

Value of Information Analysis in Environmental Health Risk Management Decisions: Past, Present, and Future

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Experts agree that value of information (VOI) analyses provide useful insights in risk management decisions. However, applications in environmental health risk management (EHRM) remain largely demonstrative thus far because of the complexity in modeling and solving VOI problems. Based on this comprehensive review of all VOI applications published in the peer-reviewed literature of such applications, the complexity of solving VOI problems with continuous probability distributions as inputs in models emerges as the main barrier to greater use of VOI although simulation allows analysts to solve more complex and realistic problems. Several analytical challenges that inhibit greater use of VOI techniques include issues related to modeling decisions, valuing outcomes, and characterizing uncertain and variable model inputs appropriately. This comprehensive review of methods for modeling and solving VOI problems for applications related to EHRM provides the first synthesis of important methodological advances in the field. The insights provide risk analysts and decision scientists with some guidance on how to structure and solve VOI problems focused on evaluating opportunities to collect better information to improve EHRM decisions. They further suggest the need for some efforts to standardize approaches and develop some prescriptive guidance for VOI analysts similar to existing guidelines for conducting cost-effectiveness analyses.

KEY WORDS: Bayesian decision theory; environmental health; risk analysis; risk management; value of information

1. INTRODUCTION

Value of information (VOI) analysis evaluates the benefit of collecting additional information to reduce or eliminate uncertainty in a specific decision making context. As noted in one of the earliest published VOI applications, “no theory that involves just the probabilities of outcomes without considering their conse-

quences could possibly be adequate in describing the importance of uncertainty to a decision maker.”^(1:26) VOI analysis makes explicit any expected potential losses from errors in decision making due to uncertainty and identifies the “best” information collection strategy as one that leads to the greatest net benefit to the decision maker (DM). The recent Presidential/Congressional Commission on Risk Assessment and Risk Management (1997) noted that “when stakes in a decision are large and the uncertainties complex, risk managers or their technical staffs may find it useful to experiment with formal value-of-information tools.”^(2:92) Although some of the earliest publications on decision analysis introduced methods for modeling and solving VOI analyses decades ago,^(1,3-7) unlike other economic analytic methods such as cost-benefit analysis and cost-effectiveness analysis, very

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few VOI applications in environmental health risk management (EHRM) exist. Moreover, recent analysis of VOI applications shows the tendency of articles to focus on demonstrating the usefulness of the VOI approach rather than on applications to actual management decisions.⁽⁸⁾

The lack of VOI applications in actual EHRM decisions appears to arise in part from the inherent complexities in modeling the underlying probabilistic risk assessment and decision analysis. The analyst must model all relevant sets of actions and information collection strategies available to the DM, capture all significant consequences of each action given all possible states of the world, value those outcomes in a common metric, and characterize important uncertainty, variability, and the accuracy of information to be collected by fitting probability distributions to available information. In addition, for VOI calculations analysts cannot model the risk assessment problem using point estimates. Instead, they must characterize the uncertainty quantitatively, and they cannot rely on a general monotonic relationship between VOI and action flexibility (i.e., increasing actions available to the DM) or the level of initial uncertainty in the prior distribution since no such relationship exists.⁽⁹⁾ With the increasing evolution of simulation strategies, examples of fairly complex VOI problems now appear in the literature. However, many analytical challenges remain and Ron Howard's prediction in 1967 still rings true: "[I]t is inevitable that in the future both technical and managerial decision-makers will employ formal logical methods in decision-making. The transition will probably be painful."^(6:60) Remarkably, no articles currently provide a comprehensive review of the existing VOI applications for EHRM issues, and this analysis fills the void by providing a critical review of 16 VOI applications.⁽¹⁰⁻²⁵⁾

2. MODELING VALUE OF INFORMATION ANALYSES

The expected value of perfect information (EVPI) represents the value of completely eliminating uncertainty (i.e., collecting information with perfect accuracy). For a risk-neutral expected utility maximizer with a linear or exponential utility function,⁽²⁶⁾ Equation (1) shows the EVPI about an uncertain input s :

$$\text{EVPI} = \int_{s \in S} \left[\max_{a \in A} u(a, s) \right] f(s) ds - \max_{a \in A} \left[\int_{s \in S} u(a, s) f(s) ds \right], \quad (1)$$

where $f(s)$ represents the probability distribution representing prior belief about the likelihood of s . The first term represents the weighted average of the utility associated with taking the optimal action for all possible values of s over the prior belief about the likelihood of s . The second term represents the expected utility from taking an action that yields the highest expected utility.

When the DM faces multiple sources of uncertainty, the expected value of perfect X information (EVPXI) (where X represents a particular uncertain model input) can yield a useful measure for determining the relative importance of resolving uncertainty between inputs. For example, if a DM faces two uncertain inputs x and y , Equation (2) gives the EVPI about x (called EVPXI):

EVPXI

$$= \int_{x \in X} \left[\max_{a \in A} \int_{y \in Y} u(a, x, y) f(y | x) dy \right] f(x) dx - \max_{a \in A} \left[\int_{y \in Y} \int_{x \in X} u(a, x, y) f(x, y) dx dy \right], \quad (2)$$

where $u(a, x, y)$ equals the utility of the DM, $f(y | x)$ gives the prior conditional probability of y given x , $f(x)$ represents the prior probability of x , and $f(x, y)$ equals the prior joint distribution of x and y . EVPXI thus becomes the difference between the expected utility from taking the optimal action based on the revelation of the exact value of one uncertain input, x , and the expected utility from the optimal decision given only the prior information. Although perhaps counterintuitive, the nonadditivity of EVPXI to yield the EVPI represents a well established property of EVPXI (i.e., the sum of EVPXI from all sources of uncertainty does not necessarily sum to the total EVPI for resolving all uncertainties simultaneously, even for independent model inputs and particularly for inputs with dependent uncertainties).^(1,6,27,28)

Since obtaining perfect information proves nearly impossible in most situations, the expected value of sample information (EVSI) or expected value of imperfect information represents the more relevant measure of information value in decision making. Calculating EVSI requires a preposterior analysis, which implies making a decision before the collection of information and receiving knowledge of the sample outcome. Bayesian updating of the probability of s for all possible sample information, t , begins with computing the posterior probability of s given observation t :

$$p(s | t) = \frac{f(s)g(t | s)}{h(t)}, \quad (3)$$

where $g(t | s)$ represents the likelihood function of observing t given a state of the world s , and $h(t)$ gives the predictive density of t :

$$h(t) = \int_{s \in S} f(s)g(t | s) ds. \quad (4)$$

Equation (5) provides the value of reducing but not eliminating uncertainty:

$$\begin{aligned} \text{EVSI} = & \int_{t \in T} \max_{a \in A} \left[\int_{s \in S} u(a, s)p(s | t) ds \right] h(t) dt \\ & - \max_{a \in A} \left[\int_{s \in S} u(a, s) f(s) ds \right]. \end{aligned} \quad (5)$$

EVSI represents the difference between the expected utility of taking the optimal action based on the posterior probability of s given experimental information t , and the expected utility from taking the optimal decision given only the prior information about s . In general, the predictive density of the information sampled (Equation (4)) lacks a closed-form solution, except for uncertainty expressed as a set of discrete probabilities or for select model structures featuring likelihood functions with conjugate priors.⁽²⁹⁾ In comparison, EVPI represents a simpler calculation than the EVSI and serves as a useful theoretical upper bound for the value of additional information for a particular decision context. All EHRM VOI analyses require several key elements including:

1. *A set of available actions and information collection strategies.* This may include discrete actions or continuous decision variables.
2. *A risk model with variability and uncertainty in its inputs clearly defined within the context of the decision.*^(27,30) This represents a particularly important aspect of EHRM models since, unlike uncertainty, true variability cannot be reduced by obtaining more information. As the National Research Council's Committee on Risk Assessment of Hazardous Air Pollutants stated: "uncertainty forces decision makers to judge how *probable* it is that risks will be overestimated or underestimated for every member of the exposed population, whereas variability forces them to cope with the *certainty* that different individuals will be subjected to risks both above and below any reference point one chooses."^(31:237) For discrete inputs, analysts may use a discrete set of value-probability pairs, while for continuous

inputs they may use distribution functions uniquely defined by parameters or empirical distribution functions based on a data set of observed values or based on subjective judgments from experts (which they characterize by employing a variety of techniques).⁽³²⁻³⁴⁾ Analysts must also characterize any dependence between multiple inputs in their joint distribution, a critical consideration in VOI analyses since dependencies significantly complicate VOI calculations.⁽³⁵⁾

3. *Values for the risk outcomes.* For societal decisions and those that impact large organizations that manage a portfolio of risks, typically analysts assume risk neutrality and rely on linear utility with valuation monetized. This necessitates often controversial choices about discounting and monetary equivalent values of health outcomes (e.g., the value of a statistical life).

All estimates for EVSI also require estimates of the accuracy of future information prior to its collection. As discussed by Brand and Small⁽³⁶⁾ within the context of the continuum from release of a substance into the environment to any ultimate health outcomes, numerous opportunities may exist to obtain information along the continuum and the actual point where the analyst obtains information may differentially impact overall uncertainty about the final result.

3. SOLVING VALUE OF INFORMATION ANALYSES

3.1. Overview

Analysts solve the simplest VOI applications (i.e., with uncertainty characterized for independent model inputs using discrete distributions) with "pencil and paper" by rolling back the decision tree. However, for more complex models they tend to use off-the-shelf software using decision trees or influence diagrams, or they write their own code to solve large and complex problems given the exponential computational effort associated with using a discrete approximation strategy. For a small number of carefully chosen models with continuous distributions, simple applications exist with closed-form solutions that yield exact values. In general, however, problems with continuous inputs prove more complex and they lead the analyst to use one of several numerical approximation methods.

The simplest numerical approximation method discretizes continuous inputs, allowing the analyst to solve problem with artificially discrete model inputs. Optimal strategies for choosing the discrete value probability pairs exist.⁽³⁷⁾ Alternatively, the analyst may use simulation, which relies on randomly sampling input values to calculate an output value for each iteration and iterating enough times to create an output distribution that offers a good approximation for making statistical inferences.⁽²⁷⁾ The quality of any approximation depends on how closely the input distributions used match the true distributions and error decreases with increasing computational effort, which remains linear in the number of uncertain inputs and number of iterations.

We illustrate differences in solution strategies using a classic example from one of the first published illustrations of the VOI approach: an optimal bidding problem by Howard.⁽⁶⁾ In this VOI problem, a company can make any bid to compete against competitors to win a contract for a project. The risk-neutral DM for the company seeks to maximize profits. The company only wins the contract if its bid comes in lower than the lowest bid of the competitors. However, since all bidders submit their offers in secret, the company lacks certainty about its competitors' bids. Given this formulation, the company maximizes the expected value of profit:

$$\text{profit} = (b - c)I(l > b), \quad (6)$$

where b represents the company's bid, c represents the uncertain cost of the project to the company, l represents the lowest bid of the competitors, and $I(l > b)$ provides an indicator variable that takes a value of 1 if the company wins the bid ($l > b$) and 0 otherwise. Howard⁽⁶⁾ characterized the two uncertain inputs with uniform distributions using a lower bound of 0 and an upper bound of 1 for the cost of the project, and a lower bound of 0 and an upper bound of 2 for the lowest bid of competitors. The analysis assumes that independence between the project cost, lowest competitor's bid, and the company's bid.

3.2. Solution Strategies

Howard⁽⁶⁾ presents the analytical solution to this VOI problem. Fig. 1 represents a schematic for solving the expected value of profit under different information schemes. With only prior information, we evaluate the expected value of profit for each possible bid

value and choose the bid that maximizes the expected profit:

$$E\{\text{profit} \mid \text{prior}\} \\ = \max_b \int_{l=0}^2 \int_{c=0}^1 (b - c)I(l > b) f(c) f(l) dc dl. \quad (7)$$

For expected value of profit with perfect information about cost, we first evaluate the expected value of profit with unknown lowest bid values, treating cost as a constant, and choose the bid that maximizes the expected profit. We then take a weighted average of the expected profit over all possible values of cost:

$$E\{\text{profit} \mid \text{cost}\} \\ = \int_{c=0}^1 \left\{ \max_b \int_{l=0}^2 (b - c)I(l > b) f(l) dl \right\} f(c) dc. \quad (8)$$

Similarly, for perfect information about lowest bid we solve:

$$E\{\text{profit} \mid \text{lowest}\} \\ = \int_{l=0}^2 \left\{ \max_b \int_{c=0}^1 (b - c)I(l > b) f(c) dc \right\} f(l) dl. \quad (9)$$

Under perfect information about both lowest bid and cost, we choose the bid that maximizes profit and then take a weighted average of the expected profit over all possible values of cost and lowest bid:

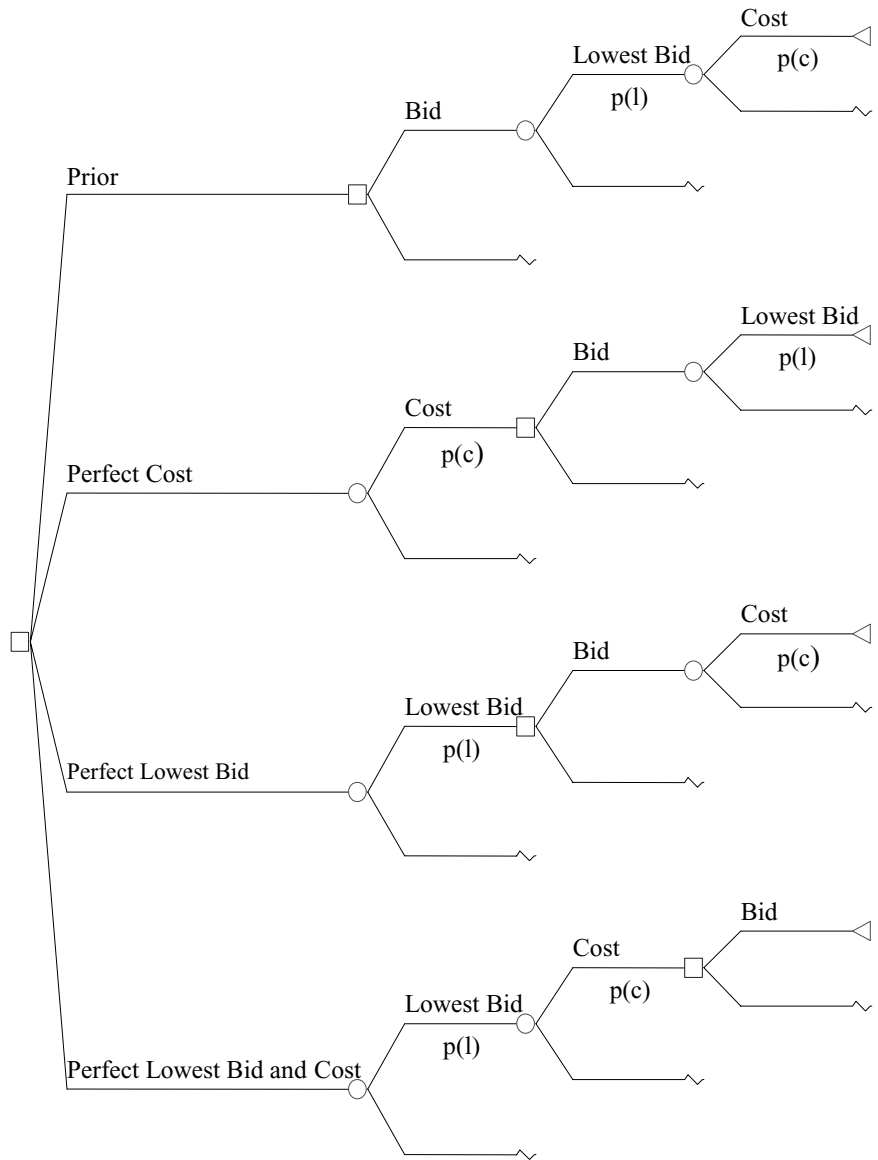
$$E\{\text{profit} \mid \text{both}\} \\ = \int_{l=0}^2 \int_{c=0}^1 \max_b \{(b - c)I(l > b)\} f(c) f(l) dc dl. \quad (10)$$

Given the nature of this problem, the first step requires using either discretization or simulation of the company's bid. We divide the continuous decision variable into increments of 0.01 from 0 to 1.99 for a total of 200 bid values (recognizing the domination of bids greater than 2 or less than 0).

3.2.1. Discretizing the Input Distributions

For the discretization approach, we demonstrate the impact of dividing the uniform distribution of each input into 10, 32, and 100 segments of equal probability, where we assign a value of the mean of each segment to generate the value-probability pairs. With the problem now entirely discrete, we calculate the

Fig. 1. Schematic of the optimal bid problem.



expected values simply with the collapsed discrete versions of Equations (7)–(10):

$$E\{\text{profit} \mid \text{prior}\} = \frac{1}{n^2} \max_{b_k} \left\{ \sum_{i=1}^n \sum_{j=1}^n (b_k - c_i) I(l_j > b_k) \right\}, \quad (11)$$

$$E\{\text{profit} \mid \text{cost}\} = \frac{1}{n^2} \sum_{i=1}^n \max_{b_k} \left\{ \sum_{j=1}^n (b_k - c_i) I(l_j > b_k) \right\}, \quad (12)$$

$$E\{\text{profit} \mid \text{lowest}\} = \frac{1}{n^2} \sum_{j=1}^n \max_{b_k} \left\{ \sum_{i=1}^n (b_k - c_i) I(l_j > b_k) \right\}, \quad (13)$$

and

$$E\{\text{profit} \mid \text{both}\} = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \max_{b_k} \{(b_k - c_i) I(l_j > b_k)\}, \quad (14)$$

where b_k represents the k th bid value, n gives the level of discretization, c_i refers to the i th discretized value

of cost, and l_j represents the j th discretized value of the lowest bid. Analysts can solve this problem using a decision tree program. However, due to the large number of bids, we used a program in S-Plus to implement the following calculation (available from the authors on request). With 200 bid values, the discretization approach requires the evaluation of $200n^2$ bid-cost-lowest bid scenarios for each equation (i.e., in this case 20,000, 204,800 and 20,000,000).

3.2.2. *Simulating the Input Distributions*

For the simulation approach, we use Latin Hypercube sampling in @Risk (Palisade Corporation) to generate 100 sets of 100 sample values for cost and lowest bid for a total 10,000 samples for each input. We then calculate output values using subsets of the samples ($n = 100, n = 1,000,$ and $n = 10,000$) in S-Plus. Calculation of expected values differs for the simulation approach since each of the n pairs of randomly sampled cost and lowest bid values yields a value of profit with probability equal to $1/n$. For the case with only prior information, we first evaluate the mean of profit for each sample given a particular bid value and then choose the bid that maximizes the expected profit:

$$E\{\text{profit} \mid \text{prior}\} = \max_{b_j} \frac{1}{n} \sum_{i=1}^n (b_j - c_i) I(l_i > b_j), \tag{15}$$

where b_j represents the j th bid value, and c_i and l_i represent the values of cost and lowest bid from sample realization i , respectively. As Equation (15) shows, the simulation approach requires the evaluation of $200n$ (i.e., 20,000, 2000,000, and 2,000,000) bid-cost-lowest bid scenarios. Similarly, Equation (16) gives the expected value of profit from perfect information about both cost and lowest bid:

$$E\{\text{profit} \mid \text{both}\} = \frac{1}{n} \sum_{i=1}^n \max_{b_j} \{(b_j - c_i) I(l_i > b_j)\}. \tag{16}$$

Finding the EVPXI requires taking one value as fixed and the other as uncertain, which adds a level of complexity. Using a discretization approach to solve Equations (12) and (13) with the simulated input values as discrete values offers one strategy, but it requires the evaluation of $200n^2$ scenarios (i.e., 200,000,000 for 1,000 iterations). In general, when using probabilistically independent input distributions with an output value function linear in the other uncertain input, then we can substitute the expected

value of the input in the output value function and solve for the EVPI of the other input as if only one uncertain input exists.^(27,38) In the case of this optimal bid problem, we can manipulate the analytical solution for the profit, given lowest bid with cost linear in profit shown in Equation (9) to get:

$$\begin{aligned} & \int_{l=0}^2 \left\{ \max_b \int_{c=0}^1 (b - c) I(l > b) f(c) dc \right\} f(l) dl \\ &= \int_{l=0}^2 \left\{ \max_b (b - E\{c\}) I(l > b) \right\} f(l) dl. \end{aligned} \tag{17}$$

Thus, for the simulation approach, we evaluate the following:

$$E\{\text{profit} \mid \text{lowest}\} = \frac{1}{n} \sum_{i=1}^n \max_{b_j} \{(b_j - \bar{c}) I(l_i > b_j)\}, \tag{18}$$

where \bar{c} refers to the mean of the sampled cost values:

$$\bar{c} = \frac{1}{n} \sum_{i=1}^n c_i. \tag{19}$$

Analysts must be careful to consider the relationship of an uncertain input to the output value function since the expected value of a function of random variables does not necessarily equal to the function of the expected values of the random variables.⁽²⁷⁾ For the expected value from perfect information about cost, we cannot substitute expected value of lowest bid given its nonlinearity in profit. By manipulating Equation (8) we get:

$$\begin{aligned} & \int_{c=0}^1 \left\{ \max_b \int_{l=0}^2 (b - c) I(l > b) f(l) dl \right\} f(c) dc \\ &= \int_{c=0}^1 \left\{ \max_b (b - c) E\{I(l > b)\} \right\} f(c) dc, \end{aligned} \tag{20}$$

where $E\{I(l > b)\}$ refers to the expected value of the indicator variable, or the probability that a particular bid will win. Therefore, for the simulation approach, we evaluate Equation (21):

$$E\{\text{profit} \mid \text{cost}\} = \frac{1}{n} \sum_{i=1}^n \max_{b_k} \{(b_j - c_i) \bar{I}_j\}, \tag{21}$$

where \bar{I}_j represents the mean of the indicator variable for each bid:

$$\bar{I}_j = \frac{1}{n} \sum_i I(l_i > b_j). \tag{22}$$

Theses nonintuitive substitutions create complexities for analysts, and in some cases analysts used incorrect

calculations. For example, in a medical decision making VOI problem with multiple independent uncertain inputs,⁽³⁹⁾ the authors calculated EVPXI as the difference between the EVPI (from resolving all uncertainties), and the EVPI from substituting the expected value of x (i.e., the $EVPI | \bar{x}$) using Equation (23):

$$EVPXI = EVPI - EVPI | \bar{x}. \tag{23}$$

In the limited case of only two uncertain, independent inputs, x and y , in a linear model:

$$EVPI | \bar{x} = \int_{y \in Y} \left\{ \max_{a \in A} u(a, \bar{x}, y) \right\} f(y) dy - \max_{a \in A} \int_{y \in Y} \int_{x \in X} u(a, x, y) f(x) f(y) dx dy. \tag{24}$$

However, the EVPI given the expected value of x equals the expected value of the utility with respect to x :

$$u(a, \bar{x}, y) = \int_{x \in X} u(a, x, y) f(x) dx. \tag{25}$$

Therefore, $EVPI | \bar{x}$ represents EVPYI or:

$$EVPXI = EVPI - EVPYI. \tag{26}$$

The only condition where both Equations (23) and (26) hold true occurs in the extremely rare case where

the EVPXI and EVPYI sum to the EVPI from resolving both uncertainties simultaneously.

3.3. Computational Insights

Table I provides the exact analytical solutions, estimates obtained from discretization of uncertain inputs into 10, 32, and 100 value-probability pairs, and estimates obtained from simulations using sample sizes of 100, 1,000, and 10,000. The analytical solution to the optimal bid problem shows that the expected value of perfect information about lowest bid (EVPLI) and of perfect information about cost (EVPCI) do not sum to the EVPI that occurs with simultaneous resolution of both uncertainties. Moreover, the analysis shows that perfect information about lowest bid appears much more valuable to the company than perfect information about cost.

The table shows that given a level of computational burden, measured by the number of scenarios that must be evaluated, the simulation approach tends to yield estimates closer to the analytical solution per calculation for this relatively simple problem with just two uncertain inputs. Better accuracy for discretization may be achieved if we use more sophisticated techniques for discretizing the distributions such as Gaussian quadrature, but this adds another dimension of computational complexity requiring the

Table I. Solutions to the Optimal Bid Problem in Howard

	Scenarios (000's)	Expected Profit				EVPI		
		Prior	Cost	Lowest	Both	Cost	Lowest	Both
Analytical	—	0.28	0.29	0.56	0.58	0.01	0.28	0.30
Discretize								
$n = 10$	20	0.32	0.33	0.55	0.58	0.01	0.24	0.26
$n = 32$	204.8	0.29	0.30	0.56	0.58	0.01	0.27	0.29
$n = 100$	2,000	0.28	0.29	0.56	0.58	0.01	0.27	0.30
Simulate (1 set)								
$n = 100$	20	0.26	0.30	0.56	0.57	0.03	0.29	0.30
$n = 1,000$	200	0.28	0.30	0.56	0.58	0.01	0.28	0.30
$n = 10,000$	2,000	0.28	0.29	0.56	0.58	0.01	0.28	0.30
Simulate (10 sets)								
$n = 100$	200							
Mean		0.29	0.30	0.56	0.57	0.01	0.27	0.29
95% CI of mean		0.27–0.30	0.30–0.30	0.56–0.56	0.567–0.581	–0.003–0.021	0.260–0.285	0.278–0.297
Range		0.26–0.32	0.30–0.30	0.558–0.560	0.558–0.589	–0.024–0.031	0.240–0.294	0.262–0.304
$n = 1,000$	2,000							
Mean		0.283	0.293	0.559	0.580	0.010	0.276	0.297
95% CI of mean		0.281–0.285	0.293–0.293	0.559–0.559	0.578–0.583	0.007–0.012	0.273–0.278	0.298–0.298
Range		0.280–0.289	0.292–0.293	0.558–0.559	0.575–0.585	0.003–0.013	0.269–0.279	0.294–0.299

Source: Reference 6.

strategies, types of VOI analyses, number of uncertain inputs, inclusion of analysis by subgroup to account for variability, types of probability distributions used, whether the analysts used expert judgment or outputs of models as a source for input distributions, and types of outcomes the analysis specifically valued.

4.1. Modeling the Set of Available Actions and Information-Collection Options

The discrete analyses^(10–14) focused on EVSI from toxicological testing in determining the carcinogenicity of chemicals. Since these articles illustrated the VOI framework for a generic management decision rather than informing a specific real decision, they considered the very limited set of only two regulatory actions (i.e., do nothing or do something). These analyses assumed that regulators would follow a hypothetical, preset, certain control decision if testing the chemical suggested carcinogenicity. Most of these analyses consider only two information-collection strategies. Two articles by Lave and Omenn^(10,11) considered the value of regulating chemicals based on the results of a short-term test for carcinogenicity compared to allowing chemicals to go unregulated. Lave *et al.*⁽¹²⁾ evaluated the benefit of conducting rodent bioassays to no testing. In contrast, Olson⁽¹³⁾ compared acting on prior information to conducting a mutagenicity test and then evaluating whether to collect bioassay information or not. Omenn *et al.*⁽¹⁴⁾ went a step further and compared 13 different approaches developed by various researchers that combine information on structure-activity relationships, short-term tests, sub-chronic rodent assays, and/or expert judgment for predicting the results of a lifetime rodent bioassay.

The analyses that discretized continuous uncertain inputs represent much more complex decision problems. The earliest two analyses focused on characterizing model uncertainty by measuring the value of perfect information, and they did not evaluate specific information-collection schemes. North and Merkhofer⁽¹⁵⁾ calculated the EVPI from resolving uncertainties in choosing four alternative strategies to control pollution emissions for three representative power plant types. Finkel and Evans⁽¹⁶⁾ described a VOI framework for environmental management and calculated the EVPI and EVPXI for dose of a hypothetical risk management problem with three alternatives.

The next three analyses, on the other hand, evaluated specific information-gathering strategies. Evans

et al.⁽¹⁷⁾ modeled homeowner's EVSXI from monitoring radon in the home in choosing one of five remediation strategies (with two of the five always dominated and not included in the final calculation). Reichard and Evans⁽¹⁸⁾ considered the EVPI, EVPXI, and EVSXI of four options for monitoring groundwater for arsenic in a decision to install or not install point of use drinking water filtration system. Taylor *et al.*⁽¹⁹⁾ assessed the EVPI and EVSI of the animal bioassay's ability to determine magnitude of cancer-causing potential above and beyond determining its carcinogenicity. Using the simplistic choice of actions (i.e., act or not), they provided general insights about the three strategies for collecting toxicological information: collect none, use only a subchronic bioassay, or conduct additional long-term bioassay.

The last analysis in this group evaluated the benefit of a two-stage approach to pollution control, which allowed the incorporation of information collected from the first stage in the decision at the second stage.⁽²⁰⁾ Chao *et al.*⁽²⁰⁾ calculated EVSI from waiting for more information to choose the optimal levels of control of both nitrogen oxides and volatile organic compounds to reduce tropospheric ozone based on dividing the level of emission reduction into five levels for each pollutant (0, 20, 40, 60, and 80%) for each stage, which yielded 625 action scenarios.

The analyses that used simulation include two that considered only two action alternatives, and two others that did not consider specific information-collections strategies. Dakins *et al.*^(21,22) evaluated the value of resolving uncertainty about PCB contamination in fish to assist in choosing the optimal level of remediation of contaminated sediments in New Bedford Harbor, MA. Dakins *et al.*⁽²¹⁾ evaluated EVPI for the model and Dakins *et al.*⁽²²⁾ conducted a preposterior analysis to evaluate the EVSI from sampling 2, 5, and 10 randomly selected flounder from New Bedford Harbor. Thompson and Evans⁽²³⁾ calculated the EVPI and EVPXI from collecting national exposure information about perchloroethylene (perc) used in dry cleaning. Unlike other applications, this analysis compared regulating perc exposure at three different levels of decision making: individual dry-cleaning facilities, by particular dry-cleaning machine category (defined by type and size), and by particular machine type, with several control options for each type of machine (a total of 11).

The remaining two analyses, in contrast, focused on optimal information collection, and simple choice of action or no action. Lin *et al.*⁽²⁴⁾ evaluated the EVSI from measuring radon concentrations in private

homes to assist in the decision to take remediation action or not, and compared four different policies for monitoring strategies at the national level. Bartell *et al.*⁽²⁵⁾ evaluated the EVSI from a screening program to prevent chronic beryllium disease (CBD) from occupational exposure. The analysis evaluates the value from resolving the uncertainty in the presence or absence of a genetic polymorphism that makes an individual susceptible to CBD. They compared three different strategies for screening to doing nothing, where a “positive” screening result leads to an intervention that would lead to either early treatment of the disease or prevention of exposure to beryllium.

4.2. Characterizing Variability and Uncertainty

The discrete analyses represented simple models that did not include any characterization of variability. They focused on characterizing the value of resolving uncertainty about a single dichotomous input (i.e., carcinogenicity of a chemical or not) and used hypothetical point estimates of prior probability of carcinogenicity and point estimates for the likelihood of test results based on empirical data. The articles by Lave and Omenn^(10,11) assumed a range of point estimates for the sensitivity and specificity of rodent bioassays in predicting human carcinogens. Lave *et al.*⁽¹²⁾ assumed point estimates for both sensitivity and specificity based on empirical evidence for each type of testing. Olson⁽¹³⁾ also used a hypothetical prior but disaggregated likelihood into the four categories of possible bioassay results established by the U.S. National Toxicology Program (NTP), and assumed discrete probability values based on available empirical evidence. Omenn *et al.*⁽¹⁴⁾ compared 13 approaches for predicting results of a lifetime rodent bioassay (carcinogenic or not) developed through the Carcinogenic Prediction Challenge sponsored by the NTP. They used a hypothetical point estimate of prior probability and the sensitivity and specificity values implied by the strategies for 44 chemicals.

The analyses that used a discretization approach tended to include only a couple of continuous uncertain inputs, with only a couple of them explicitly addressing variability.^(15,17) These studies used log-normal distributions for prior distributions and/or likelihood functions that allowed analytical solutions for the product of inputs and posterior distributions. The EVPI analysis by North and Merkhofer⁽¹⁵⁾ evaluated the value of simultaneously resolving two uncertainties in the model: how a unit of emission translates to ambient concentration and the total health

cost per unit increase in suspended sulfate concentration, which they assumed as independent in the analysis. The authors relied on their subjective judgment of extreme values to represent the 5th and 95th percentile points on the cumulative probability distribution. They provided a sketch of the cumulative distribution for the ambient sulfate concentration increment and assumed a log-normal distribution for the total health cost. To account for variability in the total health cost on local population density and fuel-burning technology, they solved the optimization problem for three types of power plants: an existing coal plant in a rural area, a new construction in a rural area, and an oil-burning plant (originally designed for coal) in an urban East Coast location.

Finkel and Evans⁽¹⁶⁾ modeled uncertainty about the health risk from a contaminant as the product of two components: dose and exposure. They assumed a log-normal distribution for both components and consequently the uncertainty about risk was also log-normal. They used hypothetical parameter values to illustrate how EVPI varied with different prior beliefs about the uncertainty in risk. Evans *et al.*⁽¹⁷⁾ used the approach established by Finkel and Evans⁽¹⁶⁾ to characterize the uncertainty in exposure to radon in individual homes and its potency in causing cancer. They developed log-normal distributions for both components using formal expert judgment elicitation. One expert set parameter values for the prior distribution for exposure based on previously available monitoring information (e.g., regional monitoring data, monitoring in a neighbor’s home), and accounted for variability in radon exposure by setting different parameter values based on region of the country and characteristic of the home. A different expert provided parameters for a log-normal distribution of excess relative risk of cancer to the general population based on epidemiological data available for occupational exposure of radon to miners. Evans *et al.*⁽¹⁷⁾ included additional variability in potential benefits from monitoring by analyzing the VOI to household of four representative demographic compositions.

Reichard and Evans⁽¹⁸⁾ used a similar approach and included two uncertain inputs characterized by a log-normal distribution: the potency of arsenic in causing cancer and the exposure to arsenic in well water. They assumed a log-normal distribution for potency, and fit a multistage dose-response model to epidemiological data to estimate the geometric standard deviation of potency. For exposure to arsenic, however, they used simulation to propagate uncertainty in a hydrogeologic model with five uncertain inputs

characterized by uniform distributions. Instead of using the output of the simulation to characterize the uncertainty in exposure they fit a log-normal distribution for use in subsequent analysis. They also used an empirically based likelihood function to characterize the error in concentration measurement.

Unlike the other analyses in this group, Taylor *et al.*⁽¹⁹⁾ included only one uncertain input in their model related to the carcinogenic potency of a chemical. They based the prior distribution of potency on the results of the first 213 NTP mouse bioassay results. Since the NTP determined only half of the chemicals yielded positive results, for the base case analysis, Taylor *et al.*⁽¹⁹⁾ characterized the potency distribution as the sum of a delta function at zero potency with a probability mass of 50%, and a log-normal distribution fit to the statistically significant test results and normalized so that the entire distribution integrates to unity. They modeled the likelihood of test results as a binomial distribution and created a matrix of values to solve for the values of the posterior distribution. Chao *et al.*⁽²⁰⁾ expressed uncertainty in current emission rates of nitrogen oxides and volatile organic compounds as uniform distributions, and uncertainty in the photochemical model expressed as a log-normal distribution. They chose distributions and parameter values to represent their judgment of the best available information. The analysis models information available after the first stage as a sampling outcome with a hypothetical point estimate for accuracy.

Analyses that use simulation as solution strategies tended to include a number of continuous uncertain inputs and used a variety of parametric distributions to characterize uncertainty, with two of these analyses explicitly including variability.^(23,24) The EVPI analysis in Dakins *et al.*⁽²¹⁾ evaluated simultaneously resolving all six uncertain inputs in the model, such as PCB concentration in the sediment, average water temperature, and growth rate of flounder. They described each input by a normal, triangle, or uniform distribution to reflect the best-available information for New Bedford Harbor collected for a previous food chain model. Dakins *et al.*⁽²²⁾ modeled the prior distribution for body burden by simulating 50 replications using the model established in Dakins *et al.*⁽²¹⁾ and focusing on only the uncertainty in the PCB body burden of flounder (derived from six uncertain inputs in the previous analysis). They conducted a preposterior analysis to evaluate the EVSI from sampling 2, 5, and 10 randomly selected flounder from New Bedford Harbor assuming a normal distribution for the likelihood of observing a particular set of body

burden measurements given a true value of total body burden. For the case of sampling five flounders, they repeated the simulation five times with different random seeds to check the robustness of the calculation using simulation results, and this study represents the only article that reports multiple simulation results.

Thompson and Evans⁽²³⁾ considered 14 uncertain inputs, such as potency of perc in causing cancer, fraction of inhaled perc metabolized, perc's lifetime in the atmosphere, and uncertainty in predictions of a Gaussian dispersion model. They characterized the inputs using log-normal, triangular, or uniform distributions to best reflect the empirical evidence. The authors also used an empirical distribution to characterize perc's potency developed in a previously published risk assessment. They developed two inputs, fraction of time spent at a dry-cleaning facility by consumers and workers, based on informal discussions with local dry cleaners. They considered variability by modeling risks to four distinct populations: dry-cleaning workers, families of workers, consumers of dry-cleaning services, and the general public from ambient exposure. Unlike all of the other analyses that use simulation, this article includes an EVPXI analysis to evaluate the relative importance of the different sources of uncertainty. In this case, the authors found that the individual EVPXIs sum to a number larger than the overall EVPI.

In the analysis by Lin *et al.*⁽²⁴⁾ the concentration of radon represented the only uncertain model input. They characterized the prior distribution using a log-normal distribution based on a hierarchical linear regression model that fit county-level explanatory variables to radon measurements, yielding parameter values that varied by county and housing type. Additionally, they accounted for variability in risk of cancer based on gender and smoking status. For the base case, the authors assumed that long-term monitoring produces an unbiased, log-normally distributed estimate of concentration such that the posterior distribution of true concentration given the measurement would also distribute log-normally, an assumption that led to some criticism that the analysts essentially confined the analysis to simple, parametric distributions.⁽⁴¹⁾

Bartell *et al.*⁽²⁵⁾ evaluated using a probabilistic risk assessment resolving the uncertainty in the presence or absence of a genetic polymorphism that makes an individual susceptible to CBD. The model included seven uncertain inputs, such as sensitivity and specificity of the genetic screening test, cost of testing, cost of genetic counseling, and risk reduction efficacy from interventions. They characterized

the model inputs using beta, triangular, and uniform distributions to reflect their best judgments of available information. They used a point estimate for the prior prevalence of susceptible individuals, and updated this value based on information collected from three screening strategies with uncertain sensitivity and specificity to estimate the posterior probability of genetic polymorphism.

4.3. Modeling and Valuing Outcomes

Table III summarizes the health outcomes evaluated by the analyses and values used to calculate VOI. As the table shows, the discrete analyses used a simple cost-benefit framework, and did not use sophisticated measures of outcomes. Most of the discrete analyses^(10–12,14) evaluated a lump sum for two consequences: a regulatory false positive assumed to impose a net cost to society from unnecessarily regulating a noncarcinogen of \$1 million, or a regulatory false negative that imposes a net cost to society from

not regulating a carcinogen of \$10 million. These analysts used the same baseline values consistently in their articles published between 1986 and 1995. None of these analyses reported a discount rate or whether the analysts adjusted the monetary values to a common year dollar. In contrast, Olson⁽¹³⁾ calculated the number of cases of cancer from a hypothetical unregulated carcinogen and valued them at \$2 million (1986 dollars) using a discount rate of zero.

The articles that discretized continuous uncertain inputs tended to develop much more sophisticated cost-benefit analyses and to use more refined measures of outcomes. North and Merkhofer⁽¹⁵⁾ identified and valued a variety of health endpoints from air pollution, including premature death (\$30,000), aggravation of heat and lung disease symptoms (\$20/day), asthma attack (\$10/case), a child's lower respiratory disease (\$75/case), and chronic respiratory disease (\$250/case). They also estimated the cost of ecological damage (\$0.015 per pound of sulfur emitted), and esthetic effects (\$0.034 per pound of sulfur emissions),

Table III. Valuation of Outcomes in EHRM VOI Analyses (Grouped by Solution Method)

Reference	Discount Rate	Year Dollar	Baseline	Low	High	Outcome
Discrete						
Lave and Omenn (1986) ⁽¹⁰⁾	NR	NR	\$10 million	–	–	Lump sum per unregulated carcinogen
Lave and Omenn (1988) ⁽¹¹⁾	NR	NR	\$10 million	–	–	Lump sum per unregulated carcinogen
Lave <i>et al.</i> (1988) ⁽¹²⁾	NR	NR	\$10 million	–	–	Lump sum per unregulated carcinogen
Olson (1990) ⁽¹³⁾	0%	1986	\$2 million	–	–	Cancer death (generic)
Omenn <i>et al.</i> (1995) ⁽¹⁴⁾	NR	NR	\$10 million	–	–	Lump sum per unregulated carcinogen
Discretized Continuous						
North and Merkhofer (1976) ⁽¹⁵⁾	NR	NR	\$30,000	–	–	Premature death of chronically ill from air pollution
Finkel and Evans (1987) ⁽¹⁶⁾	NR	NR	\$1 million	\$250,000	\$4 million	Cancer death (generic)
Evans <i>et al.</i> (1988) ⁽¹⁷⁾	3%	NR	\$3 million	\$1 million	\$10 million	Cancer death from radon
Reichard and Evans (1989) ⁽¹⁸⁾	5%	NR	\$1 million	\$50,000	\$10 million	Cancer death from arsenic
Taylor <i>et al.</i> (1993) ⁽¹⁹⁾	5%	NR	\$10 million	–	–	Cancer death (generic)
Chao <i>et al.</i> (1994) ⁽²⁰⁾	5%	1989	\$1 billion			Lump sum per ppm of ozone if peak concentration exceeds 0.12 ppm
Simulation						
Dakins <i>et al.</i> (1994) ⁽²¹⁾	0%	1985	–	–	–	No health
Dakins <i>et al.</i> (1996) ⁽²²⁾	0%	1985	–	–	–	No health
Thompson and Evans (1997) ⁽²³⁾	5%	1989	\$3 million	\$1 million	\$10 million	Cancer death from perc
Lin <i>et al.</i> (1999) ⁽²⁴⁾	5%	NR	\$210,000*	–	–	Cancer death from radon
Bartell <i>et al.</i> (2000) ⁽²⁵⁾	0–7%	NR	–	\$12,200	\$16 million	Illness and death from CBD

*Implied by the standard chosen.

Note: NR = not reported.

but did not report whether they adjusted all monetary values to a common year dollar or a discount rate.

Four analyses^(16–19) modeled dose response and exposure to estimate the risk of developing cancer from various carcinogens and used a nominal value of life ranging from \$1 million to \$10 million (with an even wider range in sensitivity analyses). Since the earliest analysis used the smallest baseline value and the latest used the largest, the difference in real value appears greater, although we cannot calculate it exactly because these analyses did not report what year dollar the values represented. The discount rate in these analyses ranged from 3 to 5%. Chao *et al.*⁽²⁰⁾ did not specifically measure a health endpoint, but instead estimated a lump-sum value of \$10 billion per year per ppm of peak ozone concentration exceeding 0.12 ppm (in 1989 dollars).

The analyses that used simulation as a solution strategy took much more varied approaches to modeling outcomes. Two analyses^(23,25) used a risk assessment approach to evaluate the net societal costs from health damages caused by exposure to perc and CBD, respectively. Thompson and Evans⁽²³⁾ valued each premature death from cancer at \$3 million in 1989 dollars, with a range of \$1 million to \$10 million in sensitivity analyses, and used a consistent discount rate of 5% in the base case analysis to account for time preference. Bartell *et al.*⁽²⁵⁾ valued a case of CBD prevented using four different estimates ranging from a low of \$12,200, which considered only future medical costs averted discounted at 7%, to a high of \$16,300,000, which represents an upper value of a statistical life ignoring disease latencies and discounting (no year dollar given).

In contrast, Lin *et al.*⁽²⁴⁾ modeled risks of cancer from radon exposure, but rather than using a specific value per life to drive the analysis, they used an action level as the benchmark, assuming that above the set level of exposure remediation should occur. They predetermined the action level based on household composition, a household's risk preference, and WTP for risk reduction. The objective focused on minimizing total cost, which included residual risk after remediation. In the base case analysis, the authors chose the EPA's action level using an annual living area average concentration of 4 pCi/L. Assuming a household consists of the average number of male and female smokers and never-smokers in the United States, the action level implies a value of \$210,000 per life. They used a discount rate of 5% to account for time preference, but did not report whether they adjusted all monetary values to a common year dollar.

In contrast, Dakins *et al.*^(21,22) focused on choosing an optimal level of dredging to minimize remediation costs while meeting a health-based standard for PCB concentration in fish set by the FDA. The analysis, which evaluated the cost of underremediation in 1985 dollars and use a 0% discount rate, assumed future perfect knowledge of the correct level of remediation, and it assumed that meeting the standard implied no residual health risk. If insufficient remediation occurred, then this required additional remediation to meet the standard and led to penalties (e.g., the fishery remained closed for longer time and additional costs of remobilizing research and remediation efforts). These studies did not account for health risks from suboptimal dredging or any health risks that would exist either during or after remediation.

5. REMAINING ANALYTICAL CHALLENGES

The VOI analyses in EHRM decisions to date represent primarily demonstrations of the usefulness of approaching a management problem using a VOI framework. While simulation appears to allow analysts to solve more complicated and realistic problems, important barriers remain.

With respect to modeling, none of the previous analyses addressed nonlinearity in risk models. Though analysts generally model cancer risks as a linear function of dose, current procedures for non-cancer risk assessment require the modeling of a threshold below which no detrimental effects occur. As discussed previously, when nonlinear inputs exist in the output value function, analysts cannot calculate the EVPXI with simulation using a simple substitution of expected value of that input. Further, none of the EHRM analyses included correlated uncertain input distributions or assessed the impact of dependence on information collected. This suggests that dealing with probabilistic dependencies remains one of the greatest challenges, as foreseen by Howard.⁽³⁵⁾

Not surprisingly, the challenges associated with obtaining appropriate data for characterizing model input distributions remain paramount.⁽³²⁾ Collecting additional information may lead to surprises that reveal incorrect basic assumptions, such as observing input values outside the bounds of prior belief in those cases where the model used overconfident priors.^(42,43) Characterizations based on expert judgment may be overly narrow due to tendency of both lay people and experts to be overconfident in their knowledge⁽⁴⁴⁾ and using more than one expert leads

to issues related to combining and weighting the information from experts and dealing with their lack of independence.⁽⁴⁵⁾

Further efforts to improve characterization and distinction of variability and uncertainty promise to continue to improve VOI applications. As the case of mandatory passenger-side airbags in motor vehicles shows, significant consequences derive from ignoring variability in a decision analysis.⁽⁴⁶⁾ In addition, since more information does not reduce true variability, a distribution that combines uncertainty and variability in VOI analyses may yield inappropriate results and analysts must use care to treat the uncertainty and variability appropriately and transparently in the context of the desired characterization of risk.^(27,47) For example, for a decision where a risk manager seeks to control the total number of cases of a disease, the analyst may collapse the variability in the population into the uncertainty in the model and that distribution should represent the uncertainty about the mean (i.e., the standard error), not the standard deviation of individuals in the population.^(23,27,32)

The review suggests that analysts may not sufficiently deal with the issue of potential errors in VOI analyses stemming from numerical approximation methods. Only one analysis, Dakins *et al.*,⁽²³⁾ reported multiple simulation results to show the robustness of the VOI estimate, although the results by Thompson and Evans⁽²⁴⁾ actually represented the mean of 10 simulations of 10,000 iterations each. As computational capabilities of personal computers increase, large simulations become increasingly faster and cheaper and consequently the time required to set up the model in many cases now represents the slowest part of the process.

Nonetheless, analysts must still evaluate the benefit of performing increasingly complex VOI analyses in the context of the actual decision⁽³¹⁾ and recognize that even without formal analysis, the VOI framework can provide helpful insights for determining the appropriate balance between taking action and waiting for more information. As Howard stated: “[O]ne of the arts of the decision analyst is the art of knowing how much and what kind of decision analysis to do. The degree of analysis can range from making simple lists to constructing giant interactive computer models. To be effective decision analysis must be ‘appropriate’: the extent of the analysis must be suitable to the means and ends of the decision-maker.”^(35:22) A National Research Council report more recently asserted that “value-of-information analysis can be of considerable use in the analytic-deliberative pro-

cess”^(48:111) and suggested that “risk characterization should be a decision-driven activity, directed toward informing choices and solving problems.”^(48:155)

Valuation issues also present important challenges for analysts. Though economists calculate a societal value for averting premature morbidity and mortality, no widely agreed value exists.^(49,50) Not surprisingly, the values analysts choose to use vary widely, leading to challenges since the choice of value of life can dramatically impact the VOI results.⁽²³⁾ In addition, empirical evidence from both revealed preference and stated preference studies show that factors such as age, income, baseline mortality risk, and latency of the risk influence the value of statistical life (VSL), but only income shows a predictable, monotonic relationship.⁽⁵¹⁾ Similarly, no consensus exists on what discount rate to use to reflect societal time preference, an important model input for decisions related to diseases that may have long latencies, and latency in cancer remains a very difficult factor to include in risk and decision analytic models.

For the societal perspective, analysts need to evaluate all relevant contexts where DMs might find utility in information. VOI can inaccurately estimate the true societal value of perfect information if positive or negative externalities arise from the information collection while not explicitly modeled in the decision. For example, toxicological information about one substance may be used for the specific regulatory decision about controlling that substance, but it may also be used in models to predict toxicity of substances with similar structures. While some analysts discuss this issue, we see no progress on developing strategies to deal with it in VOI analyses. Another continuing challenge relates to valuing nonmonetary outcomes, but perhaps one of the largest hurdles involves creating forums and processes for risk analysts and economists to collaborate more effectively in developing integrated analyses to support EHRM decisions.⁽⁵²⁾

6. CONCLUSION

Rigorous VOI analyses provide opportunities to evaluate strategies to collect information to improve EHRM decisions. This review of the methodology and applications shows that advances in computing tools allow analysts to tackle problems with greater complexity, although the literature still lacks “real” applications, probably due to a number of barriers. These barriers include the lack of guidance from EPA and others on criteria for standardizing EHRM risk and

decision analyses, the lack of consensus on values to use for health outcomes, the lack of default probability distributions for frequently used inputs, and inexperience of risk managers and communicators with using probabilistic risk results.^(27,30) In addition, it remains analytically challenging to model decisions that use all available information, deal with nonlinear inputs, and include correlation in input distributions and dependence in information collected. We suggest that an effort to bring analysts together to discuss strategies for standardizing methods might provide some useful guidance, similar to the experience in the field of clinical decision making when the U.S. Public Health Service convened the Panel on Cost-Effectiveness in Health and Medicine in 1993.⁽⁵³⁾ We expect that future reviews may also play an important role in synthesizing the experiences of analysts, encouraging analysts to publish their work in peer-reviewed journals, and demonstrating the state of the field.

ACKNOWLEDGMENTS

Dr. Yokota received support for this research from the U.S. Environmental Protection Agency, Office of Research and Development, National Center for Environmental Research, Science to Achieve Results Program (grant no. U-915561-01-1). The views expressed in this article are the authors' personal views and do not represent the views of the Office of Management and Budget or the administration.

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