Routing in Ad Hoc Networks:
A Theoretical Framework with Practical Implications

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Objective: Basic Limits on Routing Overhead

- Considering a variable topology network, such as for example a mobile ad hoc network;
  - can we characterize the variability of a network topology?
  - and relate it to the (minimum) routing overhead?
  - and hence discover some basic limits on routing?
  - …and maybe these limits could be used as reference curves (similar to Shannon's Capacity for error free communication)

Related work: Theory of Computing

- In TOC, routing typically refers to building distributed memory message passing multiprocessor systems for computing applications, where the interest primarily is in finding the computational cost (complexity) of a certain message passing i.e. routing algorithm (e.g. [1]).

- The hypercube is one of the most popular (and robust) topologies used for building and studying routing in this context (e.g. [2]).

[1] C. Gavoille, "Routing in distributed networks: Overview and open problems," ACM SIGCAT News, vol. 32, no. 1, pp. 36-52, March 2001.

[2] M.J. Kumar, L.M. Patnaik, B. Nag, "Fault-tolerant message routing in the extended hypercube," Journal of Systems Architecture (Elsevier), 44, 1998, 691-702.

Related work: Optimization

- Saha-Mukherjee [3] proposes an optimization approach to find the optimal number of clusters that minimize the total route computation cost, assuming the route computation cost per hierarchical level is some known constant \alpha_i, and a fixed traffic matrix.

- Kleinrock-Kamoun [4] proposes an optimization approach for finding the number of clusters that minimize the size of the routing table in a variable network topology

[3] D. Saha and A. Mukherjee, “Computational analysis of an optimal routing strategy in a hierarchical computer communication network,” Computer Communications,” Volume 18, Issue 7, July 1995, Pages 507-511.

[4] L. Kleinrock and F. Kamoun, “Hierarchical routing for large networks Performance evaluation and optimization,” *Computer Networks (1976), Volume 1, Issue 3, January 1977, Page 155.*

Related Work: Information Theory

- Gallager [5] proposes an information theoretic approach to find basic limits on protocol overhead for maintaining the start and stop time of messages between pairs or nodes in a communication network.

- In this set-up, the network is the “source” and the protocol is thus merely a “source encoder”.

[5] Basic Limits on Protocol Information in Data Communication Networks, IEEE Trans. on Inf. Theory, vol. 22, no. 4, July 1976, paes 385-398.

Proposed Information-Theoretic Framework

- Analyze the changes of topology as random process
  - Define the topology itself as random variable

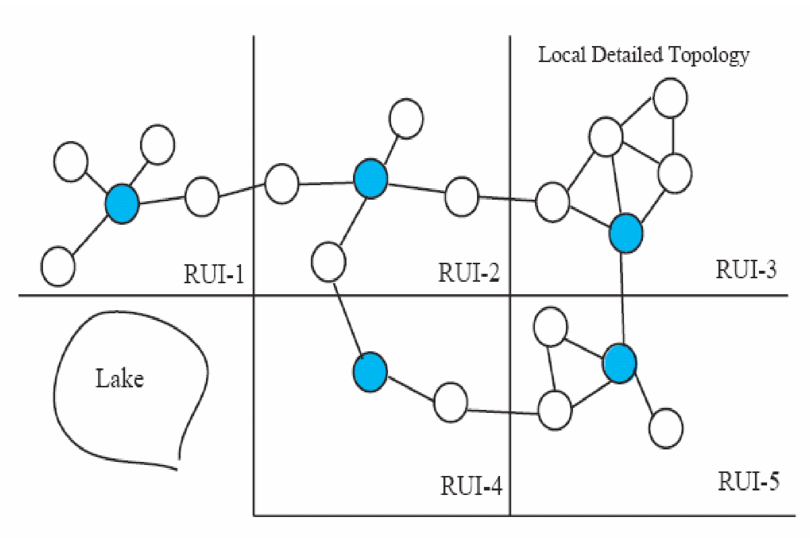
- Apply information-theoretic principles to quantify the minimum amount of overhead :
  - Routing message overhead (bits / unit time).

  - Routing memory overhead (bits)

Theoretical Framework (Cont'd)

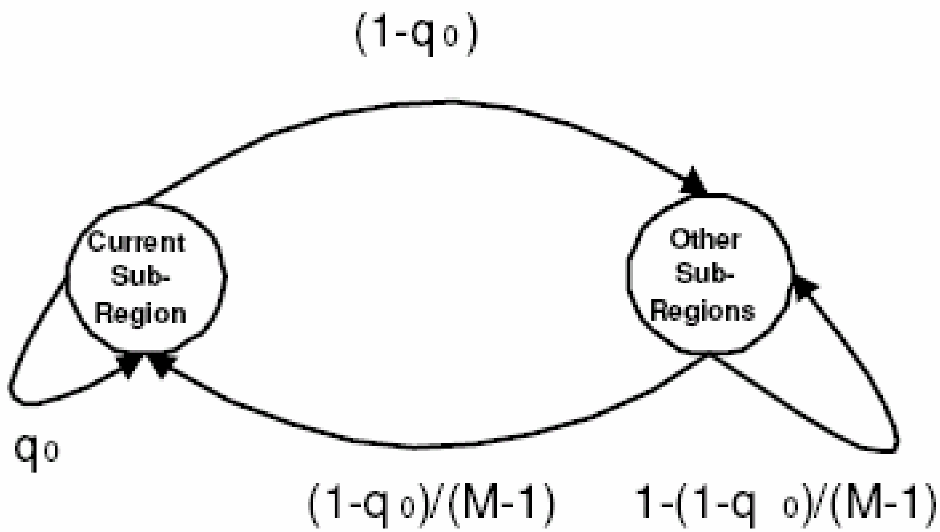
- Minimum amount of information needed to describe a change in the network topology? Entropy; Minimum Expected Codeword Length (MCL)
- Minimum amount of overhead needed to inform the cluster head of that change? Send the MCL over the shortest paths to cluster heads
- Memory is needed to support the information exchange? Topology + topology change info

Hierarchical Proactive Routing Protocol Model

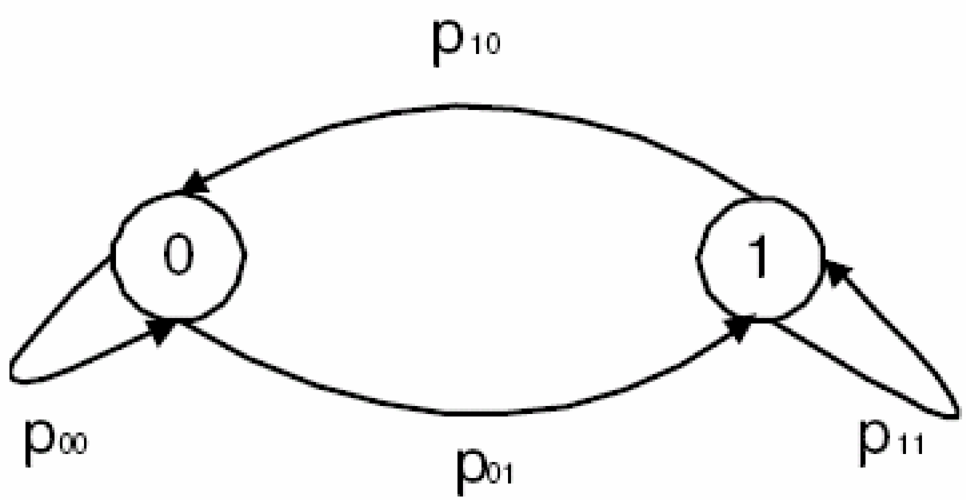


- Bounded area; N nodes; M sub-regions; Connected network.
- Each node maintains link status info by periodic hello messages at periodic intervals \tau_i;
- regular nodes inform cluser head about link changes.
- Whenever there is a change, a regular node receives the new path information from the cluster head.
- Whenever cluster membership changes, the cluster head announces this change to all other clusters at periodic intervals \tau_e.

Mobility and Link Status models



State transition diagram of node movement between clusters. q_0 is the probability that a node stays in the same cluster.



State transition diagram of the status of an arbitrary link; P_{00} (P_{11}) is the probability that a link is down (up) at next time step if the link was down (up) at previous time step.

Analysis Outline

- Topology Granularities
  - Global Ownership Topology
  - Local Ownership Topology
  - Local Detailed Topology

- Analysis – for each topology level
  - MCL based on Cardinality (i.e. all topologies equally likely)
  - MCL based on Topology Stationary Probability Distribution
  - MCL on Prediction Using Previous Topology Knowledge (given a certain mobility and link status change models)

Summary of Results

- Routing Overhead
  - R_e : Exterior Routing Overhead (exchanging local ownership topologies)
  - R_i :Interior Routing Overhead
    - R_h : overhead associated with hello messages
    - R_d : Notification of link status change to cluster head
    - R_p : Notification of new path to the regular node
- Memory Requirement
  - Cluster Head: M_c
    - Global Ownership Topology: M_{cg}
    - Local Detailed Topology: M_{cd}
  - Regular Node: M_r
    - Shortest path to cluster head

Application of the results: Network Scaling

- Given the various expressions for routing overhead, there are different methods for scaling the network (increasing N)
  - Model1: Increase N but keep g constant (hence decrease d0, the coverage radius of each node)
  - Model2: Increase N but keep $g=\Theta(\log N)$ at the critical value needed for connectivity
  - Model3: Increase N while keeping d_0 constant (hence g increases)

  Note1: These methods keep other parameters such as A (area) and M (number of clusters) constant
- Also derived the scaling laws with M

Scaling Laws

- Scaling with N

Overhead	M1:g const	M2:g_c	M3:d_0 const
R_e	$N^{\frac{3}{2}}$	$\frac{N^{\frac{3}{2}}}{\sqrt{\log N}}$	N
R_h	$N \log N$	$N \log N$	$N \log N$
R_d	$N^{\frac{5}{2}}$	$\frac{N^{\frac{5}{2}}}{\sqrt{\log N}}$	N^2
R_p	$N^{\frac{5}{2}} \log N$	$\frac{N^{\frac{5}{2}}}{\sqrt{\log N}}$	$N \log N$

Other Practical Implications

- M_{optm} minimizing memory requirement of cluster heads

$$\mathrm{M}_{optm} = \sqrt{(\ln 2)N}$$

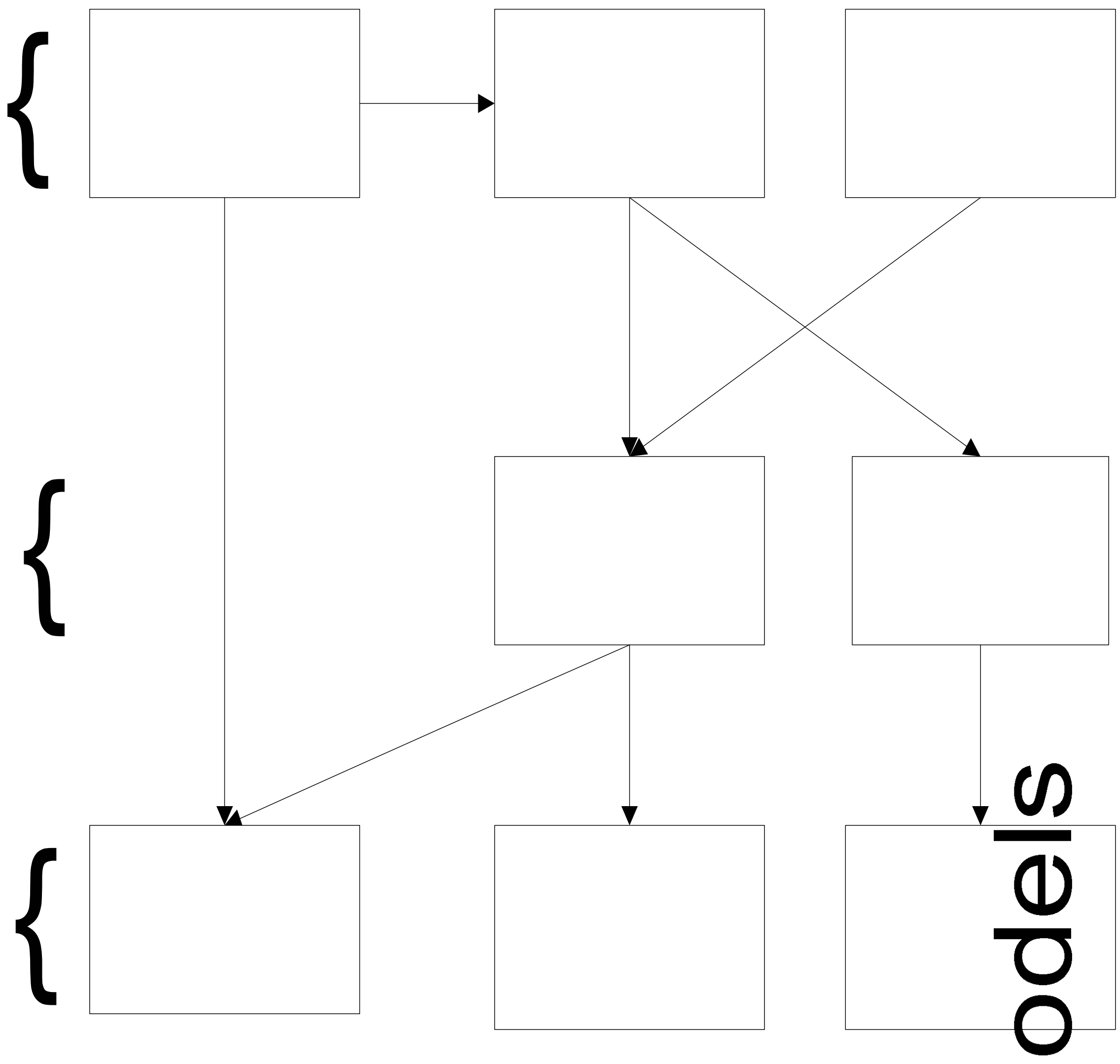
- M_{ratio} minimizing the ratio of the memories of regular vs. cluster-head node

$$2M_{ratio}^{2}(1+\ln M_{ratio}) = (\ln 2)N$$

- M_{optr} minimizing the total routing overhead in a large network (Model1)

$$M_{optr} = \sqrt[3]{\frac{\tau_e(2N+(1-p_{11})\beta^2 N\log N}{4\tau_i}}$$

Summary of the analysis



Conclusion

- Developed an information theoretic framework for quantifying hierarchical proactive routing overhead
- Derived expressions for the overhead
- Analyzed these expressions to derive scalability results
- Applied these expressions to find the cluster size that asymptotically optimize several different objectives

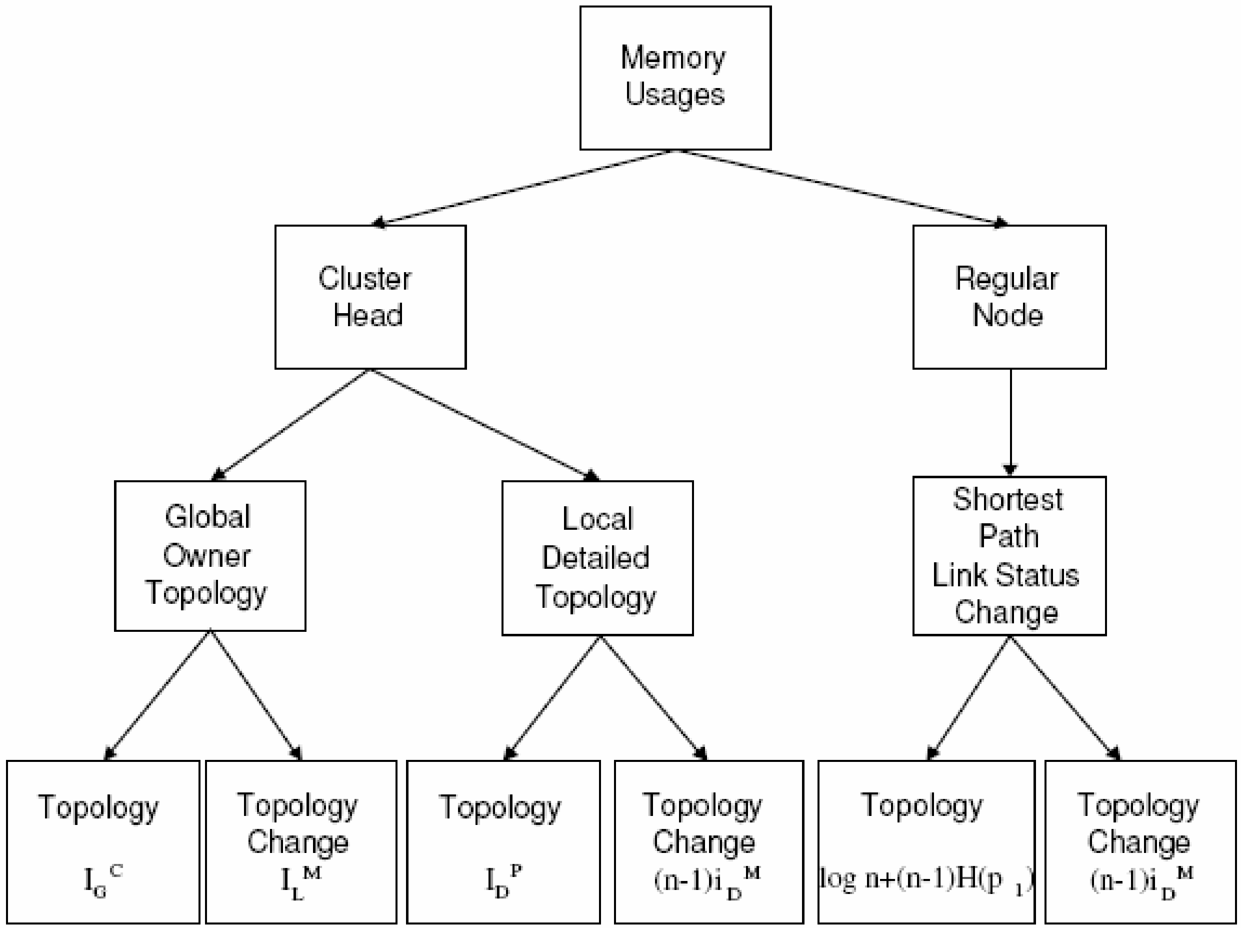
Future Work

- Reactive routing overhead (by conditioning on the traffic matrix)
- Tradeoffs between routing overhead and topology accuracy
- Extend the results to multiple hierarchies
- Dynamic cluster formation and elimination

Questions

Backup slides

Memory Requirement



Key Aspects of Hierarchical Routing

- Communication
  - Intra-cluster
  - Inter-cluster

- Maintain the topology
  - Cluster head maintains the detailed connectivity relationships (local detailed topology) within a group of nodes
  - Cluster head also maintains aggregate global information (global ownership topology) for routing beyond the limits of the cluster (i.e. inter-cluster routing)
  - Regular node maintains a path (shortest) to its cluster head

Local Detailed Topology

$$I_D^C = \frac{n(n-1)}{2} \qquad (1)$$

$$I_D^P = \frac{n(n-1)}{2}(-p_1 \log p_1 - (1-p_1)\log(1-p_1)) \qquad (2)$$
$$= \frac{n(n-1)}{2} H(p_1)$$

$$I_D^M = \frac{n(n-1)}{2}((1-p_1)H(p_{00}) + p_1 H(p_{11})) = \frac{n(n-1)}{2} i_D^M \qquad (3)$$

p_1 – steady state probability that a direct link between two arbitrary nodes exists

Mobility and Topology Changes

- Mobility induces topology changes
  - Ownership change (nodes move between clusters) → 1) Update local ownership topology; 2) Update global ownership topology
  - Connectivity change (links up/down) → 1) Update local detailed topology; 2) Possible need to update shortest path from a regular node to its cluster head