \Delta s/c, \quad \Delta s = \int_A^B \sqrt{ds^2}$$

where Δs is the integral of the metric between A and B along the worldline.

Note that if an interval is lightlike ($\Delta s=0$) the proper time measured along a lightlike interval is zero.

The distance between two events A and B in a reference frame for which they occur simultaneously is the **proper distance** :

$$\Delta L = \sqrt{(\Delta s)^2}$$