Competitive Balance vs Incentives to Win: a Theoretical Analysis of Revenue Sharing¤

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Abstract

We analyze a dynamic model of strategic interaction between the league organizing a professional sport, the teams playing the tournament organized by this league, and broadcasters competing for the rights to televise their matches. Teams and broadcasters maximize expected pro¯ts, while the league's objective may be either to maximize the demand for the sport or to maximize the teams' joint pro¯ts. Demand depends positively on competitive balance among teams and how intensively they compete to win the tournament. Revenue sharing increases competitive balance but decreases incentives to win. Under demand maximization, a performance-based reward scheme (as used by European top soccer leagues for national TV deals) may be optimal. Under joint pro¯t maximization, full revenue sharing (as used by US team sport leagues for national TV deals) is always optimal.

Keywords: Sport league, Revenue sharing, competitive balance, incentives to win.
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Revenue sharing is a controversial topic in the organization of any professional sport league. In recent years, the importance of this topic has been made even more evident by the growth in revenues American and European professional leagues fetch from the television broadcasters.[1] Not surprisingly, it has attracted the attention of professional economists (see Fort and Quirk (1995) for a comprehensive review). Surprisingly, there is few theoretical analysis of the di®erent sides of the controversy. In an attempt to shed some light on this issue, we study a dynamic model of tournament-like competition among teams and we let the body organizing the competition decide how to award prizes to winners and losers. In other words, we address the following question: how should a professional sport league allocate revenues among participating teams?

The standard argument in favor of revenue sharing in sports observes that there are large di®erences among revenues and wealth of teams. For example, Scully (1995) and Fort and Quirk (1995) provide evidence on large disparities of revenues from local TV deals and ticket sales among teams located in di®erent cities. As a consequence, richer teams tend to be more successful.[2] A mechanism which redistributes income from richer to poorer teams makes the competition more balanced, hence more enjoyable to the fans. A consequence of this argument is that revenue sharing increases demand for the sport, hence increasing the revenues of the league. Furthermore, if teams are pro¯t maximizers, revenue sharing also decreases the price teams pay for top players since their marginal value decreases. Hence, revenue sharing has a positive impact on the pro¯t of teams. On the other hand, revenue sharing provides little incentives to win. In the end, this may have a negative e®ect on demand since the lack of incentives for team owners will induce lack of incentives for players.[3] Moreover, as noticed by Daly (1992) and Fort and Quirk (1995), if teams have nothing to compete for, fans may strongly doubt the integrity of the competition on the playing ¯eld with an obvious negative e®ect on demand. Hence, revenue sharing has a negative impact of the team pro¯ts.

In this paper, we present a rigorous analysis of the opposing views in this controversy. Our starting point is a description of aggregate demand for a sporting competition. This determines how much money the league may obtain for selling the rights to broadcast the event. Then the league chooses a monetary reward scheme, knowing that its choice in°uences how team will compete in the event, hence in°uencing the aggregate demand.

Aggregate demand for a sport is ultimately determined by how much the fans enjoy the show provided by the tournament in which the teams compete. Following the literature sur-

[1] The latest reported television deals for NFL and NBA, for example, are $17.6 billion over eight years and $2.4 billion over four years respectively (see Araton 1998).

[2] See Scully (1995) for detailed evidence in American professional leagues.

[3] An example of this e®ect is given by the higher TV ratings for playo® matches when compared to regular season ones.

veyed in Fort and Quirk (1995), we assume it depends on three factors. Quality of playing talent in the sport, how hard teams are trying to prevail in the tournament, and competitive balance in the tournament. The league's quality is measured by the combined wealth of the participating teams; it re°ects the league's ability to attract talented athletes. The environment in which the league operates strongly in°uences league-wide quality. While US sport leagues are monopsonists in the market for players (i.e., only intra-league trades are observed so that league-wide talent is constant), European sport leagues operate in a competitive environment and compete for top players (inter-league trades of top players are as frequent as intra-league trades). A wealthier league (i.e., a larger total wealth of teams) attracts better players, hence having a positive e®ect on demand. Willingness to win is measured by the salaries a team pays to its Athletes. If the e®ort players produce is observable, a higher salary is the consequence of a higher e®ort. If the e®ort is not observable, higher prize when winning the competition generates a higher e®ort. Competitive balance is measured by uncertainty of the outcome; fans enjoy sporting events whose winners are not easy to predict. In other words, the more symmetric the winning chances of the teams, the more exciting the tournament is to watch. Since a team's probability of winning ultimately depends on the athletes playing for it, competitive balance also depends on a team's wealth and how much it pays its athletes.

In a dynamic setting, revenue sharing has two e®ects on demand. The ¯rst e®ect we call competitive balance: increased revenue sharing at time t increases demand at time $t + 1$ by making the teams' future winning chances more equal. This e®ect has consequences for the competitive balance at time $t + 1$ even if teams are equally wealthy at time t: a large prize today introduces an asymmetry in the probabilities of winning tomorrow. The second e®ect we call incentives to win: revenue sharing decreases demand by lowering the gain teams may obtain from winning and consequently diminishing their e®ort to win. This lowers demand since fans enjoy more e®ort from players.

In this paper, we are able to derive the optimal level of revenue sharing in a repeated tournament by analyzing the trade-o® between competitive balance and incentives to win. We consider two natural possibilities for the objective function of a professional sport league. First, we assume that the league as an independent body and assume it maximizes the revenues given by the amount of money it can obtain from television broadcaster. In our framework, this assumption is equivalent to maximizing demand for the sport. Second, we consider the league as a cartel of pro¯t maximizing ¯rms and assume it maximizes the teams' joint pro¯t (as assumed by Atkinson, Stanley and Tschirhart (1988)). Under demand maximization, a performance-based reward scheme, as used by European top soccer leagues for national TV deals (see Table 1), may be optimal[4]. Under joint pro¯ts maximization,

[4] See Hamil, Michie and Oughton (1999) for more details about England.

full revenue sharing, as used by US team sport leagues for national TV deals, is always optimal .

Our paper extends the existing literature[5] in several ways. First, we consider a multi-period model. Therefore, we are able to capture the trade-o® between pro¯ts today and pro¯ts tomorrow generated by revenue sharing. Second, we consider the possibility that a league faces competition from other leagues and that they compete for top players as is the case in Europe. Existing studies of revenue sharing consider the case of US sport leagues that do not face competition[6] Therefore, we can study the in°uence of revenue sharing at time t on league-wide talent at time t + 1.

The analysis carried out in this paper goes beyond the sports literature. Our model presents an example of a repeated moral-hazard problem between a principal and multiple agents in which the di®erence in output produced by the agents is detrimental to the principal and agents' income at time t in°uences their productivity at time t+1[7] In this setting, the principal faces a trade-o® between \output balance" among agents and incentives to produce large quantities. In a dynamic model, a principal can \invest" in output balance, i.e., lower the output at time t in order to get less di®erence in output at time t + 1. Such an investment is not possible in a static model.

The organization of the paper is as follows. Section 2 introduces the basic model and Section 3 derives its equilibrium. Section 4 to 6 consider three possible extensions. These are the problem of multi-period TV deals; the case in which teams have revenues that do not depend on the leagues' sharing policy; the situation in which teams cannot observe players e®ort (how hard they try to win). Finally, Section 6 concludes and an Appendix contains all proofs.

1 The model

In this section, we present a very simple model of the interaction between teams, leagues, and broadcasters. Many simplifying assumptions are made only to obtain a closed form solution of the model and do not appear necessary for our qualitative results. We study a two period game with four players. These are a professional sport league, the two teams

[5]El Hodiri and Quirk (1971), Atkinson, Stanley and Tschirhart (1988), Fort and Quirk (1995), and Hoen and Szymanski (1999), Vrooman (1999).

[6]Hoen and Szymanski (1999) also compare a league operating in a competitive environment and an isolated one. However, they do not study the optimal level of revenue sharing.

[7]For example, consider a situation such that there are two agents 1 and 2, the income of the principal at time t is $Min(q_{t;1}; q_{t;2})$, $q_{t;i}$ being the output of agent i at time t. Moreover, $q_{t;i}$ depends on agent i's (unobservable) e®ort, his productivity and some noise, and the productivity at time t depends on past income. (One can think of productivity as being the consequence of investment in more or less sophisticated machines.)

competing in a tournament this league organizes, and a broadcaster who pays to show this tournament to its viewers. In each period, the following sequence of moves occurs. First, the broadcaster decides how much to pay for the exclusive right to televise the sporting event. Then, the league decides how to divide this money between loser and winner of the tournament. Finally, the teams simultaneously decide how much to spend on players' incentives. At the end of period, the tournament is played, winner and loser are determined, and money is awarded.

Let K_t be the amount paid by the broadcaster in period t. Denote $W_{t,i}$ and $e_{t,i}$ the wealth of team at the beginning of period t and the effort exerted by team i at time t, respectively. The initial wealth of teams 1 and 2 are $W_{1,1}$ and $W_{1,2}$, respectively. Wealth at the beginning of period two is represented by the sum of the initial wealth and the profits realized in period 1.

The probability of winning the tournament

The outcome of the sporting event depends on the effort choices of the two teams and on their initial ability. The probability that team i wins in period t depends on its players' talent and how hard they play. Talent can be thought of as a team's ability to sign players at the beginning of the season and is measured by the team's wealth $W_{t,i}$. How hard players try to win can be thought of as effort, and is measured by the incentives necessary for players to perform during the season $e_{t,i}$. Formally, we assume the probability that team i wins in period t is

$$\begin{array}{ll} p_{t,i} = \alpha \frac{e_{t,i}}{e_{t,i}+e_{t,j}} + \beta \frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i} + e_{t,j} > 0 \\ p_{t,i} = \frac{W_{t,i}}{W_{t,i}+W_{t,j}} & \text{if} \quad e_{t,i} + e_{t,j} = 0 \end{array}$$

with $\alpha + \beta = 1$ and $i \neq j$. Quite obviously, $p_{t,j} = (1 - p_{t,i})$ since there are only two teams. The probability of winning increases with the difference in effort and the difference in wealth. When the two teams are equally wealthy and produce the same effort level, their probability of winning is $\frac{1}{2}$. One can think of $\frac{\alpha}{\beta}$ as a rough measure of how winning depends on incentives relative to initial quality. If $\frac{\alpha}{\beta} > 1$ the marginal return to effort is higher than the marginal return to wealth. Loosely speaking, in this case `trying hard is more important than being better'. The probability function we choose captures the following idea in a simple fashion. A richer team can buy better players, hence having an initial advantage. However, a poorer team can compensate this initial disadvantage by producing a higher effort level. In order to make players to produce a higher effort level, teams must reward them. Here, the effort level is measured in monetary terms.

Demand

Fans preferences determine how much they enjoy the show provided by the tournament the teams play. We assume these preferences depend on three sets of variables: overall quality of the league, competitive balance in the tournament, how hard players are trying to prevail in the competition. The league's quality is measured by the wealth of the participating teams; this reflects their ability to attract talented athletes. Competitive balance is measured by uncertainty of the outcome; fans enjoy sporting events whose winners are not easy to predict. The more competitive the league, the more symmetric the winning chances of the two teams, the more exciting the tournament is to follow. Willingness to win is measured by players' effort; it is important because fans enjoy athletes playing hard.[8]

In each period t, we assume a simple specification of demand in monetary terms D_t. Demand for sport by fans in period t is:

$$D_t = \gamma(e_{t;1} + e_{t;2}) + \delta[1 - (p_{t;1} - p_{t;2})^2] + \rho(W_{t;1} + W_{t;2}) \quad (1)$$

where $e_{t;i}$ denotes team i's effort in period t, $p_{t;i}$ its probability of winning, $W_{t;i}$ its wealth; the parameters $\gamma \in (0;1)$ and $\rho \in (0;1)$ are coefficients while $\delta > 0$ is expressed in monetary term. It represents the monetary value of one unit of competitive balance. Equation (1) can loosely be interpreted as measuring fans welfare from watching the tournament. The first term measures the importance of watching athletes "giving their best", the second measures the importance of watching a competitive tournament, and the third measures the importance of watching talented athletes. For this last term, the idea is that if there are several competing leagues, league-wide talent depends on the total wealth of teams[9]. The wealthier the teams of the considered league, the more talented the players they attract.

Since demand is expressed in monetary terms, the idea is that the broadcast of games generates income from say advertising and this income increases with the audience that watches them.

The market for TV rights is assumed to be perfectly competitive i.e., broadcasters expect zero profits in equilibrium and, in period 1, they expect to get the rights to broadcast the game in period 2 with probability zero. Hence, in each period, we have $K_t = D_t$.

[8] A possible fourth set of variables may measure fans' attachment to a team. Since we model demand for the sport, we assume that these "individual team" effects wash out in the aggregate.

[9] This assumption corresponds to the case of European sport leagues who organize domestic competitions and sign TV deals with national broadcasters. Top players often switch from one league to another, hence changing league-wide talents. Conversely, US sport leagues are in an isolated environment where league-wide talent is given and only intra-league trades occur.

1.0.1 The league

After having received K_t from broadcasters, the league decide how to allocate it between the two team at the end of the competition. We denote $K_{t;w}$ and $K_{t;l}$ ($K_{t;w} + K_{t;l} = K_t$) the amounts allocated to the winner and the loser, respectively, in period t. We consider two possible objective functions for the league.

Assumption (D) The league maximizes the demand for sport. Given that assumption of perfect competition in the broadcasting industry, this is equivalent to assuming that the league maximizes the revenues from the sale of TV rights. Hence, the league maximizes K_2 in period 2 and $K_1 + K_2$ in period 1.

Assumption (JP) The league maximizes the joint profit of the teams.

The Teams

The teams compete in a tournament whose outcome is uncertain. Since the probability of winning and the revenue to be allocated between teams in period 2 depends on the outcome of period 1. Hence, a fully rational team should consider the influence of its strategy in period 1 on the game that will be played in period 2. We do not think that this is very realistic since a league is usually made of a relatively large number of teams and the strategic influence of a specific team on the revenue of the league in the following period is small. Therefore, we start by considering the behavior of myopic teams.[10] Formally, a team's profits are:

$$\pi_{i;t} = p_{t;i} K_{t;w} + (1 - p_{t;i}) K_{t;l} - c(e_{t;i})$$

Since effort is measured in monetary terms, we assume the cost function of effort in each period is given by $c(e) = e$.

Finally, two remarks should be made. First, our model concentrates on the sale of rights to national TV network and on the allocation of these revenue between teams. Of course, teams have other sources of profits (e.g., ticket sales, sponsoring, merchandising, local TV deals). In the model, this is captured by the difference in initial wealth and this is assumed to be constant over the two periods. An alternative view is that the league is able to centralize all the revenues generated by teams and to redistribute Second, we have not modeled a market for talent. Implicitly, we assume that talent is linear in price and that teams maximize their expected profit from talent under the constraint that they cannot borrow. In such a case, if for a cost of talent equal to the total wealth of a team, the marginal profit is larger than the marginal cost, teams invest their entire wealth.

[10] The case of fully rational teams is analyzed in the Appendix.

2 The Equilibrium

In this section, we characterize the equilibrium of the game described previously. We begin by analyzing the subgame starting at the beginning of period 2. The solution concept we use is subgame perfect Nash equilibrium. Applying backward induction, we start with period 2 subgame and look at three optimization problems. First, the teams' optimal effort choices, given their wealth, the prizes decided by the league, and the TV rights. Then, the league optimal prize choice, given the TV rights, and the teams' equilibrium play that follows. Finally, the broadcaster optimal TV rights choice, given teams' and league equilibrium play. Then, we repeat a similar procedure for period 1, considering equilibrium play in the following period.

2.1 Period 2 Subgame

Formally, in period 2, team i maximizes

$$\pi_{i;2} = p_{i;2} K_{2;w} + (1 - p_{i;2}) K_{2;l} - e_{i;2} \tag{2}$$

Let $\Delta K_2 = K_{2;w} - K_{2;l}$. We have the following result.

Proposition 1 There exists an equilibrium of the effort game such that

$$e_{2;1} = e_{2;2} = \frac{\alpha \Delta K_2}{4} \tag{3}$$

Proof: See Appendix.

Proposition 1 says that the effort produced by teams increases with the difference between the prize money going to the winner and the loser. Hence, the larger the amount of revenue sharing (i.e., the smaller ΔK_2), the smaller the effort level produced by teams. We can now turn to the problem of the league.

The league maximizes demand for sport

Under the assumption that the league maximizes the demand for sport, the problem of the league in period 2 is equivalent to choosing ΔK_2 so as to maximize the effort produced by teams under the constraint that teams do not make losses. This implies that $K_{2;l} \geq e_{2;i}$. We derive the following proposition.

Proposition 2 Assume that the league maximizes the demand for sport, then

$$\Delta K_2 = \frac{K_2}{1 + \alpha/2} \tag{4}$$

Proof: see Appendix.

This result states that when the demand for sport depends on the e®ort produced by team, full revenue sharing does not lead to the maximization of demand for sport in the last period. The league always provides incentives for teams to produce e®ort. Hence ¢K_2 > 0 in equilibrium.

From Propositions 1 and 2, we can write the demand in period 2 as a function of K_2. Furthermore, the assumption of perfect competition in the broadcasting industry implies that $D_2 = K_2$. Therefore, we derive the revenue $K_2(d)$ of the league in period 2:

$$K_2(d) = \frac{\pm(2 + ®)}{2 + ®(1 ¡ °)} \; 1 ¡ -_2\frac{(W_{2;1} ¡ W_{2;2})^2}{(W_{2;1} + W_{2;2})^2} + \frac{º(2 + ®)}{2 + ®(1 ¡ °)}(W_{2;1} + W_{2;2}) \qquad (5)$$

The league maximizes teams' joint pro¯t

The joint pro¯t of teams in period 2 (¦$_2$) is given by

$$¦_2 = (° ¡ 1)(e_{2;1} + e_{2;2}) + \pm \; 1 ¡ -_2\frac{(W_{2;1} ¡ W_{2;2})^2}{(W_{2;1} + W_{2;2})^2} + º(W_{2;1} + W_{2;2}) \qquad (6)$$

It is straightforward that ¦$_2$ is decreasing in the e®ort level. Therefore, the objective of the league is to minimize the e®ort level produced by teams. Hence, we have the following result.

Proposition 3 Assume that the league maximizes teams' joint pro¯t. Then, the league chooses full revenue sharing, i.e., ¢K_2 = 0.

It follows that the revenue of the league in period 2 is

$$K_2(jp) = \pm \; 1 ¡ -_2\frac{(W_{2;1} ¡ W_{2;2})^2}{(W_{2;1} + W_{2;2})^2} + º(W_{2;1} + W_{2;2}) \qquad (7)$$

2.2 Period 1 Behavior and the Equilibrium of the Game

Given that teams are myopic, the problem they face in period 1 is identical to that faced in period 2. Hence, substituting ¢K_2 and $K_{2;l}$ by ¢K_1 and $K_{1;l}$, respectively, Proposition 1 still holds. Therefore, we can study directly the problem of the league. Given that the league is fully rational, it takes into account the in°uence of its decision in period on the game played in period 2. Furthermore, there is uncertainty about the payo®s in period 2 since $W_{2;1}$ and $W_{2;2}$ are dependent of the outcome of the competitions between the teams in period 1.

The league maximizes demand for sport

From Proposition 1, we deduce that the revenue of the league in period 2 if team 1 wins in period 1 is

$$K_2^{*}(d;1) = \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1-\beta^2\frac{(W_{1;1}-W_{1;2}+\Delta K_1)^2}{(W_{1;1}+W_{1;2}+K_1-(e_{1;1}+e_{1;2}))^2}\right] + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1;1}+W_{1;2}+K_1-(e_{1;1}+e_{1;2}))$$

while the revenue of the league in period 2 if team 2 wins in period 1 is

$$K_2^{*}(d;2) = \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1-\beta^2\frac{(W_{1;1}-W_{1;2}-\Delta K_1)^2}{(W_{1;1}+W_{1;2}+K_1-(e_{1;1}+e_{1;2}))^2}\right] + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1;1}+W_{1;2}+K_1-(e_{1;1}+e_{1;2}))$$

Therefore, in period 1, the league maximizes

$$D = D_1 + p_{1;1}K_2^{*}(d;1) + (1-p_{1;1})K_2^{*}(d;2) \tag{8}$$

Given that the two teams produce the same effort in period 1, we deduce that

$$p_{1;1} = \frac{\alpha}{2} + \beta\frac{W_{1;1}}{W_{1;1}+W_{1;2}} \tag{9}$$

and

$$D_1 = \gamma(e_{1;1}+e_{1;2}) + \delta\left[1-\beta^2\frac{(W_{1;1}-W_{1;2})^2}{(W_{1;1}+W_{1;2})^2}\right] + \nu(W_{1;1}+W_{1;2}) \tag{10}$$

From equations (9) and (10), we derive the following proposition.

Proposition 4 Assume the league maximizes the demand for sport. Then:
(i) If $\gamma(3-\gamma) > 3\nu$, there exists $\underline{\alpha} < 1$ such that for all $\alpha > \underline{\alpha}$, $\Delta K_1 > 0$.
(ii) if $\gamma(3-\gamma) < 3\nu$, full revenue sharing is optimal, i.e., $\Delta K_1 = 0$.

Proof: See Appendix.

The level of revenue sharing chosen by the league in period 1 influences its revenue in three ways. As the level of revenue sharing increases (ΔK_1 decreases), first, the revenue in period 1 decreases through a lower effort produced by teams. Second, the revenue of period 2 increases through a larger total wealth, and third, the revenue of period 2 increases through an increase in the balancedness of the league ($|p_{2;1}-p_{2;2}|$ increases). Part (i) of the proposition states that when the sensitivity of demand to effort is large relative to the sensitivity of demand to total wealth (so that $\gamma(3-\gamma) > 3\nu$), then if the sensitivity of the probability of winning to effort (α) is large enough the league sets the level of revenue sharing so as to maximize the effort level produced by teams. Hence, the league chooses partial revenue sharing ($\Delta K_1 > 0$). Part (ii) states that when the sensitivity of demand to wealth is large (so that $\gamma(3-\gamma) < 3\nu$) then the league shares its revenue evenly between teams in order to maximize the total wealth in period 2, hence the demand in period 2.

The league maximizes teams' joint profits

We know that when the league maximizes teams' joint profit, it sets $\Delta K_2 = 0$, so that $e_{2,1} = e_{2,2} = 0$. It follows that if team 1 wins in period 1, the revenue of the league in period 2 is

$$K_2^*(|p,1) = \delta\left(1 - \beta^2\frac{(W_{1,1} - W_{1,2} + \Delta K_1)^2}{(W_{1,1} + W_{1,2} + K_1 - (e_{1,1} + e_{1,2}))^2}\right) + \nu(W_{1,1}+W_{1,2}+K_1 - (e_{1,1}+e_{1,2}))$$

while if team 2 wins in period 1, the revenue of the league in period 2 is

$$K_2^*(|p,2) = \delta\left(1 - \beta^2\frac{(W_{1,1} - W_{1,2} - \Delta K_1)^2}{(W_{1,1} + W_{1,2} + K_1 - (e_{1,1} + e_{1,2}))^2}\right) + \nu(W_{1,1}+W_{1,2}+K_1 - (e_{1,1}+e_{1,2}))$$

The objective of the league in period 1 is then to maximize

$$\Pi = D_1 + p_{1,1}K_2^*(|p,1) + (1 - p_{1,1})K_2^*(|p,2) - (e_{1,1} + e_{1,2}) \tag{11}$$

Given the effort level chosen by teams as a function of $K_{1,w}$ and $K_{1,l}$, we have the following result.

Proposition 5 Assume that the league maximizes the joint profit of the teams. Then, full revenue sharing is optimal, i.e., $\Delta K_1 = 0$.

Proof: Proceeding as in the proof of Proposition 4, one shows that $\partial\Pi/\partial\Delta K_1 < 0$. □

The proposition states that the cost of an increase of the demand through a higher effort produced by teams is offset by the cost of such an effort. Hence, the league chooses ΔK_1 so that teams minimize their effort level.

3 Multi-period TV deal

So far, we have assumed that, at the beginning of each period, TV deals are negotiated for one period. Now, we consider the case in which at the beginning of period 1, the league sells the right to broadcast games for the two seasons and the payment is made at the beginning of period 1. In such a case, the league decides two things: the allocation of prizes between periods and then the allocation between the winner and the loser in each period. Such an assumption has two implications. First, at the beginning of period 1, teams know prizes to be awarded in the second period. This was not the case before since the K_2 was dependent of the winning team in the first period. Second, we do not have $D_t = K_t$, $(t = 1, 2)$. If we denote K the revenue of the league at the beginning of period 1, $K = D_1 + D_2$.

Also, note that the problem faced by teams in each period remains unchanged. Hence, the effort levels in periods 1 and 2 remain unchanged as functions of ΔK_1 and ΔK_2, respectively.

If the league maximizes the demand for sport, we are able to derive solutions in the corner cases $\alpha = 0$ and $\alpha = 1$. When the league maximizes teams' joint profits, we have a more general result.

Proposition 6 (i) Assume that the league maximizes the demand for sport. If $\alpha = 1$ and $\gamma > \delta$ then $K_1 = K$, $K_2 = 0$, and $\Delta K_1 = 2K_1/3$. If $\alpha = 1$ and $\gamma < \delta$ then $K_1 = K$, $K_2 = 0$ and $\Delta K_1 = 0$. If $\alpha = 0$, then $K_1 = K$, $K_2 = 0$ and $\Delta K_1 = 0$.
(ii) Assume that the league maximizes teams' joint profits. Then, $K_{1,w} = K_{1,l} = K/2$ and $K_2 = 0$.

Proof: See Appendix.
When deciding how to allocate money between teams and between periods, the league has to take into account two types of effects. First, the importance sensitivity of the probability of winning to effort (α) relative to its sensitivity to wealth (β). As already mentioned, the larger α, the larger the effort level produced by teams. The second effect is the importance of the sensitivity of demand to effort (γ) relative to the sensitivity of demand to wealth (δ). For a given total effort produced by teams in the two periods, the league prefers to concentrate these efforts in period 1 since it generates an increase in total wealth in period 2. Hence, for any α the league encourages effort in period 1. Also, for a given ΔK_1, the larger K_1 the smaller the difference in relative wealth between the two teams in period 2, hence the more balanced the competition. For these two reasons, the league sets $K_2 = 0$. When α is large, the choice of ΔK_1 for a given K_1 depends on the values of γ and δ. The league faces a trade-off. If ΔK_1 is large, then teams produce a high effort level in period 1 hence generating a high demand in period 1. In such a case, the total profit of teams is small since teams face a high cost for such an effort level. It follows that the total wealth in period 2 is small and so is the demand of period 2 generated by total wealth.

4 Teams have multiple sources of revenue

In this section, we assume that teams have revenues that are not submitted to possible revenue sharing by the league. For example, these revenues may come from local TV deal or from merchandising. However, we assume that these revenue are dependent of past performance, the idea being that the better a team is performing, the more attractive it is, hence the higher its revenue. Formally, we assume that the winner of the competition in period t receives $K_{t,w} + A$ with A strictly positive and independent of the degree of revenue sharing chosen by the league. As before, the loser receives $K_{t,l}$. Under such an assumption we have the following results.

Proposition 7 Assume that the league maximize the demand for sport and let

$$A^* = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \frac{(W_{2,1}-W_{2,2})^2}{(W_{2,1}+W_{2,2})^2}\right] + \frac{2\nu}{\alpha(1-\gamma)}(W_{2,1}+W_{2,2})$$

If $A > A^*$ then

$$K_2 = \frac{\gamma\alpha A}{2} + \delta\left[1 - \frac{(W_{2,1}-W_{2,2})^2}{(W_{2,1}+W_{2,2})^2}\right] + \nu(W_{2,1}+W_{2,2})$$

$\Delta K_2 = 0$ and $e_{2,i} = \alpha A/4$ $(i = 1, 2)$. If $A \leq A^*$ then

$$K_2 = \frac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1 - \frac{(W_{2,1}-W_{2,2})^2}{(W_{2,1}+W_{2,2})^2}\right] + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{2,1}+W_{2,2})$$

$$\Delta K_2 = \frac{2K_2-\alpha A}{2+\alpha} \quad e_{2,1} = e_{2,2} = \alpha(\Delta K_2 + A)/4$$

Proof: See Appendix.

When the additional source of revenue is not too large (i.e., smaller than A^* so that the league does not choose full revenue sharing), it generates a higher revenue for the league. The reason is that A affects the effort level produced by teams in two ways. The first effect is a direct one. If the amount earned by the winning team increases, it provides incentives for teams to increase their effort level. This generates an indirect effect: the league increases the level of revenue sharing so that

$$e_{2,i} = K_{2,l} = \frac{\alpha(K_2+A)}{2(2+\alpha)}$$

When the additional source of revenue is large (i.e., lager than A^*), the league chooses full revenue sharing and teams' effort level is only determined by A. Furthermore, the losing team makes a loss.

In period 1, the problem teams face is the same as in period 2. Therefore,

$$e_{1,1} = e_{1,2} = \alpha(\Delta K_1 + A)/4 \tag{12}$$

From Proposition 7 and equation (12), we deduce the following result.

Proposition 8 Assume that the league maximizes the demand for sport. If $\gamma(3-\gamma) > 3\nu$, there exist $\bar{A} > 0$ and $\underline{\alpha} < 1$ such that if $A < \bar{A}$ and $\alpha > \underline{\alpha}$, then $\Delta K_1 > 0$.

This result suggests that in a league in which revenues from TV deals represent a fraction not too large of team revenues, full revenue sharing is not damaging to effort since other source of revenues provide incentives for teams to produce effort. Conversely, in a league in which revenues from TV represent a large fraction of teams' revenues, then the league chooses a performance-based allocation.

5 Unobservable Effort

So far, we have implicitly assumed that effort produced by team players was observable, hence teams could offer effort-based compensation to players. In this section, we relax this assumption. A direct consequence is that teams can only offer performance-based contracts to players. Let $\mu_{t;i}(w)$ and $\mu_{t;i}(l)$ the fraction of the gain paid to players when team i earns $K_{t;w}$ and $K_{t;l}$, respectively. The objective of team i is to maximize

$$\pi_{t;i} = p_{t;i}(1 - \mu_{t;i}(w))K_{t;w} + (1 - p_{t;i})(1 - \mu_{t;i}(l))K_{t;l}$$

subject to $\mu_{t;i}(w) \geq 0$, $\mu_{t;i}(l) \geq 0$, and

$$e^*_{t;i} \in \text{Argmax} \;\; p_{t;i}\mu_{t;i}(w)K_{t;w} + (1 - p_{t;i})\mu_{t;i}(l)K_{t;l} \tag{13}$$

This last equation represents the incentive compatibility constraint.

Let $\Delta K_{t;i} = \mu_{t;i}(w)K_{t;w} - \mu_{t;i}(l)K_{t;l}$. Then, proceeding as in the proof of Proposition 1, one shows that the equilibrium of the effort game is such that

$$e^*_{t;i} = \text{Sup}\left\{0; \frac{\alpha(\Delta K_{t;i})^2 \Delta K_{t;j}}{(\Delta K_{t;1} + \Delta K_{t;2})^2}\right\} \tag{14}$$

with $i \neq j$. It follows that if $e^*_{t;i} > 0$, then

$$p_{t;i} = \alpha\frac{\Delta K_{t;i}}{\Delta K_{t;1} + \Delta K_{t;2}} + \beta\frac{W_{t;i}}{W_{t;1} + W_{t;2}} \tag{15}$$

From these results, we derive the following proposition about the compensation of players by teams.

Proposition 9 Assume that $\Delta K_t > 0$. There exists an equilibrium such that
(i) $\mu_{t;i}(l) = 0$ $(i = 1;2)$
(ii) If $W_{t;i} > W_{t;j}$, then $0 < \mu_{t;i}(w) < \mu_{t;j}(w)$ and $p_{t;i} > p_{t;j}$.
(iii) $\mu_{t;l}$ $(i = 1;2)$ is an increasing function of $K_{t;w}$.

We deduce that

$$e^*_{t;i} = \frac{\alpha\mu_{t;i}(w)^2\mu_{t;j}(w)K_{t;w}}{(\mu_{t;1}(w) + \mu_{t;2}(w))^2}$$

and

$$p_{t;i} = \alpha\frac{\mu_{t;i}(w)}{\mu_{t;1}(w) + \mu_{t;2}(w)} + \beta\frac{W_{t;i}}{W_{t;1} + W_{t;2}}$$

The proposition says that players are only compensated in case of success and the incentives are more important for the team with the smaller wealth. It follows that players from the wealthier team exert a lower effort. However, in equilibrium, the wealthier team has a higher probability of winning the competition. A direct consequence of (iii) is that the level of revenue sharing influences the effort level produced by teams in two ways: directly through the difference of gains between the winner and the loser, and indirectly through the compensation scheme of the players ($\mu_{t,i}(w)$).

We turn now to the problem of the league. A main differences with the case of observable effort is that teams never make losses. Hence, in period 1, the league does not have to take into account the possibility that a team will have a negative wealth if it loses in period 1. From the previous proposition we derive the following results about the level of revenue sharing in period 2.

Proposition 10 Assume that the league maximizes the demand for sport. Then:
(i) $\Delta K_2 = K_2$.
(ii) There exists $\underline{\alpha} < 1$ such that if $\alpha > \underline{\alpha}$ and $\gamma(6 - \gamma) > 36\nu$ then $\Delta K_1 > 0$.

The proposition states that, qualitatively, the results obtained in the case of observable effort still hold if this assumption is relaxed. That is, the league minimizes the level of revenue sharing in the second period and if the influence of effort on demand is large enough with respect to the influence of total wealth, then the league does not choose full revenue sharing in the first period.

6 Conclusions

We presented a theoretical model of revenue sharing in sport leagues. Our main results derive explicit conditions under which revenue sharing may be optimal. These can be summarized by looking at the relative importance of the incentive to win versus (future) competitive balance. Higher revenues sharing increases future demand through a better competitive balance, but decreases current demand through a lower effort to win from teams. If the league maximizes the demand for sport, then a performance-based reward scheme (as used by European top soccer leagues for national TV deals) may be optimal. Conversely, if the league act as a cartel and maximizes joint profits, then full revenue sharing (as used by US team sport leagues for national TV deals) is always optimal .

Our results are also interesting for the moral-hazard literature since our model presents an example of a repeated agency problem between a principal and multiple agents in which the difference in output produced by the agents is detrimental to the principal. In this setting, the principal faces a trade-off between "output balance" among agents and incentives to produce large quantities. Our results show that the principal may have incentive to "invest" in "output balance", i.e., lower the output today in order to get a lower difference in outputs tomorrow.

Appendix

Proof of Proposition 1: Assume that $\Delta K_2 > 0$. If $e_{2,j} > 0$ then the FOC of profit maximization for player i yields

$$e_{2,i} = \operatorname{Max}\left(0; \sqrt{\alpha e_{2,j}\Delta K_2} - e_{2,j}\right) \tag{16}$$

and if $e_{2,j} = 0$ then $e_{2,i} = 0$ is not a best reply to $e_{2,j}$. Therefore, equilibria are solution of the system of equation (16) A solution is given by (26). If $\Delta K_2 = 0$, then teams' expected revenue is independent of their effort level. Hence, teams' objective is to minimize the cost of effort, thus they choose $e_{2,i} = 0$. $\square$

Proof of Proposition 2: Given that $K_{2,l} = K_2 - K_{2,w}$, $\Delta K_2 = 2K_{2,w} - K_2$ and the demand for sport, the problem of the league is to choose $K_{2,w}$ so as to maximize the effort produced by teams. From Proposition 1, we derive that the league choose $K_{2,w}$ such that

$$K_2 - K_{2,w} = \alpha 2K_{2,w} - K_2/4 \tag{17}$$

Hence,

$$K_{2,w} = \frac{1+\alpha/4}{1+\alpha/2}K_2 \tag{18}$$

This implies

$$\Delta K_2 = \frac{K_2}{1+\alpha/2} \tag{19}$$

Proof of Proposition 4: Let

$$H = \frac{\Delta K_1\left((W_{1,1}+W_{1,2}) + \frac{\alpha\beta(W_{1,1}-W_{1,2})^2}{2(W_{1,1}+W_{1,2})}\right) + (\beta+\alpha/2)(W_{1,1}-W_{1,2})^2}{(W_{1,1}+W_{1,2}+K_1-\alpha\Delta K_1/2)^3}$$

$$\frac{\partial D}{\partial \Delta K_1} = -\frac{2\beta^2\delta(2+\alpha)}{2+\alpha(1-\gamma)}H + \frac{\alpha}{2}\left(\gamma - \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}\right)$$

If $\gamma(3-\gamma) > 3\nu$, then there exists $\underline{\alpha} < 1$ such that for all $\alpha > \underline{\alpha}$, $\frac{\partial D}{\partial K_1} > 0$. Conversely, if $\gamma(3-\gamma) \leq 3\nu$, then for all $\alpha \in [0,1]$ $\partial D/\partial \Delta K_1 < 0$. $\square$

Proof of Proposition 6: Assume that the league maximizes the demand for sport. If $\alpha = 1$, then the league maximizes

$$D = \frac{(\gamma-\nu)}{2}\Delta K_1 + \frac{\gamma}{2}\Delta K_2 + 2\left[\delta + \nu(W_{1,1}+W_{1,2})\right] + \nu K_1 \tag{20}$$

subject to $K = K_1 + K_2$, $\Delta K_t \leq K_{t,l}$ $(t = 1, 2)$.
If $\nu > \gamma$, then D is decreasing in ΔK_1 and increasing in ΔK_2. Hence, the league sets

$\Delta K_1 = 0$ and $\Delta K_2/4 = K_{2,l}$. Therefore, $K_{1,w} = K_{1,l} = K_1/2$, and $K_{2,l} = K_2/4$. It follows that the problem of the league is to choose K_1 and K_2 (with $K_1 + K_2 = K$) so as to maximize $D = \gamma K_2/2 + \nu K_1$. Given that $\nu > \gamma$, the league chooses $K_1 = K$ and $K_2 = 0$. Now, assume that $\gamma > \nu$. Then, the league sets $\Delta K_1/4 = K_{1,l}$ and $\Delta K_2/4 = K_{2,l}$. Then, the league maximizes

$$D = ((\gamma + \nu)K_1 + \gamma K_2)/2 \tag{21}$$

We deduce that the league chooses $K_1 = K$ and $K_2 = 0$. Furthermore, $\Delta K_1/4 = K_{1,l}$ implies $\Delta K_1 = 2K/3$.

Assume that $\alpha = 0$. Then, the demand in period 1 is not influenced by the allocation chosen by the league. It follows that the objective of the league is to maximize

$$\begin{aligned} F = {} & \frac{W_{1,1}}{W_{1,1}+W_{1,2}}\left[1 - \frac{(W_{1,1} - W_{1,2} + \Delta K_1)^2}{(W_{1,1}+W_{1,2}+K_1)^2}\right] + \frac{W_{1,2}}{W_{1,1}+W_{1,2}}\left[1 - \frac{(W_{1,1} - W_{1,2} - \Delta K_1)^2}{(W_{1,1}+W_{1,2}+K_1)^2}\right] \\ & + \nu(W_{1,1} + W_{1,2} + K_1) \end{aligned} \tag{22}$$

It is straightforward that F is increasing in K_1. Hence, the league set $K_2 = 0$. Now, $dF/d\Delta K_1 > 0$ is equivalent to

$$-2(W_{1,1} + W_{1,2})\Delta K_1 - 2(W_{1,1} - W_{1,2})^2 < 0 \tag{23}$$

Therefore, The league chooses $\Delta K_1 = 0$.

Assume that the league maximizes teams' joint profit. Given that $e_{t,1} = e_{t,2} = \alpha\Delta K_t/4$, the league maximizes

$$\begin{aligned} \Pi = {} & 2(\gamma - \nu - 1)e_{1,1} + \delta\left(\frac{\alpha}{2} + \beta\frac{W_{1,1}}{W_{1,1}+W_{1,2}}\right)\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} + \Delta K_1)^2}{(W_{1,1}+W_{1,2}+K_1-2e_{1,1})^2}\right] \\ & + \delta\left(\frac{\alpha}{2} + \beta\frac{W_{1,2}}{W_{1,1}+W_{1,2}}\right)\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} - \Delta K_1)^2}{(W_{1,1}+W_{1,2}+K_1-2e_{1,1})^2}\right] + 2(\gamma - 1)e_{2,1} + \nu K_1 \end{aligned} \tag{24}$$

subject to $K = K_1 + K_2$, $\Delta K_t \leq K_{t,l}$ $(t = 1, 2)$.
It is straightforward that Π is decreasing in the effort in period 2. Hence the league sets $\Delta K_2 = 0$. Now,

$$\begin{aligned} \frac{\partial \Pi}{\partial \Delta K_1} = {} & \alpha(\gamma - \nu - 1)/2 - \frac{\beta^2\left\{\frac{\alpha}{2}\left(\alpha(W_{1,1} - W_{1,2})^2 + 2\Delta K_1(W_{1,1}+W_{1,2}+K_1)\right)\right\}}{(W_{1,1}+W_{1,2}+K_1-2e_{1,1})^3} \\ & - \frac{\frac{\beta^2\Delta K_1}{W_{1,1}+W_{1,2}}\left(K_1(W_{1,1}+W_{1,2}) + (W_{1,1}+W_{1,2})^2 + \frac{\alpha}{2}(W_{1,1} - W_{1,2})^2\right)}{(W_{1,1}+W_{1,2}+K_1-2e_{1,1})^3} \\ & - \frac{\beta^2(W_{1,1} - W_{1,2})^2\left(\frac{K_1}{W_{1,1}+W_{1,2}} + 1 + \frac{\alpha}{2}\right)}{(W_{1,1}+W_{1,2}+K_1-2e_{1,1})^3} < 0 \end{aligned} \tag{25}$$

Hence, $\Delta K_1 = 0$. Furthermore, it is straightforward that at $\Delta K_1 = 0$, $\partial\Pi/\partial K_1 > 0$ while $\partial\Pi/\partial K_2 = 0$. Hence, we have the desired result. $\Box$

Proof of Proposition 7: Proceeding as in the proof of Proposition 1, one shows that

$$e_{2,1} = e_{2,2} = \frac{\alpha(\Delta K_2 + A)}{4} \tag{26}$$

Then, proceeding as in the proof of Proposition 2, we obtain that the league chooses

$$K_{2,w} = \mathrm{Max}\left\{\frac{4K_2 + \alpha(K_2 - A)}{2(2+\alpha)}, \frac{K_2}{2}\right\} \tag{27}$$

This implies

$$\Delta K_2 = \mathrm{Max}\left\{\frac{2K_2 - \alpha A}{2+\alpha}, 0\right\} \tag{28}$$

If $\Delta K_2 > 0$, then K_2 is given by (7) and $K_2 > \alpha A/2$ is equivalent to $A < A^*$. If $\Delta K_2 = 0$ then $K_2 = \alpha A^*/2$, then $K_2 < \alpha A/2$ is equivalent to $A > A^*$. $\square$

Proof of Proposition 8: Let

$$A_1^*(A) = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} + \Delta K_1 + A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right]$$
$$+\frac{2\nu}{\alpha(1-\gamma)}(W_{1,1} + W_{1,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$
$$A_2^*(A) = \frac{2\delta}{\alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} - \Delta K_1 - A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right]$$
$$+\frac{2\nu}{\alpha(1-\gamma)}(W_{1,1} + W_{1,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$

with

$$K_1(A) = \frac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left(1 - \beta^2\frac{(W_{1,1} - W_{1,2})^2}{(W_{1,1}+W_{1,2})^2}\right) + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,1}+W_{1,2}) \tag{29}$$

Define the functions $F_1(A)$ and $F_2(A)$ as follows

$$F_1(A) = \begin{cases} F_{1,s}(A) & \text{if } A \le A_1^* \\ F_{1,l}(A) & \text{if } A > A_1^* \end{cases}$$

$$F_2(A) = \begin{cases} F_{2,s}(A) & \text{if } A \le A_2^* \\ F_{2,l}(A) & \text{if } A > A_2^* \end{cases}$$

where

$$F_{1,l}(A) = \frac{\gamma\alpha A}{2} + \delta\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} + \Delta K_1 + A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right]$$
$$+\nu(W_{2,1} + W_{2,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$
$$F_{1,s}(A) = \frac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} + \Delta K_1 + A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right]$$
$$+\frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,1} + W_{1,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$
$$F_{2,l}(A) = \frac{\gamma\alpha A}{2} + \delta\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} - \Delta K_1 - A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right]$$
$$+\nu(W_{2,1} + W_{2,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$

$$F_{2,s}(A) = \frac{\gamma\alpha A}{2+\alpha(1-\gamma)} + \frac{\delta(2+\alpha)}{2+\alpha(1-\gamma)}\left[1 - \beta^2\frac{(W_{1,1} - W_{1,2} - \Delta K_1 - A)^2}{(W_{1,1}+W_{1,2}+K_1(A)+A-\alpha(\Delta K_1+A)/2)^2}\right] + \frac{\nu(2+\alpha)}{2+\alpha(1-\gamma)}(W_{1,1} + W_{1,2} + K_1(A) + A - \alpha(\Delta K_1 + A)/2)$$

Let

$$D = D_1 + p_{1,1}F_{1,s}(A) + (1 - p_{1,1})F_{2,s}(A)$$

where D_1 and $p_{1,1}$ are given by (10) and (9), respectively. Now, it is straightforward that there exists $\bar{A}_2$ such that if $A < \bar{A}_2$, then $F_1(A) = F_{1,s}(A)$ and $F_2(A) = F_{2,s}(A)$. Therefore, if $A < \bar{A}_2$, then proceeding as in the proof of Proposition 4, one shows that if $\gamma(3 - \gamma) > 3\nu$, there exists $\underline{\alpha}$ such that $\partial D/\partial\Delta K_1 > 0$. Let

$$\bar{A}_1 = \frac{2\delta}{\alpha(1-\gamma)}\left(1 - \beta^2\frac{(W_{1,1} - W_{1,2})^2}{(W_{1,1} + W_{1,2})^2}\right) + \frac{2\nu}{\alpha(1-\gamma)}(W_{1,1} + W_{1,2})$$

Proceeding as in the proof of proposition 7, one shows that if $A < \bar{A}_1$, then

$$\Delta K_1 = \frac{2K_1(A) - \alpha A}{2 + \alpha} \tag{30}$$

Then, then assumption of perfect competition in the broadcasting industry in period 1 (i.e., $D_1 = K_1$) implies that K_1 is given by (29). Hence, taking $\bar{A} = \mathrm{Min}(\bar{A}_1, \bar{A}_2)$, we have the desired result. $\square$

Proof of Proposition 9:

Proof of part (i). From equations (14) and (15), we derive that

$$\frac{\partial\pi_{t,i}}{\partial\mu_{t,i}(w)} = \frac{\alpha\Delta K_{t,j}K_{t,w}}{(\Delta K_{t,1} + \Delta K_{t,2})^2}[(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l}] - p_{t,i}K_{t,w} \tag{31}$$

$$\frac{\partial\pi_{t,i}}{\partial\mu_{t,i}(l)} = -\frac{\alpha\Delta K_{t,j}K_{t,l}}{(\Delta K_{t,1} + \Delta K_{t,2})^2}[(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l}] + (p_{t,i} - 1)K_{t,l} \tag{32}$$

Assume that there exists an equilibrium with $\mu_{t,i}(w) > 0$. This implies that

$$(1 - \mu_{t,i}(w))K_{t,w} - (1 - \mu_{t,i}(l))K_{t,l} > 0$$

In turn, this implies that $\partial\pi/\partial\mu_{t,i}(l) < 0$ in equilibrium. Hence, $\mu_{t,i}(l) = 0$. Now, we need to show that the system of equations

$$\frac{\alpha\mu_{t,j}(w)}{(\mu_{t,1}(w) + \mu_{t,2}(w))^2}[(1 - \mu_{t,i}(w))K_{t,w} - K_{t,l}] - p_{t,i}K_{t,w} = 0 \qquad i = 1,2 \qquad i \neq j \tag{33}$$

has a solution in $(0,1) \times (0,1)$ which satisfies the second order conditions of profit maximization.

From equation (31), it is straightforward that if $\mu_{t,i}(l) = 0$ then $\partial^2\pi_{t,i}/(\partial\mu_{t,i}(w))^2 < 0$. Now, when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0 at the same speed (so that there exists $H > 0$ such that $H < \mu_{t,i}(w)/\mu_{t,j(w)}$ $(i = 1,2$ and $i \neq j)$ as when $\mu_{t,1}(w)$ and $\mu_{t,2}(w)$ converge to 0), then the LHS of (33) goes to infinity. Furthermore, for any given $\mu_{t;i}(w) > 0$, $\alpha\mu_{t;j}(w)/(\mu_{t;1}(w) + \mu_{t;2}(w))^2$ converges to 0 as $\mu_{t;j}(w)$ converges to zero. Hence, we deduce that by continuity, there exist $\mu_{t;1}(w)$ and $\mu_{t;2}(w)$ such that the system of equations (33) has a solution in $(0;1) \times (0;1)$.

Proof of part (ii): We use a contradiction argument. Assume that $W_{t;i} > W_{t;j}$ and $\mu_{t;i}(w) \geq \mu_{t;j}(w)$. This implies that $p_{t;i} > p_{t;j}$. From (33), it follows that

$$\frac{\mu_{t;j}(w)}{\mu_{t;i}(w)} > \frac{(1 - \mu_{t;j}(w))K_{t;w} - K_{t;l}}{(1 - \mu_{t;i}(w))K_{t;w} - K_{t;l}}$$

The LHS of this inequality is smaller than 1 while the RHS is larger than 1. Hence, the inequality does not hold and if $W_{t;i} > W_{t;j}$ then $\mu_{t;i}(w) < \mu_{t;j}(w)$.
Now, $\mu_{t;i}(w) > \mu_{t;j}(w)$ implies $\pi_{t;i} > \pi_{t;j}$ follows directly from (33).

Proof of part (iii): Let $R_t = K_t/K_{t;w}$. From (33), we deduce that

$$\frac{\partial \mu_{t;i}(w)}{\partial R_t} = -\left[\mu_{t;j}(w)(2 - R_t)(\mu_{t;1}(w) + \mu_{t;2}(w))\right]^{-1} \tag{34}$$

Hence, $\mu_{t;1}(w)$ and $\mu_{t;2}(w)$ are increasing functions of $K_{t;w}$.
2

Proof of Proposition 10:
Proof of (i): From Proposition 9, we know that in each period the effort level is increasing in K_w. Hence, we only need to show that $(p_{2;1} - p_{2;2})^2$ is not increasing in ΔK_2.

$$p_{2;1} - p_{2;2} = \alpha\frac{\mu_{2;1}(w) - \mu_{2;2}(w)}{\mu_{2;1}(w) + \mu_{2;2}(w)} + \beta\frac{W_{2;1} - W_{2;2}}{W_{2;1} + W_{2;2}}$$

Let $R_t = K_t/K_{t;w}$.

$$\frac{d(p_{2;1} - p_{2;2})}{dR_2} = \frac{2(\mu_{2;2}(w)\frac{d\mu_{2;1}(w)}{dR_2} - \mu_{2;1}(w)\frac{d\mu_{2;2}(w)}{dR_2})}{(\mu_{2;1}(w) + \mu_{2;2}(w))^2} \tag{35}$$

From equation (34), we derive that $d(p_{2;1} - p_{2;2})/dR_2 = 0$. It follows that D_2 is increasing in ΔK_2 and so the leagues sets $\Delta K_2 = K_2$.

Proof of (ii). Assume that $\alpha = 1$. In such a case,

$$\mu_{2;1}(w) = \mu_{2;2}(w) = \frac{2K_{2;w} - K_2}{3K_{2;w}}$$

and

$$e_{2;1} = e_{2;2} = \frac{2K_{2;w} - K_1}{12}$$

From part (i), we know that $K_{2;w} = K_2$. We deduce that

$$K_2 = \frac{\delta + \nu(W_{2;1} + W_{2;2})}{1 - \alpha\gamma/6} \tag{36}$$

Now, consider the problem of the league in period 1. Teams face the same problem as in period 2. Hence,

$$\mu_{1;1}(w) = \mu_{1;2}(w) = \frac{2K_{1;w} - K_1}{3K_{1;w}}$$

Therefore, if team i wins in period 1, then

$$W_{2;i} = W_{1;i} + \left(1 - \frac{2K_{1;w} - K_1}{3K_{1;w}}\right) K_{1;w}$$

while if it looses,

$$W_{2;i} = W_{1;i} + K_{1;l}$$

We deduce that, in period 1, the league maximizes

$$D = (\delta + \nu(W_{1;1} + W_{1;2})(1 + \frac{6}{6 - \gamma}) + (2K_{1;w} - K_1)(\frac{\gamma}{6} - \frac{6\nu}{6 - \gamma})$$

Hence, if $\gamma(6 - \gamma) > 36\nu$ then $dD/dK_{1;w} > 0$. By continuity, we derive that there exists $\underline{\alpha}$ such that if $\alpha > \underline{\alpha}$ $dD/dK_{1;w} > 0$, $\Delta K_1 > 0$. $\square$

The case of fully rational teams

Fully rational teams take into account the impact of their action at time 1 on their wealth in period 2. Since probabilities of winning in period 2 and the revenue of the league in period 2 depend on teams' wealth, it follows that they take into the influence of their action in period 1 on $p_{2;1}(i)$ and $K_2^*(k; i)$ ($k = d; jp$ and $i = 1; 2$). In period 2, the problem of the fully rational team is identical to that of a myopic team.

Formally, in period 1, fully rational team i solves the following problem

$$\begin{aligned} Max\, p_{1;i} & \left[K_{1;w} + p_{2;i}(i)K^*_{2;w}(k; i) + (1 - p_{2;i}(i))K^*_{2;l}(k; i) - e_{2;i}(i)\right] \\ + p_{1;i} & \left[K_{1;w} + p_{2;i}(j)K^*_{2;w}(k; j) + (1 - p_{2;i}(j))K^*_{2;l}(k; j) - e_{2;i}(j)\right] - e_{1;i} \end{aligned} \tag{37}$$

with $i \neq j$, $k = d; jp$, and $e_{2;i}(m)$ represents the effort produced by team i in period 2 if team m wins in period 1 ($m = 1; 2$).

In the corner cases $\alpha = 0$ and $\alpha = 1$, we are able to derive closed form solution in the effort game played by teams. First, if $\alpha = 0$, it is straightforward that teams do not produce any effort . Hence, the problem faced by the league is identical to the case with myopic teams. Therefore, the league sets $\Delta K_1 = 0$. If $\alpha = 1$ we have the following result.

Proposition 11 (i) Assume that the league maximize the demand for sport and $\alpha = 1$. Then

$$e_{1,1} = e_{1,2} = \frac{(3-\gamma)\Delta K_1}{12}$$

If $\gamma(3-\gamma) \geq 3\nu$ the league sets $\Delta K_1 = 5K_1/6$. If $\gamma(3-\gamma) < 3\nu$ the league sets $\Delta K_1 = 0$.

Proof: If $\alpha = 1$, then

$$K_2^*(d,1) = K_2^*(d,2) = \frac{3}{3-\gamma}(\delta + \nu(W_{1,1} + W_{1,2} + K_1 - e_{1,1} - e_{1,2}))$$

and $p_{2,j}(i) = 1/2$ $(i,j = 1,2)$. Proceeding as in the proof of Proposition 1, we obtain that the equilibrium effort produced by teams in period 1 is

$$e_{1,i} = e_1 = \frac{(3-\gamma)\Delta K_1}{2[2(3-\gamma) + 3\nu]}$$

The objective of the league in period 1 is to maximize

$$D = 2\left(\gamma - \frac{3\nu}{3-\gamma}\right)e_1 + \left(1 + \frac{3}{3-\gamma}\right)(\delta + \nu(W_{1,1} + W_{1,2}))$$

Hence, if $\gamma(3-\gamma) < 3\nu$, the league sets ΔK_1 so as to minimize the effort level produced by teams, i.e., $\Delta K_1 = 0$. Conversely, if $\gamma(3-\gamma) > 3\nu$ the leagues sets ΔK_1 so as to maximize the level of effort by teams, i.e.,

$$\frac{(3-\gamma)\Delta K_1}{2[2(3-\gamma) + 3\nu]} = K_{1,l}$$

Given that $\Delta K_1 = 2K_{1,w} - K_1$ and $K_{1,l} = K_1 - K_{1,l}$, we obtain

$$K_{1,w} = \frac{5(3-\gamma) + 6\nu}{6[(3-\gamma) + \nu]}$$

We deduce that $\Delta K_1 = 5K_1/6$. By continuity, it implies that if $\gamma(3-\gamma) > 3\nu$, there exists $\underline{\alpha}$ such that if $\alpha > \underline{\alpha}$, then $\Delta K_1 > 0$. $\square$

From Proposition 11 we deduce that fully rational teams choose a lower effort level than myopic teams. The reason is that they take into account the influence of their effort in period 1 on the demand of period 2 through their wealth. It follows that by decreasing their effort level, they increase their future wealth, hence increasing the revenue of the league in period 2 and their expected gain in that period.

References

Araton H. (1998): "Bizball", New York Times Magazine, October 18 1998.

Atkinson S., Stanley L., and Tschirhart, J. (1988): \Revenue sharing as an incentive in an agency problem: an example from the National Football League", Rand Journal of Economics, 19: 27-43.

Bird, P.W.J.N. (1982): \The demand for League Football", Applied Economics, 14: 637-649.

Daly G. (1992): \The Baseball Player's Labor Market Revisited" in Sommers P. Diamonds Are Forever: The Business of Baseball, Brookings Insitutionm Washington D.C..

El Hodiri, M. and Quirk, J. (1971): \An economic analysis of a prefessionnal sport league",Journal of Political Economy, 79: 1308-1319.

Fort, R. and Quirk, J. (1992): \Pay Dirt. The Business of Professional Sports", Princeton University Press, Princeton.

Fort, R. and Quirk, J. (1995): \Cross subsidization, incentives and outcome in professionnal team sports league", Journal of Economic Literature, XXXIII:1265-1299.

Hamil, S., Michie, J. and Oughton, C. (1999): \The business of football, Agame of two halves?", Mainstream Publishing, Edinburgh.

Hoen and Szymanski S. (1999): \The americanization of European football", Economic Policy, 28:

Scully, G. W., (1995): \The Market Structure of Sports", The University of Chicago Press, Chicago.

Szymanski S. and Smith R. (1997): \The English Football Industry: Pro¯t, Performance, and Inustrial Structure", International Review of Applied Economics, 11: 135-153.

Vrooman, J. (1995): \A general theory of a professional sport league", 61: 971-990.

Country	Best/Worst
England	2.2
France	1.8
Germany	1.7
Italy	3.4

Table 1: Ratio of revenues for the season 1999-2000 is some top European soccer leagues. Source: L'Equipe.