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Subpixel Stereo Disparity for Surface Reconstruction by Utilising a Three-Dimensional Reaction-Diffusion System

keywords: stereo algorithm, subpixel, reaction-diffusion, PDEs and numerical computation

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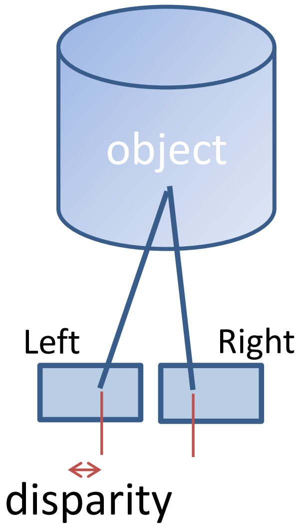
Outline

- Introduction and motivation
- Reaction-diffusion system
- Proposed algorithm
- Experimental results for the Middlebury stereo data-sets
- Conclusion

Introduction & Motivation



- Reconstruction of slanted or curved surface
- Previous algorithms:
  - Tsukahara and Hirai, *IEICE Trans. Inf. Syst.*, 1993
  - Bleyer et al., *Proc. BMVC*, 2011
- Our previous algorithm:
  - Nomura et al., *Machine Vision and Applications*, 2009
  - multiple 2D reaction-diffusion systems exclusively linked.
  - stereo corresponding problem => segmentation problem,
    not suitable for reconstruction of slanted or curved surface.
- Reaction-Diffusion Stereo algorithm with subpixel disparity in 3D domain (space and disparity).

Definition of a Reaction-Diffusion System

- Time-evolving partial-differential equation (PDE)

$$\frac{\partial u}{\partial t} = \underbrace{D_u \nabla^2 u}_{\text{diffusion}} + \underbrace{f(u)}_{\text{reaction}}$$

$u(x, t)$: distribution defined in space x and at time $t > 0$.

D_u: diffusion coefficient

∇^2: Laplacian operator on space x

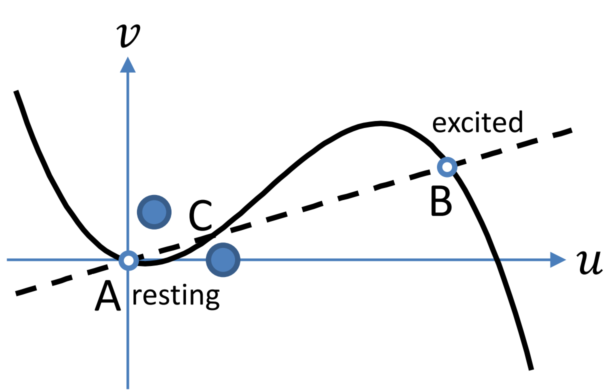
$f(u)$: reaction function

An Autonomous System: FitzHugh-Nagumo (FHN) Element

- Time-evolving ordinary-differential equation (ODE)

$$\frac{du}{dt} = f(u, v) = \frac{1}{\epsilon}[u(u-a)(1-u) - v]$$

$$\frac{dv}{dt} = g(u, v) = u - bv$$



$u(t), v(t)$: variables defined at time $t > 0$.

$f(u, v), g(u, v)$: reaction functions

a, b, ϵ: constants

A,B: stable and C: unstable

FHN Type Reaction-Diffusion System

- Diffusively coupled FHN elements:

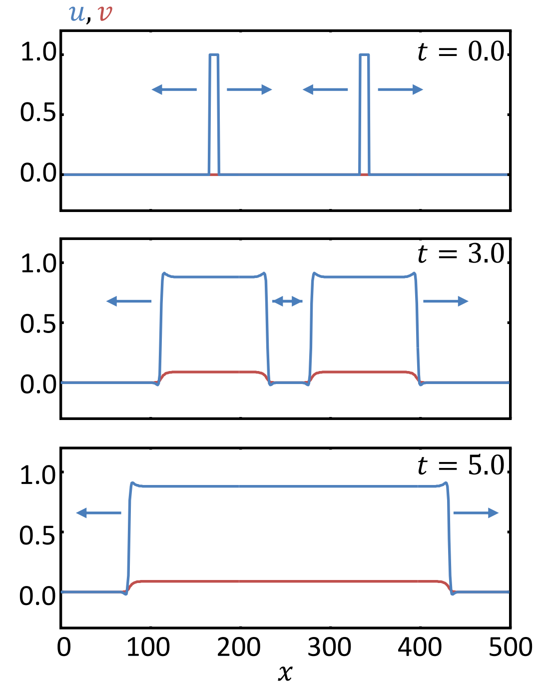
$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + \frac{1}{\epsilon}[u(u-a)(1-u) - v]$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + u - bv$$

$u(x,t),\ v(x,t)$: distributions defined in space x and at time $t > 0$.

D_u, D_v: diffusion coefficients.

=> propagating wave solution

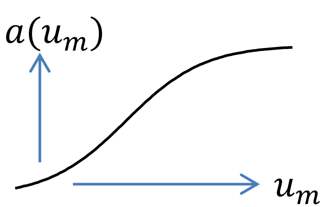


Parameter settings:
$D_u = 1.0, D_v = 3.0, a = 0.05, b = 10, \epsilon = 0.01$

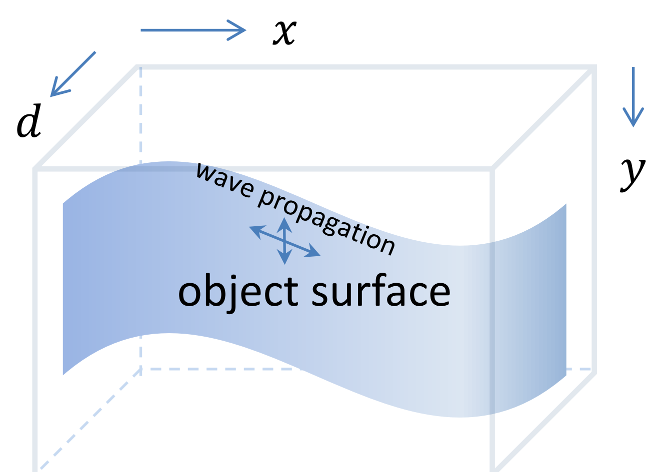
FHN Type Reaction-Diffusion System in 3D

- (u, v): defined in space (x, y) and disparity d.

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + \frac{1}{\epsilon}\left[u\big(u - a(u_m)\big)(1-u) - v\right] + \mu C_d$$
$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + u - bv$$



$a(\cdot)$: monotonically increasing function
$u_m = \max(u)$ in an inhibitory area
C_d: cross-correlation computed at a disparity d
μ: coefficient (fixed in the 1st step and varied in the 2nd step)
$u(x, y, d, t),\ v(x, y, d, t)$: distributions in a 3D domain



3D domain (x, y, d)

Finite Difference Method

- $\delta x, \delta y, \delta t$: finite differences on space and time
- δd: finite difference on disparity d

  => subpixel stereo disparity

- discretising PDEs => a set of linear eqs.

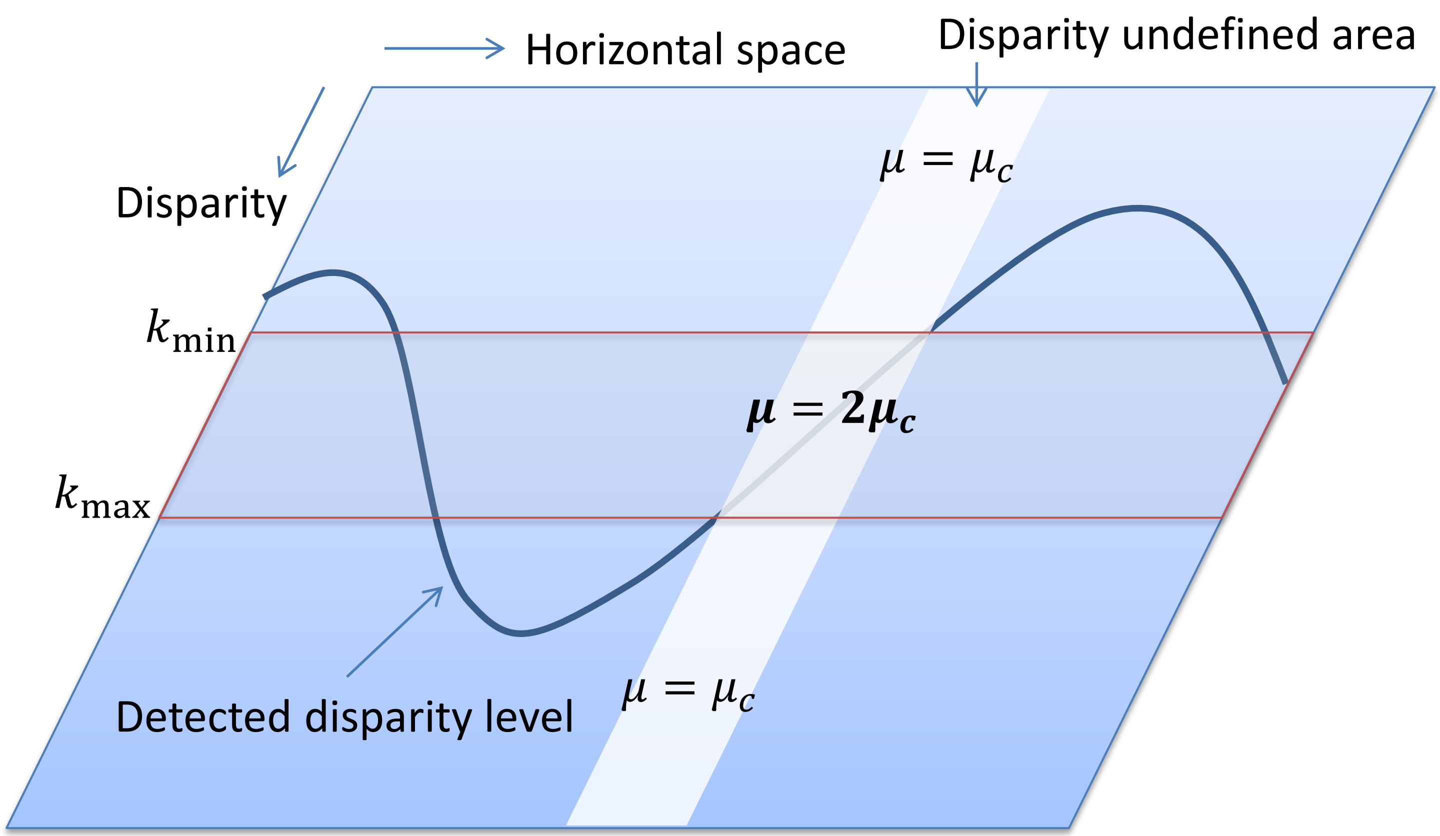
$$\frac{\partial u}{\partial t} \simeq \frac{u_{i,j,k}^{n+1} - u_{i,j,k}^{n}}{\delta t}$$

$$\frac{\partial^2 u}{\partial x^2} \simeq \frac{u_{i+1,j,k}^{n+1} - 2u_{i,j,k}^{n+1} + u_{i-1,j,k}^{n+1}}{\delta x^2}$$

$$x \simeq i\delta x, y \simeq j\delta y, d \simeq k\delta d, t \simeq n\delta t$$
$$u(x, y, d, t) \simeq u_{i,j,k}^{n}$$

Two-Step Algorithm

- Evaluation of a coefficient μ in the second step.



Algorithm Summary

- Pre-processing
  - Compute a cross-correlation function $C_{i,j,k}$ for a stereo image pair at possible subpixel disparity levels ($k = d/\delta d$).
  - Prepare initial conditions ($u_{i,j,k}^{n=0}, v_{i,j,k}^{n=0}$).
- while ($n < L_t/\delta t$) do
  - Compute ($u_{i,j,k}^{n}, v_{i,j,k}^{n}$) with the reaction-diffusion system.
  - If $n \geq \ell_t/\delta t$, evaluate $M_{i,j}^n$ and $\mu_{i,j,k}$.
  - $n \leftarrow n + 1$
- end while

L_t: finite duration of time
ℓ_t: time for the second-step
Disparity map: $M_{i,j}^n = \underset{k}{\operatorname{argmax}}\, u_{i,j,k}^n$

Stereo Images and the Ground-Truth Disparity Maps

- Available via the Middlebury stereo vision page (http://vision.middlebury.edu/stereo/)

TSUKUBA	VENUS	TEDDY	CONES
384x288 pixels 15 disparity levels	434x383 pixels 20 disparity level	450x375 pixels 60 disparity levels	450x375 pixels 60 disparity levels

Scores for the Middlebury Stereo Data-Sets (Previous* vs Proposed)

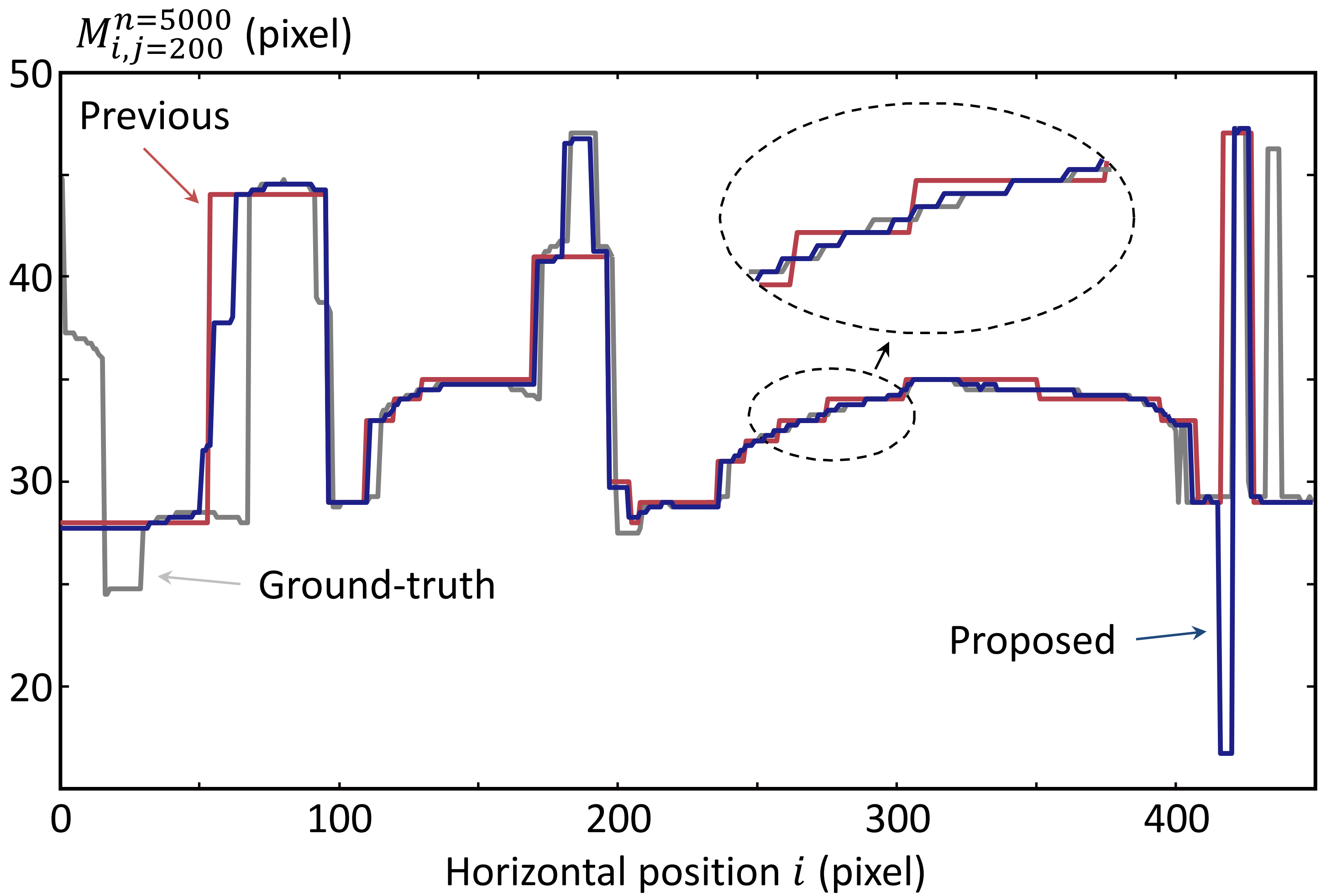
B_θ: Bad-Match-Percentage (BMP) error with threshold θ (%)
R: Root-mean-squares error (pixel)
Av. Rnk: Average Rank on the Middlebury website (September 8th, 2012).

		TSUKUBA			VENUS			TEDDY			CONES			Av. Rnk
		noc	all	ddi	noc	all	ddi	noc	all	ddi	noc	all	ddi	
Previous*	$B_{1.0}$	7.01 133	8.81 133	19.8 125	2.81 114	3.97 115	21.6 120	14.0 117	20.0 116	29.4 123	5.03 88	12.1 89	14.1 100	114.4
	$B_{0.5}$	22.8 94	24.2 98	27.6 116	10.9 106	12.0 108	25.7 111	22.5 113	29.3 114	39.0 115	10.3 63	17.4 65	22.1 87	99.2
	R	1.43	1.62	2.50	0.75	0.92	2.01	2.16	3.21	3.35	1.94	3.08	3.35	
Proposed	$B_{1.0}$	13.5 135	15.1 135	18.5 120	3.69 119	4.89 119	18.6 116	10.8 108	17.8 109	25.3 113	3.91 65	10.5 71	11.5 78	107.3
	$B_{0.5}$	19.9 79	21.4 82	24.1 98	6.32 57	7.57 66	21.8 103	13.6 50	21.8 65	30.7 68	5.04 11	12.2 16	13.8 19	59.5
	R	1.90	2.02	2.27	1.45	1.69	2.58	3.67	7.10	5.81	1.66	2.55	2.88	

*Previous: Nomura et al., *Machine Vision and Applications*, 2009.

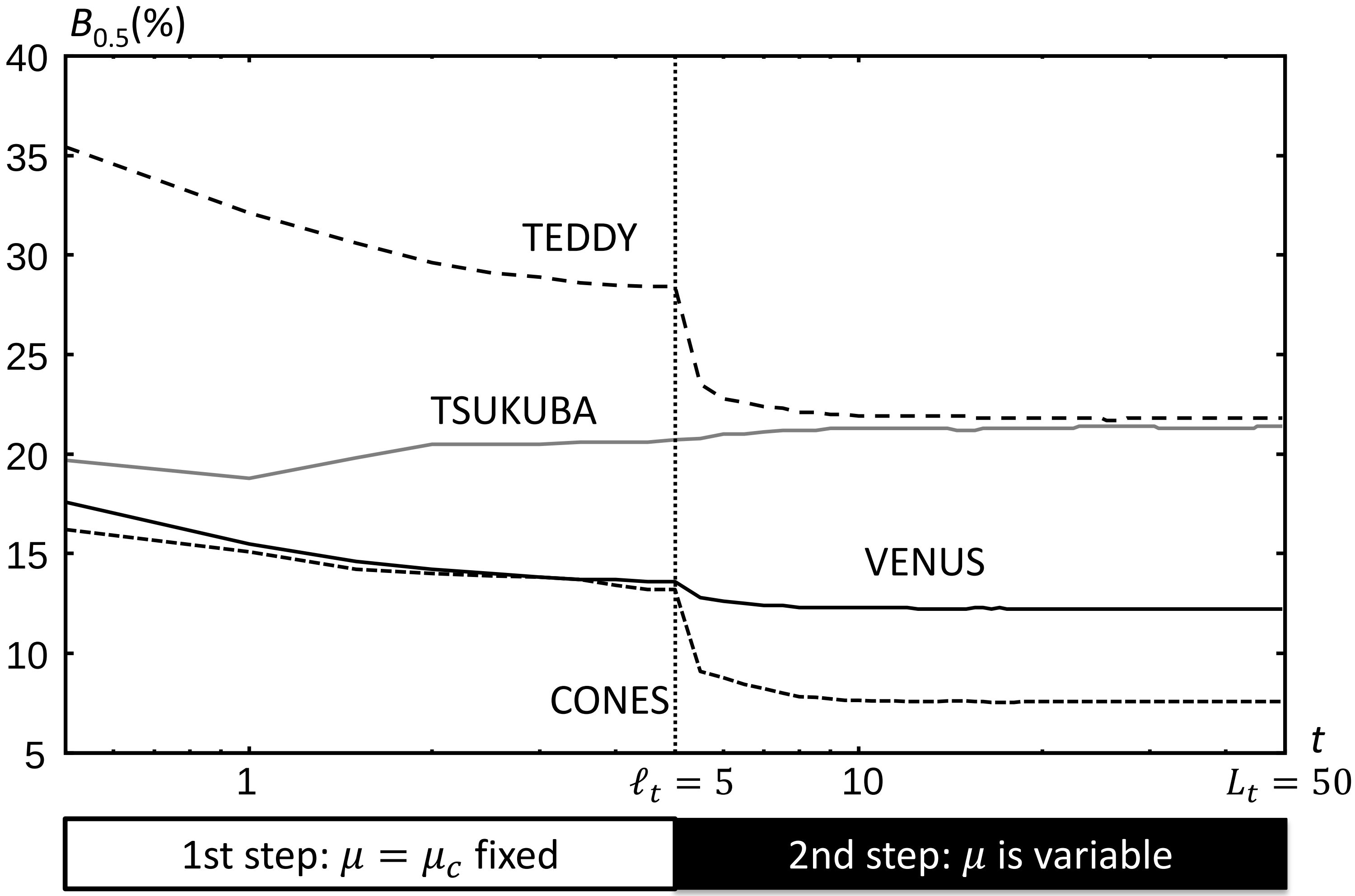
Results for the Middlebury Stereo Data-Sets

Subpixel Accuracy for CONES



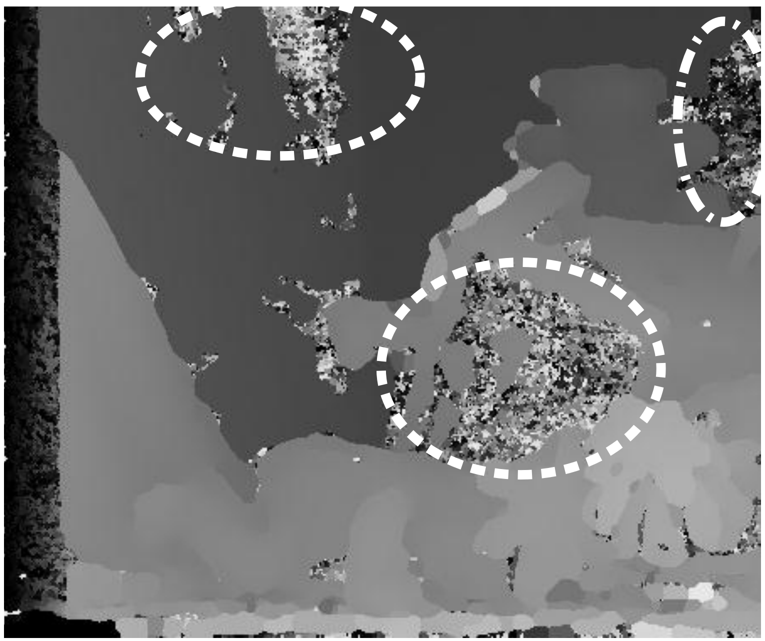
Results for the Middlebury Stereo Data-Sets

Temporal Changes of BMP errors

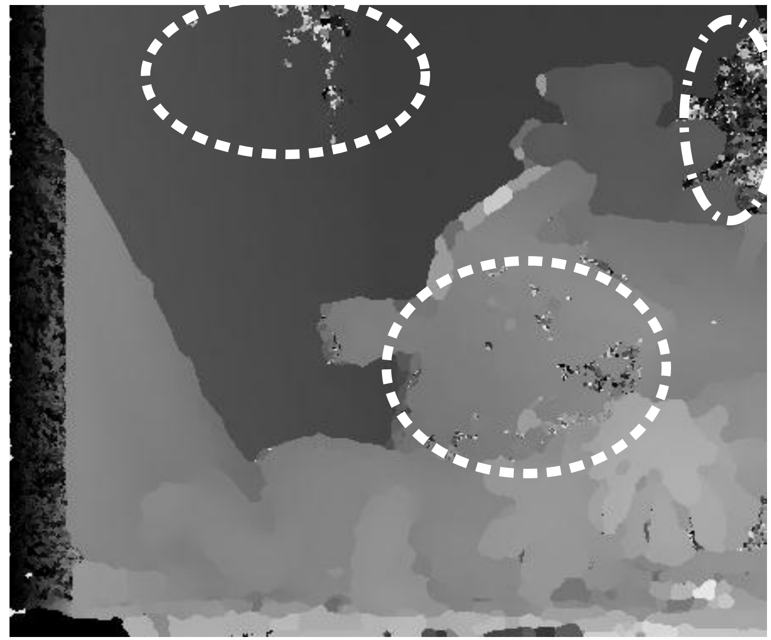


Demonstration of the Two-Step Algorithm

- The second step was switched on at $\ell_t = 5.0$.



t=5.0



t=6.0

Disparity maps obtained for TEDDY

Conclusion

- Summary of the proposed algorithm
  - a reaction-diffusion system defined in 3D domain
  - subpixel disparity with a finite difference method
  - two-step algorithm
- Results
  - effective for reconstruction of slanted or curved surface.
  - highly improved for the threshold level $\theta = 0.5$
- Future research work
  - utilising colour information of stereo images