On the use of on-line detection for maintenance of gradually deteriorating systems

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ABSTRACT: This paper deals with condition-based maintenance and non-stationary degradation process due to sudden changes. This is an attempt to propose an adaptive maintenance policy based on the on-line change detection procedure which can help to detect switches from a nominal mode to an accelerated mode in a non-informative context about the change mode time.

1 INTRODUCTION

The problem of decision making about monitoring and maintenance has been studied through many approaches and examples (Dekker 1996; Wang 2002) especially for systems described by stochastic processes. Most of the considered models deal with stationary degradation processes and a single failure mode and several extensions are possible e.g. concerning multiple failure modes (Kallen and van Noortwijk 2004) or possible use of covariates (Newby and Barker 2005). This paper deals with a non-stationary deteriorating system where the mean deterioration rate can change during a life cycle. The system begins to deteriorate according to a nominal mode and the mean deteriorating rate increases suddenly e.g. due to a change of its environment (Singpurwalla 1995). The main aim of this work is to propose a preventive maintenance policy which relies on the monitoring of the measurable system state. A maintenance decision criterion is given, based on the observed deterioration level of the system for a system which undergoes a continuous random deterioration (Pierskalla and Voelker 1979; Abdel-Hameed 1987). When the system state exceeds a fixed threshold \( L \), it is subject to non-obvious failures and its state is monitored through periodic inspections. Therefore in the presence of a non-obvious failure, the system is not known to be failed until it is revealed by inspections. When a failure is detected upon inspections, a corrective maintenance operation replaces the failed system by a new one. Moreover a preventive replacement takes place when the system state exceeds a threshold to be decided in order to avoid a failure occurrence hence a resulting period of inactivity of the system (duration between the instant of failure and the following inspection).

The choice of the inter-inspection times and the value of the preventive threshold both influence the economic performance of the maintenance policy. For example, when the inspections are costly it is not worthwhile to often inspect the system. But decreasing the number of inspections leads to increase the risk of missing a failure occurrence. For a single-mode deteriorating system, condition-based inspection/replacement (e.g. see (Grall et al. 2002; Newby and Dagg 2004; Abdel-Hameed 2004)) and continuous monitoring replacement policies (e.g. see (Park 1988; B’erenguer et al. 2003)) have been proposed. In previous works, a maintenance cost model is proposed which quantifies the costs and benefits of the maintenance strategy and permits to find the optimum balance between monitoring and maintenance efficiency. For systems subject to sudden change of mode of deterioration it is natural to adapt the maintenance decision rule according to the on-line information available about the system. An adaptive maintenance policy has been proposed in (Saassouh et al. 2005) for a continuously monitored system provided that the time of change of mode is immediately and perfectly detected.

In a context of total lack of a priori information about the time of change of mode, the main aim of this paper is to propose an adaptive maintenance policy based on an embedded optimal on-line change detection algorithm. A delayed detection of every change in the deteriorating system rate is based on the degradation levels successively observed and which takes into account the delay for detection.

In section 2 the deteriorating system is described. After a brief presentation of the on-line change detection in section 3, an adequate change detection algorithm with a short detection delay is introduced. Section 4 is devoted to the description of an appropriate
maintenance policy. In section 5, results from numerical experiments illustrate and analyze the behavior of the proposed preventive maintenance policy.

2 MODEL OF DETERIORATION

Consider a stochastically deteriorating system, for which the state at time $t$ can be summarized by a random aging variable $X_t$. In absence of repair or replacement, $(X_t)_{t \geq 0}$ is an increasing stochastic process such that:

- the initial state is $X_0 = 0$,
- the deterioration process after a time $t_0$ is independent of the deterioration before $t_0$.

In this paper, it is assumed that $(X_t)_{t \geq 0}$ is a gamma process and for all $0 \leq s \leq t$, the increment of $(X_t)_{t \geq 0}$ between $s$ and $t$, $X_t - X_s$, follows a gamma probability distribution function with shape parameter $\alpha(t-s)$ and scale parameter $\beta$. This probability distribution function can be presented as follows:

$$f_{\alpha(t-s),\beta}(x) = \frac{1}{\Gamma(\alpha(t-s))\beta^\alpha(t-s)}x^{\alpha(t-s)-1}e^{-\frac{x}{\beta}}I_{[x \geq 0]} \quad (1)$$

The two parameters $\alpha$ and $\beta$ can be estimated from deterioration data with classical statistical methods. The average deterioration speed rate is $\alpha \beta^2$ and its variance is $\alpha \beta^2$. The choice of $\alpha$ and $\beta$ allows to model various deterioration behaviors from almost deterministic to very chaotic. Note that the gamma process is a positive process with independent increments, which makes it relevant to describe the deterioration (Cooke et al. 1997). Another interest of the gamma process is the existence of an explicit probability distribution function which permits feasible mathematical developments.

The system fails when the aging variable is greater than a predetermined threshold $L$ which depends on the considered system. In this paper, we consider systems whose failures are not obvious to the user and can not be easily characterized and identified. The system can be declared as “failed” as soon as a defect or an important deterioration is present, even if the system is still functioning.

The parameters of the deteriorating system can change during a life cycle (e.g. according to the operating environment) at an unknown time $T_0$. The variable $T_0$ can be random but the fundamental hypothesis in this paper is that its probability distribution and mean remain unknown. The system can evolve according to two degradation modes denoted by $M_1$ and $M_2$. The first $M_1$ mode corresponds to the nominal evolution of the system and the second mode $M_2$ corresponds to an accelerated deterioration. These two modes can be modeled by two gamma processes with different parameters. According to the characteristic of the system, it is supposed that the mean values of the increments in the accelerated mode $M_2$ are greater than those corresponding to the nominal mode $M_1$. In each mode $M_i$, $i = 1, 2$, the process $(X_t)_{t \geq 0}$ has independent increments and for any $\Delta t > 0$ the increment $X_{t+\Delta t} - X_t$ follows a gamma law with a probability density function $f_{\alpha_i(t),\beta_i}(x)$ defined by (1).

3 DETECTION METHOD

The problem of quick detection of abrupt changes in a stochastic system on basis of sequential observations from the system and with low false alarm rate has many important applications including industrial quality control, automated fault detection in controlled dynamical systems. As it is noted in (Basseville and Nikiforov 1993), there is a large literature on the detection algorithms in complex systems but the application of these methods to a maintenance policy is never discussed. The main aim of this paper is to apply an adequate change detection algorithm to a stochastically deteriorating system described by a scalar aging variable $(X_t)_{t \geq 0}$ which summarizes its condition. As it is introduced in section 2 the parameters of the deterioration process $(X_t)_{t \geq 0}$ in change (e.g. according to the systems environment) at an unknown time $T_0$ (possibly random with an unknown probability distribution and mean). The system can then evolve according to two modes of deterioration $M_1$ and $M_2$. The two main classes of quickest detection problems are the Bayesian and non Bayesian approaches. When the observations are independent, Shiryaev (Shiryaev 1963) has formulated the problem of optimal sequential detection of the change time $T_0$ in a Bayesian framework by putting a geometric prior distribution on $T_0$ and assuming a loss of $c$ for each observation taken after $T_0$ and a loss of 1 for a false alarm before $T_0$. From the optimal stopping theory it can be shown that the Bayes rule triggers an alarm when the posterior probability that the change has occurred exceeds some fixed level. The method has been developed for more general prior distributions, random process and loss functions (Yakir 1997; Bojdecki 1979).

The framework of the paper in a non informative context about the change mode time deals with the non bayesian approach. The first algorithm in the non bayesian framework, suggested by Page in (Page 1954), is the cumulative sum algorithm (CUSUM). The asymptotic minimax (“worst case”) optimality of CUSUM has been proved by (Lorden 1971). A lower bound for the worst mean delay to detection is given and the CUSUM algorithm has been proved to reach this lower bound when the time mean before false alarm is large. The non asymptotic optimality of the non Bayesian algorithms is given in (Moustakides 1986; Ritov 1990).
In this paper the non Bayesian algorithm CUSUM is applied to the gradually deteriorating process and the asymptotical optimality of this algorithm is used to propose an adapted condition based maintenance policy. Let us recall that in the nominal mode $M_1$ the elementary random deterioration increment in a time interval $\Delta t$, i.e. $X_{t+\Delta t} - X_t$ for $t \geq 0$, follows a gamma law with shape parameter $\alpha_1 \Delta t$ and scale parameter $\beta_1$. In the accelerated deterioration mode $M_2$, $X_{t+\Delta t} - X_t$ ($t \geq 0$) follows a gamma law with respective shape and scale parameters $\alpha_2 \Delta t$ and $\beta_2$. We shall use $f_{\alpha_i \Delta t, \beta_i}$ to denote the density function of the gamma law in mode $M_i$, $i=1,2$.

Define the CUSUM stopping rule

$$N = \min \left\{ n \geq 1, \max_{1 \leq k \leq n} \sum_{t=k}^{n} \log \frac{f_{(\alpha_2 \Delta t, \beta_2)}(X_t)}{f_{(\alpha_1 \Delta t, \beta_1)}(X_t)} \geq h \right\}$$

(2)

where the constant $h$ is chosen such that $\Pr_0(N < \infty) \leq a$ or equivalently $\mathbb{E}_0(N) \geq \gamma$. As soon as the cumulative sum of the log-likelihoods exceeds the threshold $h$ it is decided that we are in mode $M_2$, i.e. the detection time of a change of mode is $t_N = N \Delta t$.

From Lorden (Lorden 1971), the stopping time of the CUSUM algorithm presented by Equation 2 minimizes the mean detection delay

$$\tau = \sup \text{esssup}_{T_0 \geq 1} \mathbb{E}(N - T_0 + 1\mid N \geq T_0, X_1, \ldots, X_{T_0})$$

(3)

We shall denote $d_{1,2}$ the Kullback-Liebler distance between $f_{\alpha_1 \Delta t, \beta_1}$ and $f_{\alpha_2 \Delta t, \beta_2}$. By (J.R. Mathiassen and Bo 2002) $d_{1,2}$ is defined by

$$d_{1,2} = \Delta t (\alpha_2 - \alpha_1) \phi(\alpha_2 \Delta t) + \alpha_1 \Delta t \log \left( \frac{\beta_1}{\beta_2} \right)$$

$$+ \alpha_2 \Delta t \left( \frac{\beta_2}{\beta_1} - 1 \right) + \log \left( \frac{\Gamma(\alpha_1 \Delta t)}{\Gamma(\alpha_2 \Delta t)} \right),$$

(4)

where $\phi(z) = \frac{\Gamma'(z)}{\Gamma(z)}$,

$$\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt, \quad \text{if } z \in \mathbb{N} \quad \Gamma(z) = (z-1)!$$

$$\Gamma'(z) = \int_0^\infty e^{-t}\ln(t)t^{z-1}dt.$$

The minimal detection delay satisfies the following relation ((Lorden 1971))

$$\tau^* = \min \tau \simeq \frac{\log a}{d_{1,2}} \quad \text{when } a \to 0.$$
As mentioned in the previous paragraph, the delay in mode $M_2$ with $\Delta t = 1$, $\alpha_1 = \beta_1 = 1$, and $\alpha_2 = 2$.

depends only on the properties of the characteristic laws of the two modes $M_1$ and $M_2$.

3.2.2 Relation between the delay and the properties of the two modes

As mentioned in the previous paragraph, the delay of detection mainly depends on the properties of the Kullback-Liebler distance $d_{12}$. When the mean and variance corresponding to $\Gamma(\alpha_1 \Delta t, \beta_1)$ and $\Gamma(\alpha_2 \Delta t, \beta_2)$ are very close the Kullback-Liebler distance $d_{12}$ is small. Hence $d_{12}$ grows as the difference between the means and variances corresponding to the two modes increases. Let us recall that the mean of a $\Gamma(\alpha, \beta)$ law is defined by $\alpha \beta$ and its variance is defined by $\alpha \beta^2$. In examples 2 and 3 from previous section, the degradation means are respectively the same for the first and the second mode but the detection delay is different. The larger delay corresponds to the larger variance in the second mode. This is due to the fact that the Kullback-Liebler distance $d_{12}$ is more sensitive to the change of variance than the change of the mean. $d_{12}$ is not necessarily constant if the difference between the means of the two modes remains constant. The relations between the detection delay and respectively the mean and variance of the gamma law in mode $M_2$ are depicted in Figures 2 and 3. It can be easily noticed the worst mean delay of detection is known for all the possible inter-inspection time values. Therefore it can be used to choose a relevant inter-inspection time $\Delta t$ from monitoring point of view as well as optimized maintenance decision rule parameters.

4 INSPECTION/REPLACEMENT POLICY

The system is not continuously monitored and its deterioration level can only be observed by inspections. An inspection reveals instantaneously, and without error, the exact state of the system. Even a “failed” state is only detected through inspections. Hence, in case of complete failure, the system remains unavailable until the next inspection.

The two possible actions for the system maintenance are the inspection and the replacement of the device, which can be either a true physical replacement or a repair but always leads to restore a “good as new system”. A corrective replacement is always performed as soon as an inspection reveals a current state greater than the failure threshold $L$.

Due to the lack of information about the possible time of change of degradation mode and for the seek of the detection method’s clarity, the system is periodically inspected at times $(t_i)_{i \in \mathbb{N}}$ with $t_i = i \Delta t$ where $\Delta t$ is a fixed parameter.

The decision about a possible replacement has to be taken at inspection times according to the current state of the system and the detected mode of deterioration. The proposed maintenance decision framework is based on a parametric decision rule. In the same way as in (Saassouh et al. 2005), several regions for the maintenance decisions are defined on the basis of a two-threshold structure which extends classical inspection/ replacement structures for single mode deteriorating systems. At each inspection time $t_i$ the possible decisions which can arise are as follows.
• If $X_t \geq L$ (system failed) then the system is correctively replaced.

• If no change of mode has been detected in the current cycle (the system is supposed to be in nominal degradation mode) and $X_t \geq M_{\text{sup}}$ the system is preventively replaced. It is still functioning but too badly deteriorated according to the nominal degradation mode.

• If a change of mode is detected at time $t_i$ or has been detected earlier in the current cycle and if $X_t \geq M_{\text{inf}}$ then the system is preventively replaced. It is still functioning but too badly deteriorated according to the accelerated degradation mode.

• In all the other cases, the decision is reported to time $t_{i+1}$.

As a consequence of the previous decision rule, if a change of mode is detected at time $t_{\text{detect}}$, the two following scenarios can arise:

• If $X_{\text{detect}} < M_{\text{inf}}$ then the system is left unchanged and a replacement is performed at time $t_{n}$ such that $X_{t_{n-1}} < M_{\text{inf}} \leq X_{t_{n}}$.

• If $X_{\text{detect}} < M_{\text{inf}}$ then the system is immediately replaced.

The two decision thresholds $M_{\text{inf}}$ and $M_{\text{sup}}$ are respectively associated to the two “limit” cases from the degradation point of view. $M_{\text{sup}}$ (respectively $M_{\text{inf}}$) has to be chosen according to the assumption of a nominal (respectively accelerated) single degradation mode. For a given value of $\Delta t$, the threshold values are optimized in order to minimize the maintenance cost as developed in the next section.

5 EVALUATION OF THE MAINTENANCE POLICY

Each intervention performed on the system entails a corresponding unit cost. Let in the sequel respectively be $C_i$ the cost of an inspection, $C_p$ the cost of preventive replacement and $C_c$ the cost of corrective replacement. Since a corrective maintenance operation is performed on a more deteriorated system, it is generally more complex and more expensive than a preventive one. Hence it is supposed that $C_p < C_c$. An additional cost is incurred by the time elapsed in the failed state at a cost rate $C_u$.

The maintenance policy is evaluated using an average long run cost rate taking into account the cost of each type of maintenance actions. Let us denote by

• $N_i(t)$ the number of inspections before $t$.

• $N_c(t)$ the number of corrective replacements before $t$.

• $N_p(t)$ the number of preventive replacements before $t$.

• $d_u(t)$ the cumulative unavailability duration of the system before $t$.

• $T$ the length of a life-time cycle.

The property of regeneration of the process $(X_t)_{t \geq 0}$ allows us to write:

$$C_\infty = \lim_{T \to \infty} \frac{E(C(t))}{E(T)}$$

where

$$C(t) = C_iN_i(t) + C_pN_p(t) + C_cN_c(t) + C_ud_u(t).$$

We know that

$$E(N_p(T)) = P(\text{cycle ends by a preventive repl.})$$

$$E(N_c(T)) = P(\text{cycle ends by a corrective repl.})$$

$$E(N_i(T)) = E\left(\frac{T}{\Delta t}\right)$$

$$E(d_u(T)) = E(T - T_L)\{1_{T_L < T}\}$$

Efficient calculations of the maintenance cost generated by the considered policy can be found e.g. in (Grall et al. 2002) for a system with a single degradation mode. Hence optimized values of the maintenance parameters $M_{\text{inf}}$ and $M_{\text{sup}}$ can be obtained by minimization of the average long run cost rate evaluated respectively with accelerated and nominal single degradation modes. The “inspection scheduling function” introduced in (Grall et al. 2002) is supposed to be constant which leads to a constant inter-inspection time $\Delta t$.

6 NUMERICAL IMPLEMENTATION

This section is devoted to the numerical implementations which bring the optimal values for the preventive threshold and the inspection intervals. To proceed to numerical implementations it is supposed that in the nominal mode $M_1$, $\alpha_1 = 1$ and $\beta_1 = 1$. As a consequence, the value of the maintenance threshold $M_{\text{sup}}$ has been taken equal to 90.2. The previous value is an optimal value which minimizes the long run maintenance cost for a single mode deteriorating system in mode $M_1$. It has been numerically optimized using single degradation mode results with $\Delta t = 4$. The numerical results have been obtained from Monte Carlo simulations. As an example, the change mode time $T_0$ has been taken as a uniformly distributed random variable, truncated on the interval $[0; 100]$. 

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Table 1. Characteristic data of the considered second degradation mode.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha_2$</th>
<th>$\beta_2$</th>
<th>$M_{inf}$</th>
<th>$\tau \Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>85.6</td>
<td>41.57</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>74.6</td>
<td>15.32</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>73.7</td>
<td>8.42</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>7</td>
<td>51.6</td>
<td>3.41</td>
</tr>
</tbody>
</table>

Table 2. Comparison of three detection configuration costs.

<table>
<thead>
<tr>
<th>Case</th>
<th>Instantaneous detection</th>
<th>Delayed detection</th>
<th>One threshold (M$_{sup}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.96</td>
<td>1.98</td>
<td>1.97</td>
</tr>
<tr>
<td>2</td>
<td>2.08</td>
<td>2.14</td>
<td>2.21</td>
</tr>
<tr>
<td>3</td>
<td>2.13</td>
<td>2.22</td>
<td>2.36</td>
</tr>
<tr>
<td>4</td>
<td>2.28</td>
<td>2.37</td>
<td>2.66</td>
</tr>
</tbody>
</table>

6.1 Policy comparison

In this part of the paper, three configurations for the maintenance policy are considered and their cost are compared.

- In the first configuration, it is supposed that the change mode detection is perfect and there is no delay between the change occurrence and the change detection. This situation can occur if we have a priori informations on the change time. Hence it is not realistic in the framework of the paper where the change time is supposed to be completely unknown.
- The second configuration corresponds to a change mode detection which take the detection delay into account. The value of the detection delay corresponding to the false alarm rate $a = 10^{-6}$ and the considered values of $\alpha_1$, $\beta_1$, $\alpha_2$ and $\beta_2$, can be extracted from the pre-defined mean time detection delay table. We can then incorporate the change detection algorithm in the numerical implementations.
- In the third configuration, the maintenance policy is applied without using the change mode detection. It is considered that the degradation process is always in the nominal mode and the preventive threshold is $M_{sup}$ for the maintenance procedure.

The first and third configurations are limit cases and for each configuration, four different accelerated modes are considered. The mode parameters $\alpha_2$ and $\beta_2$ are given in Table 1 with the associated values of the maintenance threshold $M_{inf}$ and of the mean delay time to change detection.

The maintenance costs for the different cases are presented in Table 2. It can be noticed that the maintenance cost in case of a perfect detection (without delay, config. 1) is lower than the maintenance cost in case of a standard detection (with delay, config. 2).

If the preventive threshold for each mode is different and detection delay is not too long (config. 2), the maintenance cost is lower than in case of a single threshold preventive maintenance policy (config. 3). If the same preventive threshold $M_{sup}$ is used for the two modes (config. 3), the occurrence of failures increases and the preventive replacements decrease. This policy is close to a corrective maintenance policy which is more costly.

6.2 Influence of $\Delta t$

The influence of the inter-inspection time on the global maintenance cost is difficult to analyze if the change mode time is supposed to be unknown. At a first sight, the objective was to derive a connection between the minimal detection delay and an adequate value of $\Delta t$. Several numerical experiments for various change mode time configurations have been done but do not lead to a general trend. Table 3 proposes an example of long run maintenance cost evolution as a function of $\Delta t$ for an accelerated mode $M_2$ defined by $\alpha_2=1$ and $\beta_2=7$. As in the previous section, the cost has been evaluated with Monte Carlo simulation for uniformly distributed values of $T_0$ on $[0; 100]$. It can be noticed that the probability of preventive replacements increases and the probability of corrective replacements decreases as $\Delta t$ increases. On the considered range of $\Delta t$, the maintenance cost is a decreasing function but the influence of $\tau^*$ seems not to be decisive.

7 CONCLUSION

In this article, a possible use of on-line detection algorithm in the framework of the condition-based maintenance policies has been presented for system subject to sudden change of degradation mode. The proposed maintenance decision rule takes into consideration the degradation process of a system which is only monitored with regular inspections assuming that the time of change of mode is totally unknown. From this point of view, the non-informative context considered in this paper leads to propose a generic maintenance decision rule structure.

The policy assessment strongly depends on the change mode time which is unknown. The global long run maintenance cost is considered, taking into
account the monitoring cost. The presented policy is a first attempt to study the interest of on-line change detection procedures in the maintenance decision process for gradually deteriorating systems with unknown abrupt change of mode. An extensive numerical analysis has to be done for a wide range of changes of mode. As a second step, we plan to include a priori knowledge through Bayesian approaches for the detection methods in order to be able to optimize the decision rule parameters.

REFERENCES


