RESEARCH ARTICLE

Outage, Throughput and Energy Efficiency Analysis of Some Half and Full Duplex Cooperative Relaying Schemes

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ABSTRACT

In this paper, we compare cooperative full-duplex (FD) and incremental cooperative half-duplex (HD) relaying, in terms of outage probability, throughput and energy efficiency. We consider a practical model for the FD relay, where self-interference between transmitted and received signals is taken into account. The FD transmission is based on the well known block Markov encoding scheme. For the HD transmission, incremental space-time cooperation and selection combining based on the cooperative decode-and-forward protocol are considered. Our results show that incremental cooperative HD relaying can outperform cooperative FD relaying, specially if the self-interference is non negligible, achieving a better trade-off between throughput and energy consumption. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Through cooperation among nodes even single antenna devices can achieve spatial diversity [1–3]. Probably the most known cooperative protocols are the amplify-and-forward (AF) and the decode-and-forward (DF) [1], including their variants (selective and incremental protocols [1]). Moreover, the cooperative protocols can operate either in a half-duplex (HD) or full-duplex (FD) fashion. The HD cooperative protocols are spectrally inefficient [4], in the sense that two time slots are used to transmit a message from source to destination. By its turn, in the FD mode the relay simultaneously transmits and receives in the same frequency.

The authors in [5] show that ideal (without self-interference) FD cooperative protocols achieve, in general, a higher capacity than HD cooperative protocols. In [5–8] the authors assume the best known performance achieving scheme for the relay channel, block Markov (BM) encoding, and then derive the outage probability of an ideal cooperative FD scheme. However, ideal FD operation, in which transmitted and received signals are perfectly isolated, is often not possible. In practice, transmitted power is normally much larger than received power [5], which turns the isolation a difficult task. Non-cooperative and cooperative FD schemes where self-interference is assumed between the transmitted and received signals were investigated in [9–16]. For instance, in [11] it is shown that FD relaying with self-interference is feasible even if there is a strong power leakage between the transmitted and the received signals, and that FD relaying enhances capacity when compared to multi-hop HD relaying. Similar conclusions are obtained in [14–16]. Additionally, such relaying schemes are of great interest of the industry once in band full duplex operation is being investigated for instance since Release 10 of 3GPP [17].
Recently, energy efficient schemes have become the focus of academia and industry. The objective of power consumption analysis is to propose alternatives to prolongate the battery lifetime of mobile devices as well as to reduce carbon emissions, and reduce energy consumption of the network as a whole [17, 18]. In [19–21] power consumption was analyzed for non-cooperative and cooperative networks. Moreover, the authors have accounted for different node densities and the circuitry consumption for transmitting and receiving data. In [19], the transmit power of the non-cooperative and the cooperative systems are considered to be fixed. Thus, the energy efficiency is maximized by the optimization of the packet length and the modulation order of each scheme. The performance analysis showed that the cooperative transmission outperforms the non-cooperative in terms of energy consumption when there is a great distance between the source and the destination. In [20], by establishing an acceptable limit for the packet loss, the transmit power is minimized based on the outage probability of each transmission scheme. The results show that the cooperative network can be more energy efficient than the non-cooperative network when source and destination are far apart. In [21], by defining an end-to-end throughput requirement, it is shown that incremental cooperation is more energy efficient than direct transmission and than multi-hop transmission, even at small transmission ranges. Moreover, in [22] an energy efficient analysis is carried out considering multi-hop HD and FD schemes in the AWGN relay channel. The results show that the multi-hop HD relay may require at least 50% more of the bandwidth needed by the multi-hop FD relay with the same rate and power constraints.

In this paper we analytically evaluate the outage, throughput and energy efficiency of cooperative FD relaying with interference under Rayleigh fading channel. Moreover, we adopt the Block Markov encoding at S and R [5,6]. To the best of our knowledge, such a formulation cannot be found in the literature regarding cooperative FD relaying with self-interference. While in [11,14–16] the authors consider multi-hop HD relaying, here we compare the performance of cooperative FD relaying to that of two incremental cooperative HD relaying methods: incremental redundancy space-time (IR-ST) transmission and selection combining (SC) [23]. In IR-ST the relay only cooperates if requested by the destination. Once requested, the relay sends additional parity bits, together with the source, by means of a space-time codeword, which is then appropriately combined by the destination with the first source transmission [23–29]. By its turn, in SC, which is the simplest incremental DF scheme, the relay also cooperates only if requested. In such case the relay retransmits the source message (while the source is silent), and the destination applies selection combining between the original source transmission and the relay retransmissions.

Surprisingly, our results show that incremental cooperative HD relaying can achieve a smaller outage probability and a higher throughput than cooperative FD-BM relaying with self-interference. The reason for that is twofold: i) the presence of the self-interference causes a floor in the outage probability of FD-BM relaying, which reduces the maximum achievable throughput; ii) as we consider incremental protocols for HD relaying, HD cooperation is able to achieve the same maximum throughput as ideal FD relaying (without self-interference). Thus, our main contribution is to show that, specially at a the high signal to noise ratio (SNR) or in case of high attempted information rates, incremental HD relaying presents a better trade-off between throughput and energy consumption than cooperative FD relaying with self-interference.

The rest of this paper is organized as follows. Section 2 presents the system model. Section 3 discusses the cooperative schemes considered in this paper, while Section 4 investigates the energy efficiency of the FD and HD schemes. Section 5 introduces the power and rate allocation analysis and Section 6 presents the numerical results. Finally, Section 7 concludes the paper.

2. SYSTEM MODEL

Consider a system with three cooperating terminals: source (S), relay (R) and destination (D), as shown in Fig 1. The S-D, S-R, R-R and R-D channels are all subject to quasi-static Rayleigh fading. Moreover, we account for the self-interference (dotted line in Fig. 1). We model the self-interference channel also as Rayleigh fading because we assume that such link is dominated by the scattering component of the channel, once the Line of Sight (LoS) component is considerably reduced by antenna isolation or represents a residual interference after the usage of an
interference cancellation scheme \[9,10,16\] . Additionally, we assume perfect channel state information (CSI) at the receivers. The channel noise is considered to be a complex additive white Gaussian noise (AWGN) with variance \(N_0/2\) per dimension. Without loss of generality, in this paper we assume that the noise power is \(N_0 = 1\). Moreover, we consider that R can operate either in FD or HD mode.

\[\text{Figure 1. System model with Source (S), Relay (R) and Destination (D). Notice that the relay suffers self-interference which is represented by } h_{RR}.\]

In the FD mode, S broadcasts the message \(x_S\), which is heard by both D and R. At the same time R sends a message \(x_R\) to D. Moreover, since we assume the presence of self-interference in R, the transmission from R to D interferes in the reception of the message sent by S. Following the above, the received signals at R and D can be written as:

\begin{align*}
    y_R &= \sqrt{P_S} h_{SR} x_S + n_R, \quad (1) \\
    y_D &= \sqrt{P_R} h_{RD} x_R + \sqrt{P_S} h_{SD} x_S + n_D, \quad (2)
\end{align*}

where \(h_{SD}\), \(h_{SR}\) and \(h_{RD}\) are the complex fading channel coefficients of the S-D, S-R and R-D links, respectively, while \(h_{RR}\) is the complex fading coefficient of the self-interference \[16\]. The average transmit power at S and R are \(P_S\) and \(P_R\) while \(n_R\) and \(n_D\) are the noise at R and D, while \(\kappa_{ij}\) is the path loss coefficient between \(i\) and \(j\).

In the HD mode, the transmissions are orthogonal in time, and we assume the presence of a feedback channel, so that nodes use the incremental DF protocol. In the first time slot, S broadcasts a message to R and D, so that:

\begin{align*}
    y_R &= \sqrt{P_S} h_{SR} x_S + n_R, \quad (3) \\
    y_D &= \sqrt{P_S} h_{SD} h_{RD} x_R + n_D. \quad (4)
\end{align*}

If an error is detected, D requires a retransmission. Then, two cases are considered, depending if S and R transmit concurrently in the second time slot or not. In case they do (the IR-ST method), then R cooperates with S and the received signal at D is:

\[y_D^* = \sqrt{P_S} h_{SD} x_S + \sqrt{P_R} h_{RD} x_R + n_D. \quad (5)\]

where \(x_S^*\) and \(x_R^*\) are symbols from a space-time codeword. The signals \(y_D^*\) and \(y_D\) are combined at D and a new decoding attempt is carried out. In case R retransmits in the second slot while S is silent (the SC method), then \(x_S^* = 0, x_R^* = x_R\) and the receiver applies selection combining between \(y_D^*\) and \(y_D\). Other operation modes for HD relaying could be considered, but our choice for IR-ST and SC is justified by the fact that IR-ST is a very high performance due to accumulation of the achievable rate at a cost of strict synchronization. While SC is very simple and the worst performing HD scheme based on the DF protocol \[23\]. Therefore, we are able to compare FD relaying to a sample of the most complex and high performance HD schemes and to a sample of the most simple and less performing HD methods.

\[\text{3. COOPERATIVE FD AND HD SCHEMES}\]

\[\text{3.1. Block Markov Full-Duplex Relaying}\]

The capacity for the relay channel is still an open problem. In view of this unanswered issue, the best achievable rate known in the literature is attained when the BM encoding technique is employed \[6–8,30,31\]. The Block Markov DF relaying scheme is based on the Block Markov encoding at S and R, combined with superposition coding and coding for cooperative multiple access channel and random coding \[30,\text{Chap. 15}\] and also \[5,6\] .
Next we determine the outage probability and throughput of FD-BM in the presence of self-interference.

An outage occurs when \( I_{ij} < R \), where \( I_{ij} \) is the achievable rate in the \( i \) to \( j \) link, and \( R \) is the attempted information transmission rate. We suppose complex Gaussian inputs and unitary bandwidth.

The achievable rate of FD-BM is given by the minimum of the achievable rates of the S-R link and of a MAC channel composed of the concurrent transmissions from S and R to D [5]. Therefore:

\[
I_{FD-BM} = \min(I_{SR-BM}, I_{MAC})
\]

(6)

where the achievable rate \( I_{SR-BM} \) is given by [8]:

\[
I_{SR-BM} = \log_2 \left( 1 + \left( 1 - \rho^2 \right) \gamma_R \right),
\]

(7)

while the variable \( \rho \) is the correlation coefficient between S and R messages [5] and \( \gamma_R \) is the signal to interference plus noise given by

\[
\gamma_R = \frac{|h_{SR}|^2 P_S \kappa_{SR}}{|h_{RR}|^2 P_R + 1}
\]

(8)

In [10], it was reported that in practice it may be actually impossible to perfectly estimate \( h_{RR} \) and therefore cancel out the self interference. Therefore, we have assumed in \( \gamma_R \) that the self-interference is accounted for as an additional uncorrelated noise at the relay.

The achievable rate in the multiple access channel (MAC) formed by R-D and S-D links is [5,7,8]:

\[
I_{MAC} = \log_2 \left( 1 + P_S \kappa_{SD} |h_{SD}|^2 + P_R \kappa_{RD} |h_{RD}|^2 + 2\sqrt{P_S \kappa_{SD} P_R \kappa_{RD}} \text{Re} \left( \rho h_{SD} h_{RD}^* \right) \right),
\]

(9)

with \( \text{Re}(.) \) denoting the real part and \( (.)^* \) the complex conjugate. The overall outage probability of FD-BM is [8]:

\[
\mathcal{P}_{FD-BM} = \Pr \left[ \min(I_{SR-BM}, I_{MAC}) \leq R \right]
\]

\[
= 1 - \mathcal{P}_{MAC}^C \mathcal{P}_{SR-BM}^C
\]

(10)

In order to find \( \mathcal{P}_{FD-BM} \) we first develop the complementary outage probability of the S-R link, taking into account the effect of the self-interference:

\[
\mathcal{P}_{SR-BM}^C = \Pr \left[ I_{SR-BM} \geq R \right] = \Pr \left[ \log_2 \left( 1 + \left( 1 - \rho^2 \right) \gamma_R \right) \geq R \right],
\]

(11)

thus we can write

\[
\mathcal{P}_{SR-BM}^C = \Pr \left[ \frac{|h_{SR}|^2 P_S \kappa_{SR}}{|h_{RR}|^2 P_R + 1} \geq \frac{2^R - 1}{1 - \rho^2} \right].
\]

(12)

If we define \( X = |h_{SR}|^2 P_S \kappa_{SR} \) (exponentially distributed) and \( Y = |h_{RR}|^2 P_R + 1 \) (mean shifted exponentially distributed) we can compute the distribution of \( Z = X/Y \) [32, pg. 186] given by \( f_Z(z) = \int_0^\infty y f_X(y) f_Y(y) dy \), once that this distribution is calculated, it is possible to compute the cumulative probability given in (12). Fortunately, this integral has closed form solution and its final form is given as

\[
\mathcal{P}_{SR-BM}^C = \frac{\exp \left( -\frac{\alpha}{\beta} \right) P_S \kappa_{SR} \pi_{SR}}{P_S \kappa_{SR} \pi_{SR} + \frac{(2^\alpha - 1)}{(1 - \rho^2)} P_R \pi_{RR}}
\]

(13)

where \( \pi_{SR} = E \left[ |h_{SR}|^2 \right] \) and \( \pi_{RR} = E \left[ |h_{RR}|^2 \right] \). Notice that the self-interference is taken into account into \( \gamma_R \) (parameter \( |h_{RR}|^2 \)) and at the outage probability by the parameter \( \pi_{RR} \).

The complementary outage probability of the MAC channel is [8]:

\[
\mathcal{P}_{MAC}^C = \Pr \left[ I_{MAC} \geq R \right]
\]

\[
= \frac{-(2^\alpha - 1)}{\alpha - \beta} e^{-\frac{(2^\alpha - 1)}{\beta}} - \frac{(2^\alpha - 1)}{(1 - \rho^2)} e^{-\frac{(2^\alpha - 1)}{\beta}},
\]

(14)

where \( \alpha \) and \( \beta \) are, respectively, given by:

\[
\alpha = \frac{a}{2} + \sqrt{b},
\]

(15)

\[
\beta = \frac{a}{2} - \sqrt{b},
\]

(16)

and

\[
a = (P_R \kappa_{RD} \pi_{RD} + P_S \kappa_{SD} \pi_{SD}),
\]

(17)

\[
b = \frac{a^2}{4} - P_R \kappa_{RD} P_S \kappa_{SD} \pi_{RD} \pi_{SD} \left( 1 - \rho^2 \right),
\]

(18)

\footnote{It is noteworthy that \( \rho \) can maximize the mutual information of the FD-BM scheme [5]. Once we assume CSI only at the receivers, we consider that \( \rho \) is fixed for all fading states. Thus, under this simplifying assumption, \( \rho \) is designed to minimize the outage probability which leads to \( \rho = 0 \) as we discuss later in Section 6.}
failure and S proceeds with the next data frame. Thus, the Node D verifies the retransmitted frame received from R. If the retransmission fails, D requests for a retransmission from R. Supposing an error, and that R was able to decode the message, then D verifies the message was correct. If the retransmission fails, D requests for a retransmission from R. Thus, the existence of this floor limits the performance of FD relaying, making it possible for incremental HD relaying to outperform FD-BM as we show in Section 6.

Finally, the throughput for FD-BM is:

\[ T_{\text{FD-BM}} = \mathcal{R} (1 - P_{\text{FD-BM}}). \] (20)

### 3.2. Selection Combining Half-Duplex Relaying

In the HD mode, S broadcasts a message to R and D, then R forwards it to D. Thus, the instantaneous SNR at R is \( \gamma_{\text{SR}} = |h_{\text{SR}}|^2 P_r \kappa_{\text{SR}} \). The instantaneous SNR at the S-D and R-D links are: \( \gamma_{\text{SD}} = |h_{\text{SD}}|^2 P_3 \kappa_{\text{SD}} \) and \( \gamma_{\text{RD}} = |h_{\text{RD}}|^2 P_3 \kappa_{\text{RD}} \). Note that there is no interference in the HD mode, which can be also observed in (3) and (4). The achievable rate of the S-D link is \( I_{\text{SD}} = \log_2 (1 + \gamma_{\text{SD}}) \), and its corresponding outage probability is:

\[ P_{\text{SD}} = \Pr [I_{\text{SD}} < \mathcal{R}] = \Pr [\gamma_{\text{SD}} < 2^R - 1] = 1 - \exp \left( \frac{1 - 2^R}{P_3 \kappa_{\text{SD}} \gamma_{\text{SD}}} \right). \] (21)

As the achievable rates of the S-R and R-D links are \( I_{\text{SR}} = \log_2 (1 + \gamma_{\text{SR}}) \) and \( I_{\text{RD}} = \log_2 (1 + \gamma_{\text{RD}}) \), the outage probabilities in such links, \( P_{\text{SR}} \) and \( P_{\text{RD}} \), can be written just as above, but replacing \( \kappa_{\text{SD}} \gamma_{\text{SD}} \) by \( \kappa_{\text{SR}} \gamma_{\text{SR}} \) and \( \kappa_{\text{RD}} \gamma_{\text{RD}} \), respectively, and \( P_3 \) by \( P_r \) when appropriate.

In the case of SC, after the transmission from S, node D verifies if the message was correctly received or not. Supposing an error, and that R was able to decode the message, then D requests for a retransmission from R. Node D verifies the retransmitted frame received from R. If the retransmitted frame is also in error, then D declares a failure and S proceeds with the next data frame. Thus, the outage probability of SC after cooperation is:

\[ P_{\text{SC}} = \Pr [I_{\text{SD}} < \mathcal{R}, I_{\text{RD}} < \mathcal{R}] = P_{\text{SD}} \cdot P_{\text{RD}} \] (22)

Now, we define the overall outage probability \(^5\) of SC scheme as:

\[ P_{\text{out}}^{\text{SC}} = P_{\text{SD}} (P_{\text{SR}} + (1 - P_{\text{SR}}) P_{\text{RD}}). \] (23)

Note that an outage occurs if the S-D link fails as shown in the first term or if the retransmission fails. Note that \( \Pr (I_{\text{SC}} < \mathcal{R} | I_{\text{SD}} < \mathcal{R}) = \frac{P_{\text{SD}} (1 - P_{\text{SR}}) P_{\text{RD}}}{P_{\text{SD}} (1 - P_{\text{SR}}) + P_{\text{SD}} P_{\text{RD}} (1 - P_{\text{SR}})} \) is the probability that an error occurs after a retransmission from R, given that an error occurred after the transmission from the source.

For the SC method the throughput notice that the first term in (24) comes from the direct transmission and D can attempt to decode at rate \( \mathcal{R} \). The second term comes from the cooperation from R. We recall that the factor 1/2 appears due to the HD constraint since cooperation occurs in two time slots. can be written as:

\[ T_{\text{SC}} = \mathcal{R} (1 - P_{\text{SD}}) \]

\[ + \frac{\mathcal{R}}{2} \cdot P_{\text{SD}} \cdot (1 - P_{\text{SR}}) \cdot (1 - \Pr (I_{\text{SC}} < \mathcal{R} | I_{\text{SD}} < \mathcal{R})) \]

\[ = \mathcal{R} (1 - P_{\text{SD}}) + \frac{\mathcal{R}}{2} \cdot P_{\text{SD}} \cdot (1 - P_{\text{SR}}) \cdot (1 - P_{\text{RD}}). \] (24)

### 3.3. Incremental Redundancy Space-Time Half-Duplex Relaying

In case of a requested retransmission, R and S send additional parity bits (by means of a space-time codeword) which are then appropriately combined by the destination with the first source transmission [23–29]. Thus, in case of a retransmission the overall achievable rate seen at D is \(^6\) [28]:

\[ I_{\text{IR-ST}} = \log_2 (1 + \gamma_{\text{SD}}) + \log_2 (1 + \gamma_{\text{RD}}). \] (25)

Notice that the first term in (25) comes from first transmission, while the second term represents the synchronized transmission from S and R.

Next, we derive the outage probability of IR-ST, \( P_{\text{IR-ST}} = \Pr [I_{\text{IR-ST}} < \mathcal{R}] \), given that R was able to decode the message from S in the first transmission. Since \( \gamma_{\text{SD}} \) and \( \gamma_{\text{RD}} \) are independent and exponentially distributed, we can

\(^{5}\) We define overall outage probability as the probability that accounts for all events that lead to a system outage.

\(^{6}\) It is noteworthy that if a retransmission is not required then \( I_{\text{IR-ST}} = \log_2 (1 + \gamma_{\text{SD}}) \) [26,28]. Such case is also represented on the first term of the throughput in (34).
write
\[ f_{\Gamma_M, \Gamma_D}(\gamma_{SD}, \gamma_{RD}) = \frac{1}{\gamma_{SD} \gamma_{RD}} \exp\left(-\frac{\gamma_{SD}}{\gamma_{RD}}\right) \exp\left(-\frac{\gamma_{RD}}{\gamma_{SD}}\right) \]

where \( \gamma_{SD} = P_{S^{RD}} \pi_{SD} \) and \( \gamma_{RD} = P_{R^{RD}} \pi_{RD} \) are the mean values for the variables \( \gamma_{SD} \) and \( \gamma_{RD} \), respectively. Now defining two new variables
\[ Z_1 = \Gamma_{SD} \]
\[ Z_2 = (1 + \Gamma_{SD})(1 + \Gamma_{SD} + \Gamma_{RD}) \]

Then applying the traditional methodology of changing the variables [32], it is possible to compute the Jacobian of this transformation as \( J = 1 + z_1 \). With this transformation, we can write
\[ f_{Z_1, Z_2}(z_1, z_2) = \frac{1}{1 + z_1} f_{\Gamma_M, \Gamma_D}(z_1, \frac{z_2}{1 + z_1} - 1 - z_1) \]

Note that the support now has changed from \((0 \leq \Gamma_{SD} < \infty, 0 \leq \Gamma_{RD} < \infty)\) to \((0 \leq Z_1 < \infty, (1 + Z_1)^2 \leq Z_2 \leq \infty)\). With this in mind, it is possible to write the probability density function of the target variable, \( Z_2 \), as
\[ f_{Z_2}(z_2) = \int_0^{\sqrt{z_2} - 1} f_{Z_1, Z_2}(z_1, z_2) dz_1 \]

and finally
\[ P_{IR-ST} = \int_1^{2\pi} f_{Z_2}(z_2) dz_2 \]

resulting in the final expression as
\[ P_{IR-ST} = \frac{1}{\gamma_{RD} \gamma_{SD}} \times \int_1^{2\pi} \int_0^{\sqrt{z_2} - 1} \exp\left(-\frac{z_2}{\gamma_{RD}}\right) \exp\left(-\frac{z_1 + 1}{\gamma_{SD}}\right) \frac{z_1 + 1}{z_1 + 1} dz_1 dz_2. \]

Since (32) is attained we can now determine the overall outage probability of IR-ST as
\[ P_{out}^{IR-ST} = P_{SD} \left( P_{SR} + (1 - P_{SR}) \frac{P_{IR-ST}}{P_{SD}} \right), \]

where \( \frac{P_{IR-ST}}{P_{SD}} = P \{ I_{IR-ST} < \Omega | I_{SD} < \Omega \} \) is the probability that an error occurs at D after the IR-ST transmission from S and R, given that an error occurred after the original S transmission.

Then the throughput for IR-ST is:
\[ \eta_{IR-ST} = \frac{R}{2} \left( 1 - \frac{P_{IR-ST}}{P_{SD}} \right), \]

(34)

### 4. ENERGY EFFICIENCY ANALYSIS

In this section we focus on the energy efficiency of HD and FD schemes. For that sake, we define the total energy consumption per bit of each scheme, which takes into account the required power for the transmission, the power consumption of the RF circuitry, and the bit rate. According to [33, 34] the energy consumption of the RF circuitry is much larger than the baseband processing consumption in regular nodes. Thus, we have ignored the baseband processing consumption in this paper.

The total consumed energy per bit of the direct scheme is:
\[ \varepsilon_{Dir} = \frac{P_{AMPS} + P_{RX} + P_{RX}}{R}, \]

(35)

where \( P_{AMPS} = P_S/\eta \) is the power amplifier consumption for the source transmission, \( \eta \) is the drain efficiency of the amplifier, \( P_{RX} \) and \( P_{RX} \) are the power consumed by the internal circuitry for transmitting and receiving, respectively.

In the case of the HD incremental cooperative schemes, the total consumed energy per bit depends on the outage probability on the S-D and S-R links:
\[ \varepsilon_{HD} = \frac{P_{AMPS} + P_{RX} + 2P_{RX}}{R} + P_{SD} \cdot (1 - P_{SR}) \cdot \Omega, \]

(36)

The first term in (36) corresponds to the consumed energy if D could decode the packet correctly in the first time slot, then no retransmission is required. The second term in (36) corresponds to the consumed energy when R cooperates. In the case of the SC scheme, as only R transmits in the second time slot, then \( \Omega = \frac{P_{AMPS} + P_{RX} + P_{RX}}{R} \), where \( P_{AMPS} = P_R/\eta \). On the other hand, in the IR-ST scheme both S and R transmit in the second time slot, so that \( \Omega = \frac{P_{AMPS} + P_{AMPS} + 2P_{RX} + P_{RX}}{R} \).

The additional power consumption of the retransmission
request messages is negligible, as shown in [21], and therefore is not considered here.

In the case of FD relaying, the total consumed energy per bit depends on the outage probability of the S-R link, and it is given by:

\[
E_{FD} = \frac{P_{\text{amps}} + P_{\text{tx}} + 2P_{\text{rx}}}{\mathcal{R}} + (1 - P_{SR}) \frac{P_{\text{amps}} + P_{\text{tx}}}{\mathcal{R}}.
\]

(37)

Notice that the first term in (37) corresponds to the transmission from S, which is heard by both R and D. The second term in (37) refers to the transmission from R. It is relevant to note that (37) depends only on the outage probability of the S-R link once R is always overhearing S transmission, and that R only cooperates if it was able to decode the message from S. It is also important to note that in the presence of self-interference, the outage in the S-R link in FD relaying is higher than the outage in the S-R link in HD relaying.

We considered the same RF circuitry model introduced in [35], which has the following blocks for the transmitter: digital-to-analog converter, mixer, transmit filters and frequency synthesizer, whose power consumptions are respectively given by \(P_{\text{DAC}}, P_{\text{mix}}, P_{\text{fil,tx}}\) and \(P_{\text{syn}}\). Thus, the consumed power of the transmit hardware is given by \(P_{\text{TX}} = P_{\text{DAC}} + P_{\text{mix}} + P_{\text{fil,tx}} + P_{\text{syn}}\). Moreover, at the receiver side, also according to [35], we consider the following blocks: frequency synthesizer, low-noise amplifier, mixer, intermediate frequency amplifier, receive filters and analog-to-digital converter, whose power consumptions are respectively given by \(P_{\text{syn}}, P_{\text{NA}}, P_{\text{mix}}, P_{\text{fil,rx}}, P_{\text{ADC}}\). Thus, \(P_{\text{RX}} = P_{\text{syn}} + P_{\text{NA}} + P_{\text{mix}} + P_{\text{fil,rx}} + P_{\text{ADC}}\). Based on the power consumption values found in [35], in this paper we consider the overall power consumption for transmitting and receiving as being \(P_{\text{TX}} = 97.9\, \text{mW}\) and \(P_{\text{RX}} = 112.2\, \text{mW}\), respectively, and that the drain efficiency is \(\eta = 0.35\).

5. POWER AND RATE ALLOCATION

Now, we investigate the power and rate allocation between S and R in order to establish a performance benchmark. The choice of power and rate is such that maximizes the throughput. Thus, we formalize the problem as:

\[
\max_{R, P_S^*} \mathcal{T} \quad \text{subject to} \quad P^*_S + P^*_R \leq 2P
\]

\[
\mathcal{R}_{\min} \leq \mathcal{R} \leq \mathcal{R}_{\max}
\]

(38)

where \(\mathcal{T}\) can be \(T_{\text{FD-BM}}, T_{\text{IR-ST}}\) or \(T_{\text{dir}}\), and \(P\) is the used power for direct transmission. The maximization can be performed with respect to \(\mathcal{R}\) and \(P_S^*\).

Since our goal is to compare the different HD and FD schemes, we do not focus on the proposal of a particular PA and RA solution, but we resort to numerically efficient algorithms. Moreover, performing a closed form analysis of these equations is not possible since the equations are not jointly concave for all variables [36,37]**. For RA we considered that \(\mathcal{R}\) could vary from \(\mathcal{R}_{\min} = 1\) bits/s/Hz to \(\mathcal{R}_{\max} = 10\) bits/s/Hz. At each SNR value we numerically determine the attempted rate \(\mathcal{R}\) which maximizes the throughput. Additionally, for PA we determine the values of \(P_S^*,\) and therefore \(P_R^*\). Notice that we assume \(P^*_S + P^*_R = 2P\). Thus, the two parameters \((P_S^*,\) and \(\mathcal{R}\)) are jointly numerically optimized.

6. NUMERICAL RESULTS

Next we investigate the outage, throughput and energy efficiency of FD-BM, IR-ST and SC. But first let us assume a log-distance path loss model \(\kappa_{ij} = d_{ij}^{-\alpha}\) with decay exponent \(\alpha = 4\) and \(d_{ij}\) is the distance between nodes \(i\) and \(j\). Moreover, we suppose that R is positioned in a straight line between S and D. Thus, normalizing the distance between S and D to the unit, then \(d_{SR} = 1 - d_{SR}\).

In the results we considered \(d_{SR} = 0.5\). We assume that the transmit power of S and R are the same, \(P_S = P_R\). Based on [16], we consider two levels of self-interference: the ideal case, in which \(\pi_{RR} = 0\) (-\infty dB); and the more practical case of \(\pi_{RR} = -8\, \text{dB}^{11}\).

Fig. 2.a) presents the outage probability as a function of \(\pi_{SD} = \frac{P_{\text{RX}}}{N_0} = P_S\, \kappa_{SD}\), when \(\mathcal{R} = 2\) bits/s/Hz and

**A local maximum can be found by constraining the equation to a particular interval of interest. Moreover, several efficient numerical solutions exist for such classical constrained nonlinear optimization problem [36-38]. For instance, we run a Sequential Quadratic Programming (SQP) method [38] using MATLAB.

\(^{11}\) Notice that henceforth these assumptions hold for all numerical results unless stated otherwise.
$\rho = 0$. We can notice that IR-ST outperforms the other methods\textsuperscript{11}. For FD relaying the performance decreases significantly with the increase of the self-interference. In Fig. 2.b) we consider a similar scenario, but when $R = 8$ bits/s/Hz and $\rho = 0$, where we can see that IR-ST increases its advantage over the other schemes with the increase in the attempted rate. Therefore, at least in terms of outage probability, HD is considerably superior than FD relaying. That is reasonable since in FD mode D sees a superposition of S and R signals, which increases the error probability at D. Moreover, it is clear that the self-interference results in a floor in the outage of FD-BM, compromising the performance.

In order to show the impact of $\rho$, Fig. 3 shows the outage probability versus $\rho$ for different relay positions $d_{SR}$. We assume that $\gamma_{SD} = 0$dB (black curves) or $\gamma_{SD} = 10$dB (blue curves), and that $d_{SR} \in \{1/3, 1/2, 2/3\}$. Notice that in this case $\rho = 0$ turns out to be the best choice. Therefore, next we assume that the correlation coefficient of the FD-BM scheme is null ($\rho = 0$), which turns the FD-BM scheme into a joint decoding method (at the destination). Additionally, a similar result, showing that $\rho = 0$ might be the optimum choice, was presented in [8, Figure 4], in a different context.

When the throughput is the metric to be considered, as shown in Fig. 4, we can see that FD-BM can considerably outperform the other methods. However, for higher values of $R$ and for practical values of the self-interference the performance of FD-BM considerably decreases. For $R = 8$ bits/s/Hz and $\gamma_{RR} = -8$ dB, IR-ST, SC and even the direct transmission can outperform FD-BM in terms of throughput. Moreover, we can note that in terms of throughput the simple SC scheme performs very close to IR-ST. In Fig. 5 we investigate the throughput as a function of $R$, considering fixed SNR values of $\gamma_{SD} = 0$dB and $\gamma_{SD} = 15$dB. In the case of a low SNR as in Fig 5.a), FD relaying is a better option even with self-interference. However, in the case of a higher SNR as in Fig 5.b) this is not true, as FD-BM with self-interference is outperformed by IR-ST and SC for many attempted rates $R$. The reason for this reduced performance of FB-BM is that, as we can see in (19), the self-interference results in a floor in the outage of FD-BM, which gets close to the unity as $R$ grows.

We can clearly see from the above results that there are values of SNR and $R$ for which FD relaying considerably outperforms the HD strategies while, by its turn, in some cases HD relaying becomes much more attractive. Therefore, an interesting approach to increase the performance of cooperative systems would be to adopt a hybrid strategy in which the nodes would be able to

\textsuperscript{11}The throughput of a single direct (non-cooperative) transmission is $T_{dir} = R (1 - P_{dir})$.  

Figure 2. Outage versus SNR for a) $R = 2$ bits/s/Hz and b) $R = 8$ bits/s/Hz.

Figure 3. Outage versus $\rho$ for $\gamma_{SD} = 0$dB (solid line) or $\gamma_{SD} = 10$dB (dashed line), and that $d_{SR} \in \{1/3, 1/2, 2/3\}$, for an ideal FD relay and for the case of a FD relay with self-interference.
shift between FD and HD mode according to its operating region.

![Figure 4](image-url)

**Figure 4.** Throughput (bits/s/Hz) versus SNR for a) $R = 2$ bits/s/Hz and b) $R = 8$ bits/s/Hz.

![Figure 5](image-url)

**Figure 5.** Throughput (bits/s/Hz) as a function of $R$, for HD and FD schemes when a) $\gamma_{SD} = 0$dB and b) $\gamma_{SD} = 15$dB.

### 6.1. Energy Efficiency

In order to analyse the trade-off between throughput gains and energy consumption, we normalize the throughput by the total energy consumed. The normalized throughput, given in b/s/Hz/J, is shown in Fig. 6 for $R = 2$ bits/s/Hz and $R = 8$ bits/s/Hz. Assuming that both source and relay are using the same power, i.e., $P_s = P_r$ and also the same rate, Fig. 6 show that the ideal FD-BM scheme in general outperforms the other strategies, with an exception to the IR-ST scheme which outperforms FD-BM in the low SNR region and with a high rate (Fig. 6-b)). However, recall that in practice it is still not possible to achieve the performance of ideal FD-BM, due to the presence of the self-interference. By its turn, the practical FD-BM scheme (with self-interference) is considerably outperformed by both HD relaying schemes for a high rate in the whole SNR range (Fig. 6-b)), while in the case of a low attempted rate (Fig. 6-a)) it performs better than the HD schemes. Therefore, for high attempted rates it may be considerably more efficient to use HD incremental relaying than FD relaying.

In Fig. 7-a) we analyze the trade-off between throughput and power consumption as a function of $R$ when $\gamma_{SD} = 0$dB. From the figure we can conclude that for such a low SNR value the FD-BM scheme considerably outperforms HD relaying in case of attempted rates up to 4 bits/s/Hz, even in the presence of an strong self-interference signal. On the other hand, at higher attempted information rates the IR-ST scheme becomes advantageous when compared to the FD-BM strategy, once the HD schemes has much lower outage probabilities at such high information rates. By its turn, when $\gamma_{SD} = 15$dB (Fig. 7-b)), the conclusions are significantly different as shown in Fig. 7. At this higher SNR value, the direct transmission presents itself as a better option for attempted rates up to 4 bits/s/Hz, as retransmissions are rarely required. In case of higher attempted information rates the HD schemes considerably outperform FD-BM and the direct transmission. Our analysis shows that FD relaying is feasible and it is a good approach at low SNR and/or low information rate regions. By its turn, HD schemes...
become a more interesting strategy at high-SNR and high information rate regions.

![Figure 7](image1.png)

**Figure 7.** Normalized Throughput (bits/s/Hz) versus $R$ for: (a) $\gamma_{SD} = 0$ dB and (b) $\gamma_{SD} = 15$ dB.

6.2. Joint Allocation and Energy Efficiency

Fig. 8 shows the throughput with PA and RA as a function of the SNR. From the figure we can see that the ideal FD-BM relaying outperforms IR-ST. Nevertheless, when we consider the loop interference, the performance of IR-ST becomes very competitive, specially from the mid to high SNR region.

In Fig. 9-(a) we present the power allocated to $S$ as a function of the SNR. We can notice that most of the available power is allocated to the source ($P_s^a$) in the IR-ST and FD-BM schemes, consequently less power is allocated to the relay. However, in the FD-MH scheme $P_s^a$ decreases and more power is allocated to the relay. We recall that the performance of FD-MH is also limited by the S-D link which is seen as interference at the D. Thus, reducing the transmit power at S lowers the interference level at the D. Moreover, in terms of RA, as shown in Fig. 9-(b), HD and FD schemes perform alike, except for the FD-MH scheme which employs a less aggressive RA strategy, due to the reasons discussed before.

Further, in Fig. 10 we evaluate the jointly impact of PA, RA and energy efficiency. Therefore, we can conclude that conversely to the discussion presented at the end of Section 4, if PA and RA are employed then FD relaying (even in the presence of strong self-interference) becomes more attractive from low- to high-SNR. Additionally, it is hard to notice from the figure but the direct transmission is the most energy efficient scheme in the very high SNR region. That is reasonable since in the very high SNR region the cooperation from the relay is unnecessary, only increasing the energy consumption, since the S-D link is very good.

7. FINAL COMMENTS

We analytically evaluated the performance of some cooperative FD and HD schemes in terms of outage probability, throughput and energy efficiency. In the FD scheme we took into account the self-interference caused
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- Normalized throughput (bits/s/Hz/J) as a function of the SNR when PA and RA are carried out.

by the relay transmitted signal into the relay received signal. Our results show that incremental cooperative HD schemes can outperform practical (with self-interference) cooperative FD relaying. For instance, in the case of a high SNR or of a high attempted information rate, incremental cooperative HD relaying can achieve a better trade-off between throughput and energy consumption.

ACKNOWLEDGEMENT

This work has been supported by CNPq and CAPES (Brazil), and also by the University of Oulu Graduate School, Infotech Oulu Graduate School and Academy of Finland.

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