Pricing Variance Swaps for Stochastic Volatilities with Delay and Jumps

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Outline

- Stochastic Volatilities (SV) with Delay
- Multi-Factor SV with Delay
- SV with Delay and Jumps
- Swaps
- Numerical Examples
- Conclusions

Volatility

- **Volatility** is the standard deviation of the change in value of a financial instrument with specific time horizon
- It is often used to quantify the **risk** of the instrument over that time period
- The higher volatility, the riskier the security

Types of Volatilities

- **Historical V**: standard deviation (uses historical (daily, weekly, monthly, quarterly, yearly)) price data to empirically measure the volatility of a market or instrument in the past
- **Implied V**: volatility implied by the market price of the option based on an option pricing model (smile and skew-varying volatility by strike)

Volatility Smile

- The models by Black & Scholes (continuous-time (B,S)-security market,1973) and Cox & Rubinstein (discrete-time (B,S)-security market (binomial tree), 1979) are unable to explain the **negative skewness** and **leptokurticity (fat tail)** commonly observed in the stock markets
- The famous **implied-volatility smile** would not exist under their assumptions

Coffee Options

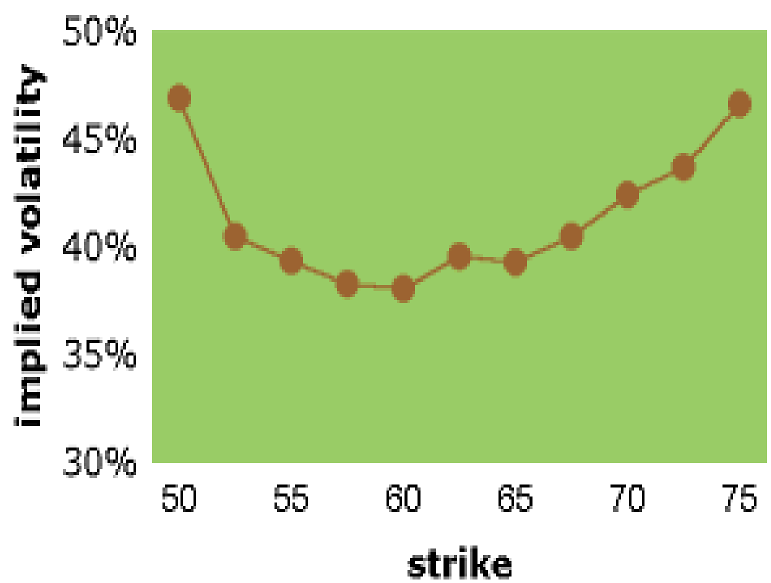
- Coffee options trade on New York's Coffee, Sugar and Cocoa Exchange (**CSCE**).

Coffee Call Option

- CSCE May 2001 coffee call option **implied volatilities** as of March 12, 2001

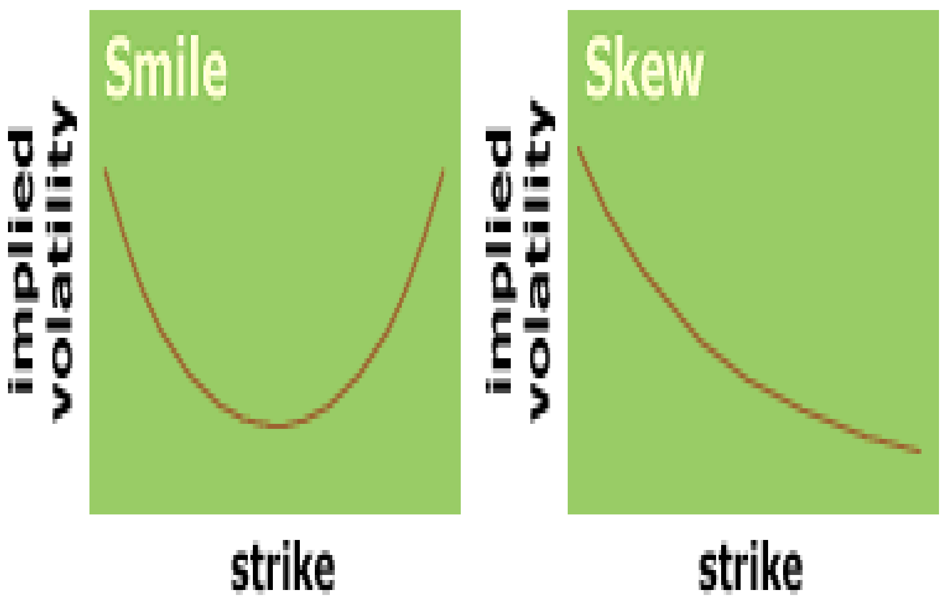
Implied Volatility: Volatility Smile

- Graph indicates implied volatilities at various strikes for the May 2001 calls based upon their March 12, 2001 settlement prices. The pattern of implied volatilities form a "smile" shape, which is called a **volatility smile**.



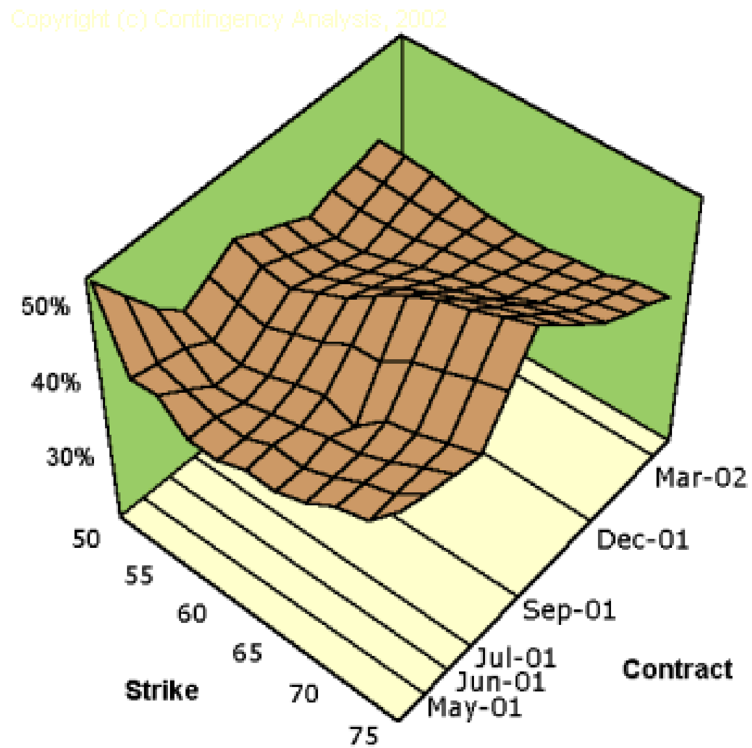
Implied Volatility: Volatility Skew

- Most derivatives markets exhibit persistent patterns of volatilities varying by strike. In some markets, those patterns form a **smile**. In others, such as *equity index options* markets, it is more of a skewed curve. This has motivated the name **volatility skew**. In practice, either the term "volatility smile" or "volatility skew" (or simply **skew**) may be used to refer to the general phenomena of volatilities varying by strike.



Implied Volatility: Volatility Surface

- Another dimension to the problem of volatility skew is that of volatilities varying by expiration. This is illustrated for CSCE coffee options. It indicates what is known as a **volatility surface**



Types of Volatilities II

- **Level-Dependent Volatility** (CEV or Firm Model)-*function of the spot price alone*
- **Local *Volatility***-*function of the spot price <u>and time</u>* (Dupire formulae, 1994)
- **Stochastic Volatility**: volatility is not constant, but a stochastic process (explains <u>smile and skew</u>)

Two Approaches to Introduce SV

- One approach-to change the clock time t to a random time T(t) (change of time)

$$\sigma W(t) \Rightarrow W(T(t))$$

- Another approach-change constant volatility into a positive stochastic process

$$\sigma \equiv \sigma(t), \quad \int_0^t \sigma^2(s)ds < +\infty$$

Stochastic Volatility: Some Models

- ARCH model (Engle (1982))

$$\ln(S_t/S_{t-1}) = \sigma_t \xi_t, \quad \xi_t \approx i.i.d.N(0,1)$$

- **Discrete SV**: GARCH model (Bollerslev (1986))

$$\sigma_n^2 = \gamma V + \frac{\alpha}{l} \ln^2(S_{n-1}/S_{n-1-l}) + (1-\alpha-\gamma)\sigma_{n-1}^2$$

- **Heston SV model** (1993)

$$d\sigma_t^2 = k(\theta^2 - \sigma_t^2)dt + \gamma \sigma_t dw_t$$

- **Mean-Reverting SV model** (Wilmott, Haug, Javaheri (2000))

$$d\sigma_t^2 = k(\theta^2 - \sigma_t^2)dt + \gamma \sigma_t^2 dw_t$$

- **Elliott and Chan** 'Option Pricing with Stochastic Volatility Driven by a Fractional Ornstein-Ohlenbeck Process'.

SV with Delay

Kazmerchuk, Swishchuk & Wu (2002)

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t)$$

One-Factor SV with Delay

The underlying asset S(t) follows the process

$$dS(t) = \mu S(t)dt + \sigma(t, S_t)S(t)dW(t)$$

$$S_t := S(t-\tau) \qquad\qquad S(t) = \phi(t), \quad t \in [-\tau, 0], \quad \tau > 0.$$

The asset volatility is defined as the solution of the following equation

$$\frac{d\sigma^2(t, S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s, S_s)\,dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t, S_t).$$

Why this equation?

From GARCH to SV with Delay

$$\sigma_n^2 = \gamma V + \alpha \ln^2(S_{n-1}/S_{n-2}) + (1-\alpha-\gamma)\sigma_{n-1}^2$$

-**discrete-time GARCH(1,1)**

$$\sigma_n^2 = \gamma V + \frac{\alpha}{l} \ln^2(S_{n-1}/S_{n-1-l}) + (1-\alpha-\gamma)\sigma_{n-1}^2$$

-**discrete-time GARCH(1,1)**
(l=1)

$$\frac{d\sigma^2(t)}{dt} = \gamma V + \frac{\alpha}{\tau} \ln^2\left(\frac{S(t)}{S(t-\tau)}\right) - (\alpha+\gamma)\sigma^2(t).$$

-**continuous-time GARCH**
(expectation of log-returns
is zero)

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

-**continuous-time GARCH**
(non-zero expectation
of log-return)

Comparison with GARCH (1,1)

GARCH (1,1)

$$\ln(S_n/S_{n-1}) = m + \sigma_n \xi_n, \quad \{\xi_n\} \sim \text{i.i.d. } N(0,1),$$
$$\sigma_n^2 = \gamma V + \alpha(\sigma_{n-1}\xi_{n-1})^2 + (1-\alpha-\gamma)\sigma_{n-1}^2$$
$$= \gamma V + \alpha(\ln(S_{n-1}/S_{n-2}) - m)^2 + (1-\alpha-\gamma)\sigma_{n-1}^2$$

Log-returns for S(t) (Ito formula)

$$\ln \frac{S(t)}{S(t-\tau)}) = \int_{t-\tau}^{t} (r - \frac{1}{2}\sigma^2(u, S(u)))du) + \int_{t-\tau}^{t} \sigma(u, S(u))dW(u)$$

Continuous-Time GARCH for SV with Delay

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

Main Features of 1-Factor SV with Delay

1) continuous-time analogue of discrete-time GARCH model

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

2) Mean-reversion

$$\frac{d\sigma_t^2}{dt} = (\alpha+\gamma)(\frac{\gamma}{\alpha+\gamma}V - \sigma_t^2)dt + \frac{\alpha}{\tau}[\int_{t-\tau}^{t} \sigma_s dw_s]^2$$

3) Market is complete (W is the same as for the stock price)

$$dS(t) = \mu S(t)\, dt + \sigma(t,S_t) S_t\, dW(t)$$

4) Incorporate the expectation of log-returns

$$\ln \frac{S(t)}{S(t-\tau)}) = \int_{t-\tau}^{t} (r - \frac{1}{2}\sigma^2(u,S(u)))du) + \int_{t-\tau}^{t} \sigma(u,S(u))dW(u)$$

Equation for the Expectation of Variance

Equation for the Variance

$$\frac{d\sigma^2(t, S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s, S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t, S_t).$$

Equation for the Expectation

$$\frac{dv(t)}{dt} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds - (\alpha+\gamma)v(t)$$

$$v(t) := E_{\mathcal{P}*}[\sigma^2(t, S_t)].$$

Solution of the Equation for the Expectation of Variance (1FSV)

Equation to be solved

$$\frac{dv(t)}{dt} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds - (\alpha + \gamma)v(t)$$

Stationary solution

$$v(t) \equiv X = V + \frac{\alpha\tau(\mu - r)^2}{\gamma}$$

General solution

$$v(t) \approx X + Ce^{-\gamma t}$$

$$C = v(0) - X = \sigma_0^2 - V - \frac{\alpha\tau(\mu - r)^2}{\gamma}.$$

General Solution

Integro-differential equation with delay

$$\frac{dv(t)}{dt} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds - (\alpha + \gamma)v(t)$$

General solution

$$\begin{aligned} v(t) &= E_{P^*}\left[\sigma^2(t, S_t)\right] \\ &\approx V + \frac{\alpha\tau(\mu - r)^2}{\gamma} + \left(\sigma_0^2 - V - \frac{\alpha\tau(\mu - r)^2}{\gamma}\right)e^{-\gamma t} \end{aligned}$$

Multi-Factor SV Models

- **One-Factor SV Models** (*all above-mentioned*): 1) incorporate the leverage between returns and volatility and

  2) reproduce the 'skew' of implied volatility
- However, it *fails to match either the high conditional kurtosis of returns* (**Chernov *et. al*. (2003)**) or *the full term structure of implied volatility surface* **(Cont &Tankov (2004))**
- **Adding jump components** in returns and/or volatility process, or considering **multi-factor SV models** are two primary generalizations of one-factor SV models

Multi-Factor SV Models

- **Chernov et al. (2003):** used efficient method of moments to obtain comparable empirical-of-fit from affine jump-diffusion models & two-factor SV family models
- **Molina et al. (2003):** used a Markov Chain Monte Carlo method to find strong evidence of two-factor SV models with well-separated time scales in foreign exchange data
- **Cont &Tankov (2004):** found that jump-diffusion models have a fairly good fit to the implied volatility surface
- **Fouque et al. (2000):** found that two-factor SV models provide a better fit to the term structure of implied volatility than one-factor SV models by capturing the behaviour at short and long maturities
- **Swishchuk (2006):** introduced two-factor and three-factor SV models with delay (incorporating mean-reverting level as a random process (GBM, OU, Pilipovich or continuous-time GARCH(1,1) model))

Advantages and Disadvantages of Multi-Factor SV Models

- Multi-Factor SV models do not admit in general explicit solutions for option prices
- But have direct implications on hedges
- Comparison: class of **jump-diffusion models (Bates (1996))** enjoys <u>closed-form solutions</u> for option prices *but the complexity of hedging strategies increases due to jumps*
- There is no strong empirical evidence to judge the overwhelming position between jump-diffusion models and multi-factor SV models

Multi-Factor SV with Delay

One-Factor SV with Delay-----> $$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t}\sigma(s,S_s)dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

- **Multi-Factor-Mean SV with Delay-mean-reversion level V is a stochastic process**

V->V (t)-stochastic process

- V (t) - geometric Brownian motion (GBM) (two-factor)
- V (t) - Ornstein-Uhlenbeck (UE) process (two-factor)
- V (t) - Pilipovich one-factor (two-factor)
- V(t) – Pilipovich two-factor process (three-factor)

2-Factor SV with Delay: GBM Mean-Reversion (GBMMR)

$$\begin{cases} \dfrac{d\sigma^2(t,S_t)}{dt} = \gamma V_t + \dfrac{\alpha}{\tau}\left[\displaystyle\int_{t-\tau}^{t} \sigma(s,S_s)\,dW(s)\right]^2 \\ \qquad\qquad\qquad\qquad - (\alpha+\gamma)\sigma^2(t,S_t), \\ \dfrac{dV_t}{V_t} = \xi\,dt + \beta\,dW_1(t). \end{cases}$$

2-Factor SV with Delay (GBMMR): Equation for the E and Solution

Equation for the E

$$\frac{dv(t)}{dt} = \gamma V_0 e^{(\xi-\lambda\beta)t} + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)\,ds - (\alpha+\gamma)v(t).$$

where $v(t) := E_{P*}\sigma^2(t, S_t)$.

Solution

$$v(t) \approx X + Ce^{-\gamma t} + (\xi - \lambda\beta)\gamma V_0 \times \left[\frac{X}{\xi - \lambda\beta}(e^{(\xi-\lambda\beta)t} - 1) + \frac{C}{\xi - \lambda\beta - \gamma}(e^{(\xi-\lambda\beta)t} - e^{\gamma t})\right]$$

2-Factor SV with Delay: OU Mean-Reversion (OUMR)

$$\begin{cases} \dfrac{d\sigma^2(t,S_t)}{dt} = \gamma V_t + \dfrac{\alpha}{\tau}\left[\displaystyle\int_{t-\tau}^{t} \sigma(s,S_s)\,dW(s)\right]^2 \\ \qquad\qquad\qquad\qquad - (\alpha+\gamma)\sigma^2(t,S_t). \\ dV_t = \xi(L - V_t)\,dt + \beta\,dW_1(t). \end{cases}$$

2-Factor SV with Delay (OUMR): Equation for the E and Solution

Equation for E

$$\frac{dv(t)}{dt} = \gamma\left(e^{-\xi t}\left(V_0 - \left(L - \frac{\lambda\beta}{\xi}\right)\right) + \left(L - \frac{\lambda\beta}{\xi}\right)\right) + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)\,ds - (\alpha+\gamma)v(t)$$

Solution

$$v(t) \approx X + Ce^{-\gamma t} + \xi\gamma\left(V_0 - \left(L - \frac{\lambda\beta}{\xi}\right)\right) \times \left[\frac{X}{\xi}(e^{-\xi t} - 1) + \frac{C}{\xi+\gamma}(e^{-\xi t} - e^{\gamma t})\right]$$

2-Factor SV with Delay: Pilipovich 1-Factor Mean-Reversion (OFMR)

$$\begin{cases} \dfrac{d\sigma^2(t,S_t)}{dt} = \gamma V_t + \dfrac{\alpha}{\tau}\left[\displaystyle\int_{t-\tau}^{t} \sigma(s,S_s)\,dW(s)\right]^2 \\ \qquad\qquad\qquad\qquad - (\alpha+\gamma)\sigma^2(t,S_t), \\ dV_t = \xi(L - V_t)\,dt + \beta V_t\,dW_1(t). \end{cases}$$

2-Factor SV with Delay (Pilipovich 1FMR): Equation for the E and Solution

Equation for E

$$\frac{dv(t)}{dt} = \gamma\left(e^{-(\xi+\lambda\beta)t}\left(V_0 - L\frac{\xi}{\xi+\lambda\beta}\right) + L\frac{\xi}{\xi+\lambda\beta}\right)\Bigg) + \alpha\tau(\mu - r)^2$$

$$+ \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)\,ds - (\alpha+\gamma)v(t)$$

Solution

$$v(t) \approx X + Ce^{-\gamma t} + \frac{\gamma\xi}{\xi+\lambda\beta}\Bigg[\left(X(\frac{V_0(\xi+\lambda\beta)}{\xi} - L\right)$$

$$\times (1 - e^{-(\xi+\lambda\beta)t}) + XLt$$

$$+ \frac{C(\frac{V_0(\xi+\lambda\beta)}{\xi} - L)}{\xi+\lambda\beta+\gamma} + \frac{CL}{\gamma}(e^{\gamma t} - 1)\Bigg].$$

3-Factor SV with Delay: Pilipovich 2-Factor Mean-Reversion (2FMR)

$$\begin{cases} \dfrac{d\sigma^2(t, S_t)}{dt} = \gamma V_t + \dfrac{\alpha}{\tau}\left[\displaystyle\int_{t-\tau}^{t} \sigma(s, S_s)\, dW(s)\right]^2 \\ \qquad\qquad\qquad\qquad\qquad\qquad - (\alpha + \gamma)\sigma^2(t, S_t), \\ dV_t = \xi(L_t - V_t)\, dt + \beta V_t\, dW_1(t), \\ dL_t = \beta_1 L_t\, dt + \eta L_t\, dW_2(t). \end{cases}$$

3-Factor SV with Delay (Pilipovich 2FMR): Equation for the E and Solution

Equation for E

$$\frac{dv(t)}{dt} = \gamma(e^{-(\xi+\lambda\beta)t}V_0 + \frac{\xi+\lambda\beta}{\xi+\lambda\beta+\beta_1}L_0(e^{(\beta_1-\lambda_1\eta)t} - e^{-(\xi+\lambda\beta)t})) + \alpha\tau(\mu-r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)\,ds - (\alpha+\gamma)v(t)$$

Solution

$$\begin{aligned} v(t) \approx\ & X + Ce^{-\gamma t} - (\xi+\lambda\beta)\gamma V_0\left[\frac{X}{\xi+\lambda\beta}(1-e^{-(\xi+\lambda\beta)t})\right. \\ & \left. + \frac{C}{\xi+\lambda\beta+\gamma}(e^{\gamma t} - e^{-(\xi+\lambda\beta)t})\right] + L_0\frac{\xi+\lambda\beta}{\xi+\lambda\beta+\beta_1} \\ & \times\left[X(e^{(\beta_1-\lambda_1\eta)t} - e^{-(\xi+\lambda\beta)t}) + \frac{C(\beta_1-\lambda_1\eta)}{(\beta_1-\lambda_1\eta-\gamma)}\right. \\ & \left.\times(e^{(\beta_1-\lambda_1\eta)t} - e^{\gamma t}) + \frac{C(\xi+\lambda\beta)}{(\xi+\lambda\beta+\gamma)}(e^{\gamma t} - e^{-(\xi+\lambda\beta)t})\right] \end{aligned}$$

Main Features of All the Solutions for MFSVD

- 1) Contains solution of one-factor SV with Delay
- 2) Contains additional terms due to the new parameters (more factors-more parameters)
- **Solution (MFSVD)=Solution (1FSVD) + Additional Terms (Due to the stochastic mean-reversion)**

Variance Swaps

Forward contract-*an agreement to buy or sell something at a future date for a set price (forward price)*

Variance swaps are forward contract on future realized stock variance

Realized Continuous Variance

Realized (or Observed) Continuous Variance:

$$\sigma_R^2(S) := \frac{1}{T}\int_0^T \sigma^2(s)ds,$$

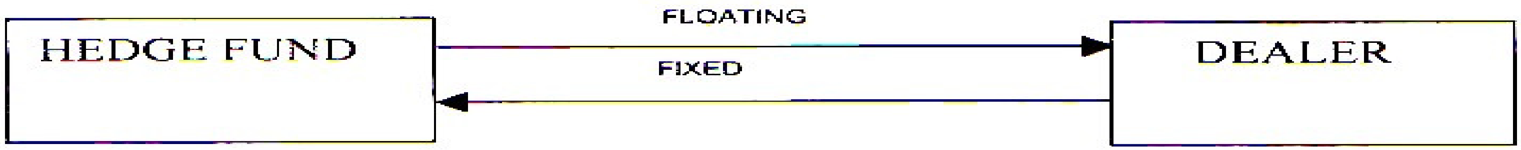
where $\sigma(t)$ **is a stock volatility,**

T is expiration date or maturity.

Why Trade Volatility (Variance)?

- Volatility Swaps allow investors to **profit** from the risks of an increase or decrease in future volatility of an index of securities or to **hedge** against these risks.
- If you think current volatility is low, for the right price you might want to take a position that profits if volatility increase.

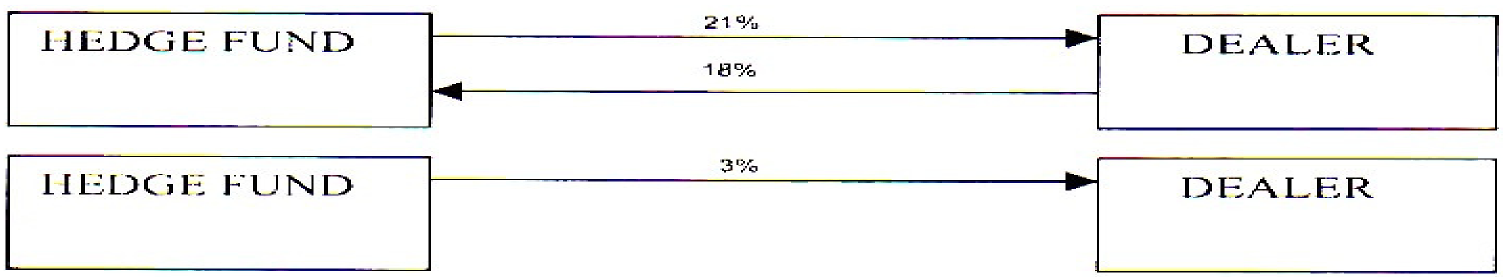
How does the Volatility Swap Work?



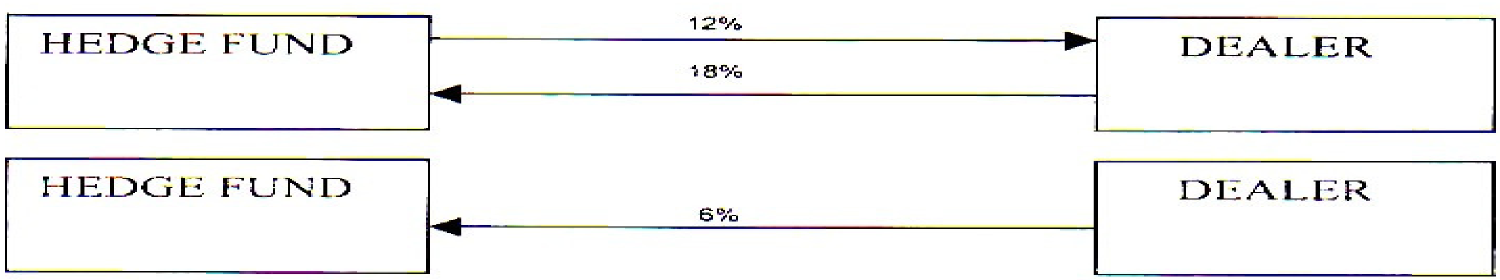
Fixed leg = strike price
Floatting leg = realized volatility

SCENARIOS

A – The volatility increases:



B – The volatility decreases:



Payoff of Variance Swaps

A Variance Swap is a forward contract on realized variance.

Its payoff at expiration is equal to

$$N(\sigma_R^2(S) - K_{var})$$

N is a notional amount ($/variance);
K$_{\mathbf{var}}$ is a strike price

Valuing of Variance Swap for Stochastic Volatility with Delay

Value of Variance Swap (present value):

$$P = e^{-rT} E_{P*}[\sigma_R^2(S) - K_{var}]$$

where **E_{P*}** *is an expectation (or mean value),* ***r*** *is interest rate*.

To calculate variance swap we need only $E_{P*}[\sigma^2(t, S_t)]$ **,**

$$\sigma_R^2(S) := \frac{1}{T}\int_0^T \sigma^2(u, S(u-\tau))du.$$

Valuing of Variance Swap for One-Factor SV with Delay in Stationary Regime

$$\frac{dv(t)}{dt} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds - (\alpha + \gamma)v(t)$$

$$v(t) = E_{P^*}[\sigma^2(t)] = V + \alpha\tau(\mu - r)^2/\gamma.$$

$$\begin{aligned} E_{P^*}[Var(S)] &= \frac{1}{T}\int_0^T E_{P^*}[\sigma^2(t)]dt \\ &= V + \alpha\tau(\mu - r)^2/\gamma. \end{aligned}$$

$$\mathcal{P}^* = e^{-rT}[V - K + \alpha\tau(\mu - r)^2/\gamma].$$

Valuing of Variance Swap for One-Factor SV with Delay in General Case

$$\frac{dv(t)}{dt} = \gamma V + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds - (\alpha+\gamma)v(t)$$

$$v(t) \approx X + Ce^{-\gamma t} = V + \alpha\tau(\mu - r)^2/\gamma + Ce^{-\gamma t}$$

$$C = v(0) - X = \sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma.$$

In this way

$$v(t) = E_{P*}[\sigma^2(t)] \approx V + \alpha\tau(\mu - r)^2/\gamma + (\sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma)e^{-\gamma t}.$$

Valuing of Variance Swap for One-Factor SV with Delay in General Case

We need to find $E_{P^*}[Var(S)]$:

$$\begin{aligned} E_{P^*}[Var(S)] &= \frac{1}{T}\int_0^T E_{P^*}[\sigma^2(t)]dt \\ &\approx \frac{1}{T}\int_0^T [V + \alpha\tau(\mu - r)^2/\gamma + (\sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma)e^{-\gamma t}]dt \\ &= V + \alpha\tau(\mu - r)^2/\gamma + (\sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma)\frac{1 - e^{-\gamma T}}{T\gamma}. \end{aligned}$$

$$\mathcal{P}^* = e^{-rT}[V - K + \alpha\tau(\mu - r)^2/\gamma + (\sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma)\frac{1 - e^{-\gamma T}}{T\gamma}]$$

Comparison of SV in Heston Model with SV with Delay

Heston Model (1993)

$$\begin{cases} \dfrac{dS_t}{S_t} &= r_t dt + \sigma_t dw_t^1 \\ d\sigma_t^2 &= k(\theta^2 - \sigma_t^2)dt + \gamma \sigma_t dw_t^2, \end{cases}$$

Comparison of SV in Heston Model with SV with Delay II

Swap for SV in Heston Model

$$E\{V\} = \frac{1 - e^{-kT}}{kT}(\sigma_0^2 - \theta^2) + \theta^2,$$

Swap for SV with Delay

$$E\{V\} \approx \frac{1 - e^{-\gamma T}}{\gamma T}(\sigma^2(0, \phi(-\tau)) - V - \alpha\tau(\mu - r)^2/\gamma) + [V + \alpha\tau(\mu - r)^2/\gamma]$$

When tau=0 (the same expression as above):

$$E\{V\} = \frac{1 - e^{-\gamma T}}{\gamma T}(\sigma^2(0, \phi(-\tau)) - V) + V.$$

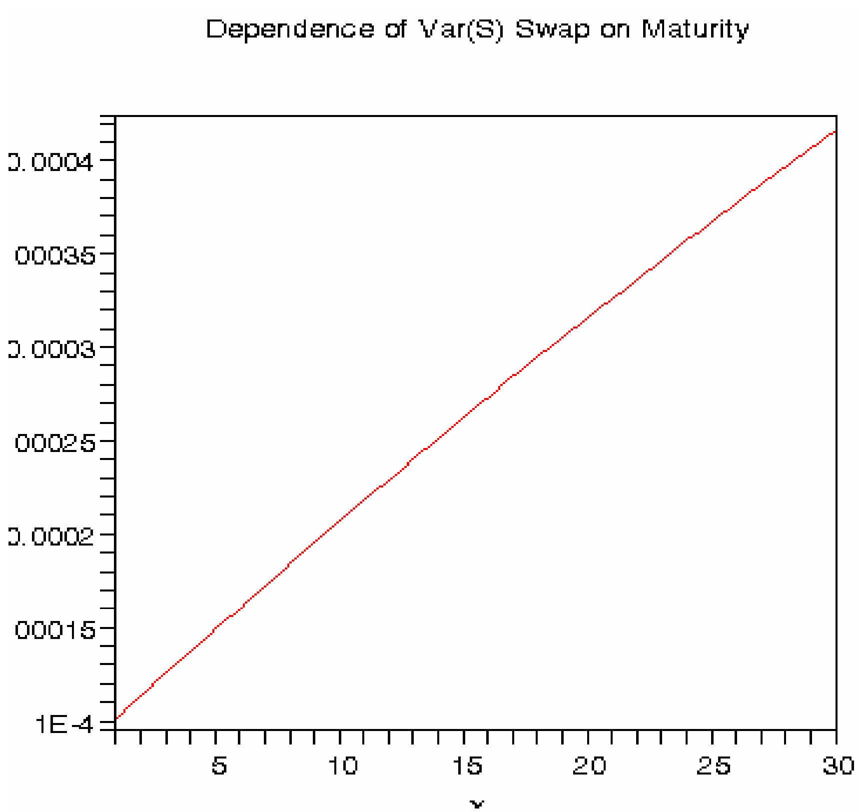
Numerical Example 1: S&P60 Canada Index (1997-2002)

Table 1

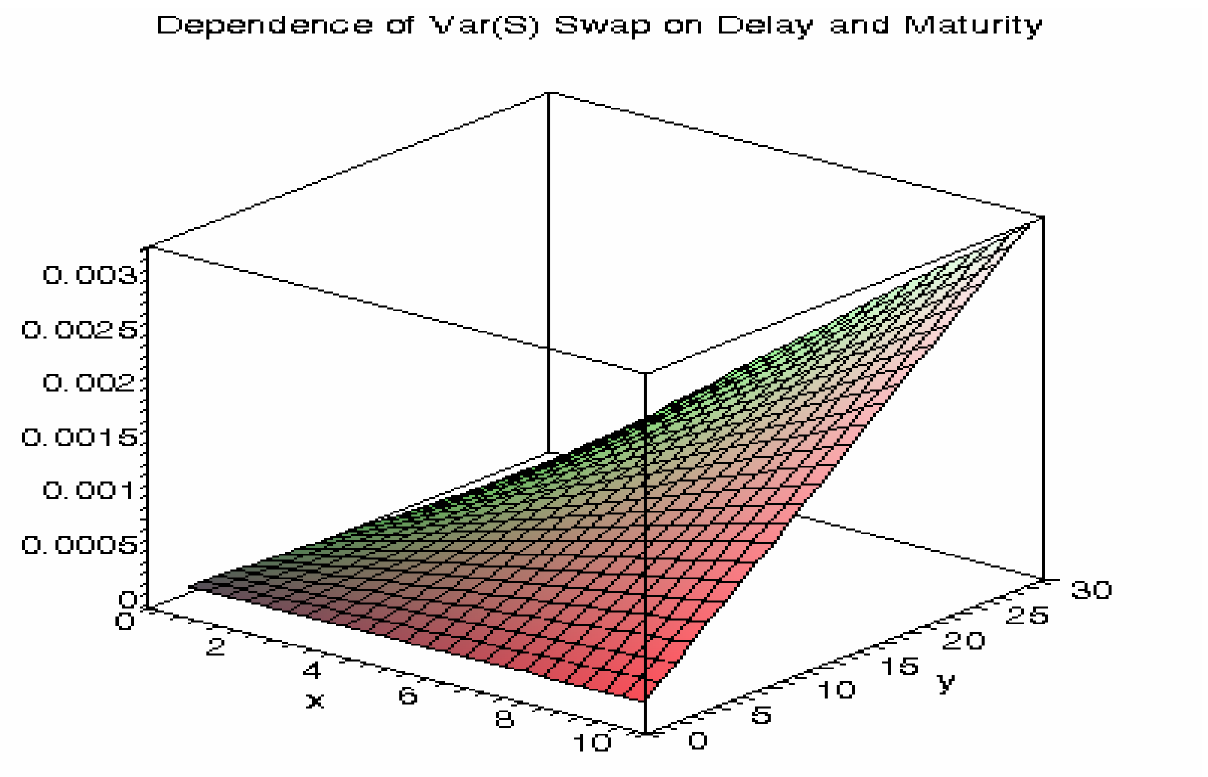
Statistics on Log Returns $S\&P60$ Canada Index	
Series:	LOG RETURNS $S\&P60$ CANADA INDEX
Sample:	1 1300
Observations:	1300
Mean	0.000235
Median	0.000593
Maximum	0.051983
Minimum	-0.101108
Std. Dev.	0.013567
Skewness	-0.665741
Kurtosis	7.787327

$$
\begin{aligned}
E\{Var(S)\} &= V + \frac{1-\gamma}{\gamma}\alpha\tau(\mu - r)^2 + (\sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma)\frac{1-e^{-\gamma T}}{T\gamma} \\
&= 0.0002 + ((1 - 0.0124)/0.0124) \times 0.0604 \times (0.0002 - 0.02)^2 \\
&+ (0.0001 - 0.0002 - 0.0604 \times (0.0002 - 0.02)^2/0.0124) \times \frac{1-e^{-0.0124}}{0.0124} \\
&= 0.000102.
\end{aligned}
$$

Dependence of Variance (Realized) Swap for One-Factor SV with Delay on Maturity (S&P60 Canada Index)



Variance (Realized) Swap for One-Factor SV with Delay (S&P60 Canada Index)



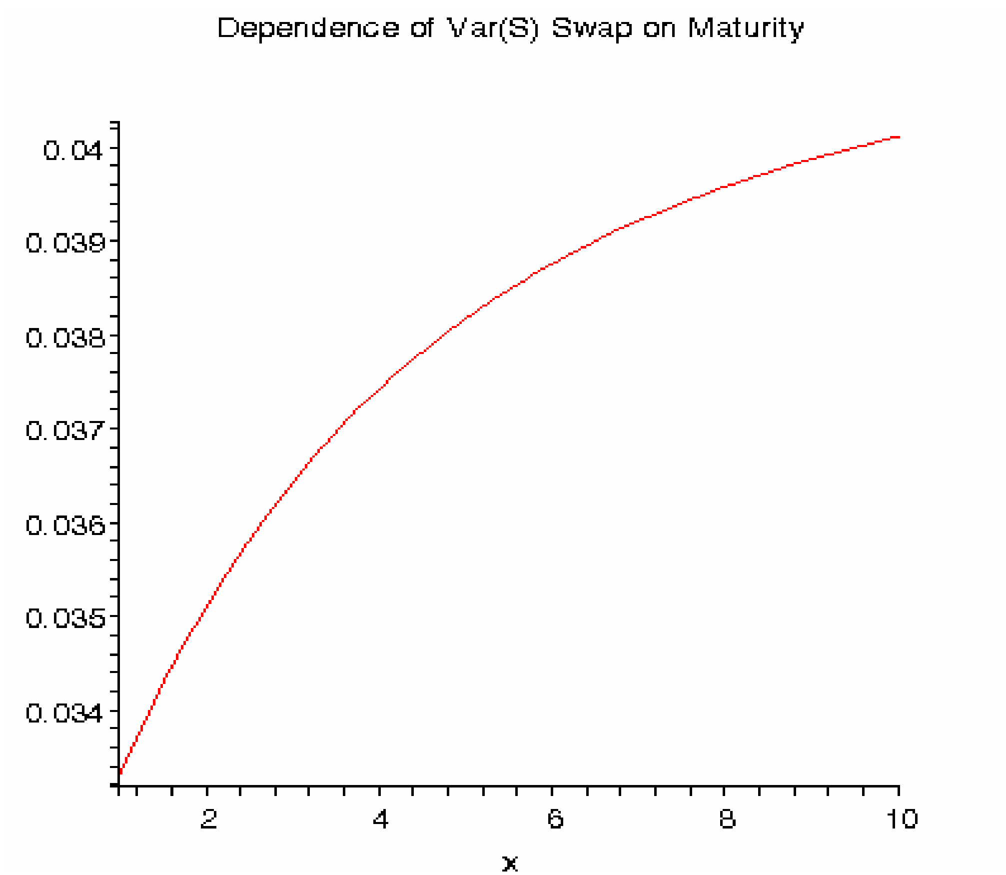
Numerical Example 2: S&P500 (1990-1993)

Table 3

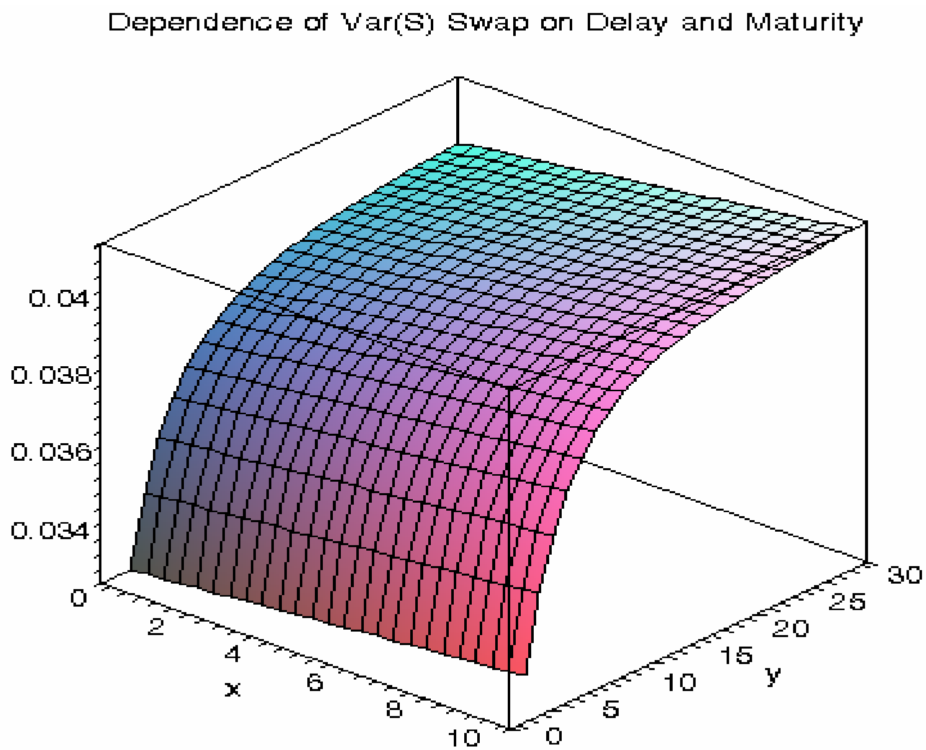
Statistics on Log Returns $S\&P500$ Index	
Series:	LOG RETURNS $S\&P500$ INDEX
Sample:	1 1006
Observations:	1006
Mean	0.000263014
Median	8.84424E-05
Maximum	0.034025839
Minimum	-0.045371484
Std. Dev.	0.00796645
Sample Variance	6.34643E-05
Skewness	-0.178481359
Kurtosis	3.296144083

$$
\begin{aligned}
E\{Var(S)\} &= V + \tfrac{1-\gamma}{\gamma}\alpha\tau(\mu-r)^2 + (\sigma_0^2 - V - \alpha\tau(\mu-r)^2/\gamma)\tfrac{1-e^{-\gamma T}}{T\gamma} \\
&= 0.004038144 + ((1-0.511)/0.511) \times 0.3828 \times 14 \times (0.000263 - 0.02)^2 \\
&+ (0.000063 - 0.04038144 - 0.3828 \times 14 \times (0.000263 - 0.02)^2/0.511) \\
&\times \tfrac{1-e^{-0.511}}{0.511} \\
&= 0.00774376584.
\end{aligned}
$$

Dependence of Variance (Realized) Swap for One-Factor SV with Delay on Maturity (S&P500)



Variance (Realized) Swap for One-Factor SV with Delay (S&P500 Index)



Numerical Example 1: S&P60 Canada Index

$$
\begin{aligned}
X &= V + \frac{\alpha\tau(\mu-r)^2}{\gamma} = 0.0002 \\
C &= \sigma_0^2 - V - \alpha\tau(\mu - r)^2/\gamma = 0.007.
\end{aligned}
$$

Expectation of Variance and the Price of Variance Swap for 1-Factor SV with Delay

$$v(t) \approx X + Ce^{-\gamma t} = V + \frac{\alpha\tau(\mu - r)^2}{\gamma} + Ce^{-\gamma t}$$

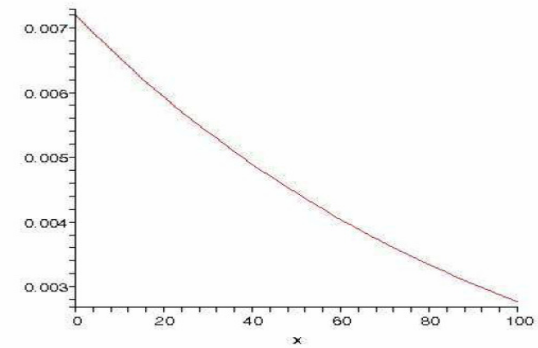


FIGURE 1: Variance of one-factor SV with delay (formula (10)).

$$\mathcal{P}^* \approx e^{-rT}\Bigg[V - K + \frac{\alpha\tau(\mu - r)^2}{\gamma} + \left(\sigma_0^2 - V - \frac{\alpha\tau(\mu - r)^2}{\gamma}\right)\frac{1 - e^{-\gamma T}}{T\gamma}\Bigg]$$

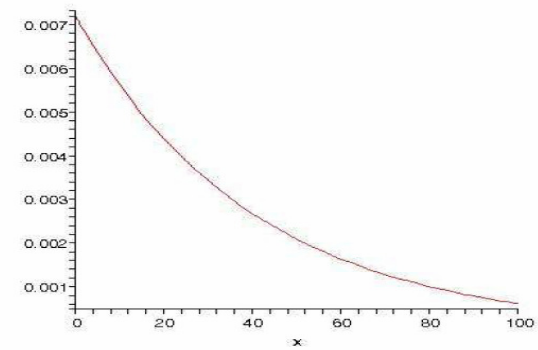


FIGURE 2: The price of variance swap for one-factor SV with delay (formula (13)).

E(Variance) and The Price of Variance Swap for SV with Delay (GBMMR)

$$v(t) \approx X + Ce^{-\gamma t} + (\xi - \lambda\beta)\gamma V_0 \times \left[\frac{X}{\xi - \lambda\beta}(e^{(\xi-\lambda\beta)t} - 1) + \frac{C}{\xi - \lambda\beta - \gamma}(e^{(\xi-\lambda\beta)t} - e^{\gamma t})\right]$$

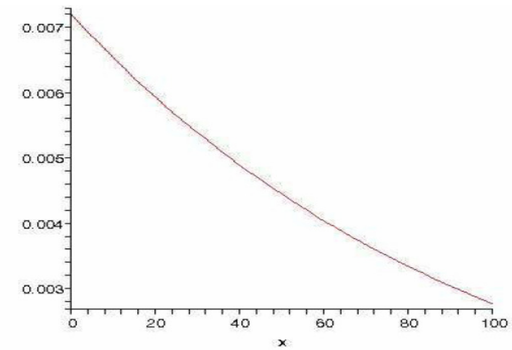


FIGURE 3: Variance of two-factor SV with delay and with GBM mean-reversion (formula (17)).

$$\mathcal{P}^* \approx e^{-rT}\left\{\left[X - K + C\frac{1 - e^{-\gamma T}}{T\gamma}\right] + \frac{(\xi - \lambda\beta)\gamma V_0}{T}\left[\frac{X}{(\xi - \lambda\beta)}\left(\frac{e^{(\xi-\lambda\beta)T} - 1}{(\xi - \lambda\beta)} - T\right) + \frac{C(e^{(\xi-\lambda\beta)T} - 1)}{(\xi - \lambda\beta)(\xi - \lambda\beta - \gamma)} - \frac{C(e^{\gamma T} - 1)}{\gamma(\xi - \lambda\beta - \gamma)}\right]\right\}.$$

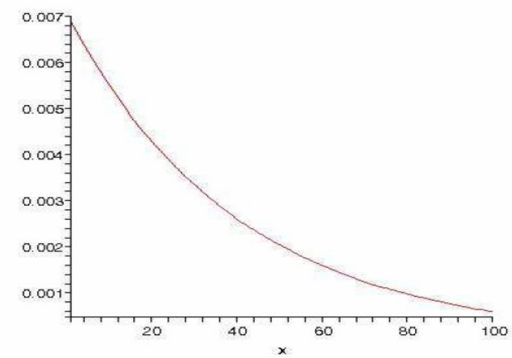


FIGURE 4: Price of variance swap for two-factor SV with delay and with GBM mean-reversion (formula (19)).

Variance and The Price of Variance Swap SV with Delay (OUMR)

$$v(t) \approx X + Ce^{-\gamma t} + \xi\gamma\Big(V_0 - \Big(L - \frac{\lambda\beta}{\xi}\Big)\Big)$$
$$\times \Big[\frac{X}{\xi}(e^{-\xi t} - 1) + \frac{C}{\xi + \gamma}(e^{-\xi t} - e^{\gamma t})\Big]$$

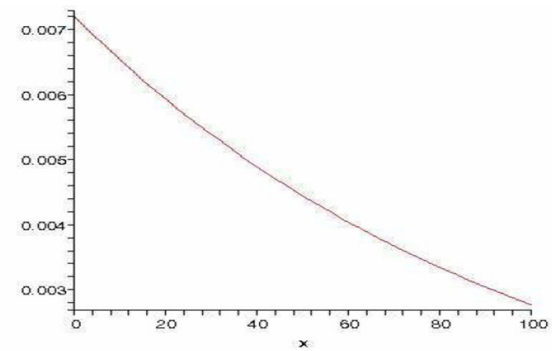


FIGURE 5: Variance of two-factor SV with delay and with OU mean-reversion (formula (22)).

$$\mathcal{P}^* \approx e^{-rT}\Big\{\Big[X - K + C\frac{1 - e^{-\gamma T}}{T\gamma}\Big] + \frac{\xi\gamma(V_0 - (L - \frac{\lambda\beta}{\xi}))}{T}$$
$$\times \Big[\frac{X}{\xi}\Big(\frac{e^{-\xi T} - 1}{\xi} + T\Big) + \frac{C(e^{-\xi T} - 1)}{\xi(\xi + \gamma)} + \frac{C(e^{\gamma T} - 1)}{\gamma(\gamma + \xi)}\Big]\Big\}$$

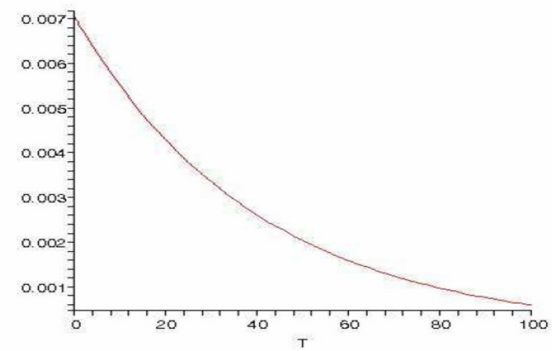


FIGURE 6: Price of variance swap for two-factor SV with delay and with OU mean-reversion (formula (24)).

Variance and The Price of Variance Swap for SV with Delay (Pilipovich 1FMR)

$$v(t) \approx X + Ce^{-\gamma t} + \frac{\gamma\xi}{\xi+\lambda\beta}\Big[\Big(X(\frac{V_0(\xi+\lambda\beta)}{\xi} - L\Big) \times (1-e^{-(\xi+\lambda\beta)t}) + XLt + \frac{C(\frac{V_0(\xi+\lambda\beta)}{\xi} - L)}{\xi+\lambda\beta+\gamma} + \frac{CL}{\gamma}(e^{\gamma t}-1)\Big].$$

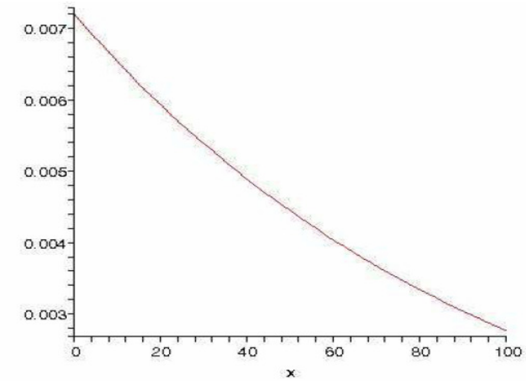


FIGURE 7: Variance of two-factor SV with delay and with Pilipovich one-factor mean-reversion (formula (27)).

$$\mathcal{P}^* \approx e^{-rT}\Big\{\Big[X - K + C\frac{1-e^{-\gamma T}}{T\gamma}\Big] + \frac{\gamma\xi}{(\xi+\lambda\beta)T} \times \Big[X\Big(\Big(\frac{V_0(\xi+\lambda\beta)}{\xi} - L\Big)\Big(\frac{e^{-(\xi+\lambda\beta)}-1}{\xi+\lambda\beta} - T\Big)\Big) + \frac{XLT^2}{2} + \frac{C(\frac{V_0(\xi+\lambda\beta)}{\xi} - L)}{\xi+\lambda\beta+\gamma} \times \Big(\frac{e^{\gamma T}-1}{\gamma} + \frac{e^{-(\xi+\lambda\beta)T}-1}{\xi+\lambda\beta}\Big) + \frac{CL}{\gamma}\Big(\frac{e^{\gamma T}-1}{\gamma} - T\Big)\Big]\Big\}$$

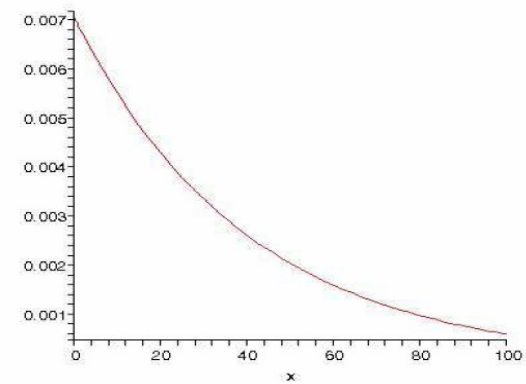


FIGURE 8: Price of variance swap for two-factor SV with delay and with Pilipovich one-factor mean-reversion (formula (28)).

Variance and The Price of Variance Swap for SV with Delay (Pilipovich 2FMR)

$$v(t) \approx X + Ce^{-\gamma t} - (\xi + \lambda\beta)\gamma V_0 \left[\frac{X}{\xi + \lambda\beta}(1 - e^{-(\xi+\lambda\beta)t}) + \frac{C}{\xi + \lambda\beta + \gamma}(e^{\gamma t} - e^{-(\xi+\lambda\beta)t}) \right] + L_0 \frac{\xi + \lambda\beta}{\xi + \lambda\beta + \beta_1} \times \left[X(e^{(\beta_1 - \lambda_1\eta)t} - e^{-(\xi+\lambda\beta)t}) + \frac{C(\beta_1 - \lambda_1\eta)}{(\beta_1 - \lambda_1\eta - \gamma)} \times (e^{(\beta_1-\lambda_1\eta)t} - e^{\gamma t}) + \frac{C(\xi + \lambda\beta)}{(\xi + \lambda\beta + \gamma)}(e^{\gamma t} - e^{-(\xi+\lambda\beta)t}) \right]$$

$$\begin{aligned}\mathcal{P}^* \approx e^{-rT} \Bigg\{ \left[X - K + C\frac{1 - e^{-\gamma T}}{T\gamma} \right] - \frac{(\xi + \lambda\beta)\gamma V_0}{T} \\ \times \left[\frac{X}{(\xi + \lambda\beta)} \left(\frac{e^{-(\xi+\lambda\beta)T} - 1}{(\xi + \lambda\beta)} + T \right) \right. \\ \left. + \frac{C(e^{-(\xi+\lambda\beta)T} - 1)}{(\xi + \lambda\beta)(\xi + \lambda\beta + \gamma)} + \frac{C(e^{\gamma T} - 1)}{\gamma(\gamma + \xi + \lambda\beta)} \right] \\ + \frac{(\xi + \lambda\beta)L_0}{(\xi + \lambda\beta + \beta_1)T} \left[\frac{X(e^{(\beta_1 - \lambda_1\eta)T} - 1 - (\beta_1 - \lambda_1\eta)T)}{\beta_1 - \lambda_1\eta} \right. \\ + \frac{X(e^{-(\xi+\lambda\beta)T} - 1 + (\xi + \lambda\beta)T)}{(\xi + \lambda\beta)} \\ + C\left(\frac{\beta_1 - \lambda_1\eta}{\beta_1 - \lambda_1\eta - \gamma} \left(\frac{e^{(\beta_1-\lambda_1\eta)T} - 1}{\beta_1 - \lambda_1\eta} \right) - \frac{e^{\gamma T} - 1}{\gamma} \right) \\ \left. + \frac{\xi + \lambda\beta}{\xi + \lambda\beta + \gamma} \left(\frac{e^{-(\xi+\lambda\beta)T} - 1}{(\xi + \lambda\beta)} + \frac{e^{\gamma T} - 1}{\gamma} \right) \right] \Bigg\}.\end{aligned}$$

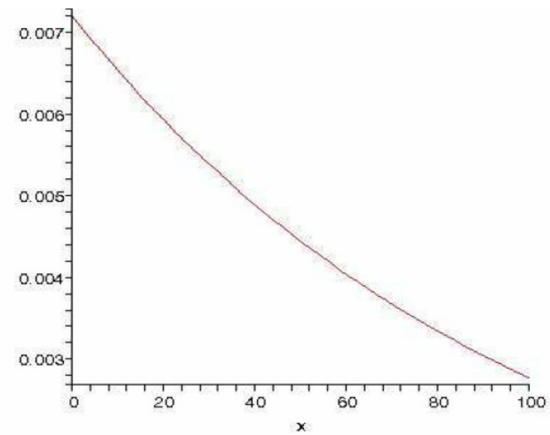


FIGURE 9: Variance of three-factor SV with delay and with Pilipovich two-factor mean-reversion (formula (31)).

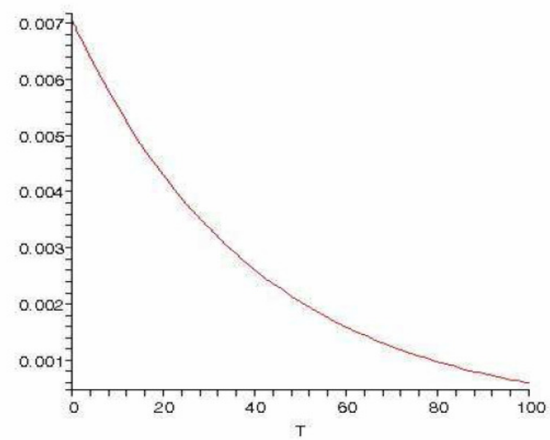


FIGURE 10: Price of variance swap for three-factor SV with delay and with Pilipovich two-factor mean-reversion (formula (33)).

Comparison

One-Factor

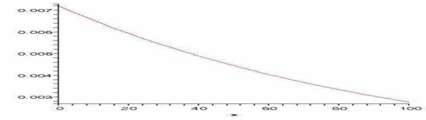


FIGURE 1: Variance of one-factor SV with delay (formula (10)).

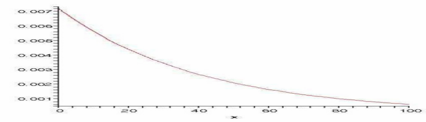


FIGURE 2: The price of variance swap for one-factor SV with delay (formula (13)).

2-F(GBMMR)

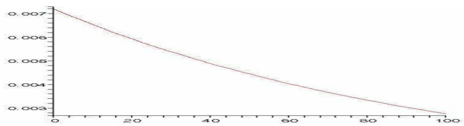


FIGURE 3: Variance of two-factor SV with delay and with GBM mean-reversion (formula (17)).

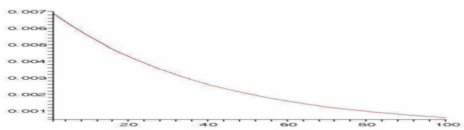


FIGURE 4: Price of variance swap for two-factor SV with delay and with GBM mean-reversion (formula (19)).

2-F(OUMR)

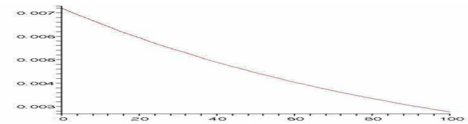


FIGURE 5: Variance of two-factor SV with delay and with OU mean-reversion (formula (22)).

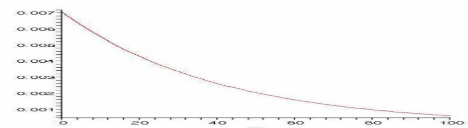


FIGURE 6: Price of variance swap for two-factor SV with delay and with OU mean-reversion (formula (24)).

2-F(Pilipovich 1FMR)

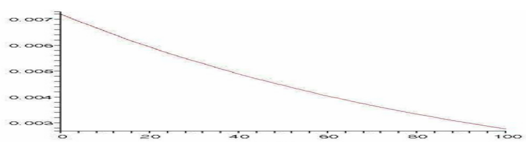


FIGURE 7: Variance of two-factor SV with delay and with Pilipovich one-factor mean-reversion (formula (27)).

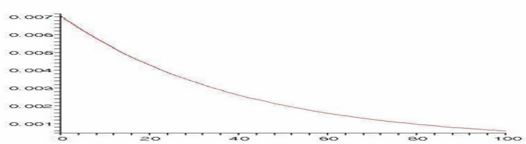


FIGURE 8: Price of variance swap for two-factor SV with delay and with Pilipovich one-factor mean-reversion (formula (28)).

3-F(Pilipovich 2FMR)

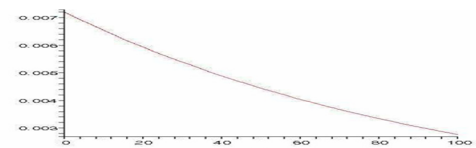


FIGURE 9: Variance of three-factor SV with delay and with Pilipovich two-factor mean-reversion (formula (31)).

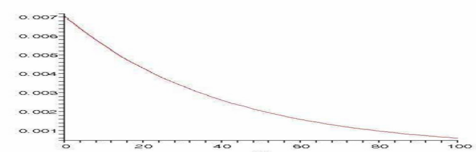


FIGURE 10: Price of variance swap for three-factor SV with delay and with Pilipovich two-factor mean-reversion (formula (33)).

Conclusion I

- There is no big difference *between One-Factor SV with Delay and Multi-Factor SV with Delay*
- One-Factor SV with Delay *catches almost all the features of Multi-Factor SV with Delay*
- One-Factor SV with Delay *is Similar to the SV in Heston Model (at least for variance swaps)*

SV with Delay and Jumps

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s) + \int_{t-\tau}^{t} y_s dN(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t)$$

SV with Delay without Jumps

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)\,dW(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t).$$

SV with Delay and Jumps (Simple Poisson Process Case)

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\left[\int_{t-\tau}^{t} \sigma(s,S_s)dW(s) + \int_{t-\tau}^{t} dN(s)\right]^2 - (\alpha+\gamma)\sigma^2(t,S_t)$$

Equation for The Mean of Variance

$$v(t) = \mathbb{E}^*[\sigma^2(t, S_t)]$$

$$\begin{aligned}\frac{dv(t)}{dt} \quad = \quad & \gamma V + \alpha\lambda + \alpha\lambda^2\tau - 2\alpha\lambda\tau(\mu - r) + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds \\ & -(\alpha + \gamma)v(t)\end{aligned}$$

Stationary Solution

$$
\begin{aligned}
v(t) \equiv X &= V + \left[\alpha\lambda + \alpha\lambda^2\tau - 2\alpha\lambda\tau(\mu - r) + \alpha\tau(\mu - r)^2\right]/\gamma \\
&= V + \frac{\alpha}{\gamma}\left[\lambda + \tau(\lambda - \mu + r)^2\right]
\end{aligned}
$$

Stationary Solution without Jumps

$$v(t) \equiv X = V + \frac{\alpha\tau(\mu - r)^2}{\gamma}$$

Price of Var Swap in Stationary Case

$$P = e^{-r(T-t)} \left\{ V - K + \frac{\alpha}{\gamma} \left[\lambda + \tau(\lambda - \mu + r)^2 \right] \right\}$$

General Solution

$$
\begin{aligned}
v(t) &\approx X + Ce^{-\gamma t} \\
&= V + \frac{\alpha}{\gamma}\left[\lambda + \tau(\lambda - \mu + r)^2\right] + Ce^{-\gamma t}
\end{aligned}
$$

$$C = \sigma_0^2 - V - \frac{\alpha}{\gamma}\left[\lambda + \tau(\lambda - \mu + r)^2\right]$$

Price of Var Swap in General Case

$$P \approx e^{-r(T-t)}\left[X - K + \left(\sigma_0^2 - X\right)\frac{1 - e^{-\gamma T}}{\gamma T}\right]$$

$$\begin{aligned} X &= V + \left[\alpha\lambda + \alpha\lambda^2\tau - 2\alpha\lambda\tau(\mu - r) + \alpha\tau(\mu - r)^2\right]/\gamma \\ &= V + \frac{\alpha}{\gamma}\left[\lambda + \tau(\lambda - \mu + r)^2\right] \end{aligned}$$

Compound Poisson Process Case

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\Big[\int_{t-\tau}^{t} \sigma(s,S_s)dW^*(s) + \int_{t-\tau}^{t} y_s dN(s) - (\mu - r)\tau\Big]^2$$
$$-(\alpha+\gamma)\sigma^2(t,S_t)$$

Equation for the Mean of Variance

$$\frac{dv(t)}{dt} = \gamma V + \alpha\lambda + \alpha\lambda^2\tau - 2\alpha\lambda\tau(\mu - r) + \alpha\tau(\mu - r)^2 + \frac{\alpha}{\tau}\int_{t-\tau}^{t} v(s)ds$$
$$-(\alpha + \gamma)v(t)$$

Stationary Solution

$$\begin{aligned} v(t) \equiv X &= V + \left[\alpha\lambda(\xi^2+\eta) + \alpha\lambda^2\tau\xi^2 - 2\alpha\lambda\tau\xi(\mu - r) + \alpha\tau(\mu - r)^2\right]/\gamma \\ &= V + \frac{\alpha}{\gamma}\left[\lambda(\xi^2+\eta) + \tau(\lambda\xi - \mu + r)^2\right] \qquad (\end{aligned}$$

Price of Swap in Stationary Case

$$P = e^{-r(T-t)}\left\{V - K + \frac{\alpha}{\gamma}\left[\lambda(\xi^2 + \eta) + \tau(\lambda\xi - \mu + r)^2\right]\right\}$$

General Solution

$$\begin{aligned} v(t) &\approx X + Ce^{-\gamma t} \\ &= V + \frac{\alpha}{\gamma}\left[\lambda(\xi^2 + \eta) + \tau(\lambda\xi - \mu + r)^2\right] + Ce^{-\gamma t} \end{aligned}$$

$$C = \sigma_0^2 - V - \frac{\alpha}{\gamma}\left[\lambda(\xi^2 + \eta) + \tau(\lambda\xi - \mu + r)^2\right]$$

Price of Swap in General Case

$$P \approx e^{-r(T-t)}\left[X - K + (\sigma_0^2 - X)\frac{1 - e^{-\gamma T}}{\gamma T}\right]$$

$$\begin{aligned} X &= V + \left[\alpha\lambda(\xi^2 + \eta) + \alpha\lambda^2\tau\xi^2 - 2\alpha\lambda\tau\xi(\mu - r) + \alpha\tau(\mu - r)^2\right]/\gamma \\ &= V + \frac{\alpha}{\gamma}\left[\lambda(\xi^2 + \eta) + \tau(\lambda\xi - \mu + r)^2\right] \qquad (\text{} \end{aligned}$$

More General case

$$\frac{d\sigma^2(t,S_t)}{dt} = \gamma V + \frac{\alpha}{\tau}\Big[\int_{t-\tau}^{t}\sigma(s,S_s)dW^*(s) + \int_{t-\tau}^{t} y_s dN(s) - (\mu - r)\tau\Big]^2$$
$$-(\alpha+\gamma)\sigma^2(t,S_t) \qquad (38)$$

where $W^*(t)$ is a Brownian motion, $N(t)$ is a Poisson process with intensity λ and y_t is the jump size at time t. We assume that $\mathbb{E}[y_t] = A(t)$, $\mathbb{E}[y_s y_t] = C(s,t), s < t$ and $\mathbb{E}[y_t^2] = B(t) = C(t,t)$, where $A(t), B(t), C(s,t)$ are all deterministic functions. Note that the change of measure do not change the Poisson intensity λ and the distribution of jump size y_t, since they are independent to the Brownian motion.

Equation for the Mean of Variance

$$\frac{dv(t)}{dt} = \gamma V + \frac{\alpha}{\tau}\Big[\int_{t-\tau}^{t} v(s)ds + \lambda \int_{t-\tau}^{t} B(s)ds + \lambda^2\big(K(t,\tau) + G\big)$$

$$+(\mu - r)^2\tau^2 - 2\lambda\tau(\mu - r)\int_{t-\tau}^{t} A(s)ds\Big] - (\alpha + \gamma)v(t)$$

General Solution

$$v(t) \approx \frac{1-e^{-\gamma t}}{\gamma} v'(0) + \left[\frac{\alpha}{\gamma}(1-e^{-\gamma t}) + 1\right] v(0) - \frac{\alpha}{\gamma\tau}\int_{-\tau}^{0} v(s)[1-e^{-\gamma(t-s-\tau)}]ds$$
$$+\frac{1}{\gamma}\int_{0}^{t} h(s,\tau)[1-e^{-\gamma(t-s)}]ds + C. \tag{49}$$

$$C = \frac{\alpha}{\gamma\tau}\int_{-\tau}^{0} v(s)[1-e^{\gamma(s+\tau)}]ds.$$

Numerical Example

Table 1

Statistics on Log Returns $S\&P60$ Canada Index	
Series:	LOG RETURNS $S\&P60$ CANADA INDEX
Sample:	1 1300
Observations:	1300
Mean	0.000235
Median	0.000593
Maximum	0.051983
Minimum	-0.101108
Std. Dev.	0.013567
Skewness	-0.665741
Kurtosis	7.787327

Constant V and E^*[v]

$$
\begin{aligned}
& V + \frac{\alpha}{\gamma}\left[\lambda(\xi^2+\eta) + \tau(\lambda\xi - \mu + r)^2\right] \\
= \; & 0.0002 + 0.0604/0.0124 \times \Big[0.0115 \times \left[(-0.003)^2 + 0.0035\right] \\
+ \; & (0.0115 \times (-0.003) - 0.0002 + 0.0124)^2 \Big] \\
= \; & 0.0023.
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}^*[v] \approx \; & V + \frac{\alpha}{\gamma}\left[\lambda(\xi^2+\eta) + \tau(\lambda\xi - \mu + r)^2\right] \\
& + \left\{\sigma_0^2 - V - \frac{\alpha}{\gamma}\left[\lambda(\xi^2+\eta) + \tau(\lambda\xi - \mu + r)^2\right]\right\} \frac{1-e^{-\gamma T}}{\gamma T} \\
= \; & 0.0023 + (0.0001 - 0.0023) \times \frac{1-e^{-0.0124}}{0.0124} \\
= \; & 0.0001136.
\end{aligned}
$$

Delivery Price and Maturity

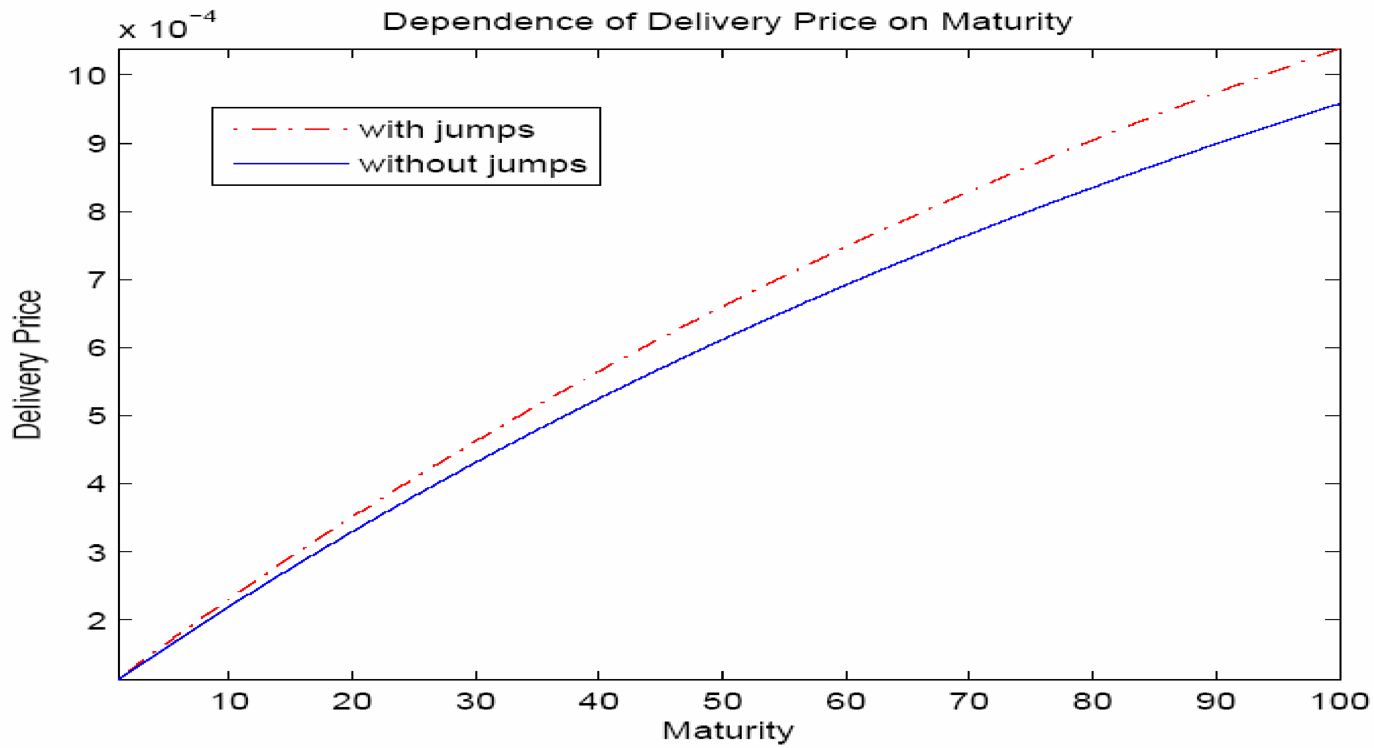


Figure 1: Dependence of Delivery Price on Maturity ($S\&P60$ Canada Index).

Delivery Price and Delay

Figure 2: Dependence of Delivery Price on Delay ($S\&P60$ Canada Index).

Delivery Price and Jump Intensity

Figure 3: Dependence of Delivery Price on Jump Intensity ($S\&P60$ Canada Index).

Delivery Price, Delay and Jump Intensity

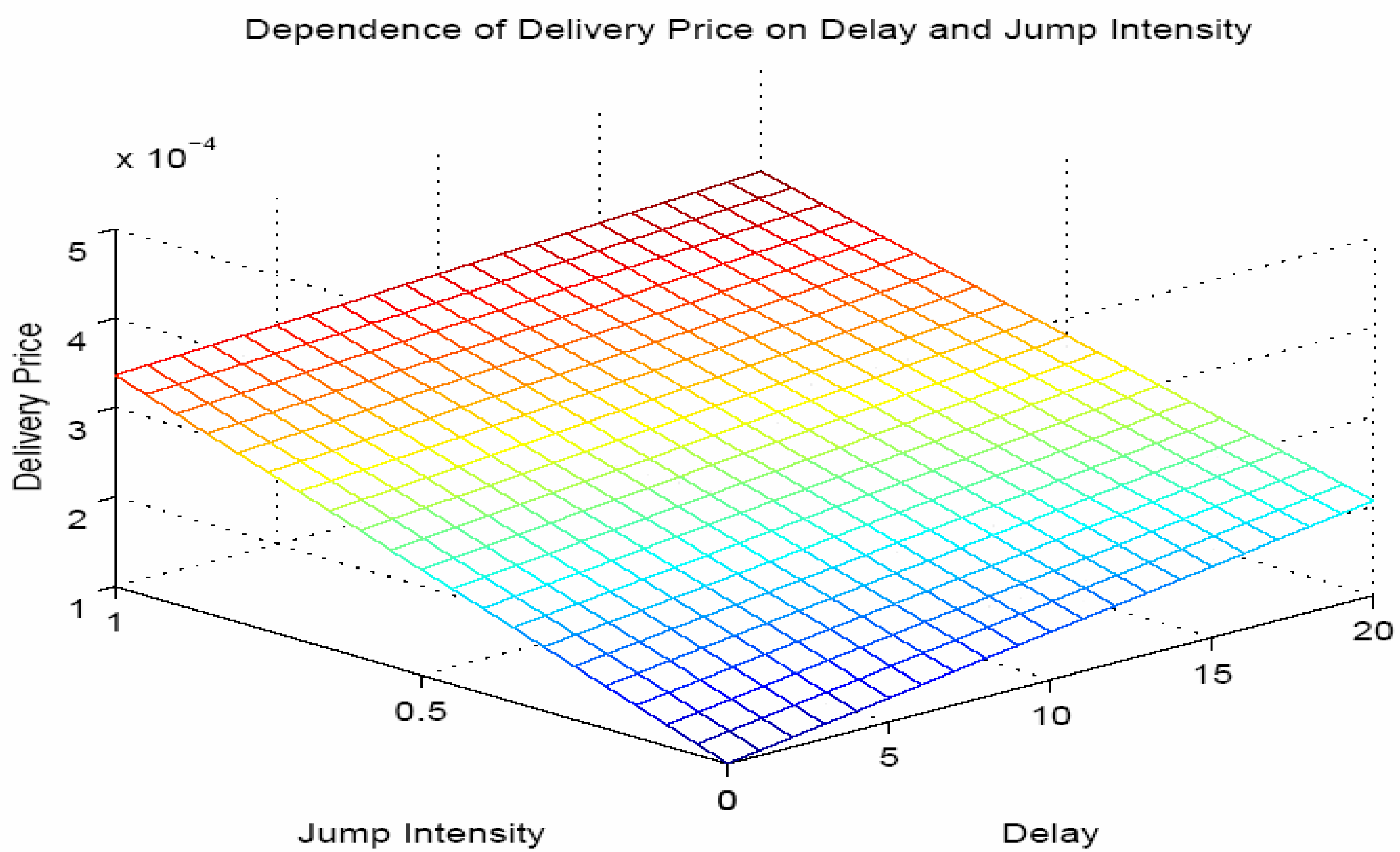


Figure 4: Dependence of Delivery Price on Delay and Jump Intensity ($S\&P60$ Canada Index).

Delivery Price, Delay and Maturity

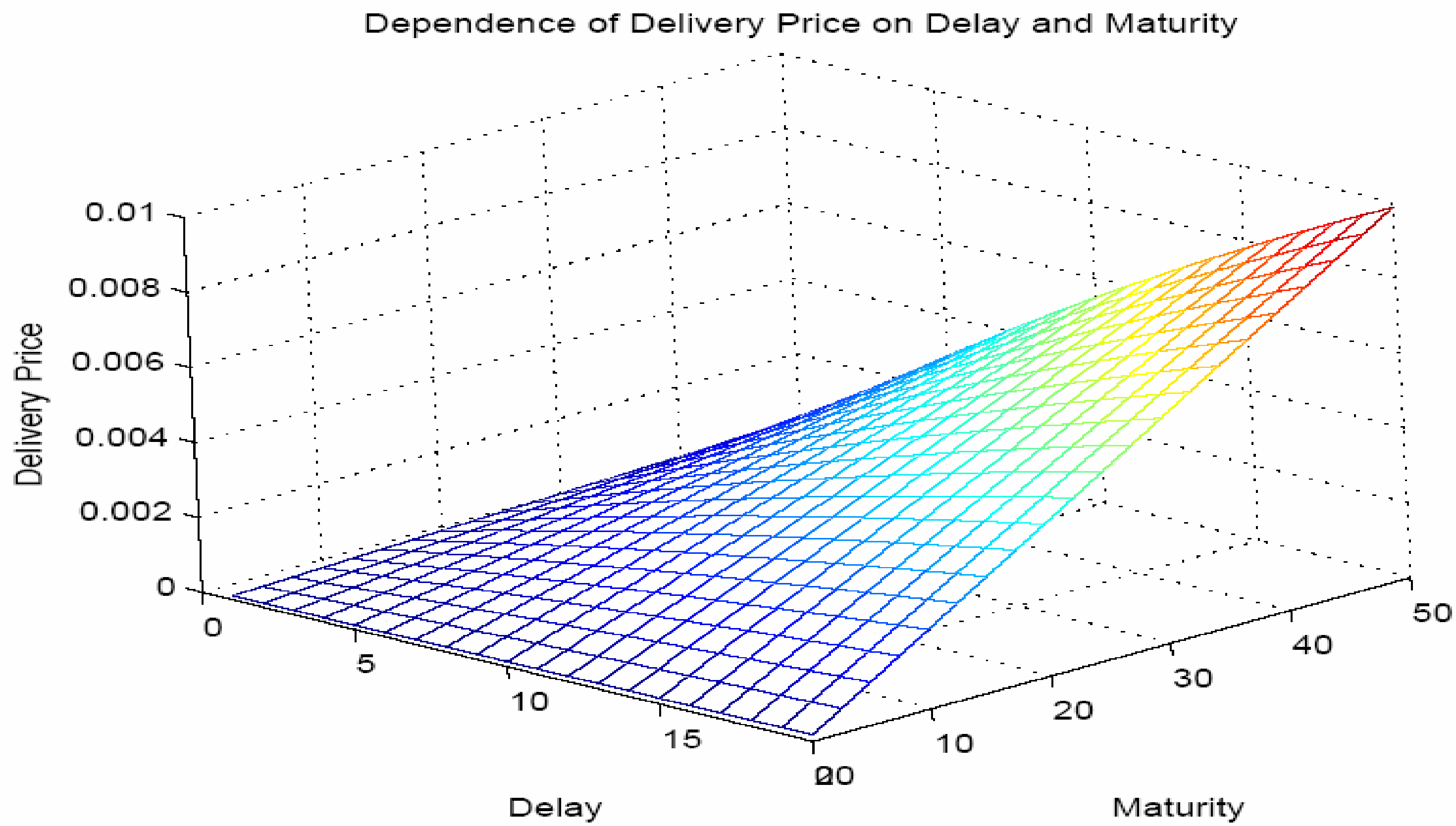
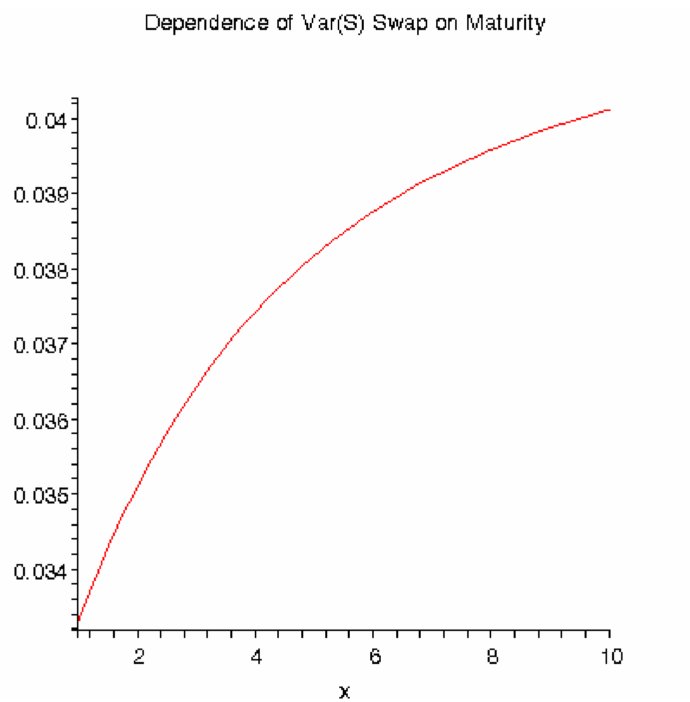


Figure 5: Dependence of Delivery Price on Delay and Maturity ($S\&P60$ Canada Index).

Delivery Price, Maturity and Jump Intensity

Figure 6: Dependence of Delivery Price on Jump Intensity and Maturity ($S\&P60$ Canada Index).

Comparison (1F&Jumps)



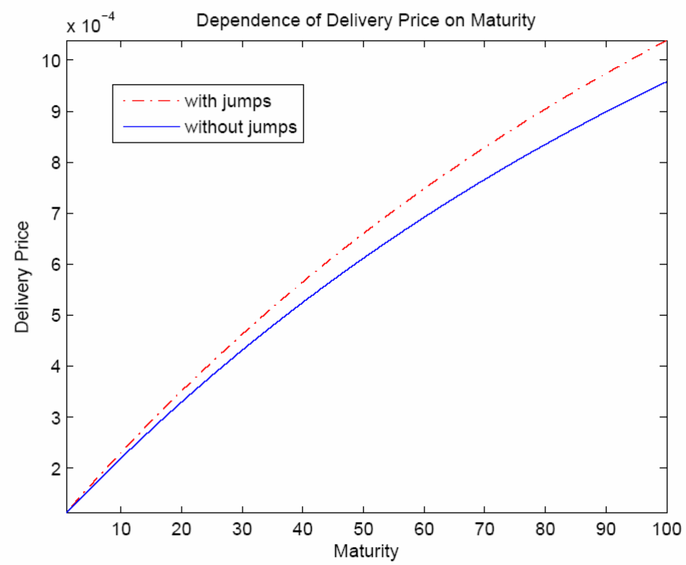


Figure 1: Dependence of Delivery Price on Maturity ($S\&P60$ Canada Index).

Conclusion II

- There is no big difference *between One-Factor SV with Delay, Multi-Factor SV with Delay and 1F SV with Delay & Jumps*

- One-Factor SV with Delay *catches all the features of Multi-Factor SV with Delay and 1F SV with Delay & Jumps*

Publications

- '*The Pricing of Options for Security Markets with Delay*' (Kazmerchuk, Sw, Wu), **Mathematics and Computers in Simulations**, 2007, v. 75/3-4 pp. 69-79
- '*Continuous-time GARCH model for Stochastic Volatility with Delay*' (Kazmerchuk, Sw, Wu), **CAMQ**, 2005, v. 3, No. 2
- '*Modeling and Pricing of Variance Swap for Stochastic Volatility with Delay*' (Sw), **Wilmott Magazine**, Issue 19, September 2005
- '*Modeling and Pricing of Variance Swaps for Multi-Factor Stochastic Volatilities with Delay*' (Sw), **CAMQ**, 2006, v. 14, No. 4
- 'Pricing Variance Swaps for Srochastic Volatilities with Delay and Jumps' (Sw, Xu, L.), Quantitative Finance (submitted), 2007

The End

- Thank You for Your Attention!
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