Quaternion Fourier transform based alpha-rooting method for color image measurement and enhancement

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A R T I C L E   I N F O

Article history:
Received 25 August 2014
Received in revised form 31 October 2014
Accepted 25 November 2014
Available online 3 December 2014

Keywords:
Image enhancement
Histogram equalization
Alpha-rooting
Discrete quaternion Fourier transform

A B S T R A C T

The aim of this paper is to present a transform-based framework for color image enhancement which processes all three color components (R,G,B) simultaneously. In this framework, the image enhancement procedure is supported by a two-dimensional discrete quaternion Fourier transform and the new multi-frequency band $\alpha$-rooting concept. Moreover, we introduce a simple measure to evaluate the quality of enhanced color images. Additionally, we demonstrate the performance of the proposed algorithms against recently proposed methods which separately process color channels. Preliminary results show that the presented algorithm out-performs other enhancement methods such as the traditional Fourier transform-based alpha-rooting and the multi-scale Retinex algorithms. The proposed algorithms are fast and simple in implementation and design which makes them very practical for image enhancement.

Published by Elsevier B.V.

1. Introduction

The goal of image enhancement techniques is to improve characteristics and quality of an image in such a way that the resulting image will look better than the original image when compared against a specific criteria [1–6]. Image enhancement plays a crucial role in image processing applications such as digital photography, medical image analysis, computer vision, remote sensing, object recognition, optical character recognition, fingerprint recognition, industrial automation, face recognition, and scientific visualization [4–17].

Recently, several image enhancement algorithms have been exploited [1–64]. They can be categorized into two key classes: spatial-domain processing and transform-domain processing [1,2,29–47]. The spatial-domain methods directly operate on pixels, and most representative of those methods are based on gray-level histogram modifications [1,14,15,27], histogram equalization [28–30], on human visual systems [1–9], on unsharp masking [3,8], on ratio image methods [9,10], on fuzzy entropy [11], on empirical decomposition based methods [12,13], partitioned iterated function systems [38], linear filters [39], and others [40–43,50]. We separately mention the class of complex Retinex methods [60–64], which uses filtering with different Gaussian kernels and additional post-processing stages for adjusting colors. The transform-domain methods directly operate on transforms of the image such as the Fourier, wavelet, and cosine transforms by analyzing and modifying the spectral coefficients of the transforms. The most popular transform-based techniques can be classified as (a) $\alpha$-rooting methods [10,16,22,23,25,36,47–49], and (b) transform histogram matching methods [31–33]. Each of them has its own advantages and limitations. For example, the advantages of frequency domain processing techniques are (1) low complexity of computations and (2) easy viewing and manipulation of the frequency composition of the image without direct reliance on spatial
Motivations of this paper are the following: (a) contrast perception keeps constant at very high luminance and/or low spatial frequency, where this independence of luminance is called Weber’s Law \([1,2,35]\); (b) contrast perception and the color saturation increase with luminance (Hunt’s effect \([36,37]\) and Steven’s effect \([45,49]\)). This is generally true, except at very high luminance or very low frequency. Furthermore, (c) at low spatial frequency, contrast sensitivity increases linearly with frequency \([44]\), and (d) it is natural to process all three color components (R, G, B) simultaneously which can help us to capture the inherent correlation between the components, and which may not generate color artifacts or blending. The final motivation is the development of a simple image enhancement metric. The formulation of image quality is the main factor when choosing the best algorithm or parameters for enhancement of color images. The basic limitation of developing enhancement algorithms is in finding a simple quantitative measure of image quality \([36–44,47–49]\). Therefore, an important step in direct image enhancement approach is to create a suitable image enhancement measure. The improvement found in the resulting images after enhancement is often very difficult to measure. This problem becomes more apparent when the enhancement algorithms are parametric and one needs (a) to choose the best parameters; (b) to choose the best transform among a class of unitary transforms; (c) to automate the image enhancement procedure.

In this paper, the following strategies have been used to solve the above-mentioned color image enhancement problems: we combine quaternion tools including the Fourier transform properties and alpha-rooting concepts, to develop the quaternion multi-frequency band alpha-rooting method. The expected advantages of the proposed method are the following: (1) the three color components in the (R,G,B) are simultaneously processed, (2) the visibility of many details in dark areas and in bright areas is improved without any iteration over previous approaches, and (3) it can be used universally in color image enhancement together with any transform-based techniques such as the DCT, Haar, wavelet and other transforms. Quaternion numbers are the generalization of complex numbers, and color images could be transformed using quaternion Fourier transforms \([53–55,69]\). The recently developed concepts of a quaternion Fourier transform, quaternion convolution, and quaternion correlation, which are based on quaternion algebra, have been found to be useful in video image processing \([56,67,68]\), in computer graphics \([66,69–71]\), in bioinformatics \([72–75]\), computer vision \([76,77]\), in navigation systems \([78]\), etc.

The rest of the paper is organized as follows: Section 2 introduces some preliminaries of the quaternion arithmetic and the two-dimensional discrete quaternion Fourier transform (2-D DQFT). Section 3 describes an image performance evaluation measure for gray-scale images and presents a new enhancement measure for color images. The main multi-frequency band alpha-rooting concept for color image enhancement is then described. Different examples of application of the proposed methods for enhancing color images are given in this section, and the performance of commonly used color image enhancement algorithms is compared with recently proposed algorithms. Examples of application of the 2-D DQFT based \(\alpha\)-rooting in the XYZ color model are also given. Finally, some conclusions are made in Section 4.

### 2. Background

In this section we present the arithmetic of quaternion numbers and describe the two-dimensional separable discrete quaternion Fourier transform.

#### 2.1. Quaternion arithmetic

The concept of the quaternion was introduced by Hamilton in 1866 \([51]\). A quaternion number \(Q = (q_1, q_2, q_3, q_4)\) has a real part and three imaginary parts and it can be written as \(Q = q_1 + q_2\hat{i} + q_3\hat{j} + q_4\hat{k}\), where \(q_1, q_2, q_3, q_4\) are real numbers. Here, \(i, j, k\) and \(k\) are basic elements such that \(ij = -ji = k\), \(jk = -kj = i\), \(ki = -ik = j\), and \(i^2 = j^2 = k^2 = ijk = -1\).

A quaternion number has a real part and three imaginary parts and can be written as \(Q = q_1 + (q_2\hat{i} + q_3\hat{j} + q_4\hat{k})\). The number \(q_1\) is considered to be the real part of \(Q\) and its “imaginary” part is \((q_2\hat{i} + q_3\hat{j} + q_4\hat{k})\). The quaternion modulus of \(Q\) is defined as \(|Q|^2 = q_1^2 + q_2^2 + q_3^2 + q_4^2\), and the conjugate equals \(\overline{Q} = q_1 - (q_2\hat{i} + q_3\hat{j} + q_4\hat{k})\). Addition and multiplication of quaternion numbers are associative as in the familiar algebra. The multiplication is, however, not commutative due to the product rule of its basic elements. Thus, \(Q_1Q_2 \neq Q_2Q_1\) in quaternion arithmetic.

The quaternion can be used for three-entry or four-entry vector analysis \([34]\). In recent years, quaternions have been utilized increasingly in color image processing. When using quaternion based operations in color image processing, each color triple is treated as a whole unit, or a vector \([54,56]\). This is a main advantage of the quaternion-type representation in color imaging when comparing the traditional method of separately processing color channels of the image.

#### 2.2. 2-D Discrete quaternion fourier transform

The discrete quaternion Fourier transform treats each color pixel \(f(n, m)\) as a single pure quaternion number:

\[
f(n, m) = Q = 0 + R(n, m)\hat{i} + G(n, m)\hat{j} + B(n, m)\hat{k}.
\]

The three imaginary parts are referred to as the three main components, \(R(\hat{e}d)\), \(G(\hat{e}en)\), and \(B(\hat{e}ue)\), of the pixel in the RGB color space. The real part of such a quaternion number is zero. By using quaternion operations, higher color information accuracy can be achieved because a color is treated as an entity. The advantage of using quaternion based operations is that each color triple is treated as a whole unit, instead of processing each color channel independently.

The quaternion Fourier transform was first defined by El [52,53], and later many practical works related to the
2-D DQFT and its applications in color image processing were presented in [54–56]. Since the quaternion multiplication is not commutative, the definition of the 2-D DQFT is not unique. For a color image \( f_{n,m} \) of size \( N \times M \), the pair of two-sided direct and inverse DQFTs is defined as

\[
F_{p,s} = \sum_{n=0}^{N-1} M_{n=0}^{M-1} W_j f_{j,n} W_k m_s = \sum_{n=0}^{N-1} M_{n=0}^{M-1} f_{n,m} W_k m_s
\]

\[
f_{n,m} = \frac{1}{NM} \sum_{p=0}^{N-1} \sum_{s=0}^{M-1} W_j f_{p,s} W_k m_s
\]

where \( p, n = 0: (N–1) \) and \( m, s = 0: (M–1) \). The basis functions are defined by the exponential coefficients:

\[
W_j = \cos(2\pi j/N) - j \sin(2\pi j/N) \quad \text{and} \quad W_k = \cos(2\pi k/M) - k \sin(2\pi k/M)
\]

The components \( F_{p,s} \) of the transform are quaternion numbers with real and imaginary parts:

\[
F_{p,s} = (F_{p,s})_1 + (F_{p,s})_2 i + (F_{p,s})_3 j + (F_{p,s})_4 k
\]

Each multiplication \( Q = W_j f_{n,m} W_k m_s \) is calculated by

\[
Q = Q_x + iQ_y + jQ_z + kQ_w = [\cos(q) - j \sin(q)] [0 + ib + jc + kd] [\cos(y) - k \sin(y)]
\]

where the angles \( q = (2\pi)/Nnp \) and \( y = (2\pi/M)ms \). Here, the quaternion number \( 0 + ib + jc + kd \) stands for \( f_{n,m} \). In matrix form this multiplication is defined by

\[
\begin{pmatrix}
Q_x \\
Q_y \\
Q_z \\
Q_w
\end{pmatrix} = \begin{pmatrix}
\cos(q) & \sin(q) & \cos(y) & \sin(y) \\
\sin(q) & \cos(q) & \sin(y) & -\cos(y) \\
\cos(q) & -\sin(q) & \cos(y) & \sin(y) \\
\sin(q) & \cos(q) & \sin(y) & \cos(y)
\end{pmatrix} \begin{pmatrix}
b \\
c \\
d \\
e
\end{pmatrix}
\]

(3)

As an example, Fig. 1 shows a color image of size 256 × 256 in part (a). In part (b), the magnitude of the 2-D 256 × 256-point DQFT, \( |F_{p,s}| \), is shown, which is calculated by

\[
|F_{p,s}| = \sqrt{(F_{p,s})_1^2 + (F_{p,s})_2^2 + (F_{p,s})_3^2 + (F_{p,s})_4^2}
\]

The transform is periodic and its components have been shifted cyclicly by the vector \((128, 128)\) to the center of the lattice of frequency points \( X = (p, s); \ p, s = 0: 255\). For comparison, the magnitude of the 2-D 256 × 256-point DFT is shown in part (c) and it is defined as \( M_{p,s} = \sqrt{R_{p,s}^2 + G_{p,s}^2 + B_{p,s}^2} \), where \( R_{p,s}, G_{p,s}, \) and \( B_{p,s} \) are the 2-D DFTs of the red, green, and color components of the image, respectively. The point-wise ratio of these magnitudes, \( |F_{p,s}|/M_{p,s} \), is shown in part (d).

The 2-D DQFT is separable and can be calculated by using the column-row algorithm over each image component in the quaternion field. Therefore, the fast Fourier transform can be used in calculations. In the \( N=M \) case, this calculation requires \( 4N^2 \) real valued \( N \)-point DFTs. When \( N = 2^r \) and integer \( r > 1 \), the number of operations of multiplication in the real valued \( N \)-point DFT equals \( N^2/2r(r – 3) + 2 \). The total number of such operations for the 2-D DQFT equals \( 3N^2(r – 3) + 12N \).

For an image \( f_{n,m} \) presented by pure quaternion numbers, the number of these operations can be reduced to \( 9/4N^2 (r – 3) + 9N \).

3. New color image enhancement measurement

This section introduces a simple color image enhancement metric. The measure of quality of images can be used for selecting optimal processing parameters for image enhancement. We first consider the quantitative measure of image enhancement described in [47,48] that relates to Weber’s law of the human visual system.

3.1. Gray-level image enhancement measure

The goal of image enhancement is to improve the visual appearance of the image, or to produce the most visually pleasing image. The difficulty in developing an image enhancement technique lies in quantifying a criterion for enhancement. Image enhancement is a complex problem because the contrast is difficult to quantify, and contrast “optimality” is a perceptual concept. In the investigation of contrast enhancement based on the human visual system, the authors point out that there is generally no criterion for identifying how much enhancement is adequate at each location of the image. Without a generally applicable measure of contrast, contrast enhancement cannot be formulated as an optimization problem [18]. In general, it is difficult to select optimal processing parameters of image enhancement without an efficient measure or an automatic procedure for image enhancement.

The analysis of the existing transform-based image enhancement techniques [45–48] shows that to select optimal processing parameters and measure the quality of images, the quantitative measure of image enhancement that relates to Weber’s law of the human visual system can serve as a building criterion for image enhancement [36,49]. Intuitively, it is reasonable to expect that an image enhancement measure with values at given pixels should depend strongly on the values at pixels that are close by and weakly on those that are further away, and also that this measure should be related with the human visual system. Agaian, Panetta, and Grigoryan proposed a modification of Weber’s and Fechner’s laws [19,20], which is called EME [47,48]. Weber established a visual law, argued that the human visual detection depends on the ratio, rather than difference, between the light intensity value \( f(x,y) \) and \( f(x,y) + df(x,y) \). The Weber definition of contrast was used to measure the local contrast of a single object. Fechner’s law proposes the following relationship between the light intensity \( f(x,y) \) and brightness:

\[
B(x,y) = k \ln \left( \frac{f(x,y)}{F_{max}} + k' \ln \left( \frac{F_{max}}{F_{min}} \right) \right)
\]

(4)

where \( k' \) is a constant, and \( F_{min} \) and \( F_{max} \) are the “absolute threshold” and “upper threshold” of the human eye, respectively [21].

The EME measure of enhancement is defined as follows. A discrete image \( f_{n,m} \) of size \( N_1 \times N_2 \) is divided by \( k_1k_2 \) blocks of size \( L_1 \times L_2 \) where integers \( k_i = \lfloor N_i/L_i \rfloor, i = 1, 2 \), where \( \lfloor \cdot \rfloor \) denotes the floor function. The quantitative
measure of enhancement of the gray-scale image processed by $\Phi$ transform, $M_{a,\Phi}: (f_{n,m}) \rightarrow (f_{n,m})$, is defined by the following function $EME_{a}(\hat{f}) = EME_{i,\Phi,a}(\hat{f})$:

$$EME_{a}(\hat{f}) = \frac{1}{k_1k_2} \sum_{k_1=1}^{k_1} \sum_{k_2=1}^{k_2} 20 \log_{10} \left( \frac{\max_{k_1,k_2}(\hat{f})}{\min_{k_1,k_2}(\hat{f})} \right).$$  \hspace{1cm} (5)

Here, $\max_{k_1,k_2}(\hat{f})$ and $\min_{k_1,k_2}(\hat{f})$ respectively are the maximum and minimum of the image $f_{n,m}$ inside the $(k_1,k_2)$th block, and $a$ is a parameter, or a vector parameter, of the enhancement algorithm. In calculation of this function, the blocks with zero $\min_{k_1,k_2}(\hat{f})$ can be skipped or 1 can be added to the image, to avoid zero values of $\log_{10}(\hat{f})$ function. In the $(k_1,k_2)$th block, the ratio in the logarithm

$$r_{k_1,k_2} = \log_{10} \left( \frac{\max_{k_1,k_2}(\hat{f})}{\min_{k_1,k_2}(\hat{f})} \right) = \log_{10} \left[ \max_{k_1,k_2}(\hat{f}) \right] - \log_{10} \left[ \min_{k_1,k_2}(\hat{f}) \right],$$

shows the range of intensities of the image $\hat{f}$ in the logarithmic scale. Therefore, $EME_{a}(\hat{f})$ determines the average-block scale of intensities in the image after processing by $\Phi$ transform.

$EME_{a}(\hat{f})$ is called a measure of enhancement, or a measure of improvement, of the image $\hat{f}$ with respect to the transform $\Phi$. We define $a_{0}$ such that $EME_{a_{0}}(\hat{f}) = \max EME_{a}(\hat{f})$ is the best (or optimal) $\Phi$ transform-based image enhancement parameter. Experimental results show that the discrete Fourier transform can be considered as optimal when compared with the cosine, Hartley, Hadamard, and other transforms [48,49]. Therefore, the enhancement measure $EME_{a}(\hat{f})$ is considered with respect to the Fourier transform. When $\Phi$ is the identity transformation, the $EME(\hat{f})$ is called the enhancement measure of the image $f$.

3.2. EME measure application: examples

We now consider a few results of image enhancement with calculated measures of EME. This measure is used to quantitatively define the quality of the image. In most cases, when image quality is improved the value of EME increases (the case when the image is degraded with a noise is not considered, since a noise may increase the value of EME). As an example, Fig. 2 shows three images in parts (a)–(c), with corresponding values of EME. One can see that the image in (b) with $EME=2.69$ has better quality than the image in (a) for which $EME=1.69$. The high quality image is in part (c) and it has an enhancement measure $EME=5.31$.

The EME measure is useful when selecting the best parameters for image enhancement by the Fourier and
other unitary transforms, $\Phi$. We now consider the known method of $\alpha$-rooting [25,36–49]. The transform coefficients, $F_{p,s}$, of the image $f_{p,m}$ are modified by the factors $C(p,s) = A|F_{p,s}|^{\alpha - 1}$, where $A > 0$ is a constant and $\alpha$ runs in the interval $(0, 1)$. Fig. 3 shows the image in part (a) and the curve $EME(\alpha) = EME(\alpha)$ of the measure of enhancement in (b) which was calculated by using blocks $(7 \times 7)$. The image enhancement by the $\alpha$-rooting method has been parameterized by $\alpha$ varying in the interval $(0.2, 1)$. The curve has a maximum at the point $\alpha_0 = 0.88$. The $\alpha$-rooting enhancement of the original image when $\alpha = 0.88$ is illustrated in (c). The enhancement equals $EME_{0.88}(\alpha) - EME(\alpha) = 19.61 - 17.11 = 2.50$.

The experimental results show that the discrete Fourier transform can be considered as the optimal, when compared with the cosine, Hartley, Hadamard, and other transforms [48,49]. Therefore, the enhancement measure $EME_{\Phi}(\alpha)$ will be considered with respect to the Fourier transform, i.e., when $\Phi = F$.

When processing a color image by $\alpha$-rooting in the traditional way, i.e., separately by the color channels, such an optimal $\alpha$ parameter can be found for each channel. As an example, Fig. 4 shows the result of such processing for the “rider” image in part (a). The enhancement measures are $EME(R) = 13.22$, $EME(G) = 12.66$, and $EME(B) = 15.14$ for the red, green, and blue channels, respectively. The corresponding “best” values of $\alpha$ are 0.82, 0.80, and 0.92. The enhancement measures are $EME_{0.82}(R) = 17.79$, $EME_{0.80}(G) = 16.72$, and $EME_{0.92}(B) = 16.06$. The color image composed after enhancing each color channel by the “best” $\alpha$-rooting is shown in part (b). The blue channel of the original image has the highest EME value, and it could be reduced by using values other than $\alpha = 0.92$. One such example, when the EME of the blue channel is reduced to $EME_{0.81}(B) = 13.04$, is shown in Fig. 5 in part (b) together with the original image in (a).

3.3. Color image enhancement measure

Different values of the parameter $\alpha$ for color channels can be found which produce a good quality color image. The process of finding such values is complicated. Note that each measure of image enhancement, $EME(\alpha)$, is calculated in the spatial domain after processing the image in the frequency domain. The color image is considered as the whole unit composed by non-independent colors. Therefore, instead of calculating three functions $EME_a(f_R)$, $EME_a(f_G)$, and $EME_a(f_B)$, we propose the following enhancement measure of the color image $\hat{f} = (\hat{f}_R, \hat{f}_G, \hat{f}_B)$:

$$EMEC(\hat{f}) = \frac{1}{k_1 k_2} \sum_{k=1}^{k_1} \sum_{l=1}^{k_2} 20 \log_{10} \left[ \frac{\max_{k,l}(\hat{f}_R, \hat{f}_G, \hat{f}_B)}{\min_{k,l}(\hat{f}_R, \hat{f}_G, \hat{f}_B)} \right].$$  (6)

The image is in the RGB color space, i.e., $\hat{f} = (\hat{f}_R, \hat{f}_G, \hat{f}_B)$. Thus, the maximum and minimum values of the image $\hat{f}$ in the $(k, l)$-th block are calculated as $\max(\hat{f}) = \max(\hat{f}_R, \hat{f}_G, \hat{f}_B)$ and $\min(\hat{f}) = \min(\hat{f}_R, \hat{f}_G, \hat{f}_B)$. This concept of enhancement measure, EMEC, can be used when processing the image by the traditional 2-D DFT, as well as with other transforms including the 2-D DQFT.

Our experimental results show that the EMEC measure can be considered as a characteristic measuring of the quality of color images, and it can be used for selecting “the best” parameters for image enhancement. As an example, Fig. 6 shows the color image in different stages of processing in parts (a)–(c). One can see that the higher the quality of the image is, the larger the value of the EMEC measure. The color image in (b) with the measure $EMEC=3.81$ has better quality, than the image in (a) for which $EMEC=2.48$. Here, the image with best quality is in part (c) and its enhancement measure is $EMEC=12.50$.

Now we consider the $[\alpha, \alpha_r, \alpha_b]$–rooting method where the color channels of the image are separately processed by the $\alpha$-rooting with values equal to $\alpha_r$, $\alpha_g$, and $\alpha_b$, for the red, green, and blue colors, respectively. The above enhancement measure $EMEC(\hat{f})$ becomes the vector function $EMEC_{\alpha, \alpha_r, \alpha_b}(\hat{f})$. For the original “rider” image, $EMEC(f) = EMEC_{1,1,1}(f) = 21.28$. To select the vector parameter $\alpha$ for color image enhancement, one can take the values with high enhancement measure. For the above image, we can process only red and green channels as the main colors on the image to be changed. Fig. 7 shows the mesh of the enhancement measure $EMEC_{\alpha_r, \alpha_b}(\hat{f})$. The high values of the enhancement measure are along the perimeter of the region $(0.80, 1) \times (0.80, 1)$. For instance, the enhancement with $EMEC=23.00$ is achieved in the point $(0.84, 0.92)$. Fig. 8 shows such an enhancement in part (b) together with the original image in (a).

We can construct a similar mesh $EMEC_{\alpha_r, \alpha_g, \alpha_b}(\hat{f})$ for the case when the green channel is not processed and $\alpha$-rooting is applied only for red and blue channels. Another mesh of enhancement measure $EMEC_{\alpha_r, \alpha_b}(\hat{f})$ can be calculated for the case when the red channel stays without the change, and the green and blue channels are separately processed by $\alpha$-rooting. Then, we can take points $(\alpha_r, 1, \alpha_b)$ and $(1, \alpha_g, \alpha_b)$ with high values of the measure for each of these two cases.
and process the image with these values. For instance, we can take the following such points: (0.85, 1, 0.93) and (1, 0.81, 0.95), respectively. Fig. 9 shows the [0.85, 1, 0.93]-rooting enhancement in part (a) and the [1, 0.81, 0.95]-rooting enhancement in (b).

In addition, we consider the case when the vector parameter \( \alpha = (\alpha_r, \alpha_g, \alpha_b) \) is on the diagonal in the region \((0, 1)^3\). Fig. 10 shows the [0.88, 0.88, 0.88]-rooting enhancement \( f_{n,m} \) when applied on the original image \( f_{n,m} \) in part (a). In part (b), the [0.88, 0.88, 0.88]-rooting \( g_{n,m} \) was
calculated on the negative image \( g_{n,m} = 255 - f_{n,m} \), and the image was reconstructed as \( f_{n,m} = 255 - g_{n,m} \).

3.4. Quaternion transform-based image enhancement

In this section, a general framework based on the 2-D discrete quaternion Fourier transform for color image enhancement is presented. The idea of the transform-based method of image enhancement is applied for the color images mapped into the imaginary subspace of quaternion numbers, following a 2-D discrete unitary transform of the image (for instance, the 2-D discrete Fourier transform). Then the spectral coefficients are manipulated, and the inverse transform is performed, as
shown in Fig. 11. Here, $M$ is a function of coefficients of the Fourier transform.

Together with the alpha-rooting, other transform-based image enhancement methods also can effectively be used in image enhancement. These techniques include the weighted $\alpha$-rooting, modified unsharp masking, and filtering which are all motivated by the human visual response [5–9]. The main advantages of transform-based image enhancement techniques are a low complexity of computations, high quality of enhancement, and the critical role of fast unitary transforms in digital image processing.

In the $\alpha$-rooting image enhancement, the magnitude of the Fourier transform of the gray-scale image is transformed as $|F_{p,s}| \rightarrow M(|F_{p,s}|) = |F_{p,s}|^{\alpha}$ for each sample $(p, s)$, and the parameter $\alpha$ is taken to be in the interval $(0, 1)$. The parameter $\alpha$ can also be considered as a function which takes different values on different frequency bands. As shown in [47,48], the frequency bands separated by circles are effective when processing the image by the Fourier transform. For color images, we will consider the similar division in the frequency domain when using the 2-D discrete quaternion Fourier transform.

**Algorithm 1.** Multi-frequency band alpha-rooting algorithm.

1. Input a color image $(f_{m,n})$.
2. Use RGB model, or map color R,G,B space into another color space such as XYZ, ….
3. Perform a 2-D discrete quaternion transform of the color image.
4. Separate the frequency region into a several bands.
5. Multiply the transform coefficients, $F_{p,s}$, by different quaternion factors $C(p,s) = C|F_{p,s}|^{\alpha-1}$ on different bands.
6. Use the quality measure $EMEC(\alpha)$ to choose the best parameters $\alpha \in (0, 1)$.
7. Perform the inverse 2-D discrete quaternion transform.
8. Output is the enhanced color image.

The use of the 2-D DQFT in the $\alpha$-rooting adopted to the case of color images is much promised. Preliminary results show that the application of the 2-D DQFT plus $\alpha$-rooting method can effectively be used for enhancing color images. Examples of application of the proposed method on different color images and comparisons with the traditional monochrome-method 2-D DFT based $\alpha$-rooting are given in the next subsection. The $\alpha$-rooting image enhancement technique can be used to enhance edge information, sharp features and low contrast images. The proposed algorithms are simple to apply and design which makes them practical. The experimental results illustrate the performance of these algorithms compared with recently proposed methods.

3.5. Experimental results

In this section, we consider a few examples of color image enhancement by using $\alpha$-rooting. The magnitude $|F_{p,s}|$ of the 2-D DQFT is modified as $|F_{p,s}|^{\alpha}$, and then the inverse 2-D DQFT is calculated. To select values of $\alpha$ for image enhancement, first we analyze the enhancement functions $EME_{\alpha}$ for all three channels which we call $E_{r}(\alpha) = EME_{\alpha}(f_{R})$, $E_{g}(\alpha) = EME_{\alpha}(f_{G})$, and $E_{b}(\alpha) = EME_{\alpha}(f_{B})$. As an example, Fig. 12 shows these three functions for the “rider” image in part (a). The $EME$ function for the green channel reaches its maximum at the point 0.82. At the point 0.92, the red channel has a high value, and the maximum of the function $EME$ for the blue channel is at the point $\alpha = 0.95$. For comparison, the curve of the function $E_{g}(\alpha) = EME_{\alpha}(f_{g})$ for the same image is shown in part (b). The maximum of this function is at the point $\alpha = 0.9180$. The results of 0.82-, 0.92-, and 0.95-rooting of the image are shown in Fig. 13 in parts (a), (b), and (c), respectively. The result of “optimal” $\alpha$-rooting of the image, for $\alpha = 0.9180$, is shown in (d). A vagueness in choosing optimal values of $\alpha$ when separately working with three color channels is replaced with the unique value of this parameter when working with the color image in the quaternion field.

We also consider an example with the color “couple” image. Fig. 14 shows this image of size $256 \times 256$ in part (a) and the graph of three enhancement functions $E_{r}(\alpha)$, $E_{g}(\alpha)$, and $E_{b}(\alpha)$ in (b) when $\alpha \in (0.3, 1)$. At the point $\alpha = 0.80$, these three functions have approximately the

![Fig. 11. General block-diagram of the quaternion Fourier transform-based image enhancement.](image)

![Fig. 12. (a) Enhancement functions for three channels and (b) color enhancement function of the image.](image)
same numbers. The result of the 0.80-rooting by the 2-D DQFT is shown in (c), and the 0.90-rooting is shown in (d).

Fig. 15 shows the curve of the function \( E_\alpha(\alpha) \) for the couple image in part (a). The maximum of this function is at the point \( \alpha = 0.7570 \). The 0.7570-rooting of the image is shown in (b). Many other values of \( \alpha \) between this maximum and 1 result in a high quality enhancement of the image. As an example, the 0.85-rooting of the image is shown in (c).

3.5.1. Other color models

The method of \( \alpha \)-rooting with the 2-D DQFT can also be used for images presented in other color models instead of the RGB color space. For example, we consider the XYZ color space [6]. Fig. 16 shows the 0.77-rooting of the “couple” image in the XYZ color space in part (a) and this enhanced image transformed into the RGB color space in (b). The curve of the function \( E_\alpha(\alpha) = \text{EMEC}_\alpha(f) \) for the couple image presented in the XYZ color space is given in (c). The maximum of the function EMEC is at the point \( \alpha = 0.77 \) which is close to the point \( \alpha = 0.757 \) found for the RGB color space.

We also consider another example. Fig. 17 shows the enhancement in part (b) together with the original image in (a). The enhancement measure function \( E_\alpha(\alpha) = \text{EMEC}(\alpha) \) for this color image is shown in part (c). Fig. 18 shows such an enhancement of the image shown in Fig. 17(a). This image in the XYZ color space is shown in part (a). The measure of enhancement of the image is 10.1933. The image enhanced by the \( \alpha \)-rooting when \( \alpha = 0.92 \) is shown in b, and its transformation to the RGB space is shown in c. The enhancement measure of the image in b for this selected \( \alpha \) is 16.3767 and this measure is 20.9700, after transforming the image in RGB color space. The enhancement function EMEC calculated in the XYZ color space is shown in d. At the point \( \alpha = 0.92 \), the EMEC function has a maximum, as the same function in the RGB space. It is interesting to note that after transforming the 0.92-rooting image in the RGB space, the measure of this image is larger than the enhancement 19.8317 obtained in the \( \alpha \)-rooting in RGB color space.

Now we compare the above results of enhancement of the “couple” image with the known Retinex algorithms. Fig. 19 shows the original image in part (a), along with the result of the multiscale retinex algorithm in (b), which is described in [61,62], when using the MATLAB-based code with script multi_scale_retinex.m with parameters normalization = 1 and hsz = [7,15,21] for filtering with Gaussian filters. One can see that the processed image in (b) requires additional stages for adjusting colors. We also consider the results of enhancement of the “couple” image with two retinex algorithms which are given in [63,64]. Fig. 20 shows the results of the McCann99 Retinex code with the script retinex_mccann99.m in parts (a) and (b) when the number of iterations equals 2 and 20, respectively. The result of Frankle-McCann Retinex code with the script retinex_frankle_mccann.m after 20 iterations is given in (c). In addition, the image enhancement by scaling the DCT coefficients [65] is shown in (d) when using blocks of size 8 \times 8. It should be noted that since the “couple” image is very dark, all above results of \( \alpha \)-rooting and enhancement measures have been calculated through negative images. In other words, the following implementation of the \( \alpha \)-rooting is considered:

\[
\tilde{f}_{n,m} \rightarrow \tilde{g}_{n,m} = (255 - \hat{f}_{n,m})^{\alpha - \text{rooting}} \rightarrow \hat{g}_{n,m} \rightarrow \tilde{f}_{n,m} = (255 - \hat{g}_{n,m}).
\]

Fig. 21 shows the “landed satellite” image of size 1024 \times 577 in part (a) along with the EME functions for
Fig. 14. (a) The original image, (b) enhancement functions for three channels of the image, and the $\alpha$-rooting by the 2-D DQFT for (c) $\alpha=0.80$ and (d) $\alpha=0.90$.

Fig. 15. (a) Enhancement functions for three channels of the image. The $\alpha$-rooting by the 2-D DQFT for (b) $\alpha=0.7570$ and (c) $\alpha=0.85$. 
the red, green, and blue channels in (b). These three functions have maximums at the points $\alpha=0.83$, 0.85, and 0.92, respectively. The results of image enhancement by the 0.83- and 0.92-rooting are shown in (c) and (d), respectively. In the processed images, minute details in the dark regions of the original image are emphasized, in particular those located at the base of the satellite and front plate, in the shadow of the satellite where the cable
lays on the ground, and on the uniform of the officer approaching the satellite. With the exception of the parachute, which is a very light region with sharp intensity transitions, all details are either preserved or emphasized in the processed images. The ratio of regions of increased acuity to decreased acuity is then 4 to 1. Therefore, both the 0.83- and 0.92-rootings are considered to be enhanced from the original image. Instead of using the maximums of the enhancement functions for color channels, we can use the EMEC measure for image enhancement. Fig. 22 shows the graph of the enhancement measure $E_q(\alpha)$ for the image in part (a) along with the enhanced images in (b) and (c) when the $\alpha$-rooting was applied for $\alpha = 0.895$ and 0.95, respectively.

Fig. 23 shows the color image of size $322 \times 212$ in part (a) and the result of enhancement by the retinex method in (b). The color image processed by the histogram equalization of each color channel is shown in (c) and the 0.96-rooting image enhancement in (d).

As an example, Fig. 24 shows the color image in part (a) and the result of the multi-scale retinex in (b). Three color channels $R$, $G$, and $B$ were processed by three Gaussian functions $y_{\sigma_k}$, $k=1, 2, 3$, with parameters $\sigma_1 = 7, \sigma_2 = 15$, and $\sigma_3 = 21$.

Fig. 25 shows the result of retinex when the image was transformed to the HSV color space [6] and only the Value channel was processed by the multi-scale retinex method, while preserving the first two channels Hue and
Saturation, and then transformed the obtained image back to the RGB color space. One can notice very different results of the multi-scale retinex applied to the same image in two color models. The processing of the image through the HSV color model looks better.

Fig. 26 shows the multi-scale retinex image in part (a) and its three color channels in (b)–(d). Fig. 27 shows the original image in part (a) and its three color channels in (b)–(d). One can notice that the smooth images of R and B channels become better and many details can be seen and these images well emanated from the background. It looks like the red (and green) image sinks in the background and shows many details.

3.6. Multi-frequency band $\alpha$-rooting

Many methods of image enhancement, including the $\alpha$-rooting, can effectively be applied when processing differently the image in frequency bands. All examples given above referred to the particular case when the absolute values of the 2-D DQFT were modified by $\alpha$-rooting with the same value of $\alpha$ for all frequencies. This is a particular case of the general approach when all frequencies are divided by zones or bands and different values of $\alpha$ are used for processing the 2-D DQFT in these bands. The application of such multi-band alpha-rooting for grayscale images is described in [1–5,48,49].

As an example, we consider the “rider” image of size $441 \times 271$ and the division of the frequency lattice $X_{441,271} = ((p, s); \ p = 0:440, \ s = 0:270)$ by four bands, as shown in Fig. 28 in part (a). The bands are separated by the circles with the center at (220,135). It should be noted that the values of the 2-D DQFT in the corners of the frequency rectangle are not complex conjugate i.e., $F_{N-pM-s} \neq F_{p,s}$, where $p = 1:N -1, \ s = 1:M -1$. The radii of the circles are $r_1 = 90, \ r_2 = 150, \ r_3 = 200, \ \text{and} \ \ r_4 = 260$, and the frequency bands, $B_k, \ k = 1:4$, are defined as

$$B_k = ((p, s); \ r_k - 1 \leq \sqrt{(220 - p)^2 + (135 - s)^2} \leq r_k),$$

where $r_0 = 0$. The 2-D DQFT of the image is modified by the $\alpha$-rooting with the values of $\alpha$ equal to $\alpha_1 = 0.95, \ \alpha_2 = 0.90, \ \alpha_3 = 0.85, \ \text{and} \ \alpha_4 = 0.80$, for the bands B1, B2, B3, and B4, respectively. The result of image processing is shown in (b).

For the “couple” image of size $256 \times 256$, Fig. 29 shows in part (a) the division of the frequency lattice $X_{256,256} = ((p, s); \ p, \ s = 0:255)$ by five bands. The radii of the frequency bands $B_k$,
These bands were processed by $\alpha$-rooting with parameters $\alpha_1 = 0.90$, $\alpha_2 = 0.87$, $\alpha_3 = 0.85$, $\alpha_4 = 0.82$, and $\alpha_5 = 0.90$. The result of processing the image is shown in (b).

Fig. 30 shows in part (a) the division of the frequency lattice $X_{212,322}$ by five bands defined by the circles with radii equal to 50, 90, 120, 160, and 220. The $\alpha$-rooting over these frequency bands was processed with the parameters of $\alpha$ equal to 0.92, 0.93, 0.94, 0.96, and 0.95; the result of processing the image is shown in (b).

For the "roller" image, the frequency lattice $X_{240,320}$ was divided by five bands, as shown in Fig. 31 in part (a). The radii of the frequency bands $B_k$, $k=1:5$, are 44, 76, 108, 140, and 201, respectively. Five frequency bands were
processed by parameters $\alpha$ equal 0.80, 0.92, 0.95, 0.97, and 0.98, and the result of processing the image is shown in (b).

3.7. Choosing optimal weights of colors components

The color pallet of the $\alpha$-rooting can be improved or changed by weighting the colors of the image, i.e., by simple mapping $[R, G, B] \rightarrow [w_1R, w_2G, w_3B]$ where $w_1, w_2, w_3$ are positive coefficients such that $w_1 + w_2 + w_3 = 1$. For example, Fig. 32 shows the color couple image in part (a) and the 0.80-rooting enhancement in (b). The weighed 0.80-rooting $[0.4R, 0.4G, 0.2B]$ is shown in (c), when the blue channel was weighed by the coefficient twice smaller than the red and green channels. The similar weight reduction for the red channel, i.e., the weighed 0.80-rooting $[0.2R, 0.4G, 0.4B]$ is shown in (d).
3.8. Multi-scale $\alpha$-rooting

A few $\alpha$-rooting enhancements can be combined in one color image to obtain the enhanced image. As an example, Fig. 33 shows the color couple image in part (a) which was calculated from three $\alpha$-rootings, namely 0.4-, 0.7-, and 0.9-rootings, and then combined with the linear coefficients 0.2, 0.2, and 0.6, respectively. The result of the
Fig. 28. (a) 2-D frequency lattice divided by four bands and (b) band $\alpha$-rooting by the 2-D DQFT.

Fig. 29. (a) 2-D frequency lattice divided by five bands and (b) band $\alpha$-rooting by the 2-D DQFT.

Fig. 30. (a) 2-D frequency lattice divided by five bands and (b) band $\alpha$-rooting by the 2-D DQFT.

Fig. 31. (a) 2-D frequency lattice divided by five bands and (b) band $\alpha$-rooting by the 2-D DQFT.
A linear combination of 0.3-, 0.75-, and 0.8-rootings with the linear coefficients 0.3, 0.4, and 0.3, respectively is shown in (b).

**Fig. 32.** (a) The image, (b) the 0.80-rooting by the 2-D DQFT, (c) [0.4R, 0.4G, 0.2B]- and (d) [0.2R, 0.4G, 0.4B]-weighed 0.80-rooting enhancements.

**Fig. 33.** The linear combinations of (a) 0.4-, 0.7-, and 0.9-rootings and (b) 0.3-, 0.75-, and 0.8-rootings.

**Fig. 34** shows the girl image of size 236 × 360 in part (a). The left part of the image can clearly be seen, but not the right part where many details of the image are in the
darkening. To improve the quality of this image, one can try different values of $\alpha$ in the rooting algorithm. Examples of such enhancement when $\alpha = 0.80$, 0.88, and 0.96 are shown in parts (b), (c), and (d), respectively.

3.9. Cascade multi-band $\alpha$-rooting

The preliminary experimental results show that the method of $\alpha$-rooting by the 2-D DQFT can effectively be used together with the method of scaling the DC coefficients. As an example, Fig. 35 shows the enhanced image by $\alpha$-rooting of this image in part (a) when $\alpha = 0.90$. The 0.80-, 0.87-, and 0.92-rootings were combined with the linear coefficients $1/6$, $8/6$, and $-1/2$, respectively. For comparison, the result of the multiscale retinex algorithm is shown in (b). The multiscale retinex enhancement in (b) produced a gamut closer to the original image than the $\alpha$-rooting by the 2D-DQFT enhancement in (a). This is due to light regions in the original image transforming to high intensities through the (a) enhancement, resulting in a loss of detail over the original light regions. Details in dark regions of the original image are more apparent in the (a) enhancement than in that of (b) as can be seen in the region containing the eyes of the girl. Heavy saturation causes a loss of details in (b) as can be seen in the trees on the left side of the image. Additionally, the saturation in (b) causes an artificial appearance of the hues in the original image. Due to the higher degree of achieved detail and

Fig. 34. (a) The image (image16.jpg) and (b) 0.80-rooting, (c) 0.87-rooting, and (d) 0.92-rooting by the 2-D DQFT.

Fig. 35. (a) The 0.90-rooting by the 2-D DQFT and (b) enhanced image after scaling the DC coefficients.
preservation of hues in the original image by the enhancement in (a), we determine that the $\alpha$-rooting by the 2D-DQFT outperforms the multiscale retinex enhancement for this image. We also note that gamma correction may further improve the appearance of (a). This image after scaling the DC coefficients in all 8 × 8 blocks is shown in (b).

4. Conclusion

We have presented a new transform-based color image enhancement concept. Particularly, we have introduced (a) a 2-D discrete quaternion Fourier transform-based $\alpha$-rooting algorithm, (b) a multi-scale $\alpha$-rooting, (c) a cascade multi-band $\alpha$-rooting, (d) a multi-frequency band $\alpha$-rooting, and (e) a color image quality evaluation measure. A number of experimental results have been given which illustrate the performance of the proposed $\alpha$-rooting algorithms against the recently proposed algorithms such as the multiscale retinex algorithm developed by Štruc and Pavešić, the McCann99 retinex code, the Frank–McCann retinex code, and the commonly used Fourier transform-based $\alpha$-rooting algorithm. The proposed concepts of the color image enhancement measure and alpha-rooting algorithm can be used effectively not only for the RGB color model but also for the other color models such as the XYZ model.

References


