Automated Support for the Investigation of Paraconsistent and Other Logics

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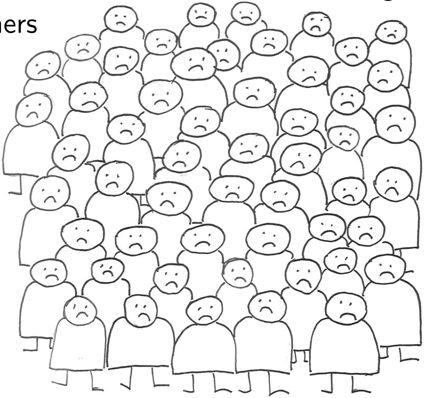
Joint Work with: A. Ciabattoni, O. Lahav and A. Zamansky

Software Engineers

Philosophers



Logicians

Algebraists

Why non-classical logics?

- Non-classical logics are logics different from classical logic
- Examples: intuitionistic logic, modal logics, paraconsistent logics, ...
- Various applications, e.g. in computer science: knowledge representation, automated reasoning, program synthesis, formal verification, ...

Investigating non-classical logics

The usefulness of logics depends on two essential components:

(i) a corresponding *analytic* calculus, i.e. where proof search proceeds by step-wise decomposition of the formulas

- e.g., prerequisite for the development of automated reasoning methods

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The usefulness of logics depends on two essential components:

(i) a corresponding *analytic* calculus, i.e. where proof search proceeds by step-wise decomposition of the formulas

- e.g., prerequisite for the development of automated reasoning methods

(ii) an *intuitive* semantics that provides insight into the logic and allows to reason about specific properties

State of the art

How to find an analytic calculus and useful semantics for a logic?

- Tailored to a specific logic
  $\Rightarrow$ Similar procedures have to be carried out often
  $\Rightarrow$ Error-prone task
  $\Rightarrow$ (Too?) Many papers are written on similar results
  $\Rightarrow$ Many open problems
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$\Rightarrow$ Uniform procedure and automated support are desirable!

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- Applications in CS: integration of information from multiple sources, negotiations among agents with conflicting goals, . . .

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- Applications in CS: integration of information from multiple sources, negotiations among agents with conflicting goals, ...
- Within classical logic, inconsistency leads to trivialization of the knowledge base as everything becomes derivable:

$$A, \neg A \vdash B$$

- *Paraconsistent logic* is a logic which allows "contradictory" but non-trivial theories.
- E.g., reflect consistency by a unary operator $\circ$ with the intuitive meaning $\circ A$: "A is consistent".

$$A, \neg A \nvdash B \qquad \Rightarrow \qquad A, \neg A, \circ A \vdash B$$

Examples: C-Systems

Positive fragment of classical propositional logic CL^+ extended with

$(\mathbf{n_1})$	$\psi \vee \neg\psi$	$(\mathbf{n_2})$	$\psi \supset (\neg\psi \supset \varphi)$
$(\mathbf{c})$	$\neg\neg\psi \supset \psi$	$(\mathbf{e})$	$\psi \supset \neg\neg\psi$
$(\mathbf{n}^{\mathbf{l}}_{\wedge})$	$\neg(\psi \wedge \varphi) \supset (\neg\psi \vee \neg\varphi)$	$(\mathbf{n}^{\mathbf{r}}_{\wedge})$	$(\neg\psi \vee \neg\varphi) \supset \neg(\psi \wedge \varphi)$
$(\mathbf{n}^{\mathbf{l}}_{\vee})$	$\neg(\psi \vee \varphi) \supset (\neg\psi \wedge \neg\varphi)$	$(\mathbf{n}^{\mathbf{r}}_{\vee})$	$(\neg\psi \wedge \neg\varphi) \supset \neg(\psi \vee \varphi)$
$(\mathbf{n}^{\mathbf{l}}_{\supset})$	$\neg(\psi \supset \varphi) \supset (\psi \wedge \neg\varphi)$	$(\mathbf{n}^{\mathbf{r}}_{\supset})$	$(\psi \wedge \neg\varphi) \supset \neg(\psi \supset \varphi)$
$(\mathbf{b})$	$\psi \supset (\neg\psi \supset (\circ\psi \supset \varphi))$	$(\mathbf{r}_{\diamond})$	$\circ(\psi \diamond \varphi) \supset (\circ\psi \vee \circ\varphi)$
$(\mathbf{k})$	$\circ\psi \vee (\psi \wedge \neg\psi)$	$(\mathbf{i})$	$\neg\circ\psi \supset (\psi \wedge \neg\psi)$
$(\mathbf{o}^{\mathbf{1}}_{\diamond})$	$\circ\psi \supset \circ(\psi \diamond \varphi)$	$(\mathbf{o}^{\mathbf{2}}_{\diamond})$	$\circ\varphi \supset \circ(\psi \diamond \varphi)$
$(\mathbf{a}_{\diamond})$	$(\circ\psi \wedge \circ\varphi) \supset \circ(\psi \diamond \varphi)$	$(\mathbf{a}_{\neg})$	$\circ\psi \supset \circ\neg\psi$

E.g., **BK**: $CL^+ + (n_1) + (b) + (k)$

A modular approach for C-Systems

A. Avron, B. Konikowska, and A. Zamansky, LICS 2012:

Starting from a Hilbert system:

1. Extract semantics
   $\Rightarrow$ needs ingenuity!
2. Use the semantics to generate a corresponding sequent calculus

Can we do more and in an automated way?

An automated procedure for ...

Axioms containing either ...

- **one** occurrence of $\star(\psi \diamond \varphi)$
  (and possible occurrences of $\star\psi, \star\varphi$ and $\psi, \varphi, \ldots$), or
- **one** occurrence of $\star\star\psi$
  (and possible occurrences of $\star\psi$ and $\psi, \varphi, \ldots$), or
- **at least one** occurrence of $\star\psi$
  (and possible occurrences of $\star\psi$ and $\psi, \varphi, \ldots$)

with $\star = \star_1 \mid \ldots \mid \star_n$ new unary connectives, and $\diamond = \wedge, \vee, \supset$.

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$(\mathbf{c})$	$\neg\neg\psi \supset \psi$	$(\mathbf{e})$	$\psi \supset \neg\neg\psi$
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$(\mathbf{a}_{\diamond})$	$(\circ\psi \wedge \circ\varphi) \supset \circ(\psi \diamond \varphi)$	$(\mathbf{a}_{\neg})$	$\circ\psi \supset \circ\neg\psi$

. . . and infinitely many more axioms . . .

Automated support for paraconsistent and other logics

A. Ciabattoni, O. Lahav, LS and A. Zamansky, LFCS 2013:

Starting from a Hilbert system:

1. Generate a corresponding sequent calculus
   - Transform Hilbert axioms (containing new unary connectives) into logical rules
   - Add the logical rules to the base calculus LK^+
2. Extract semantics from the generated calculus and use them to reason about the calculus

From axioms to logical rules: LK^+

$$\psi \Rightarrow \psi$$

$$\frac{\psi, \varphi, \Gamma \Rightarrow \Delta}{\psi \wedge \varphi, \Gamma \Rightarrow \Delta}\ (\wedge, l) \qquad \frac{\Gamma \Rightarrow \Delta, \psi \quad \Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \psi \wedge \varphi}\ (\wedge, r)$$

$$\frac{\psi, \Gamma \Rightarrow \Delta \quad \varphi, \Gamma \Rightarrow \Delta}{\psi \vee \varphi, \Gamma \Rightarrow \Delta}\ (\vee, l) \qquad \frac{\Gamma \Rightarrow \Delta, \psi, \varphi}{\Gamma \Rightarrow \Delta, \psi \vee \varphi}\ (\vee, r)$$

$$\frac{\Gamma \Rightarrow \Delta, \psi \quad \varphi, \Gamma \Rightarrow \Delta}{\psi \supset \varphi, \Gamma \Rightarrow \Delta}\ (\supset, l) \qquad \frac{\psi, \Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \psi \supset \varphi}\ (\supset, r)$$

$$\frac{\Gamma \Rightarrow \Delta, \psi \quad \psi, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ (cut)$$

$+$ structural rules

From axioms to logical rules: A transformation procedure

Transformation procedure based on 2 key concepts:
(A. Ciabattoni, N. Galatos, and K. Terui, LICS 2008)

1. Invertibility of the (logical) rules of LK^+: $\wedge, \vee, \supset$
2. Ackermann Lemma
   - allows the formula to "jump" from one side of the sequent to the other side in the premises

$$A \Rightarrow B \qquad \text{equivalent to} \qquad \frac{B, \Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta}$$

$$A \Rightarrow B \qquad \text{equivalent to} \qquad \frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, B}$$

From axioms to logical rules: Example

Example

Axiom ($n^l_\wedge$)	$\neg(\psi \wedge \varphi) \supset (\neg\psi \vee \neg\varphi)$
Invertibility	$\dfrac{}{\Rightarrow \neg(\psi \wedge \varphi) \supset (\neg\psi \vee \neg\varphi)}$
Invertibility	$\dfrac{}{\neg(\psi \wedge \varphi) \Rightarrow \neg\psi \vee \neg\varphi}$
Ackermann Lemma	$\dfrac{}{\neg(\psi \wedge \varphi) \Rightarrow \neg\psi, \neg\varphi}$
Equivalent logical rule	$\dfrac{\Gamma, \neg\psi \Rightarrow \Delta \qquad \Gamma, \neg\varphi \Rightarrow \Delta}{\Gamma, \neg(\psi \wedge \varphi) \Rightarrow \Delta}$

From axioms to logical rules: Generated rules

Type 1: $\dfrac{\mathcal{Q}}{\Gamma, \star(\psi \diamond \varphi) \Rightarrow \Delta}$ $\dfrac{\mathcal{Q}}{\Gamma \Rightarrow \star(\psi \diamond \varphi), \Delta}$

Type 2: $\dfrac{\mathcal{P}}{\Gamma, \star\star\,\psi \Rightarrow \Delta}$ $\dfrac{\mathcal{P}}{\Gamma \Rightarrow \star\star\,\psi, \Delta}$

Type 3: $\dfrac{\mathcal{P}}{\Gamma, \star\psi \Rightarrow \Delta}$ $\dfrac{\mathcal{P}}{\Gamma \Rightarrow \star\psi, \Delta}$

- Premises $\mathcal{Q}$ contain (a subset of) $\{\psi, \star\psi, \varphi, \star\varphi\}$
- Premises $\mathcal{P}$ contain (a subset of) $\{\psi, \star\psi\}$

Properties of the generated calculi

- Adding *logical* rules: analytic calculus?
- $\Rightarrow$ "global" check is needed

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Definition (M. Baaz, O. Lahav and A. Zamansky, 2012)

A partial non-deterministic matrix (PNmatrix) $\mathcal{M}$ consists of:

(i) a set $\mathcal{V}_{\mathcal{M}}$ of truth values,

(ii) a subset of $\mathcal{V}_{\mathcal{M}}$ of designated truth values, and

(iii) a truth-table $\diamond_{\mathcal{M}} : \mathcal{V}^n_{\mathcal{M}} \to P(\mathcal{V}_{\mathcal{M}})$ for every n-ary connective $\diamond$.

PNmatrices generalise the notion of non-deterministic matrices (A. Avron and I. Lev, 2001) by allowing *empty sets* in the truth tables (i.e., $P(\mathcal{V}_{\mathcal{M}})$ instead of $P(\mathcal{V}_{\mathcal{M}})\backslash\emptyset$).

Extracting semantics: PNmatrices

- Truth values: tuples with size $= \#$ of unary connectives $+1$
- New rules reduce the level of non-determinism

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Type 2: $\dfrac{\mathcal{P}}{\Gamma, \star_i \star_j \psi \Rightarrow \Delta}$ determine truth tables for $\star_j$

Type 1: $\dfrac{\mathcal{Q}}{\Gamma, \star(\psi \diamond \varphi) \Rightarrow \Delta}$ determine truth tables for $\diamond$

Example: PNmatrices

- CL^+ with one new unary connective $\neg$:
  $\langle x, y\rangle$ where $x = 1$ iff ψ is "true", and $y = 1$ iff $\neg\psi$ is "true":

$$\mathcal{V}_{\mathcal{M}} := \{\langle 0,0\rangle, \langle 0,1\rangle, \langle 1,0\rangle, \langle 1,1\rangle\}$$

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- Rules introducing unary connectives reduce $\mathcal{V}_{\mathcal{M}}$, e.g.:

$$\frac{\Gamma, \psi \Rightarrow \Delta}{\Gamma \Rightarrow \neg\psi, \Delta}$$

$$\mathcal{V}_{\mathcal{M}} := \{\langle 0, 0 \rangle, \langle 0, 1 \rangle, \langle 1, 0 \rangle, \langle 1, 1 \rangle\}$$

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$\neg$	
$\langle 0, 1\rangle$	$\{\langle 1, 0\rangle, \langle 1, 1\rangle\}$
$\langle 1, 0\rangle$	$\{\langle 0, 1\rangle\}$
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Semantics at work

Theorem

All our logics are decidable.

Proof.

From the generated calculi we can extract a finite PNmatrix. $\square$

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The calculus is "analytic" iff the corresponding PNmatrix does not have empty sets in the truth tables of the connectives.

Semantics at work

Theorem

All our logics are decidable.

Proof.

From the generated calculi we can extract a finite PNmatrix. □

Theorem

The calculus is "analytic" iff the corresponding PNmatrix does not have empty sets in the truth tables of the connectives.

What to do if there is an empty set?

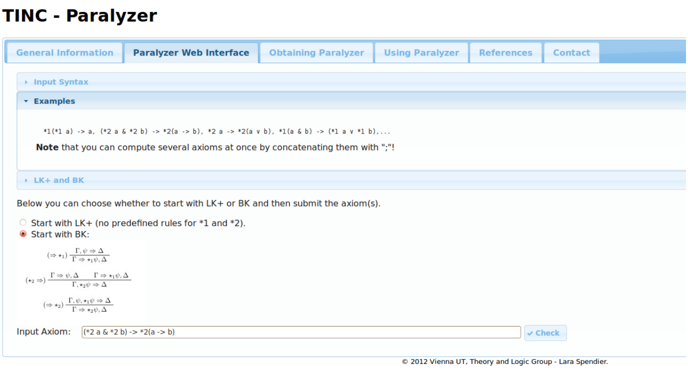
- Transform the PNmatrix into a *finite family* of Nmatrices
- Apply the method in (A. Avron J. Ben-Naim, and B. Konikowska, 2006) and produce a *family* of cut-free sequent calculi.

But ...

do you really want to do all of this by hand?

Our system: Paralyzer

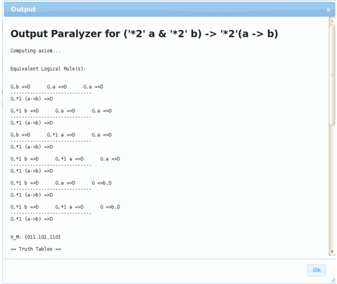
- PARAconsistent logic anaLYZER
- http://www.logic.at/staff/lara/tinc/webparalyzer/paralyzer.html
- Input: Axioms according to the grammar



Our system: Paralyzer

Output:

- Syntax: sequent calculus for the logic
- Semantics: truth tables (using PNmatrices)



An extension: An automated procedure for ...

(work in progress)

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- **one** occurrence of $\bar{\bullet}(\psi \diamond \varphi)$
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- **one** occurrence of $\bar{\bullet} \star \psi$
  (and possible occurrences of $\bar{\bullet}\psi$ and $\psi, \varphi, \ldots$), or
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with $\star = \star_1 \mid \ldots \mid \star_n$ new unary connectives, $\bar{\bullet}$ any finite concatenation of unary connectives and $\diamond = \wedge, \vee, \supset$.

Examples: L8

Positive fragment of classical propositional logic CL^+ extended with ($\diamond \in \{\vee, \wedge, \supset\}$):

$(\mathbf{c_1})$	$\neg^2\neg\psi \supset \neg\psi$	$(\mathbf{e_1})$	$\neg\psi \supset \neg^2\neg\psi$
$(\mathbf{e}^{\mathbf{l}}_{\diamond_2})$	$\neg^2(\psi \diamond \varphi) \supset (\neg^2\psi \diamond \neg^2\varphi)$	$(\mathbf{e}^{\mathbf{r}}_{\diamond_2})$	$(\neg^2\psi \diamond \neg^2\varphi) \supset \neg^2(\psi \diamond \varphi)$
$(\mathbf{o}^{\mathbf{l}}_{\wedge_1})$	$\neg(\psi \supset \varphi) \supset (\psi \wedge \neg\varphi)$	$(\mathbf{o}^{\mathbf{r}}_{\wedge_1})$	$(\psi \wedge \neg\varphi) \supset \neg(\psi \supset \varphi)$
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$(\mathbf{o}^{\mathbf{l}}_{\supset_1})$	$\neg(\psi \vee \varphi) \supset (\neg\psi \wedge \neg\varphi)$	$(\mathbf{o}^{\mathbf{r}}_{\supset_1})$	$(\neg\psi \wedge \neg\varphi) \supset \neg(\psi \vee \varphi)$

And similar axioms for **L16**, **L32**, $\ldots$ **L**2^{n+2} (N. Kamide)

An extension: From axioms to logical rules

Same algorithm:

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An extension: Extracting PNmatrices

Similar extraction:

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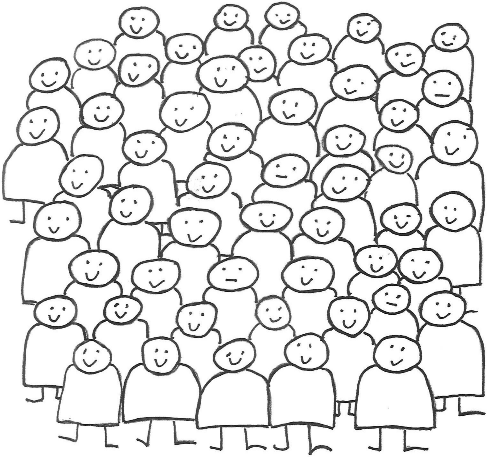
- **More** truth values: tuples with size $=$ concatenations of unary connectives $+1$

Summary

We subsume many existing results:

- **Analytic** calculi and **intuitive** semantics for many logics
- Tool (Paralyzer) for the **automated** generation of syntax and semantics



Some open questions

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- Extension of the grammar and the procedure

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