Parameter estimation in dielectrometry measurements

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Abstract

Interdigital dielectrometry is used for non-destructive evaluation of material properties. Spectroscopic analysis of frequency-dependent dielectric properties increases sensitivity and selectivity of measurement of many physical variables (e.g. moisture, porosity, density, and aging status) that are correlated with changes of dielectric properties. This paper presents a convenient algorithmic methodology of analysis and visualization of properties of frequency-dispersive materials. Analytical expressions for the simple case of a Maxwell parallel-plate capacitor are developed and contrasted with parametric numerical models of interdigital sensor probes. The visualization of these parametric spaces helps develop intuition and understanding of dielectric measurement results in more complex cases. In particular, loss of sensitivity in different frequency regimes, interpretation of negative transcapacitance values, and byproducts of fast inversion procedures are discussed.

Experimental data with fluid and solid dielectric samples is presented for illustration of algorithms, proof of principle, and a basic discussion of multiple wavelength measurement sensitivity. References to descriptions of applications of the presented techniques to practical problems are provided. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Dielectrometry; Complex permittivity; Parameter estimate; Interdigitated electrodes; Dielectric properties

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1. Introduction

Planar interdigital electrodes are used for sensing applications in many technical fields, including dielectrometry sensors [1], chemical sensors [1,2], humidity sensors [3,4], surface acoustic wave transducers [5], and micro-electromechanical devices [6]. One promising application that has been under development during the last 2 decades is interdigital \( \omega - \kappa \) (frequency–wavenumber) dielectrometry [7–15]. A family of sensor geometrical designs and algorithms has been developed during the last 3 years based on the generic three-wavelength sensor concept. In this case, the term “wavelength” refers to the spatial periodicity of interdigital structure. In other words, the sensor spatial wavelength is equal to the distance between neighboring fingers of the same electrode. The work reported here is a logical continuation of previous research [7,11,13,14,16].

The main goal of interdigital dielectrometry is to indirectly and non-destructively measure various physical properties of insulating materials. This goal is achieved by measuring spatial distributions of dielectric permittivity and conductivity throughout the volume of the material under test, and then relating these distributions to the physical variables of interest. Oftentimes, a carefully planned series of experiments must be conducted to generate a mapping from the dielectric to other physical properties. Physical variables that can be measured in this way include, but are not limited to, moisture content, porosity, density, structural integrity, chemical content, surface roughness, and aging status.

Each wavelength of an interdigital sensor measures the electrical admittance (conductance and capacitance) at the terminals. Multi-wavelength interdigital sensors can thus approximately measure profiles of multiple layers or smoothly stratified profiles by combining measurements from interdigital sensors of different periodicity. Applications of this concept to moisture diffusion process monitoring are discussed in [17–19]. From our multi-wavelength measurements together with theoretical analysis it becomes possible to calculate the permittivity and conductivity of an unknown dielectric layer. However, the inverse problem to calculate the permittivity and conductivity from terminal admittance measurements is time-consuming. The purpose of our paper is to develop an approach that tabulates the entire discretized permittivity–conductivity space in terms of terminal conductance and capacitance for each wavelength of our three-wavelength sensors. By storing these precomputed values in the computer, the inverse problem can be done in real-time by interpolation.

1.1. Forward and inverse problems

Non-destructive measurement methods are often treated in the framework of inverse problem theory. In our case, the forward problem is defined as an electromagnetic calculation problem where transcapacitance and transconductance between the sensor electrodes are calculated as functions of known material properties and geometric parameters of the electrode system. Given enough information about material properties and geometric dimensions, this problem can
be solved numerically, analytically, or through calibration, depending on the complexity of the task (see Fig. 1).

Assuming that the electric scalar potential distribution varies sinusoidally with time as
\[ \Phi(x, y, z, t) = \text{Re} \hat{\Phi}(x, y, z)e^{j\omega t}, \]  
the forward problem for a uniform lossy dielectric solves the complex Laplace’s equation:
\[ \nabla \cdot (\sigma + j\omega\varepsilon)\nabla \Phi(x, y, z) = 0, \]  
where \( \Phi(x, y, z) \) is complex amplitude of the electric scalar potential distribution, \( \omega \) is the angular frequency of excitation, \( \sigma \) is conductivity, and \( \varepsilon \) is the real part of dielectric permittivity of the material under test. The boundary conditions for a pair of electrodes in the short-circuit measurement mode are
\[ \Phi = \begin{bmatrix} \hat{V}_D \\ 0 \end{bmatrix}, \]  
where \( \hat{V}_D \) is the complex amplitude of the driving excitation voltage on electrode one, and electrode two is held at zero voltage.

One way of solving the forward problem is to use a continuum model [8]. From the electroquasistatic field point of view, in a homogeneous dielectric, the electric scalar potential of the field excited by the driven electrode is a solution to Laplace’s equation. At any constant \( z \) position, the electric field distribution far away from the sensor edges is periodic in the \( x \) direction and assumed uniform in the \( y \) direction. For a homogeneous dielectric of semi-infinite extent, the scalar potential can be written as an infinite series of sinusoidal Fourier modes of fundamental spatial wavelength \( \lambda \) that decays away in the \( z \) direction. It is also possible to solve the forward problem with commercial finite-element software [20], with finite-difference techniques, or by using analytical approximations [21].

For most applications, the inverse problem is inherently more difficult. It requires solving for unknown properties given a known subset of material and geometrical properties plus the measured transcapacitance and transconductance. The forward problem has a unique solution, whereas the inverse problem does not necessarily...
have a unique solution. Furthermore, even if a unique and exact mathematical solution exists for a given set of input values, it may have no resemblance to the true physical parameters because of the effects of measurement noise.

To demonstrate this approach we present three-dimensional plots relating dielectric permittivity $\varepsilon$ and conductivity $\sigma$ to terminal transcapacitance $C_{12}$ and transconductance $G_{12}$ for each wavelength for a single homogeneous layer of semi-infinite extent. Future work will expand this method to multiple layers of finite thickness. For easier graphical use to determine accurate values we also present iso-contour plots. Because the relationships between $(\varepsilon, \sigma)$ and $(C_{12}, G_{12})$ cannot be expressed in closed form for interdigital dielectrometry, we precede our development with the case of a two-layer lossy dielectric system between parallel-plate electrodes as then all solutions can be developed in closed form. In addition to providing motivation and an introduction to the similar analysis for interdigital electrodes, the parallel-plate geometry is of value in its own right as a way to estimate the properties of an unknown lossy dielectric layer in a two-layer system.

The fundamental feature of $\omega-\kappa$ interdigital dielectrometry is the one-sided application of sets of interdigital electrodes of different spatial periodicity to measure properties of dielectric materials at different depths from the material surface. Apart from the property-versus-depth profiling features, $\omega-\kappa$ dielectrometry is similar to other dielectrometry measurement methodologies because it uses electrodes excited by a time-varying voltage signal. Indeed, admittance spectroscopy is commonly used to measure properties of frequency-dispersive materials. However, the electrode structure in more conventional techniques is usually either parallel plates or coaxial cylinders. Simple shapes offer the advantage of having closed-form algebraic solutions for the interelectrode admittance. When dealing with interdigital electrodes, the task of computing the admittance matrix of the sensor is more difficult. Finite element simulation allows the computation of the admittance matrix given the inhomogeneous distribution of material properties and the sensor geometry in several seconds using modern personal computers. While this computational time is acceptable for the forward problem (computation of electric fields and terminal admittance matrix), it may be prohibitively longer for the inverse problem of determining unknown material properties or geometry from dielectrometry measurements.

The algorithms that are used for the estimation of material properties from solving the inverse problem are computationally expensive because model-based material property estimation (as opposed to straightforward calibration) is required in order to extract the maximum amount of information about the material. Even modern computers do not have the speed required for online or manual measurements if each measurement were to be followed by a full-scale computer simulation of the tested region.

One efficient way to speed up the process is to compute the response of the sensor for all possible values of the measured parameters with a reasonable degree of discretization. Then, each measurement point can be interpolated using the results of the prior calculations. With a small number of unknown parameters, this approach works reasonably well. However, as the number of unknown parameters grows, the
computational space quickly becomes unacceptably large. Several techniques are available to reduce the number of points that have to be computed in advance. One of the techniques explored in this paper exploits the property that the frequency-dependent terminal admittance of a homogeneous ohmic non-dispersive dielectric material can be predicted from a single frequency measurement \[1,14\]. The proposed normalization allows mapping the results of measurements at one frequency to the results of measurements at a different frequency. As illustrated later in the text, this technique is applicable for evaluation of material properties even when they are frequency dispersive. Non-dispersive material properties always map onto the same point in the base computational space, while the frequency-dispersive material properties map onto different points in the base space for different frequencies \[15\]. Although the presented methodology is valid for any geometry, parallel-plate electrode cases that have analytical solutions are first considered in this paper for illustrative purposes.

2. Maxwell capacitor

The purpose of this section is to introduce our inverse problem solving approach in the framework of an easily understood parallel-plate capacitor (PPC). Since analytical solutions are available for the PPC case, they help to develop intuition and understanding of the three-wavelength interdigital sensor case in the next section. Unfortunately, the closed form analytical solution for a set of interdigital electrodes is known only for the simplest configurations \[22,23\], that do not include the presence of dissimilar slightly conducting materials and backplane proximity. These solutions are not adequate for the configuration analyzed here.

Let us first consider the simplest case of a Maxwell capacitor, which has the two series dielectric topology similar to the interdigital sensor structure considered in the next section, but has an easily understood analytical solution. Fig. 2 shows the schematic of the PPC test cell and equivalent lumped-element circuit. The lower layer is a perfect insulator \((\sigma_b = 0)\). Suppose that the dielectric permittivity of the lower layer, \(\varepsilon_b\) is known, and our task is to determine the dielectric properties of the upper layer, \(\varepsilon_a\) and \(\sigma_a\), from parallel-plate admittance measurements. This is a realistic arrangement that exists when the parallel-plate test cell is in air and the test dielectric

![Fig. 2. Maxwell PPC and equivalent circuit.](image-url)
is inserted between the electrodes. Except for possible interfacial effects not considered here, it makes no difference whether the air gap is on one side of the dielectric or both sides.

One can express the admittance as a complex number, and represent the real part of it as conductance \( G \); and the imaginary part as the product of the angular frequency \( \omega \) of a sinusoidal voltage excitation times the capacitance \( C \):

\[
Y = Y_r + jY_i = G + j\omega C.
\]

It is possible to relate the material properties and geometric parameters of the Maxwell capacitor test cell to the terminal conductance and capacitance as

\[
\frac{G}{\omega C_b} = \frac{(\sigma_a S/a\omega)(\epsilon_b S/b)}{(\epsilon_a S/a + \epsilon_b S/b)^2 + (\sigma_a S/a\omega)^2}
\]

\[
= \frac{G/(\omega C_b)}{(1 + C_a/C_b)^2 + (G_a/(\omega C_b))^2}
\]

and

\[
\frac{C}{C_b} = \frac{(\epsilon_a S/a)(\epsilon_a S/a + \epsilon_b S/b) + (\sigma_a S/(a\omega))^2}{(\epsilon_a S/a + \epsilon_b S/b)^2 + (\sigma_a S/a\omega)^2}
\]

\[
= \frac{(C_a/C_b)(1 + C_a/C_b) + (G_a/(\omega C_b))^2}{(1 + C_a/C_b)^2 + (G_a/(\omega C_b))^2},
\]

where \( S \) is the area of the PPC and the equivalent conductance and capacitances are

\[
C_a = \frac{\epsilon_a S}{a}, \quad C_b = \frac{\epsilon_b S}{b}, \quad G_a = \frac{\sigma_a S}{a}.
\]

This formulation is the forward problem of computing terminal capacitance \( C \) and conductance \( G \) from properties and dimensions of each layer. The determination of an unknown \( C_a \) and \( G_a \) in terms of the known \( C_b \) and from terminal measurements of \( C \) and \( G \) is obtained by inverting (5) and (6) to

\[
\frac{G_a}{\omega C_b} = \frac{G/(\omega C_b)}{(G/(\omega C_b))^2 + (1 - C/C_b)^2}
\]

and

\[
\frac{C_a}{C_b} = \frac{(C/C_b)(1 - C/C_b) - (G/(\omega C_b))^2}{(G/(\omega C_b))^2 + (1 - (C/C_b))^2}.
\]

We can visualize the functions described in (5)–(9) by discretizing the solution space determined by the unknown variables. Fig. 3(a) shows the solution space of \( G/(\omega C_b) \) that corresponds to the forward problem solutions of (5). The ratio \( C_a/C_b \) varies linearly from 0 to 20 and the ratio \( G_a/(\omega C_b) \) varies logarithmically from \( 10^{-3} \) (high frequency) to \( 10^3 \) (low frequency). Such a wide range of values is needed to accommodate the wide range of frequencies in dielectrometry spectroscopy studies (in our experiments the frequency is usually varied from 10 kHz to 0.005 Hz). This
and all subsequent plots use a logarithmic scale for non-dimensionalized conductance of the general form $G/(\omega C_b)$. The normalized terminal impedance component ratio $G/(\omega C_b)$ remains $<1$ (negative on the logarithmic scale) for all frequencies. It assumes its highest values when $G_a/(\omega C_b)$ is close to 1 (0 on logarithmic scale), that is, when the capacitive and conductive currents in the dielectric “a” are of comparable magnitude. Fig. 3(b) shows a contour plot of the same solution space as is shown in Fig. 3(a). Both plots indicate that the real part of the normalized terminal admittance attains its highest values when the conductance $G_a$ is of the same order of magnitude as the product $\omega C_b$ and approaches zero when they are significantly different. When the layer “a” is essentially insulating ($G_a/(\omega C_b) \ll 1$), the entire system becomes a capacitor consisting of two capacitors formed by the layers “a” and “b” in series. In the other extreme, when the layer “a” is essentially a conductor ($G_a/(\omega C_b) \gg 1$), it can be seen as the extension of the upper
electrode, and the entire system reduces to the capacitor formed by the layer “b”. The terminal conductance approaches zero at high and at low frequency limits.

Keeping in mind the previous discussion, one can interpret the solution space for the imaginary part of the normalized terminal admittance, $C/C_b$, that is shown in Fig. 4(a) as a three-dimensional function of the normalized layer “a” properties and also repeated in Fig. 4(b) as a contour plot. High values of capacitance $C_a$ have the same effect on the total capacitance value as high values of $G_a$: they bring $C/C_b$ closer to unity. Again, the terminal conductance approaches zero at a high frequency limit.

The inverse problem relations of (8) and (9) are plotted in Figs. 5 and 6, allowing determination of unknowns $G_a/\omega C_b$ and $C_a/C_b$ in terms of measured terminal variables $C/C_b$ and $G/\omega C_b$. The dielectric permittivity $\varepsilon_a$ and conductivity $\sigma_a$ can be determined directly from $C_a$ and $G_a$ in parallel-plate geometry using (7).

Fig. 4. A three-dimensional view (a) and a contour plot (b) of the forward problem solution space for the normalized terminal capacitance in terms of normalized capacitance and conductance of the upper “a” layer.
Now, after the reader has developed an understanding of our approach to solving the inverse problem with the simple Maxwell capacitor example, we turn our attention to the device of our primary interest, the three-wavelength interdigital sensor. Again, the task is to determine dielectric properties of materials under test using our knowledge of sensor geometry and its measured circuit response.

The top view of a three-wavelength sensor is shown in Fig. 7. Three sets of copper electrodes are deposited on the common Teflon substrate and connected to an interface circuit through the leads shown at the bottom. The design of this sensor and relevant experimental data have been described in our recent publications.

3. Three-wavelength interdigital sensor

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The top view of a three-wavelength sensor is shown in Fig. 7. Three sets of copper electrodes are deposited on the common Teflon substrate and connected to an interface circuit through the leads shown at the bottom. The design of this sensor and relevant experimental data have been described in our recent publications.
The three-wavelength design has been selected because we believe that accurate measurement of moisture profiles in power transformer pressboard requires at least three wavelengths. We have shown that with this sensor design we can measure smoothly varying moisture profiles in 1 mm thick pressboard [27,28].

The fact that a two-dimensional modeling of sensor electrodes is usually sufficient is one of the major advantages of interdigital structures. Fig. 8 shows the equivalent circuit of the sensor superimposed onto the schematic view of a half-wavelength cell. Note that each wavelength has an opposite conducting guard plane at the bottom of the substrate. For each wavelength, a follower op-amp drives the guard plane at the substrate bottom at the voltage \( V_G = V_s \), thus eliminating any current between the sensing and the guard electrode. The complex voltage gain of the sensor is then

\[
\hat{G} = \frac{\hat{V}_S}{\hat{V}_D} = \frac{G_{12} + j\omega C_{12}}{G_{12} + j\omega C_{12} + j\omega C_L}
\]  

Fig. 6. A three-dimensional view (a) and a plot of the inverse problem solution space iso-contours (b) for the normalized capacitance of the upper "a" layer in the Maxwell capacitor of Fig. 2.

[15,16,20,24–26]. The three-wavelength design has been selected because we believe that accurate measurement of moisture profiles in power transformer pressboard requires at least three wavelengths. We have shown that with this sensor design we can measure smoothly varying moisture profiles in 1 mm thick pressboard [27,28].

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\]  

Fig. 6. A three-dimensional view (a) and a plot of the inverse problem solution space iso-contours (b) for the normalized capacitance of the upper "a" layer in the Maxwell capacitor of Fig. 2.
where transconductance $G_{12}$, transcapacitance $C_{12}$, and known load capacitor $C_L$ are as shown in the circuit model in Fig. 8. This equation connects the terminal circuit elements to the voltage divider gain, but does not provide the relationship between the terminal circuit elements and material properties. Had such a relationship existed in an analytical form, there would be no need for the technique presented in this paper.

The excitation frequency in most of our experimental data varies from 0.005 Hz to 10 kHz, since this frequency range covers the transition frequency between the capacitive and conduction current dominant modes for materials with electrical conductivity on the order of $10^{-14}$–$10^{-9}$ S/m. Transformer oil and transformer pressboard usually fall into this range.

The numerical simulations were performed with Maxwell software by Ansoft Corp. Each sensor element was modeled as a half-wavelength cross-section of infinite
depth. Earlier Ref. [16] contains details of the simulation approach. The plots in Figs. 9–12 of the solution spaces are computed for the 5 mm wavelength of a three-wavelength sensor assuming an infinitely thick homogeneous material under test next to the sensor with Teflon substrate \((\varepsilon_r = 2.1)\) 254 \(\mu\)m thick. Similar plots describe the responses of the 1 and 2.5 mm wavelengths. They follow the same pattern that was used for the Maxwell capacitor. Fig. 9(a) shows the three-dimensional view of the 5 mm wavelength forward problem solution space for the real part of the normalized terminal transadmittance (conductance) per unit length computed with finite-element software. The normalization is with respect to the product of the dielectric permittivity of the Teflon substrate \((\varepsilon_1/\varepsilon_0 = 2.1)\) and the frequency of excitation. Thus, introducing the \(n\) subscript to denote the normalized value of transconductance and transcapacitance, we obtain

\[
C_n = C_{12}/\varepsilon_s
\]  

(11)
and

\[ \log_{10} G_n = \log_{10} \frac{G_{12}}{\varepsilon_a \omega} \]  

(12)

A more conventional way to represent loss would be to use the loss factor \( \varepsilon'' = \sigma / \omega \), but our representation was chosen to relate to our experimental data which is usually presented in terms of conductivity. At this point, the reader may notice that we switched independent variables of the forward problem from \( (C_a, G_a, \text{ and } C_b) \) to \( \varepsilon_a, \sigma_a \text{ and } \varepsilon_b \), respectively). These variables are almost equivalent for the parallel-plate case, being related to each other as shown in Fig. 2 and Eq. (7). The same dependence for the interdigital capacitor is not trivial. The equivalent circuit elements were selected for the parallel-plate case to make (8) and (9) more compact. Obviously, the dependence of terminal circuit elements on the material properties...
would have exactly the same shape as plots in Figs. 3 and 4, differing only by the geometric factors $S/a$ and $S/b$.

Fig. 9(b) shows a contour plot of the same solution space. Fig. 10 shows the forward problem solution space for the normalized transcapacitance per unit length. Notice that the transcapacitance may take on negative values at low frequencies. Due to a continuous conduction path between the electrodes, this plot has fundamentally different shape from that shown in Fig. 3.

Unlike the Maxwell capacitor example, an exact closed form solution is also not available for the terminal transadmittance of the interdigital sensor. In order to visualize the inverse functions of interest, the solution for the forward program is stored in computer memory in the form of a table that consists of columns $e$, $\sigma$, $G_{12}$, and $C_{12}$. Figs. 11(a) and 12(a) show the three-dimensional representation of the

![Graphical representation of the inverse problem solution space for the normalized material conductivity.](image-url)
inverse problem solution space for the entire range of values of normalized permittivity and conductivity values of the material under test. Each calculation point is shown as a dot in a three-dimensional space. The uniqueness of solution is evident for all combinations of the variable values. Figs. 11(b) and 12(b) show contour plots for the same solution space. The datapoint marked explicitly in these figures is discussed later in the paper when we present experimental data in Section 5.

When the permittivity $\varepsilon$ of the material under test is high, most of the field lines connecting the sensing and the driven electrodes go through the test material rather than through the substrate. In that case, the response of the sensor approaches that of a parallel-plate sensor, and the inverted value of $\varepsilon$ becomes almost independent of transconductance, as can be seen in Fig. 12(a).

The measurement data is interpolated using these solution spaces of $C_{12}$ and $G_{12}$ to find values of material properties $\varepsilon_2$ and $\sigma_2$. The solution spaces for the interdigital
The sensors presented in this section are not dependent on measurement electronics and are only functions of the sensor geometry and material properties. The responses of Maxwell capacitor and interdigital sensor are similar at high frequency and differ significantly at low frequency because the Maxwell capacitor with $\sigma_b = 0$ has no DC conduction path whereas the interdigital sensor has a DC conduction path through the lossy material under test.

The measurements at different excitation frequencies can be mapped onto corresponding solution spaces presented in this section. If the material is not frequency-dispersive, the entire frequency sweep maps onto a single point in the non-dimensional solution space. A properly executed measurement of dielectric properties of dispersive materials at different frequencies mapped onto the same non-dimensional solution space produces a sequence of points that move from one measurement point to another forming a curve that falls entirely within the boundaries of the solution space. The drift of data for non-dispersive material measurements indicates shortcomings in the physical model of the simulated measurement setup. For example, the presence of a non-modeled electrochemical double layer at the electrode-dielectric interface results in very high values of effective dielectric permittivity at low frequencies due to double layer capacitance.

### 4. Frequency conversion of calculated results

The instructions included in this section can serve as a reference on how to use the calculated tables. An alternative to this approach would be a multivariable fit of tabulated data. Both approaches are conceptually valid, but the multivariable fit is left for future investigation for the following reasons. As discussed in the following section, the table look-up with local function interpolation approach has been selected here because of easily automated computer implementation. In other words, a multivariable fit requires different functions for different phenomena that affects experimental and theoretical data. For example, the presence of an electrical double-layer requires the addition of equivalent circuit elements to the system. The response of an interdigital sensor is quite different from a classic Debye response because of the distributed nature of the equivalent circuit, which complicates its mathematical representation. For example, it is not obvious how to select the function that results in negative values of transcapacitance at low frequency, which can be seen in the lower right corner of Fig. 10(b). For an additional discussion of the nature of the negative transcapacitance values seen in Fig. 10, see [16].

Consider an interdigital sensor immersed into a homogeneous material with unknown dielectric permittivity $\varepsilon_2$ and conductivity $\sigma_2$. Suppose that at some well-chosen reference frequency $f_r$, the capacitance and conductance was tabulated for all $\varepsilon_2$ and $\sigma_2$. Then consider a test on an unknown sample whose properties are not known but are to be determined. From a practical point of view, we would like to run an experiment at a drive frequency such that there is appreciable phase in the response to be able to accurately distinguish $\sigma_2$ and $\varepsilon_2$. Furthermore, the material may be dispersive and as such its properties will change with frequency. For these
reasons the test might not be conducted at the reference frequency. Nevertheless, given the measurement frequency $f_i$ and the response and tabulated results from computer simulation at the reference frequency $f_r$ we can estimate the parameters $\sigma_2$ and $\varepsilon_2$ at frequency $f_i$. Specifically, for each $G_{12} - C_{12}$ (transconductance–transcapacitance) pair determined at frequency $f_i$, one can identify the corresponding $\varepsilon_2$ and $\sigma_2$ at the reference frequency that would give this response. Note that $\sigma_i = \sigma_r(f_i/f_r)$ so that $\sigma_i = \sigma_r$ at the frequency $f_i$ can readily be obtained.

Let us introduce the $\sigma - f$ normalization which allows one to find the values of $G_{12}$ and $C_{12}$ for any value frequency $f$ and any given pair of $\varepsilon$ and $\sigma$. For capacitances, the normalization is

$$C[f_i, \sigma_i f_i/f_r, \varepsilon] = C[f_i, \sigma, \varepsilon],$$

(13)

where $\sigma_i$ is the conductivity of interest, $f_i$ is the measurement frequency of interest, and $f_r$ is the reference frequency. Similarly, for conductances, the normalization is

$$G[f_r, \sigma_i f_r/f_i, \varepsilon] = \frac{f_r}{f_i} G[f_i, \sigma, \varepsilon].$$

(14)

To best visualize the variation of conductance with conductivity, it is convenient to plot the conductance normalized to $\sigma_i/\sigma_r$, where $\sigma_r$ can be chosen arbitrarily.

The following algorithm is used for estimation of complex dielectric permittivity from measurements with the interdigital sensor:

1. Convert the values of gain and phase measured at frequency $f_i$ to $G_{12}$ and $C_{12}$ using (10).
2. Normalize to $f_r$ by applying (13) and (14).
3. By interpolation, find the values of $\varepsilon$ and $\sigma$ on the calibration surfaces that correspond to the reference frequency values of $G_{12}$ and $C_{12}$.
4. Calculate the conductivity value at measurement frequency $f_i$ by applying the relationship $\sigma_i = \sigma_r(f_i/f_r)$.

These steps are performed for each distinct measurement frequency separately. Note that this procedure does not require any iterations and extensive computations, and so the inversion is performed nearly instantly once the look-up table at the reference frequency has been computed.

The range of calibration should be selected so that the ratio $\omega \varepsilon / \sigma$ is between $10^{-3}$ and $10^3$. Outside this range the influence of one of the variables on the output is insignificant. The following section includes the example of using experimental data with non-dimensional property plots.

5. Experimental data

5.1. Experimental setup

A simple validation measurement was conducted using our standard experimental setup shown in Fig. 13. The controller provides a sinusoidal signal from 0.005 Hz to
10 kHz and performs front-end data acquisition and signal processing. The dielectrometry interface is necessary to provide high impedance input for the sensing electrode, generate shielding signal for the sensor backplane and possible experimental enclosures, and boost the sensor output signal level for A/D conversion without waveform distortion. The shielding signal is necessary for the voltage divider measurement scheme, where the voltage on the guarding electrodes must be equal to the voltage on the sensing electrode. The schematics of the flexible three-wavelength sensor can be seen in detail in Fig. 7. The measurements with liquids also involve a standard parallel-plate sensor cell with guard ring electrodes to verify liquid permittivity values.

5.2. Measurements with corn oil

This section introduces an experimental data example to illustrate the performance of the algorithmic approach described here. Measurements with the three-wavelength sensor of Fig. 7 were taken with a sample of corn oil at 21.5°C. The sensor was completely submerged into oil, therefore the test material can be assumed of infinite thickness. Fig. 14 shows measured transconductance (a) and transcapacitance (b) in the frequency range from 0.005 Hz to 10 kHz for each of the three wavelengths. Notice that for the same interdigital electrode meander length the high frequency capacitance increases as the spatial wavelength decreases. According to our PPC measurements, as well as literature data [29], corn oil does not exhibit significant frequency dispersion in this frequency range. Therefore, traditional PPC
measurements would have resulted in a completely flat response for both conductance $G_{12}$ and capacitance $C_{12}$. However, due to the fringing field geometry of our sensor design, they are functions of frequency. The negative values of transcapacitance at low frequencies are valid and attributed to the terminal active drive measurement scheme, as explained in [16].

In the next step, the transcapacitance and transconductance values are converted to material dielectric properties $\varepsilon$ and $\sigma$. Since we work with a homogeneous dielectric fluid, the dielectric permittivity and conductivity estimated by each of the three wavelengths should be approximately the same between individual wavelengths for each frequency of excitation without additional calibration. Fig. 15 shows the results of parameter estimation of corn oil dielectric permittivity and conductivity for all wavelengths. Sensitivity analysis shows that the error of conductivity estimation increases with frequency, and the error of capacitance estimation reduces with frequency (at least for our electroquasistatic frequency range). The selection of
wavelength pairs for two-layer property estimation is also sensitivity-dependent [30]. In addition, the effects of the electrochemical double layer at the electrode-corn oil interface influence the validity of estimated values of bulk relative dielectric permittivity at low frequency. The parameters of electrochemical double layer in oil-electrode systems have been previously studied in [31]. The estimates are only given in regions where the algorithm is expected to produce sufficiently accurate results. Therefore, the divergence of the individual wavelengths data in the high frequency end of the conductivity spectrum of Fig. 15(a) is due to an increasing measurement error, and the divergence in the low frequency end of the spectrum is due to electrochemical double layer effects. The corn oil is practically not frequency dispersive even though the terminal characteristics in Fig. 14 are strongly frequency-dependent. The sudden jump of dielectric permittivity values at around 1Hz in

Fig. 15. Estimated conductivity (a) and relative dielectric permittivity (b) of corn oil from measurements shown in Fig. 14 for frequency ranges where measurement error is acceptably low.
Fig. 15 is artificial and corresponds to the change of the electrical excitation waveform generation sequence in the measurement electronics. Ideally, all three data points at each frequency should overlap. The demonstrated accuracy is acceptable for most applications explored in our research projects. Additional calibration procedures are available to increase accuracy when necessary.

Notice that these data points, acquired at different frequencies were all estimated using the same solution spaces for a 1 Hz excitation for each wavelength like those shown in Figs. 9–12 (for the 5 mm wavelength). To clarify this transformation, consider a specific datapoint for the 5 mm wavelength. At a frequency of 0.1 Hz, the measured conductance is \( G_{12}^M = 5.97 \text{ pS} \) and capacitance is \( C_{12}^M = 2.67 \text{ pF} \). The normalized values are calculated next according to (11) and (12). Both measured values are multiplied by 2 because the meander length of each sensor wavelength is 0.5 m and the transconductances and transcapacitances are calculated on a per meter length basis.

\[
\log_{10}(2G_{12}^M/\varepsilon_0\omega) = \log_{10}\left(\frac{2 \times 5.97 \times 10^{-12}}{2.1 \times 8.85 \times 10^{-12} \times 2 \pi 0.1}\right) = 0.01 \tag{15}
\]

and

\[
2C_{12}^M/\varepsilon_0 = \frac{2 \times 2.67 \times 10^{-12}}{2.1 \times 8.85 \times 10^{-12}} = 0.287. \tag{16}
\]

These non-dimensional values are plotted on the theoretical plots of Figs. 11(b) and 12(b) and after interpolation of the contour plots yield estimated non-dimensional values of \( \log_{10}(\sigma/\varepsilon_0\omega) = 0.4 \) and \( \varepsilon/\varepsilon_0 = 1.48 \), or, converted back to dimensional values, conductivity \( \sigma = 29.3 \text{ pS} \) and relative dielectric permittivity \( \varepsilon_{sr} = 3.11 \), in agreement with the estimated values in Fig. 15.

Fig. 16. Parametric plot of real \((G_{12})\) and imaginary \((\omega C_{12})\) parts of interelectrode transadmittance for each sensor wavelength measured in Fig. 15 plotted against each other with frequency as an independent parameter varying from 1 to 0.08 Hz.
For the sake of completeness of graphical data representation, we can also include in Fig. 16 a plot of measured real ($G_{12}$) versus imaginary ($\omega C_{12}$) parts of the interelectrode transadmittance. The frequency changes from 1 Hz on the left to about 0.09 Hz on the right of the data spread for each wavelength. The variation of $G_{12}$ increases with the wavelength size, indicating that the deviation from the parallel-plate sensor response increases as the ratio of the wavelength to substrate thickness increases. In other words, the experimental data shows that the influence of the actively driven backplane on the sensor response is stronger when it is closer to the interdigital electrodes, as expected.

5.3. Matching fluid approach

If various secondary effects are neglected, it is possible to estimate the dielectric properties of irregularly shaped solid materials by positioning them between the electrodes of the test cell, which can be, for example, a PPC or a coaxial cylindrical capacitor. If the dielectric permittivity of the solid material under test is uniform, it can be determined in the following way. Two identical test cells are positioned in the same container filled with some kind of a dielectric liquid. One cell contains only this liquid, and the other cell contains the solid material under test surrounded by the same dielectric liquid. The capacitances between the corresponding pairs of electrodes are equal when the dielectric permittivity of the liquid is equal to the dielectric permittivity of the solid sample. Initially, the capacitances of each cell are different, but by varying the dielectric permittivity of the liquid, one can bring them to be equal. The easiest way to estimate the dielectric permittivity of the solid dielectric is to gradually mix two miscible dielectric liquids with approximately equal specific gravity, as was done in [32] to measure the dielectric permittivity of various grains. For this type of measurement, it is necessary that the relative dielectric permittivity of the solid dielectric should be between the dielectric permittivities of the two liquids.

Similar to the liquid displacement approach described in [33], the two-fluid method provides a potentially higher accuracy of measurements, especially with irregularly shaped material surfaces. This project evaluated the applicability of two-fluid method to determine $\varepsilon_r$ with a greater accuracy than that could be achieved otherwise. Two nearly identical 2.5 mm wavelength sensors, A and B (identical in terms of metallization and capacitance) were submersed in a liquid composed of 2000 ml transformer oil ($\varepsilon_r = 2.2$) and castor oil ($\varepsilon_r = 4.5$) (as shown in Fig. 17). A solid sample (Lexan for our measurement in Fig. 17) was placed on one of the two single wavelength sensors (A). The sensors were placed on a second piece of Lexan ($\varepsilon_r = 3.0$) to ensure good contact with the solid Lexan sample since the bottom of the basin was flexible and not flat. Additionally, a PPC was used to give another measure of the permittivity of the liquid. The castor oil was mixed into the transformer oil until both sensors measured the same capacitance (adjusted for the small difference in metallization) and permittivity. At this point, a value for the permittivity of the solid sample could be obtained. Although the only necessary stopping criterion is
that the measured dielectric permittivity be the same, the equality of the capacitance provided an additional check.

Fig. 18 shows the change of apparent relative dielectric permittivity computed from measurements under the assumption that the sensor head is in contact with a homogeneous dielectric medium. The two lines intersect at the correct value of the Lexan relative dielectric permittivity $\varepsilon_r = 3.0$ when the amount of added castor oil is equal to 1000 ml.

The estimation of the relative dielectric permittivity plotted in Fig. 18 requires either calibration of interdigital sensors or a model-based inverse problem algorithm. It is also possible to bypass modeling by adding the measurement data from the PPC to the analysis. The ratio of capacitances of the two interdigital sensors is plotted in Fig. 19 against the relative dielectric permittivity measured by the PPC. The intersection of the measured capacitance ratio curve with the equal capacitance curve gives the estimate of the relative dielectric permittivity of material under test.
(\(\varepsilon_r = 2.95\)). The equal capacitance curve comes from the measurement of the capacitance of each interdigital sensor in air; the values are not exactly 1.0 because the sensors are not completely identical due to inherent manufacturing imprecision of flexible circuits.

6. Multiple wavelength measurement sensitivity

When the interdigital sensor is pressed against a solid dielectric, the contact conditions play a major role in measurement error analysis, because the electric field is high in the near electrode region. Modeling of the gap between the solid sample and the substrate due to electrode thickness has been addressed in our earlier publication [19]. However, in practical measurements, even larger effective gap may be introduced due to the surface roughness of both the sensor and the sample. We model this situation with the effective air gap \(h\).

The major interest that drives multiple wavelengths measurement technique development stands from the ability to determine several material properties with a single device. Such an approach also implies a possibility to increase measurement accuracy of properties of interest by reducing the effect of measurement perturbations. The following example shows that filling the gap between the sensor head and the solid material under test may improve selectivity of the measurement. The gap is modeled here as an ideally flat one. This representation is adequate for purely capacitive cases with no significant conduction currents. Fig. 20 shows lines of equal capacitance computed with finite element simulation for a specific point in a two-variable (\(\varepsilon_r\) and \(h\)) solution space. The values of relative dielectric permittivity \(\varepsilon_r\) and equivalent air gap \(h\) can be found from two measurements with different spatial
wavelengths, different ambient media, or both. The angle between the iso-capacitance lines should be as close as possible to 90° in order to provide the best selectivity with respect to both variables, \( \varepsilon_r \) and \( h \).

![Fig. 20. Calculated lines of equal capacitance for different levels of perturbation from the operating point (\( \varepsilon_r = 3.0, h = 30 \mu m \)), for each of the three wavelengths shown in Fig. 7 with air, \( \varepsilon_r = 1.0 \), (A) and castor oil \( \varepsilon_r = 4.5 \), (C) filling the gap. The values for each iso-capacitance line are shown in Table 1.]

<table>
<thead>
<tr>
<th>Combination</th>
<th>Permittivity</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1–A2.5</td>
<td>2.6</td>
<td>-13.33</td>
</tr>
<tr>
<td>A1–A5</td>
<td>2.9</td>
<td>-3.33</td>
</tr>
<tr>
<td>A2.5–A5</td>
<td>2.8</td>
<td>-6.67</td>
</tr>
<tr>
<td>A1–C5</td>
<td>2.95</td>
<td>-1.67</td>
</tr>
<tr>
<td>A2.5–C2.5</td>
<td>3.0</td>
<td>0.00</td>
</tr>
<tr>
<td>A2.5–C1</td>
<td>3.1</td>
<td>3.33</td>
</tr>
<tr>
<td>C1–C2.5</td>
<td>2.75</td>
<td>-8.33</td>
</tr>
<tr>
<td>C1–C5</td>
<td>3.05</td>
<td>1.67</td>
</tr>
<tr>
<td>C2.5–C5</td>
<td>30.2</td>
<td>6.67</td>
</tr>
</tbody>
</table>

The accuracy of measurement generally improves with the increasing angle between the iso-capacitance lines shown in Fig. 20.
One of the ambient media in this simulation is castor oil, marked with the letter C in Fig. 20, and the other one is air, marked with the letter A. Since the dielectric permittivity of air ($\varepsilon_r = 1$) is smaller than that of Lexan ($\varepsilon_r = 3$), all three iso-capacitance lines given in Table 1 lie in the first and third quadrants (if the origin is placed at the operation point, where all lines intersect). Similarly, since the dielectric permittivity of castor ($\varepsilon_r = 4.5$) oil is larger than that of Lexan, all three iso-capacitance lines for castor oil lie in the second and fourth quadrants. Therefore, it is theoretically impossible to obtain a $90^\circ$ angle between iso-capacitance lines for the same media. Also, since the absolute value of the slope of the iso-capacitance lines increases with spatial wavelength, it is reasonable to expect that the accuracy of measurement is better for a combination of most different spatial wavelengths, 1.0 and 5.0 mm wavelength for our case.

Table 2 shows the results of measurements with selected pairs of spatial wavelengths for the same case. The measurements confirm the general trend of improved accuracy when the angle between the iso-capacitance lines approaches $90^\circ$.

7. Conclusions

Previously utilized algorithms for material characterization with interdigital dielectrometry required iterative guesses of material properties followed by computation of the admittance matrix associated with the sensor structure. Some applications require faster property evaluation than can be achieved with this approach. A fast algorithm suitable for real-time implementation of interdigital dielectrometry is presented. The speed is achieved through a non-iterative calculation of material properties based on look-up tables found in advance. Simulated calibration of an interdigital sensor is performed with the finite-element software. A normalization approach that allows reduction of the number of unknown variables is proposed. Simple analytical examples serve to illustrate the validity of the general approach.

In addition, the uniqueness of the solution for the inverse problem of material characterization is demonstrated for the case of a homogeneous tested material. Local function approximation is used to relate scaled values of calibration planes to the properties of tested materials. Future publications will demonstrate application of the presented algorithm to extensive dielectrometry measurements.

Two conceptually similar techniques for improvement of potential accuracy of interdigital dielectrometry measurements with solid dielectrics have been evaluated. Both techniques require immersion of the sensor and the solid material under test into a liquid dielectric. The liquid fills the gap formed due to a non-ideal contact between the sensor and the material surface. The difference between dielectric constants of the liquid dielectric and air provides additional measurement information about the gap and can be used to reduce the effect of the gap measurement perturbations on estimated values of properties for materials of interest. This approach is useful for rough surfaces and non-contact measurements as well as for very accurate measurements of material properties. Future work that
builds on this feasibility study should include a more rigorous and statistically extensive analysis of measurement sensitivity for different conditions and practical cases of industrial and scientific applications.

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