LOAD SHUFFLING AND TRAVEL TIME ANALYSIS OF A MINILOAD AUTOMATED STORAGE AND RETRIEVAL SYSTEM WITH AN OPEN-RACK STRUCTURE

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Abstract
Automated storage and retrieval systems (AS/RSs) are major material handling systems, which have been widely used in distribution centers and automated productions. This paper studies the characteristics of a miniload AS/RS with an open-rack structure and unidirectional-upward mobile loads within the rack. In this warehousing system, the stacker crane is used for the retrieval operations and shuffling procedures, while the storage operations are carried out by separate devices namely, storage platforms. The proposed AS/RS has one storage platform for each rack to unload several loads at the same time into the rack. The stacker crane shuffling processes are used to provide more available locations for increasing the storage capacity of the storage platform. Heuristic algorithms and models are developed for load shuffling processes and travel time of the storage platform. Subsequently, travel time of the proposed AS/RS is analyzed under randomized storage policy and by using Monte Carlo simulation. The results show that the proposed models and algorithms are reliable for the design and analysis of this kind of AS/RS.

Keywords: Automated storage and retrieval systems (AS/RS); Open-rack structure; Load shuffling; Travel time; Monte Carlo simulation

1. INTRODUCTION
Automated storage and retrieval systems (AS/RSs) are computer-controlled storage systems that can automatically store and retrieve the loads [1]. The basic components of the AS/RS are storage racks, stacker cranes (storage/retrieval or S/R machines), input/output (I/O) stations, and interface conveyors [2]. Miniload AS/RS is used to handle small loads (individual parts or supplies) that are contained in small containers, bins or drawers in the storage system [3]. In conventional AS/RSs, stacker cranes are used to store and retrieve loads into or from the storage cells. Performance of a conventional AS/RS can be enhanced when the ratios of storage and retrieval operations are approximately equally distributed and in this case, a single-shuttle stacker crane can operate up to dual command cycle. For an AS/RS, the throughput performance (i.e., the number of storages or retrievals performed per period) can be increased by minimizing the system travel time which also results in reducing AS/RS operating costs [4]. One strategy to increase the throughput performance of an AS/RS is to handle more loads at one time. In a balanced system, the ratios of storage and retrieval operations are equally distributed and therefore the inbound work-flow is equal to the outbound work-flow [5-6]. Under a balanced situation, for instance, a single-shuttle stacker crane can operate up to dual command cycle (i.e., one storage operation and one retrieval operation are performed in a cycle) [7]. It is important to recognize that the possibility of performing dual command cycles depends on the availability of both storage and retrieval requests [8]. However, considering dynamic nature and realistic operating characteristic of an AS/RS, the ratios of storage and retrieval operations are not distributed equally during certain time slots, and the system operates under an unbalanced situation [6]. A perfectly balanced system is a very idealized situation which is unlikely to occur in real storage systems [5]. Another strategy which can result in minimizing the AS/RS travel time and consequently increasing its throughput performance is to use the load shuffling procedures. An AS/RS needs to store and retrieve loads in the shortest possible time period. In order to retrieve loads as quickly as possible, a solution is to shuffle (pre-sort/rearrange) the loads to specified locations to minimize the response time of retrieval. However, very little information about load shuffling can be found in the literature [9].
2. OPEN-RACK SYSTEM FOR MINILOAD AS/RS

In this paper, the open-rack AS/RS with unidirectional-upward mobile loads within the rack (Figure 1) is analyzed, in which the stacker crane is only used for the retrieval operations and the storage operations are carried out by separate devices, namely, storage platforms (see Vasili et al., [10] for details). Under such circumstances, the system is enabled for handling several incoming items at the same time (under both balanced and storage-oriented unbalanced situations), thereby increasing the system performance.

![Illustrations](image)

Figure 1: Illustrations of (a): Open-Rack AS/RS and (b): Open-Rack structure

2.1. LOAD SHUFFLING IN OPEN-RACK

In this section the following notations are used:

- \( N_l, N_b \) number of levels and bays of an open-rack AS/RS, respectively
- \( N \) total number of cells in the rack \((N_l \times N_b)\)
- \( SP \) storage platform
- \( T_p, V_p \) travel time and average speed of storage platform, respectively
- \( M_p \) movement or the distance traversed by the storage platform
- \( \nu, \nu_h \) speed of stacker crane for vertical and horizontal movement, respectively
- \( VL, HL \) height and length of the rack, respectively
- \( H_h, H_s \) height of handover station and standard containers, respectively
- \( T_v \) the time to reach the top of the rack vertically
- \( T_h \) the time to reach the end of the rack horizontally
- \( d, L_c \) spaces between standard containers and width of bays, respectively
- \( \delta, \rho \) ratio for storage operations and shape factor, respectively
- \( \alpha, b \) ratio for storage and shape factor, respectively

Two generic examples of loads shuffling in open-rack structure are illustrated in Figure 2. Consider four loads are available to be stored. Based on the arrangement of the loads in Figure 2a, since there are empty storage locations in all four bays, the SP beneath the bays is loaded with four containers. In the subsequent step, the SP unloads the containers into the bays in the rack (Figure 2b). However, based on the arrangement of the loads in Figure 2c, the bay 4 is fully occupied and for the storage of four loads into the rack two runs of storage platform are required, which results in higher operation time. In this case, in order to increase the storage capacity of SP the following method can be used. Since the stacker crane and conveyor can operate simultaneously, while the conveyor releasing the four loads on the SP, the stacker crane moves one of the loads from bay 4 to the either bay 2 or bay 3 which have more than one empty location (Figure 2c). Therefore, it can be observed than the shuffling operation of one loads by the stacker...
crane, will cause to the simultaneous transferring of four loads into the handover station. In the subsequent step, the SP stores these four loads into the rack in one run (Figure. 2d). In order to obtain the upper bound and lower bound of the distance traversed by SP, consider that there is an empty cell in level \( i \) of a particular bay. Figure 3 illustrates different steps of the load shuffling in one bay of the open rack AS/RS. Considering Figure 3, lower bound and upper bound values of \( M_p \) for each load in a bay to be transferred to its immediate upper neighbor locations are,

$$ \text{Lower Bound } M_p = H_h + (N_t - 1)d + (H_a + \delta) \quad (1) $$

$$ \text{Upper Bound } M_p = H_h + (H_a + d) + (H_a - \delta), \quad (2) $$

3. TRAVEL TIME ANALYSIS FOR THE OPEN-RACK AS/RS

3.1. ASSUMPTIONS

1) The stacker crane operates on single command basis. The dwell-point positions for the stacker crane and the SP are the output station and the lowest point of the handover station, respectively;
2) There are no technical problems for the construction of the proposed open-rack AS/RS;
3) The output station is located at the lower left-hand corner of the rack. The rack input point is handover station, which is the lowest level of the rack.
4) The accelerations and decelerations of the stacker crane and SPs and the load transfer times can be ignored without affecting the relative performance of the control policies;
5) Randomized storage assignment is used. All the requests are served on FCFS basis.
6) There are no concurrent movements of the stacker crane and SP for different requests.

3.2. ANALYSIS

The objective is to shuffle the loads and at the same time minimize the travel time of storage operation. Therefore, the lower bound \( M_p \) is used to obtain the total travel time of the SP for the storage operation,

$$ T_p = \frac{2}{V_p}[H_h + (N_t - 1)d + (H_a + \delta)]. \quad (4) $$

As different expressions must be used to obtain the expected travel time for a storage operation and a retrieval operation, it is necessary to distinguish the operation type in order to obtain the formula to describe the expected travel time [11]. The formula is,

$$ E[T] = P(s)E[T_s] + P(r)E[T_r], \quad (5) $$

where \( T, T_s \) and \( T_r \) are random variables. \( T \) is the cycle time for the stacker crane and SP to complete an operation. \( T_s \) indicates the time spent if the current job is storage, while \( T_r \) is the time used for a retrieval operation. Obviously, \( E[T] \) denotes the expected travel time for one operation. \( E[T_s] \) gives the expected travel time if the current job is a storage operation and \( P(s) \) is the probability of the current job to be a storage operation. \( E[T_r] \) and \( P(r) \) are similarly defined for the case of retrieval. By definition, \( P(r) = 1 - P(s) \). Assume that the ratio for storage operations is \( \alpha \) in an arbitrary finite job sequence thus,

$$ P(s) = \alpha \quad \text{and} \quad P(r) = 1 - P(s) = 1 - \alpha \quad \text{and} \quad E[T] = \alpha.E[T_s] + (1 - \alpha).E[T_r]. \quad (6) $$
Under randomized storage assignment, the probability of accessing any cell is identical. Let $N$ denotes the total number of cells in the rack. Therefore, assuming $t_\text{s}$ as a random variable to indicate the two-way travel time between the output station and a randomly chosen location in the rack and using the Chebychev travel time (i.e., the travel time of the stacker crane is the maximum of the isolated horizontal and vertical travel times), the stacker crane mean or expected retrieval time can be expressed,

$$E[t_{r,i}] = \frac{2}{N} \sum_i \sum_j \max \left[ \frac{T_h}{N_b}, i, \frac{T_v}{N_i} (j - 1) \right],$$

where $1 \leq i \leq N_b$, $1 \leq j \leq N_i$.

Note that, the SP performs the storage operation when it achieves a predefined number of loads, which is referred to as “storage batch size”. Let $\rho$ represent the size of this batch, where $1 \leq \rho \leq N_b$. Since the batch storage is used, in order to avoid the waiting time in the storage platform, the following method is used. If there is any request for the retrieval of a specific item which is inside the handover station, the storage platform should perform the storage operation immediately, regardless of achieving to the full $\rho$. An illustration of required movements for an immediate storage operation is presented in Figure 4. Same color arrows in Figure 4 represent concurrent movements. Therefore, there are two types of retrieval operation in the open-rack AS/RS. The first is a normal retrieval operation, where the stacker crane retrieves a load from inside the rack. The second is retrieval operation of a load which is inside handover station and consequently it contains an immediate storage operation. Hence, based on Eqs. (4) and (7), Eq. (8) represents the expected retrieval time of open-rack AS/RS under randomized storage assignment.

$$E[T_r] = f.E[t_{r,i}] + (1-f).E[\max(t_{s,r}, t_{p}) + t_{l,r}]$$

where $t_{s,r}$ is a random variable which denotes the one-way travel time between the output station and a random location in the level 1 of the rack. $f$ indicates the probability that a request from inside the rack takes place and $(1-f)$ represents the probability that a request from inside the handover station takes place. In the case of storage, as mentioned earlier, the operation of the conveyor in the open-rack AS/RS (Figure 5), to some extent is similar to the carousel storage systems. According to Groover [3] the circumference of conveyor rail ($C$ ) is given by following expression, where, $L$ is the length of the conveyor; $H$ is height of the conveyor; $L_{cart}$ is the length of each cart and $d_c$ is the space between carts.

$$C = 2(L - H) + \pi H$$

Also

$$L_c = L_{cart} + d_c$$

$$\pi H = 2.L_c$$

Unlike the retrieval operation, for the storage operation the system enable to handle more than one load per cycle. Hence, depending on the number of loads to be stored, the conveyor handles $\rho_1$ loads in each cycle. Let $C_b$ denotes the distance traversed by all the loads on the conveyor, up to the point where all carts have released their loads, and $t_{c,b}$ represents the time spent for this traversing. The measurement of the distance for obtaining $C_b$ is started from input station and finished at the point where all the carts have met all the bays. Assuming $v_c$ as the speed of the conveyor (where acceleration/deceleration effects are ignored) hence,

$$C_b = N_b, L_c + (\rho_1 - 1), L_c = L_c(N_b + \rho_1 - 1), \quad \text{and} \quad t_{c,b} = L_c/v_c(N_b + \rho_1 - 1).$$

Meanwhile, the stacker crane expected travel time for performing $Z$ shuffling operation sequentially is equal to the expected time for a single-command cycle and $(2Z - 1)$ travel-between times. Therefore,

$$E[T_{sa}] = E[SC] + (2Z - 1)E[TB].$$

where $E[SC]$ is the expected single command cycle travel time, $E[TB]$ is the expected travel time between two randomly selected locations (see Bozer and White [12]).
4. SIMULATION STUDY

In this study, for the simulations Monte Carlo simulation method is used under MATLAB®, MICROSOFT EXCEL 2007 and Monte Carlo Add-In for EXCEL. The simulation uses randomized number generators for \( i \) and \( j \) to choose a new destination for each new operation. Then the Chebychev travel time is used (as in Eqs. 7 and 8) to obtain the stacker crane retrieval operation time for this randomized destination. The same method is used for the case of storage operations to obtain the stacker crane shuffling operations time and then the response time for the storage operation of batches of loads are calculated (as in Eqs. 4, 9 and 8). For each operation, the probability that the preceding operation is a storage is set to be \( \alpha \) and \((1 - \alpha)\) for retrieval operation. A series of 100,000 jobs (which is considerably large compared with the number of cells in an AS/RS rack) is executed in each experiment to simulate the infinite batch of jobs. Finally, the travel time of system is calculated through obtaining the average of all simulated results (as in Eq. 5). The specifications which are used are such that \( r_{\text{max}} = 0.65 \text{ m}; \ r_{\text{min}} = 0.35 \text{ m}; \ L_c = 0.48; \ H_a = 0.05 \text{ m}; \ d = 0.01 \text{ m}; \ \delta = 0.01 \text{ m}; \ \nu_v = 0.50 \text{ m/s}; \ h_v = 1.00 \text{ m/s}; \ v_c = 1 \text{ m/s}; \) the total number of cells in the rack \((N_t \times N_b)\) is 600; \( \rho \) is equal to \( N_b \) and \( V_p = 0.04 \text{ m/s}. \) partial of the results are shown in Table 1.

Table 1: Simulation Results for expected travel time of the system when \( \alpha = 0.5 \)

<table>
<thead>
<tr>
<th>No. of tiers</th>
<th>No. of bays</th>
<th>Cells in rack</th>
<th>Shape factor, ( b )</th>
<th>Travel time (Sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30</td>
<td>600</td>
<td>1.0000</td>
<td>12.78</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
<td>600</td>
<td>0.5625</td>
<td>13.85</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
<td>600</td>
<td>0.3600</td>
<td>15.97</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>600</td>
<td>0.2500</td>
<td>18.37</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>600</td>
<td>0.1600</td>
<td>22.20</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>600</td>
<td>0.0900</td>
<td>28.68</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>600</td>
<td>0.0625</td>
<td>33.98</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>600</td>
<td>0.0400</td>
<td>41.89</td>
</tr>
</tbody>
</table>

Figures 6 and 7 show the influences of shape factor \( b \) and \( \alpha \) on the expected travel time and throughput performance of the system, respectively. The throughput performance of the system is defined here as the reciprocal of the average travel time for the system to handle a job [11]. From Figures 6 and considering the general trend of travel time results, it can be observed that, the travel time increases as the shape factor becomes smaller. The reason for the increase in travel time is due to the increase in the response time of the retrieval operations. In other words, the shape factor has inverse relationship with the average retrieval time and therefore the average retrieval time increases as the shape factor becomes smaller. However, in the open-rack AS/RS, by design, the average storage time is rather small compared with the average retrieval time. Clearly, the average storage time is not a significant factor affecting the trend of average retrieval time through different shape factors, and the travel time is mainly influenced by the average retrieval time as the shape factor becomes smaller. Similarly, all the variations of throughput performance results through different values of \( b \) and \( \alpha \) (Figure 7) can be justified based on the inverse relationship between the throughput performance and the expected travel time.

5. CONCLUSION

In this study, an open-rack AS/RS with unidirectional-upward mobile loads within the rack was investigated. Using this mechanism, the average handling time for a batch of jobs can be greatly reduced. The advantages of this AS/RS include high throughput, more flexible AS/RS rack configuration and high fault tolerance. However, the application of this mechanism to the storage of heavy products may be limited. Heuristic algorithms and models were developed for load shuffling and travel time of the storage platform, respectively, and the expected travel time of the proposed AS/RS was analyzed under randomized storage assignment and by using Monte Carlo simulation. The results indicate that the proposed models and algorithms are reliable for the design and analysis of this kind of AS/RS. Some recommendations for further studies to expose the potentials of the open-rack AS/RS are to study the policies for request sequencing and the policies for storage assignment under multiple platforms.
REFERENCES


